Exam Groupwork

4.

y	0	1	2	3	4	5	6
f(y)	?	1.3	?	-2.9	?	?	2.2
Ans	0	1.3	-2.2	-2.9	0	2.9	2.2

✓ i) $f(0) = 0$

✓ ii) $f(-1) = -1.3$

✓ iii) $f(2017) = 1.3$

✓ iv) $f(2) = -2.2$

✓ v) $f(4) = 0$

Let's try to fill out the table first

iv) $f(2)$: Use the fact that $f(t)$ is odd and periodic:

$f(t) = -f(-t)$, $f(t) = f(t+c)$.

We know $f(6) \Rightarrow f(-6) = -f(6) = -2.2$

$f(-6) = f(-6+8) = f(2) \Rightarrow f(2) = -2.2$

$f(5)$:

$f(-3) = -f(3) = 2.9 \Rightarrow f(-3) = f(-3+8) = f(5) = 2.9$

i) $f(0) = -f(-0)$; only possible if $f(0) = 0$

v) $f(4) = f(4-8) = f(-4)$, but $f(4) = -f(-4)$ as well $\Rightarrow f(4) = 0$

ii) $f(-1) = -f(1) = -1.3$

iii) We can see that $2017/8 = 252$ remainder of $1 \Rightarrow f(1+2016) = 1.3$ ↗ $252 \times 8 = 2016$ (252 periods)

4b) $q(x) = 3e^{(x-5)^2}$ and $r(x) = e^{x^2/4}$

$r(x) \longrightarrow q(x)$ thru transformations

Order of transformations

① Horizontal: Stretch, Reflect, Shift

② Vertical: Stretch, Reflect, Shift

We can see that there are horizontal stretches/compressions (to get rid of the $\frac{1}{4}$) and shifts (to get $(x-5)$). We also have a vertical streteh

Let's go step by step:

1. $e^{\frac{(x)^2}{4}} \xrightarrow{\text{horizontal compression by } \frac{1}{2}} e^{\frac{(2x)^2}{4}} = e^{\frac{4x^2}{4}} = e^{(x)^2}$

2. $e^{(x)^2} \xrightarrow{\text{horizontal shift by 5 to the right}} e^{(x-5)^2}$

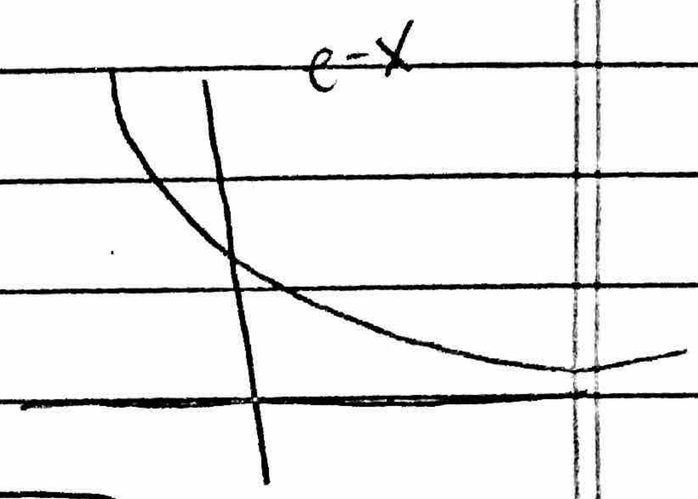
3. $e^{(x-5)^2} \xrightarrow{\text{vertical stretch by a factor of 3}} 3e^{(x-5)^2} = q(x)$

3a) $M(t) = A + Be^{kt}$ $\quad M(0) = 185°F$, $M(5) = 140°F$

Horizontal asymptote: $\lim_{t\to\infty} M(t) = 68°F$

We know that k should be < 0 since this is a decaying exp. function

$\lim_{t\to\infty} M(t) = \lim_{t\to\infty} A + Be^{kt} = \boxed{A = 68°F}$



$M(t) = 68 + Be^{kt}$

$M(0) = 68 + B = 140 \Rightarrow \boxed{B = 185 - 68 = 117}$

$M(t) = 68 + 117e^{kt}$

$M(5) = 68 + 117e^{5t} = 140$

$117e^{5k} = 140 - 68 = 72$

$e^{5k} = 72/117$

$5k = \ln\left(\frac{72}{117}\right)$

$\boxed{k = \ln(72/117)/5}$

b) $C(t) = 68 + 100e^{-0.05t}$

$C(t) = 131 \Rightarrow 68 + 100e^{-0.05t} = 131$

$100e^{-0.05t} = 131 - 68 = 63$

$e^{-.05t} = 63/100$

$-.05t \ln(e) = \ln\left(\frac{63}{100}\right)$

$t = \ln\left(\frac{63}{100}\right) \cdot \frac{-1}{.05} = -20\ln\left(\frac{63}{100}\right)$