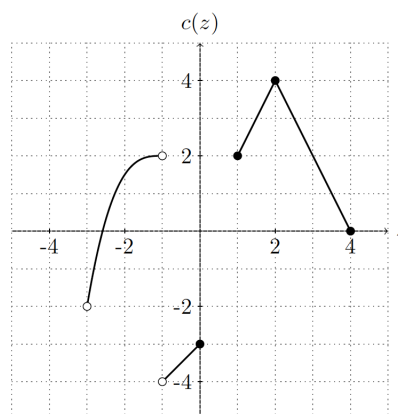Write legibly, show work and indicate your final answers. No books, notes, etc. are permitted. This is double sided. Good luck!

1. (*14 points*) Consider the functions $a(y)$, $b(w)$, and $c(z)$ given below:

$c(z)$

y	-10	-4	-1	1	3	4
$a(y)$	4	-2	2	-4	0	3

$$b(w) = \begin{cases} 1.5w + 8 & \text{for } -5 \le w < -1 \\ -4 \cdot 2^{-w} & \text{for } 1 \le w \le 5. \end{cases}$$

(a) (*3 points*) Find the domain of $c(z)$. Express your answer in interval notation or with inequalities.

The domain of $c(z)$ is _____

(b) (*3 points*) Find the range of $b(w)$. Express your answer in interval notation or with inequalities.

The range of $b(w)$ is _____

(c) (*4 points*) Calculate the following or write "UNDEFINED" if the quantity is not defined. Simplify your answer.

   i. $(a(-1))^{-1} = $ _____

   ii. $a(a(-10)) = $ _____

   iii. $c(b(-5) + 2.5) = $ _____

   iv. $b^{-1}(2) = $ _____

(d) (*4 points*) Using only the information given, find all the solutions to each of the equations below. Simplify your answers below, but leave them in **exact form**. If an equation has no solution, write "NO SOLUTION" in the blank.

   i. $c(a(y)) = 2$

$y = $ _____

   ii. $b(w) = a(3)$

$w = $ _____

2. (*6 points*) Chump is on his yacht, enjoying his annual vacation. After finishing a bottle of Martinelli's sparkling apple cider, he tosses the empty bottle into the ocean. The trajectory of the bottle is a parabola. When the bottle is a horizontal distance of x meters away from Chump, it is $H(x)$ feet above the level of the yacht deck, where $H(x) = -x^2 + \frac{\pi}{2}x + \frac{1}{2}$

(a) (*4 points*) Use the method of completing the square to put $H(x)$ into vertex form. **Your answer must be exact**, and you must *show all your work, step by step*, to get full credit.

$$H(x) = \underline{\hspace{3cm}}$$

(b) (*2 points*) What was the maximum height of the bottle? Give your answer in **exact form**.

The maximum height was $\underline{\hspace{3cm}}$ above the level of the yacht deck.