3. [11 points] Yolko Ono purchases a serving of her favorite TV dinner, *Chuck's Caterpillar Chop and Gravy*, from Crowger's, her local supermarket chain. At home, she heats up the frozen dish in the microwave oven. Right out of the oven, the temperature of the meal is 185 °F. After 5 minutes, the meal cools to 140 °F. If left out on the counter, the meal will eventually cool to room temperature, 68 °F. **Please leave your answers in exact form for all parts of this problem.**

a. [7 points] Let $M(t) = A + Be^{kt}$ be the temperature of the meal (in degrees Fahrenheit) t minutes after it leaves the oven. Using the information given, find the values of A, B, and k.

Solution: We first solve for A and B using the value of $M(t)$ at $t = 0$ and the limiting value as t tends to infinity.

$$68 = \lim_{t\to\infty} M(t) = A$$

$$185 = M(0) = A + B = 68 + B$$

$$B = 185 - 68 = 117$$

Now we solve for k using the value of $M(t)$ at $t = 5$.

$$140 = M(5) = 68 + 117e^{k5}$$

$$e^{k5} = \frac{140-68}{117} = \frac{72}{117}$$

$$5k = \ln\frac{72}{117} \implies k = \frac{1}{5}\ln\frac{72}{117}$$

$A =$ 68

$B =$ 117

$k =$ $\frac{1}{5}\ln\frac{72}{117}$

b. [4 points] Yolko has poured a cup of hot coffee into a thick mug. The temperature of the coffee (in degrees Fahrenheit) t minutes after she pours the coffee is given by the function $C(t) = 68 + 100e^{-0.05t}$. Yolko has a sensitive beak and wants to drink the coffee when it is at 131 °F. How long does she have to wait before she can drink it?

Solution: We want to find the value of t such that $C(t) = 131$. Using the formula for $C(t)$, we get

$$131 = 68 + 100e^{-0.05t}$$

$$e^{-0.05t} = \frac{131-68}{100} = \frac{63}{100}$$

$$t = \frac{\ln\frac{63}{100}}{-0.05} = 20\ln\frac{100}{63}$$

She will have to wait $20\ln\frac{100}{63}$ minutes.

4. [11 points]

a. [5 points] Suppose that $f(y)$ is **odd** and is **periodic** of period 8 with domain $(-\infty, \infty)$. Some of its values are given in the table below.

y	0	1	2	3	4	5	6
$f(y)$	?	1.3	?	-2.9	?	?	2.2

Find the following values of f. If it is not possible to find the value specified using the information given, write NOT POSSIBLE. *You do not have to show any work for this problem.*

(i) $f(0) =$ 0

(ii) $f(-1) =$ -1.3

(iii) $f(2017) =$ 1.3

(iv) $f(2) =$ -2.2

(v) $f(4) =$ 0

b. [6 points] Suppose that $q(x) = 3e^{(x-5)^2}$ and $r(x) = e^{x^2/4}$. List the transformations you need to apply to the graph of $y = r(x)$ to transform it to that of $y = q(x)$. Fill each space with either a number or one of the phrases below, as appropriate.

SHIFT IT HORIZONTALLY TO THE RIGHT	SHIFT IT HORIZONTALLY TO THE LEFT	SHIFT IT VERTICALLY UPWARDS	SHIFT IT VERTICALLY DOWNWARDS
COMPRESS IT HORIZONTALLY	STRETCH IT HORIZONTALLY	COMPRESS IT VERTICALLY	STRETCH IT VERTICALLY

To get the graph of $y = q(x)$ starting with the graph of $y = r(x)$,

first, we SHRINK IT HORIZONTALLY by 0.5,

and then we SHIFT IT HORIZONTALLY TO THE RIGHT by 5,

and then we STRETCH IT VERTICALLY by 3.

OR

first, we SHIFT IT HORIZONTALLY TO THE RIGHT by 10,

and then we SHRINK IT HORIZONTALLY by 0.5,

and then we STRETCH IT VERTICALLY by 3.