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Quiz 7 Sol

1a) Ant running on outer edge of the track.
Runs at a constant speed of 4.8 cm/sec
for 5 mins

This is an arc length type of problem:
$s = r\theta$

angular distance corresponds to θ in the arc length eqn

$$\frac{4.8\,cm}{sec} \cdot \frac{60\,sec}{min} \cdot 5\,mins = 1440\,cm = s$$

$r = 16\,cm$ for outer track

$$s = r\theta \Rightarrow 1440 = 16\theta \Rightarrow \boxed{\theta = \frac{1440}{16} = 90 \text{ radians}}$$

* θ in arc length eqn is always in radians! *

b) Inner edge for total angular distance of

$$\frac{27\pi}{5} = \frac{20\pi}{5} + \frac{7\pi}{5} = 4\pi + \frac{7\pi}{5} = 2\cdot 2\pi + \frac{7\pi}{5}$$

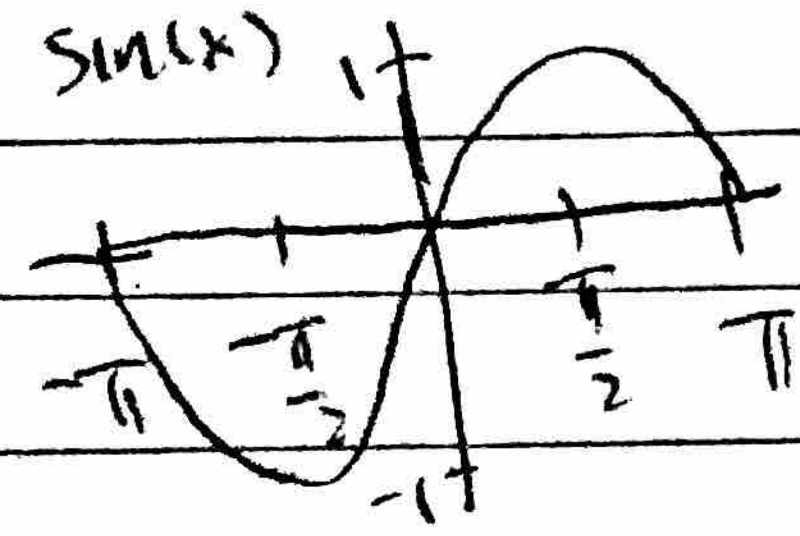
$2\pi = 1$ revolution $\Rightarrow$ $2\cdot 2\pi = 2$ revolutions

Additional distance: $\frac{7\pi}{5}$

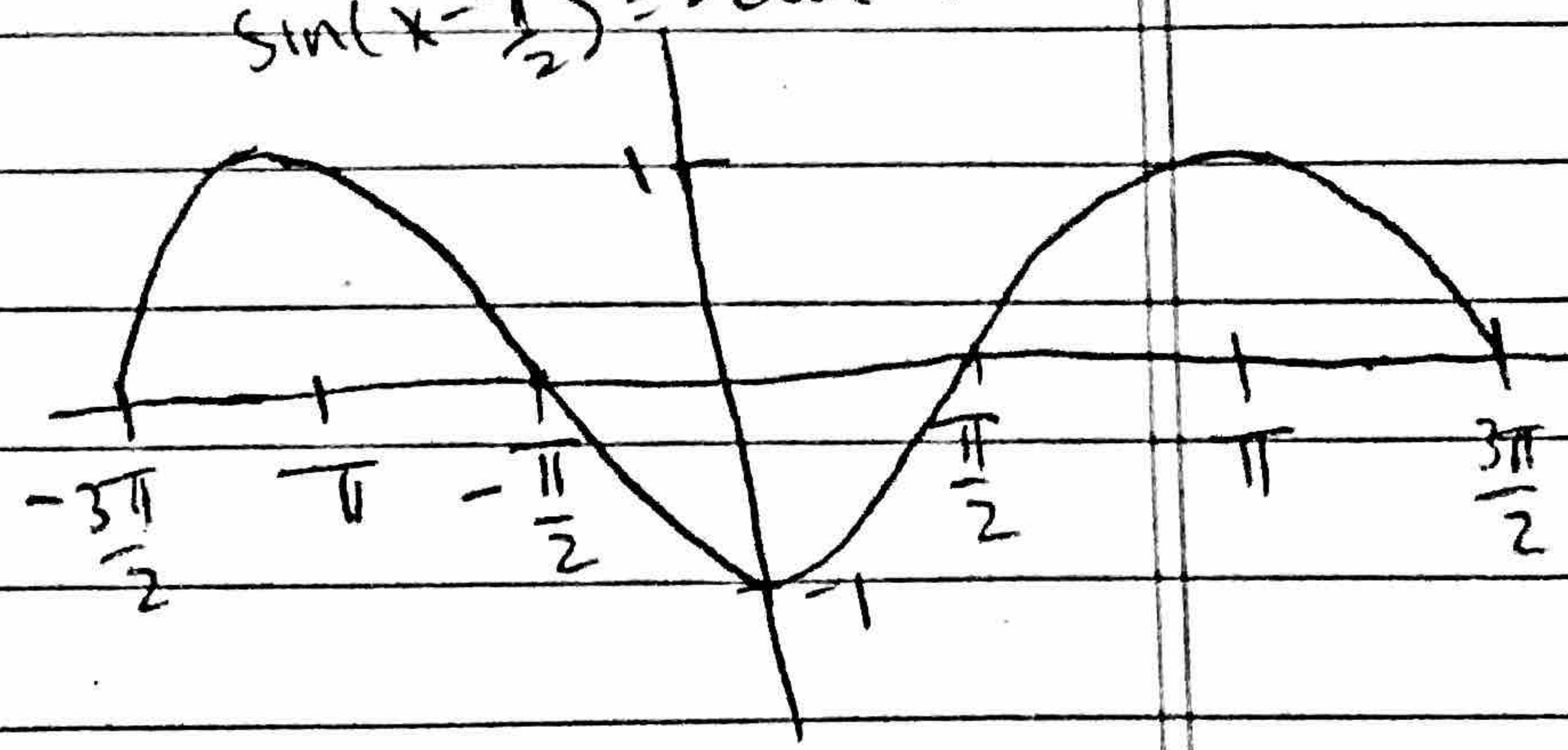
$\boxed{\text{Passes starting pt twice and runs additional } \frac{7\pi}{5} \text{ radians}}$

2.

a) $f(x) = \sin(x - \frac{\pi}{2})$



h. shift to the right by $\frac{\pi}{2}$

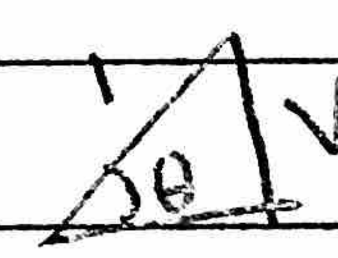


Try to memorize these small h. shifts of sine and cosine. You could also use your calculator and notice that $\sin(x - \frac{\pi}{2}) = -\cos(x)$

$\boxed{-\cos(x) = \text{even function}}$

b) $0 \leq \theta \leq 90$ $\boxed{v = \sin(\theta)}$, $\cos(180 + \theta)$?

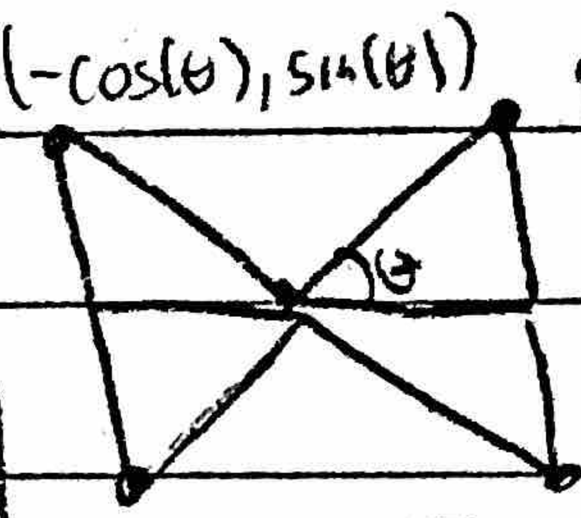
$\sin(\theta)$ corresponds to vertical coordinate
$v = r\sin(\theta)$, however $r = 1$!



Using pythagorean identity: $a^2 + b^2 = c^2$, $a = v$, $b = \cos(\theta)$, $c = 1$

$$b = \sqrt{c^2 - a^2} = \sqrt{1 - v^2}$$

$P(90+\theta) = (-\cos(\theta), \sin(\theta))$ $P(\theta) = (\cos(\theta), \sin(\theta))$



$\boxed{P(180+\theta) = (-\cos(\theta), -\sin(\theta))}$ $P(270+\theta) = (\cos(\theta), -\sin(\theta))$

$$\Rightarrow \boxed{\cos(180 + \theta) = -\cos(\theta) = -\sqrt{1 - v^2}}$$

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d) ~~Test Ant~~ Grows in half the time ⇒ grows to times faster than regular ant

Regular Ant = $A(t)$

Test Ant = $B(t)$

$\Rightarrow B(t) = A(2t)$

c) $A(x)$ has V.A at $x=5$

$B(x) = 3A(3x-6)+1$ has V.A at?

Only horizontal changes will affect where the vertical asymptote is at! (A vertical change affects where the horizontal asymptote is at)

$A(x)$: V.A at $x=5$

$B(x) = 3A(3x-6)+1$

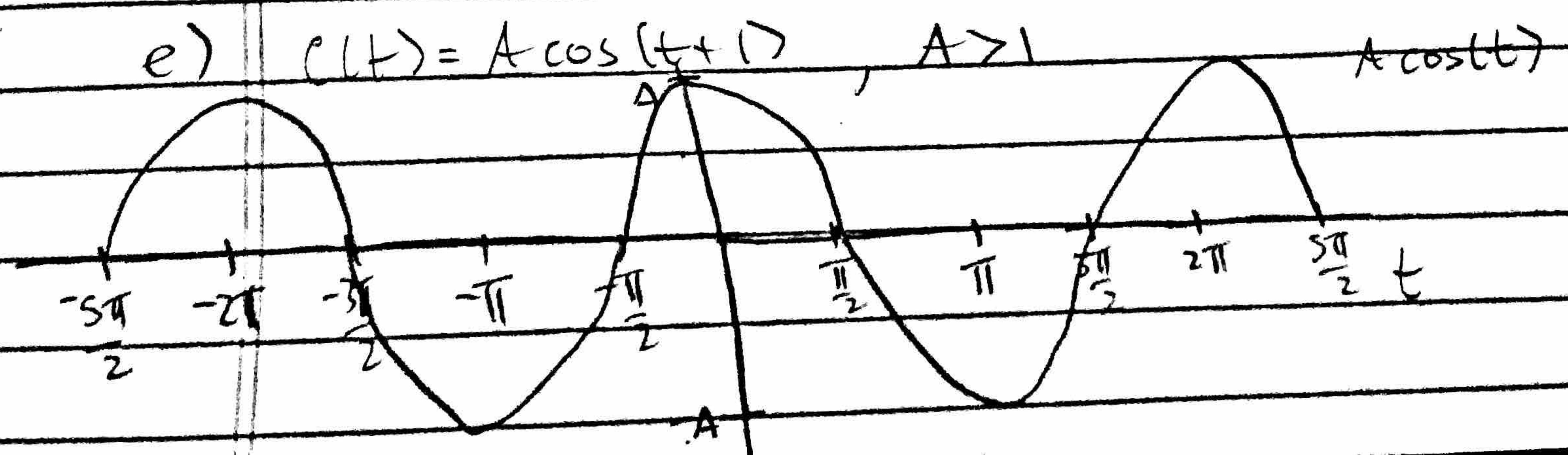
Solve this equation: $\boxed{3x-6=5}$ This lets us follow the point thru the transformation

$3x-6=5$

$3x = 11$ ⇒ none of the above

$x = 11/3$

e) $C(t) = A\cos(t+1)$, $A>1$



We can see that $A\cos(t)$ is concave down between $[-\frac{5\pi}{2}, -\frac{3\pi}{2}]$ and $[\frac{3\pi}{2}, \frac{5\pi}{2}]$

($\cos(t)$ is concave down near its max, concave up near its min; same with $\sin(x)$)

If shift $A\cos(t)$ to the left by 1,

$[-\frac{5\pi}{2}, -\frac{3\pi}{2}] \Rightarrow -\frac{5\pi}{2} \le t \le -\frac{3\pi}{2}$

$[\frac{3\pi}{2}, \frac{5\pi}{2}] \Rightarrow \frac{3\pi}{2} \le t \le \frac{5\pi}{2}$

replace t with $t+1$

$$-\frac{5\pi}{2} \le t+1 \le -\frac{3\pi}{2} \rightarrow -\frac{5\pi}{2} - 1 \le t \le -\frac{3\pi}{2} - 1$$

$$-\frac{3\pi}{2} \le t+1 \le \frac{5\pi}{2} \rightarrow -\frac{3\pi}{2} - 1 \le t \le \frac{5\pi}{2} - 1$$