Math 115, Section 068 - Quiz 2
Date: 9/28/18

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Write legibly, show work and indicate your final answers. No books, notes, etc. are permitted. Calculators are permitted. This is double sided. Good luck!

1. (10 *points*) Find all solutions to

$$4 - 5\sin(\frac{\pi}{2}x - \frac{\pi}{6}) = 2 \quad \text{for } 0 \le x \le 5$$

Your answers must be found algebraically and in exact form

$4 - 5\sin(\frac{\pi}{2}x - \frac{\pi}{6}) = 2$

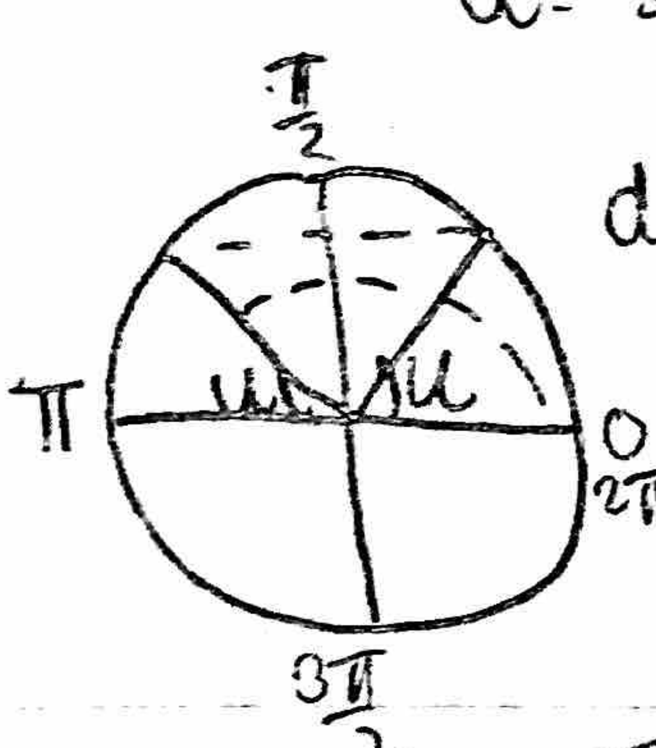
$-5\sin(\frac{\pi}{2}x - \frac{\pi}{6}) = -2$

$\sin(\frac{\pi}{2}x - \frac{\pi}{6}) = 0.4$

Let $u = \pi x_1 - \frac{\pi}{6}$

$\sin(u) = 0.4$

$u = \sin^{-1}(0.4) \approx .411$



dashed angle = 2^{nd} "fake" sol

$z = \pi - u = \pi - \sin^{-1}(0.4)$

$z = 2.73$

Convert back to X

$u = \sin^{-1}(0.4) = \frac{\pi}{2}x_1 - \frac{\pi}{6}$

$\sin^{-1}(0.4) + \frac{\pi}{6} = \frac{\pi}{2}x_1$

$\frac{2}{\pi}[\sin^{-1}(0.4) + \frac{\pi}{6})] = x_1$

$x_1 \approx .935$

$z = \pi - \sin^{-1}(0.4) = \frac{\pi}{2}x_2 - \frac{\pi}{6}$

$\frac{7\pi}{6} - \sin^{-1}(0.4) = \frac{\pi}{2}x_2$

$\frac{2}{\pi}[\frac{7\pi}{6} - \sin^{-1}(0.4)] = x_2$

$x_2 = 2.071$

Now Add/subtract multiples of period to solutions to find other solutions in interval:

$x_1 \approx .935$

$P = \frac{2\pi}{B}, \quad B = \frac{\pi}{2}$

$x_1 = \frac{2}{\pi}[\sin^{-1}(0.4) + \frac{\pi}{6}]$

$P = \frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$

$x_3 = \frac{2}{\pi}[\sin^{-1}(0.4) + \frac{\pi}{6}] + 4$

Answer: $x =$ x_1, x_2, x_3

Solutions: $x_1 = \frac{2}{\pi}[\sin^{-1}(0.4) + \frac{\pi}{6}]$

$x_2 = \frac{2}{\pi}[\frac{7\pi}{6} - \sin^{-1}(0.4)]$

$x_3 = x_1 + 4$

$= \frac{2}{\pi}[\sin^{-1}(0.4) + \frac{\pi}{6}] + 4$

2. (10 *points*) Consider the rational function r defined by:

$$r(x) = \frac{3(x-\sqrt{2})(\pi x+7)^2(x+1)}{(x+1)(x-\sqrt{3})}$$

For all of the following parts of this problem, leave your answers in exact form.

(a) (2 points) What is the domain of $r(x)$?

Domain: $(-\infty,-1)\cup(-1,\sqrt{3})\cup(\sqrt{3},\infty)$

Answer: $(-\infty,-1)\cup(-1,\sqrt{3})\cup(\sqrt{3},\infty)$

(b) (2 points) Find the equations of all vertical asymptotes of $r(x)$. If there are none, write *NONE*.

$x=\sqrt{3}$

Answer: $x=\sqrt{3}$

(c) (2 points) Let $p(x) = 3x^2 + 1.2x - 5$. Find the equations of all horizontal asymptotes of $\frac{r(x)}{p(x)}$. If there are none, write *NONE*. Show your work or reasoning to justify your answer.

$$\lim_{x\to\infty} \frac{3(x)(\pi x)^2(x)}{x(x)(3x^2+1.2x)} \approx \lim_{x\to\infty} \frac{3\pi^2 x^4}{3x^4+1.2x^3} \approx \lim_{x\to\infty} \frac{x^4(3\pi^2)}{x^4(3+\frac{1.2}{x})} = \frac{3\pi^2}{3} = \pi^2$$

Answer: $y=\pi^2$

(d) (3 points) If $q(x) = \frac{2e^{kx}}{1+2^x}$, find all values of k so that $\lim_{x\to\infty} q(x) = 0$. If there are none, write *NONE*. Show your work or reasoning to justify your answer.

$\lim_{x\to\infty} \frac{2e^{kx}}{1+2x} = 0$ if $k<\ln(2)$ since this will make $(e^k)<2^x$

$e^k<2 \Rightarrow k<\ln(2)$

Answer: $k<\ln(2)$