Solving Trigonometric Eqns

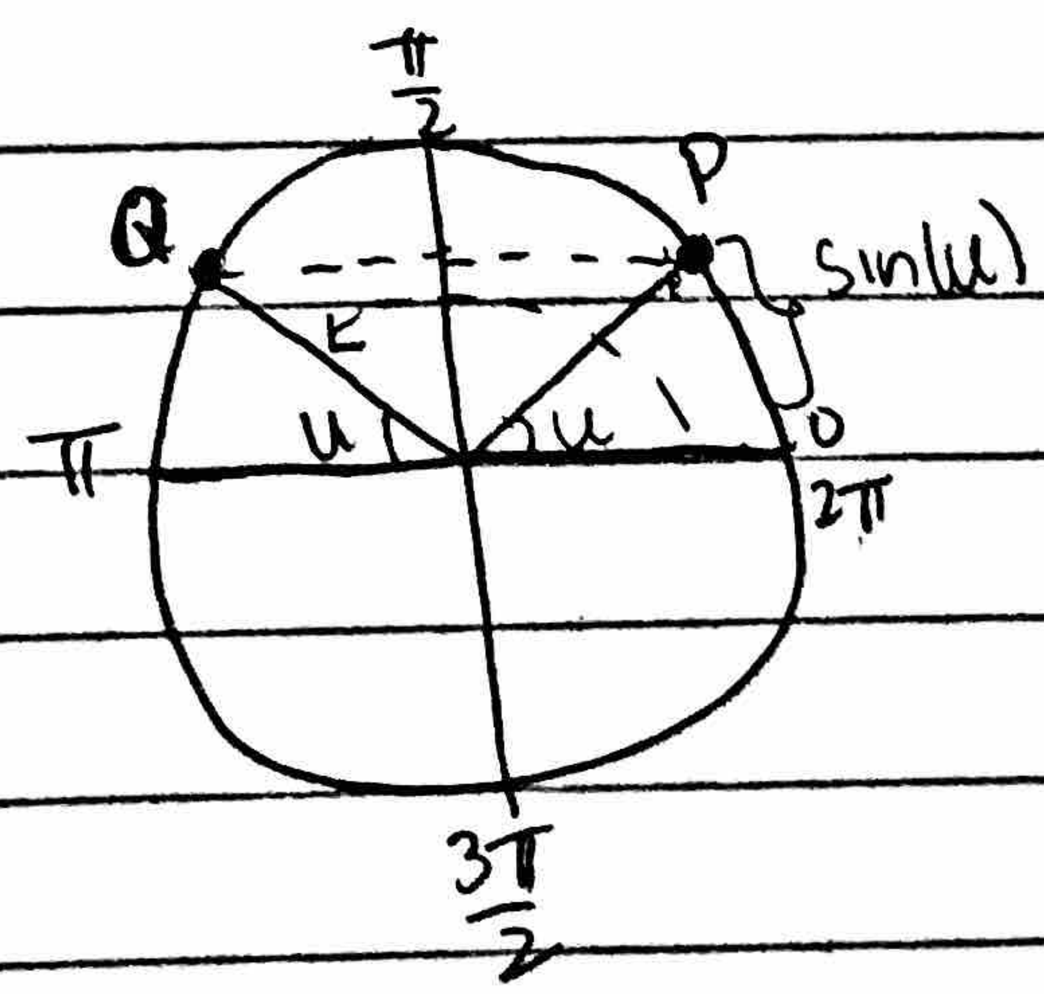
Solving Sine Equations

$2 = 5\sin(3x)$ $[-\pi, \pi]$

$\frac{2}{5} = \sin(3x)$

It is possible to solve this eqn since $\frac{2}{5}$ is within the range of $\sin(3x)$

Let $u = 3x_1$. u is our "fake" solution and x_1 is our "real" solution.



$\frac{2}{5} = \sin(u)$

$u = \sin^{-1}\left(\frac{2}{5}\right) \approx 0.4115$

Now that we found our first solution, let's find our 2^{nd} "fake" solution.

By extending a horizontal line from point P to the other side of the unit circle.

Now, we want to find the angle that brings us to that extension point, Q.

You may notice that the angle denoted by the dashed line is $\boxed{\pi - u \text{ radians}}$.

Therefore, the 2^{nd} fake solution is

$$z = \pi - u = 3x_2$$

$$z = \pi - \sin^{-1}\left(\tfrac{2}{5}\right) = 3x_2$$

Now, let's convert our fake solutions into terms of x_1 & x_2

$$u = \sin^{-1}\left(\tfrac{2}{5}\right) = 3x_1 \Rightarrow x_1 = \frac{\sin^{-1}\left(\frac{2}{5}\right)}{3} \approx .137 \text{ radians}$$

$$z = \pi - \sin^{-1}\left(\tfrac{2}{5}\right) = 3x_2 \Rightarrow x_2 = \frac{\pi - \sin^{-1}\left(\frac{2}{5}\right)}{3} \approx .9100 \text{ radians}$$

Now we add/subtract the period to these solutions to find the rest of the solutions in the interval $[-\pi, \pi]$, $\pi \approx 3.1415$ radians

Period $= \frac{2\pi}{3}$

$-2\pi/3$

$$\frac{\sin^{-1}\left(\frac{2}{5}\right)}{3} - \frac{2\pi}{3}, \quad \frac{\pi}{3} - \frac{\sin^{-1}\left(\frac{2}{5}\right)}{3}, \quad \frac{\sin^{-1}\left(\frac{2}{5}\right)}{3}, \quad \frac{\pi - \sin^{-1}\left(\frac{2}{5}\right)}{3}, \quad \frac{2\pi}{3} + \frac{\sin^{-1}\left(\frac{2}{5}\right)}{3}$$

$-2\pi/3$ $\quad$ $+2\pi/3$

These are the five solutions in the interval. You might wonder why we didn't add $\frac{2\pi}{3}$ to $\frac{\pi - \sin^{-1}\left(\frac{2}{5}\right)}{3}$. This is because it would bring us outside of the interval we're concerned with.

Solving Tan Eqns

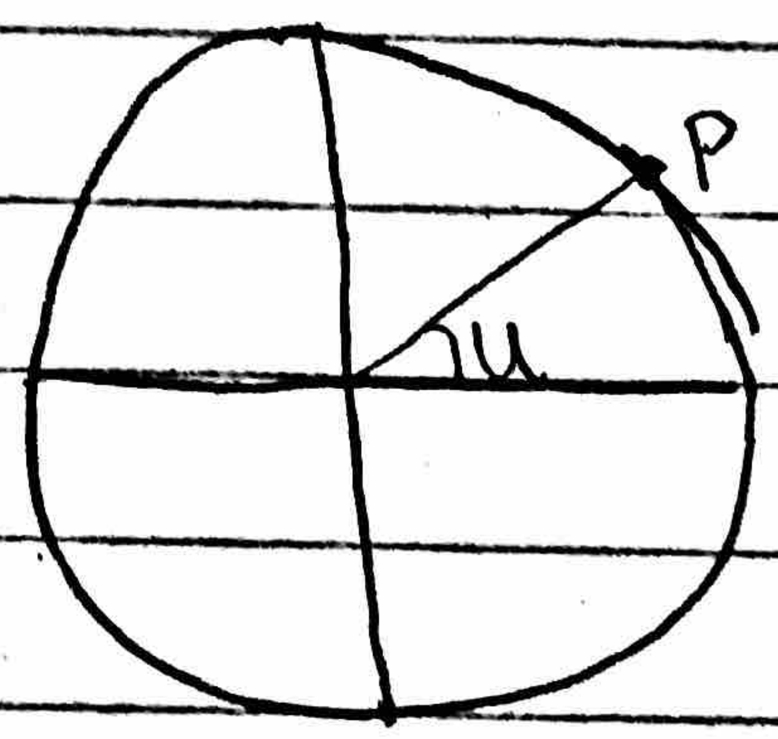
$8 = 4\tan(5x)$ $[0, \pi]$

$2 = \tan(5x)$

Let $u = 5x$,

$2 = \tan(u)$

$u = \tan^{-1}(2) \approx 1.107$



We have our first solution! However, we don't need to find a 2nd solution when working with tan (x). All you have to do is add and subtract the period to the real solution.

$P = \frac{2\pi}{5} \Rightarrow$ Solutions: $\tan^{-1}(2)$, $\tan^{-1}(2) + \frac{2\pi}{5}$

$+2\pi/5$