1.4 Groupwork

1. $e^{x+8} = 2^{7x-6}$.

$\ln(e^{x+8}) = \ln(2^{7x-6})$ * $\ln(e^x) = x$ *

$x+8 = (7x-6)\ln(2)$

$x+8 = 7x\ln(2) - 6\ln(2)$

$x - 7x\ln(2) = -6\ln(2) - 8$

$x(1-7\ln(2)) = -6\ln(2) - 8$

$x = \frac{-6\ln(2) - 8}{1 - 7\ln(2)}$

2. $\log(2x^2 - 1) = 0$

$10^{\log(2x^2-1)} = 10^0$

$2x^2 - 1 = 10^0 = 1$ * $10^{\log(x)} = x$, $10^0 = 1$ *

$2x^2 = 2$

$x^2 = 1 \Rightarrow x = \pm 1$

3. $\ln(11 \cdot e^p) = -14p + 2017$

$e^{\ln(11 \cdot e^p)} = e^{-14p+2017}$

$11 \cdot e^p = e^{-14p+2017}$ * $e^{\ln(x)} = x$ *

$11 \cdot e^p = e^{-14p} \cdot e^{2017}$ * Look up exponent laws

$e^{15p} = e^{2017}$

$\ln(e^{15p}) = \ln(e^{2017}/11)$

$15p \cdot \ln(e) = \ln(e^{2017}) - \ln(11)$ → * $\ln(A/B) = \ln(A) - \ln(B)$, * $\ln(A^p) = p(\ln A)$.

$15p = 2017 - \ln(11)$

$p = \frac{2017 - \ln(11)}{15}$

3 (Alternate way).

$$\ln(11\cdot e^p) = -14p + 2017$$
$$\ln(11) + \ln(e^p) = -14p + 2017$$
$$\ln(11) + p\ln(e) = -14p + 2017$$
$$p = -14p + 2017 - \ln(11)$$
$$15p = 2017 - \ln(11)$$
$$p = \frac{2017 - \ln(11)}{15}$$

7. $\log(10^x + 1) = \pi$

$$10^{\log(10^x+1)} = 10^{\pi}$$
$$10^x + 1 = 10^{\pi}$$
$$10^x = 10^{\pi} - 1$$
$$\log(10^x) = \log(10^{\pi} - 1)$$
$$x = \log(10^{\pi} - 1)$$

8. Population of squirrels is growing exponentially and has increased by 11% in two years. How long will it take (in years) for the population to triple in size? What is the continuous growth rate?

$P(t) = P_0 a^t$

$t = 2 \Rightarrow \quad 1.11P_0 = P_0 a^2$ * Initial population grew by 11% in 2 yrs *

$1.11 = a^2$

$(1.11)^{1/2} = a \quad \Rightarrow \quad P(t) = P_0 (1.11)^{t/2} = P_0\left((1.11)^{1/2}\right)^t$

Now, to find how long it takes to triple:

$3P_0 = P_0 (1.11)^{t/2} \Rightarrow 3 = (1.11)^{t/2} \rightarrow \ln(3) = \frac{t}{2}\ln(1.11) \rightarrow t = \frac{2\ln(3)}{\ln(1.11)}$ yrs

Continuous growth rate?

Recall that $a = (1.11)^{1/2}$

Also recall that from a couple lectures ago...

$$a = e^K \longrightarrow K = \ln(a)$$

$$\Rightarrow K = \ln(1.11^{1/2}) = \frac{1}{2}\ln(1.11)$$

$$K = .05218 \approx 5.22\%$$