Math 115, Section 068 - Quiz 1 **Name:** Donald Glover
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Write legibly, show work and indicate your final answers. No books, notes, etc. are permitted. Calculators are permitted. This is double sided. Good luck!

1. (5 *points*) Fill out the following table of log properties. In the second column, write out the corresponding rule as if we were using the natural log (ln).

Log Property	Natural Log Property
$\log(1) = 0$	$\ln(1) = 0$
$\log(10) = 1$	$\ln(e) = 1$
$\log(AB) = \log(A) + \log(B)$	$\ln(AB) = \ln(A) + \ln(B)$
$\log(\frac{A}{B}) = \log(A) - \log(B)$	$\ln(\frac{A}{B}) = \ln(A) - \ln(B)$
$\log(B^u) = u\log(B)$	$\ln(B^u) = u\ln(B)$
$\log(10^u) = u$	$\ln(e^u) = u$
$10^{(\log u)} = u$	$e^{\ln u} = u$

2. (10 *points*) The numbers, in thousands, of men $M(t)$ and women $W(t)$ on Logistic Island t years after January 1st, 2015 are given by the formulas

$$M(t) = \frac{150}{5 + 95(1.5)^{-3t}}, \qquad W(t) = \frac{A}{2 + 100e^{-kt}}$$

where $A, k > 0$ are constants.

(a) (2 *points*)What was the population of men on Logistic Island on January 1st, 2015?

$$M(0) = \frac{150}{5 + 95(1.5)^{-3(0)}}$$

Answer: 1500

$$M(0) = \frac{150}{5+95} = \frac{150}{100} = 1.5 \text{ thousand}$$

Turn page

(b) (6 *points*) When was the number of men on the Logistic Island equal to two thousand? Your answers needs to be ***exact***. Show all of your of work.

$2 = \frac{150}{5+95(1.5)^{-3t}} \rightarrow 2(5+95(1.5)^{-3t}) = 150$

$10 + 190(1.5)^{-3t} = 150 \longrightarrow (1.5)^{-3t} = \frac{140}{190} = \frac{14}{19}$

$\ln(1.5^{-3t}) = \ln(\frac{14}{19}) \rightarrow -3t\ln(1.5) = \ln(14/19)$

$t = \frac{\ln(14/19)}{-3\ln(1.5)}$

Answer: $t = \frac{\ln(14/19)}{-3\ln(1.5)}$

(c) (2 *points*) Knowing that $\lim_{t\to\infty} W(t) = 50$, find the value of A.

$W(t) = \frac{A}{2+100e^{-kt}}$, $\lim_{t\to\infty} \frac{A}{2+100e^{-kt}} = 50 \Rightarrow \frac{A}{2} = 50 \Rightarrow A = 100$

Answer: $A =$ 100

3. (5 *points*) An apple farmer wants to assess damage done by a plague to the trees in his orchard. In order to do so, he installs cameras on a couple of small flying robots to film the damage done by the plague to the trees. Let $f(t)$ and $s(t)$ be the height above the ground (in feet) of the first and second robot t seconds after they started recording.

Let $f(t) = 4 - 3\cos(\frac{\pi}{5}t - \frac{2\pi}{5})$. Find the time(s) at which the first robot is 6 feet above the ground for $0 \le t \le 12$. Your answer(s) should be **exact**. Show all of your work.

$\boxed{\text{Set } f(t) = 6}$

$4 - 3\cos(\frac{\pi}{5}t - \frac{2\pi}{5}) = 6$

$\cos(\frac{\pi}{5}t - \frac{2\pi}{5}) = -\frac{2}{3}$

$u = \frac{\pi}{5}t_1 - \frac{2\pi}{5}$

$\cos(u) = -\frac{2}{3}$

$u = \cos^{-1}(-\frac{2}{3})$

2nd fake solution: $z = -u = \frac{\pi}{5}t_2 - \frac{2\pi}{5}$

$z = -\cos^{-1}(-\frac{2}{3})$

Convert back to t_1, t_2

$u = \cos^{-1}(-\frac{2}{3}) = \frac{\pi}{5}t_1 - \frac{2\pi}{5}$

$\cos^{-1}(-\frac{2}{3}) + \frac{2\pi}{5} = \frac{\pi}{5}t_1$

$\frac{5}{\pi}\cos^{-1}(-\frac{2}{3}) + 2 = t_1 \approx 5.1061$

$z = -\cos^{-1}(-\frac{2}{3}) = \frac{\pi}{5}t_2 - \frac{2\pi}{5}$

$\frac{2\pi}{5} - \cos^{-1}(-\frac{2}{3}) = \frac{\pi}{5}t_2$

$2 - \frac{5}{\pi}\cos^{-1}(-\frac{2}{3}) = t_2 \approx -1.1061$ $\boxed{\text{Not in interval}}$

Add period: Period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{5}} = 10$

Solutions in interval $0 \le t \le 12$

$\frac{5}{\pi}\cos^{-1}(-\frac{2}{3}) + 2$, $t_2 + 10 = 12 - \frac{5}{\pi}\cos^{-1}(-\frac{2}{3})$

Answer: $t = \frac{5}{\pi}\cos^{-1}(-\frac{2}{3}) + 2,\ 12 - \frac{5}{\pi}\cos^{-1}(-\frac{2}{3})$