Problem set 5

PPHA 31102 Statistics for Data Analysis II: Regressions

Jay Ballesteros

February 15, 2026

Part II

Question 1

Consider the following simplified duopoly game. Each firm simultaneously chooses whether to Cooperate or Defect. Payoffs are shown in the table below, and given as (π_1, π_2):

	Cooperate	Defect
Cooperate	$(100, 100)$	$(0, 150)$
Defect	$(150, 0)$	(X, X)

Suppose the firms interact infinitely many times, with common discount factor $\delta \in (0, 1)$. They attempt to sustain cooperation using a grim trigger strategy: both firms cooperate unless a defection occurs, in which case both defect forever after.

(a) (5 points)

What is the present value of cooperating in each time period, assuming that the other firm will also cooperate in each time period?

If both firms cooperate in each period, each firm earns the $\pi = 100$ per period. The present value is for this scenario is:

$$PV_C = \frac{100}{1 - \delta}$$

(b)

What is the present value of defecting today, assuming that the other firm will cooperate today, but defect in each time period in the future?

If a firm defects today while the other cooperates, it earns 150. Under grim trigger, both firms then defect forever, earning

$$PV_D = 150 + \frac{\delta X}{1-\delta}$$

(c)

Suppose $X = 0$. For what values of δ can cooperation be sustained?

Cooperation is sustainable under grim trigger if $PV_C \geq PV_D$. With $X = 0$,

$$\frac{100}{1-\delta} \geq 150$$

Solving $100 \geq 150(1-\delta)$, we get $\delta \geq \frac{1}{3}$

Therefore, cooperation can be sustained for $\delta \geq \frac{1}{3}$.

(d)

Suppose $X = 50$. For what values of δ can cooperation be sustained?

Cooperation is sustainable if $PV_C \geq PV_D$. With $X = 50$:

$$\frac{100}{1-\delta} \geq 150 + \frac{50\delta}{1-\delta}$$

Multiplying by $(1-\delta)$: $100 \geq 150(1-\delta) + 50\delta$, we get $\delta \geq \frac{1}{2}$

Cooperation can be sustained for $\delta \geq \frac{1}{2}$.

(e)

Consider your answer to the previous two questions. Does a higher X make cooperation more likely or less likely? Provide a concise explanation.

A higher X makes cooperation less likely. When $X = 0$, cooperation requires $\delta \geq \frac{1}{3}$. When $X = 50$, cooperation requires $\delta \geq \frac{1}{2}$.

In this case, X represents the punishment payoff from mutual defection. A higher X translates to a less severe punishment, reducing the incentive to cooperate, as the cost of defection is lower.