Problem set 7

PPHA 32300 Microeconomics and Public Policy I

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1. George is planning his breakfast for the week.

He has utility function:

$$U(Q_A, Q_T) = Q_A^{1/2} Q_T^{1/2}$$

where Q_A is the quantity of avocados and Q_T is the quantity of toast. The price of avocados is P_A, the price of toast is P_T, and George has a budget of M to spend on breakfast.

(a) Solve for the tangency condition.

Considering the marginal utilities:

$$MU_A = \frac{1}{2} Q_A^{-1/2}$$

$$MU_A = \frac{1}{2} Q_T^{-1/2}$$

$$\frac{Q_T^{-1/2}}{Q_A^{-1/2}} = \frac{P_A}{P_T}$$

(b) What is the demand function for avocados? What is the demand function for toast?

Considering the tangency condition:

$$Q_T = Q_A \left(\frac{P_A}{P_T} \right)^2$$

Substituting into the budget constraint:

$$M = P_A Q_A + P_T \left[Q_A \left(\frac{P_A}{P_T} \right)^2 \right]$$

$$M = P_A Q_A + \frac{P_A^2}{P_T} Q_A$$

$$M = Q_A P_A \left(1 + \frac{P_A}{P_T} \right)$$

Thus, the demand function for avocados is:

$$Q_A = \frac{M}{P_A\left(1 + \frac{P_A}{P_T}\right)}$$

Substituting back to find the demand function for toast:

$$Q_T = \frac{M}{P_T\left(1 + \frac{P_T}{P_A}\right)}$$

(c) (5 points) Are avocados normal goods? Justify your answer by mathematical derivation.

2. Chloe makes $500 per week and spends all her income on books and tea. Books cost $25 each, and Chloe buys 16 each week. Tea costs $5 per cup, and Chloe buys 20 cups. When Chloe's income falls to $450 per week, she cuts her consumption of books by 3 books and purchases 5 more cups of tea. Find the income elasticity of the demand for books; are books normal goods or inferior good for Chloe? Find the income elasticity of the demand for tea; is tea a normal good or an inferior good for Chloe?

The income elasticity of the demand for books is:

$$E_B = \frac{\Delta Q_B}{\Delta M} \cdot \frac{M_{initial}}{Q_{Binitial}} = \frac{-3}{-50} \cdot \frac{500}{16} = 1.875$$

Considering the income elasticity of demand for books is positive, books are normal goods for Chloe.

The income elasticity of the demand for tea is:

$$E_T = \frac{\Delta Q_T}{\Delta M} \cdot \frac{M_{initial}}{Q_{Tinitial}} = \frac{5}{-50} \cdot \frac{500}{20} = -2.5$$

Since the income elasticity of demand for tea is negative, tea is an inferior good for Chloe.

3. You are the alderman for the 20th ward of Chicago. In your ward, there are vacant lots that you are in charge of selling. Currently, the city sells a full lot for $150,000. Buyers can purchase a little or a much of a lot as the wish. For example, if someone buys half of a lot, they would pay $75,000.

Utility:

$$U(Q_R, Q_G) = 500,000Q_R - 250,000Q_R^2 + 2Q_G$$

(a) Solve for a buyer's demand for real estate and demand for all other goods.

Considering budget, Q_G in terms of Q_R:

$$Q_G = M - P_R Q_R$$

Substituting into utility function:

$$U(Q_R) = 500,000 Q_R - 250,000 Q_R^2 + 2(M - P_R Q_R)$$

Taking the derivative with respect to Q_R and setting to zero:

$$\frac{dU}{dQ_R} = 500,000 - 500,000 Q_R - 2P_R = 0$$

Solving for Q_R:

$$Q_R = 1 - \frac{2P_R}{500,000}$$

Substituting back to find Q_G:

$$Q_G = M - P_R \left(1 - \frac{2P_R}{500,000}\right) = M - P_R + \frac{2P_R^2}{500,000}$$

(b) When the city is selling land for $150,000 per lot, how much real estate does each buyer purchase? How about all other goods?

Substituting $P_R = 150,000$ into the demand functions:

$$Q_R = 1 - \frac{2(150,000)}{500,000} = 0.4$$

$$Q_G = M - 150,000 + \frac{2(150,000)^2}{500,000} = M - 150,000 + 90,000 = M - 60,000$$

(c) With the upcoming opening of the Obama Foundation headquarters, the city is going to raise the price of real estate to $200,000. Now how much real estate and other goods will each buyer purchase?

Substituting $P_R = 200,000$ into the demand functions:

$$Q_R = 1 - \frac{2(200,000)}{500,000} = 0.2$$

$$Q_G = M - 200,000 + \frac{2(200,000)^2}{500,000} = M - 200,000 + 160,000 = M - 40,000$$

(d) Using the Lasperyres approach, find out how much residents' incomes would have to rise in order to ensure they can afford their old bundle.

The original bundle consists of $Q_R = 0.4$ and $Q_G = M - 60,000$. So the cost at the new prices is:

$$Cost = P_R \cdot Q_R + Q_G = 200,000 \cdot 0.4 + (M - 60,000) = 80,000 + M - 60,000 = M + 20,000$$

(e) Given a choice between the compensated income and new prices you discovered in 3d and a return to old prices, what would consumers prefer?