Problem Set 3

PPHA 32400 Microeconomics and Public Policy I
Winter 2026

Instructions

(i) This problem set is due **Friday, Jan 30th, at 11:59pm**.
   - Late problem sets will not be accepted.
(ii) This problem set is divided into two parts:
   - Part I includes *ungraded* practice problems that students can optionally complete to deepen their understanding of the material. **Do not submit your answers.**
   - Part II includes *graded* problems. **You have to submit your answers**. You are allowed to collaborate in small groups for this part. However, **each student must submit their own individual write-up**. Your submission should reflect your personal understanding of the material, and therefore must be written in your own words. Please show your work, including calculations and clear, legible diagrams.
(iii) **Submit Part II electronically on Gradescope as a PDF:**
   - Your solutions must be submitted as typewritten PDF files. Examples of software that can be used includes Microsoft Word (which has a built-in equation editor) or LaTeX, a typesetting software.
   - To create diagrams, we recommend using a tablet, online software, or even drawing the diagram (neatly!) by hand and then add it to your file using a camera or scanner.
   - Ensure that you correctly tag your questions on Gradescope. Each improperly tagged question will be subject to a penalty equal to 10% of the question's value.

Part I: Practice Problems (Do NOT submit answers)

1. Consider a monopolist with two types of consumers in their market:

   - **High-type consumers:** 100 consumers, each willing to pay up to $10 for one unit.
   - **Low-type consumers:** N consumers, each willing to pay up to $4 for one unit.

   Each consumer demands at most one unit of the good. The monopolist can choose between two pricing strategies. They can set a high price $p = 10$ and sell only to high-type consumers, or they can set a low price $p = 4$ and sell to both high- and low-type consumers.

   The monopolist has marginal cost $MC = 0$ and fixed cost $FC = 0$. Note that these are simplifying assumptions meant to ensure that a monopolist's profit is equal to their revenue.

   (a) Suppose that $N = 100$, what is the monopolist's optimal pricing strategy? *(Hint: find their revenue/profit under both strategies, and compare them.)*
   If they price their goods at $p = 10$, they sell only to high-types, and earn $100 \times 10 = 1000$.

   If they price their goods at $p = 4$, they sell to both types, and earn $200 \times 4 = 800$

   They will price their goods at $p = 10$

   (b) Suppose $N = 200$, what is the monopolist's optimal pricing strategy?
   If they price their goods at $p = 10$, they sell only to high-types, and earn $100 \times 10 = 1000$.

   If they price their goods at $p = 4$, they sell to both types, and earn $300 \times 4 = 1200$

   They will price their goods at $p = 4$

   (c) For what values of N will the monopolist attempt to sell to both types i.e., set the price at $p = 4$?
   If they price their goods at $p = 10$, they sell only to high-types, and earn $100 \times 10 = 1000$.

   If they price their goods at $p = 4$, they sell to both types, and earn $(100 + N) \times 4 = 400 + 4N$

   They will price their goods at $p = 4$ if their profits from doing so exceed the profits from setting their price at $p = 10$

   $$400 + 4N > 1000 \Rightarrow 4N > 600 \Rightarrow N > 150$$

   The monopolist will set their price at $p = 4$ as long as the population of low types exceeds 150.

   (Technically, the monopolist is indifferent between selling to both types and only selling to high types at $N = 150$, so $N \geq 150$ is also a correct answer to this question. The substantive conclusions of models rarely ever hinge on this distinction, and we will not penalize students for using $\geq$ instead of $>$ or vice-versa. We use $>$ and $<$ in this course mainly for clarity.)

2. Frank is a handyman, meaning that he does plumbing, carpentry, and various odd jobs for hire. He has two types of customers, with different demand curves:

   High-demand customers: $Q = 24 - 0.3P \Longleftrightarrow P = 80 - \frac{10}{3}Q$
   Low-demand customers: $Q = 10 - 0.1P \Longleftrightarrow P = 100 - 10Q$

   where Q is the number of hours Frank works for the customer and P is Frank's wage in dollars per hour. The marginal cost of supplying each hour of work is $20.

(a) Suppose that Frank can distinguish between high-demand and low-demand consumers. What price should he charge high-demand consumers? How many hours of work will they demand at that price?

We know that $P = 80 - \frac{10}{3}Q$, which implies $MR = 80 - \frac{20}{3}Q$. Profit is maximized at $MR = MC \Rightarrow 80 - \frac{20}{3}Q_h = 20 \Rightarrow Q_h = 9$ (hours). The hourly rate is $P_h = 80 - \frac{10}{3}(9) = 50$.

(b) Suppose that Frank can distinguish between high-demand and low-demand consumers. What price should he charge low-demand consumers? How many hours of work will they demand at that price?

We know that $P = 100 - 10Q$, which implies $MR = 100 - 20Q$. Profit is maximized at $MR = MC \Rightarrow 100 - 20Q_l = 20 \Rightarrow Q_l = 4$ (hours). The hourly rate is $P_l = 100 - 10(4) = 60$.

Suppose Frank has difficulty telling high-demand from low-demand customers, so he decides to use price discrimination to make different types of customers self-select into the most profitable pricing scheme. He sets a price for one-hour service, but also offers a quantity discount for customers willing to buy multiple hours of service in advance. Frank hopes that high-demand customers will self-select into the discounted plan, and that low-demand customers will choose to buy individual hours rather than the discounted plan.

(c) Given his goal, what price should Frank set for individual hours of service? What price and minimum quantity should he set for the discounted plan?

Frank is trying to channel high-demand consumers into purchasing their profit-maximization level of 9 hours at $50 per hour, and the low-demand consumers into purchasing 4 hours at $60 per hour. Thus, the individual hourly rate should be $60, while the discounted price should be $50 subject to a minimum of 9 hours.

(d) Consider high-demand customers. If they paid Frank by the hour, how many hours would they demand? What is their consumer surplus in this case? If they purchase the discounted plan, what is their consumer surplus? Will they purchase the discounted plan?

At $60 per hour, high-demand customers would purchase $Q = 24 - 0.3P = 24 - 0.3 \times 60 = 6$ (hours).
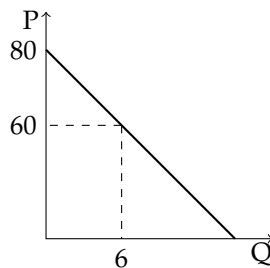

Figure 1: High-demand: Consumption under per-hour plan

From Figure 1 we can see that consumer surplus is $CS = \frac{1}{2} \times (80 - 60) \times 6 = 60$.

From Figure 2 we see that, when they buy the discounted plan, their $CS = \frac{1}{2} \cdot (80 - 50) \cdot 9 = 135$. Since $135 > 60$, they would prefer the discounted plan.

(e) Consider low-demand customers. If they paid Frank by the hour, how many hours would they demand? What is their consumer surplus in this case? If they purchase the discounted plan, what is their consumer surplus? Will they purchase the discounted plan?
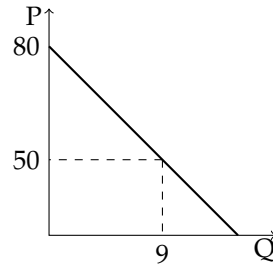
Figure 2: High-demand: Consumption under discounted plan

If they paid Frank by the hour, they would purchase $Q = 10 - 0.1P = 10 - 0.1(60) = 4$ (hours).

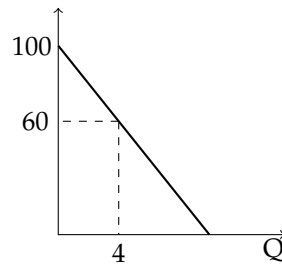From Figure 3, we see that their consumer surplus is $Q = \frac{1}{2} \cdot (100 - 60) \cdot 4 = 80$.

Figure 3: Low-demand: Consumption under per-hour plan

Analyzing their consumption under the discounted plan is a little trickier, because at $50 they would buy less than 9 hours. Let's start by asking ourselves two questions.

First, how many hours would they have purchased at $P = 50$? The answer is $Q = 10 - 0.1(50) = 5$

Second, at what price would they buy 9 hours? The answer is given by $P = 100 - 10(9) = 10$.

Change in welfare is shown in Figure 4. The consumer gains the green area minus the red area.
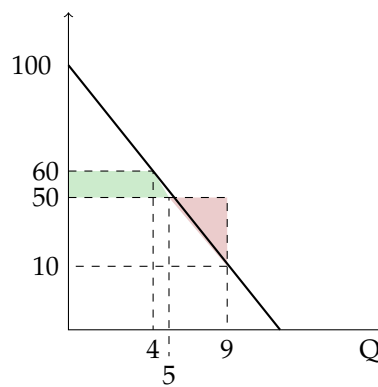

Figure 4: Low-demand: Welfare under discounted plan

The green area is a rectangle plus a triangle, and has the area

$$\underbrace{(60-50)\times 4}_{\text{Rectangle}}+\underbrace{\frac{1}{2}\times(60-50)\times(5-4)}_{\text{Triangle}}=40+5=45$$

The red area is a triangle with area

$$\frac{1}{2}\times 40\times 4=80$$

Thus the gain is $45-80=-35$ so the consumer would be worse off if they took this option.

(f) Based on your answers to (d) and (e), will the plan be successful in making customers self-select into the most profitable plan for the service?
Yes – low-demand consumers would choose to purchase the per-hour plan, and high-demand consumers would choose to purchase the discounted plan.

Part II: Problems (Submit your answers)

This section is meant to build on the lessons you learn by attempting Part I; we therefore strongly recommend that you attempt Part I before attempting this section.

For each of the answers below, make a good faith attempt to answer each question and sub-question to the best of your ability. Provide complete answers and show your work, including explanations where appropriate. Failure to explain your answers may be penalized.

You should feel free to be parsimonious when typesetting math, including what is necessary for a reader to understand your solution, but no more. You do not need to include algebra, simplification, each step of a derivation etc.

1. (10 points) Consider a market where a monopolist can provide two versions of a product: Cool and Generic. There are two types of consumers: Hipsters and Normies. The monopolist cannot tell them apart and must rely on versioning to induce them to self-select. Consider the willingness to pay of each type presented below:

	Generic	Cool
Hipster	$5	$10
Normie	$3	$6

   The Generic version of the product is priced at $3. Let the price of of the Cool version be p. *(Recall that a person's consumer surplus is their willingness to pay for a good minus the price they pay for the good.)*

   (a) (4 points) What is the consumer surplus that Hipsters get from buying the Generic version? What is the consumer surplus that Hipsters get from buying the Cool version? For what values of p do Hipsters prefer the Cool version? *(Hint: your answer to the final sub-question will take the form of an inequality – either $p < A$ or $p > A$ where A is some constant.)*

   (b) (4 points) What is consumer surplus that Normies get from buying the Generic version? What is the consumer surplus that Normies get from buying the Cool version? For what values of p do Normies prefer the Generic version?

   (c) (2 points) What are the prices p such that Hipsters self-select into buying the Cool version and Normies self-select into buying the Generic version?

2. (16 points) ClayBrick is a corporation selling proprietary video and audio editing software, targeting creative people such as filmmakers and musicians. For simplicity, assume there is a single filmmaker and a single musician, and their willingness to pay is shown in the table below:

	Video editor	Audio editor	Bundle
Filmmaker	300	100	400
Musician	100	300	400

   Assume that the marginal cost in this market is $MC = 0$

   (a) (4 points) Suppose ClayBrick were to sell each piece of software separately. For each good, what is the profit maximizing price?

   (b) (2 points) Consider your answer to Question 2a. What is the producer surplus, consumer surplus, and total surplus in this market?

   (c) (4 points) Suppose ClayBrick were to sell its software as a bundle. What is the profit maximizing price? What is the producer surplus, consumer surplus and total surplus in this market?

(d) (3 points) Imagine a world where the audio editor does not exist, and ClayBrick sells video editors. William Fence is an entrepreneur looking to make some money in tech. How much would Mr. Fence be willing to invest in the development of an audio editor? How much would ClayBrick be willing to invest in the development of an audio editor?* You may assume that Mr. Fence owns no other software, and that ClayBrick can sell software as bundles.

(e) (3 points) Explain, in your own words, why a firm with market power in market A might be willing to fund research and development in a different market B. Base your explanation on your answer above. Keep your response intuitive, non-technical and under 60 words.

---

*Hint: Agents are willing to invest up to X in the development of new software, where X is the amount of profit they expect to gain from it. What is Mr. Fence's expected profit from the sale of video editors? What is ClayBrick's expected profit from the sale of a bundle?