Laboratory Assignment 5
Amplitude Modulation

PURPOSE

In this lab, you will explore the use of digital computers for the analysis, design, synthesis, and simulation of an amplitude modulation (AM) system. In AM systems, an information signal is multiplied by a sinusoid at a much higher frequency. This technique makes it possible to transmit the information signal over the air via electromagnetic waves; in essence, using a higher-frequency signal to "carry" the information signal. You should be familiar with AM radio, which uses this technique; the frequency indicated on your AM radio dial for each station is the carrier frequency.

In this experiment, you will explore how modulation, particularly AM, impacts the signal spectrum and how this information is used to simulate the modulation and demodulation of an audio signal. In the process, you will learn about distortion resulting from under-sampling, called aliasing, and how to prevent it. This will be particularly important in simulating an AM system as modulated signals have substantially higher-frequency components than audio signals. From this assignment, you should gain an appreciation for some of the practical trade-offs in designing AM system technologies and in using sampled CT signals for AM system simulation.

Submission Instructions: Solutions to the questions in the "Lab Assignment" section should be handed in as part of your Lab 1 report. All reports should include the MATLAB code with comments and a description of what you have learnt with this lab exercise. **Due Date of Submission is 29th November 2019 for all sections.** The answers to each problem should include the methodology/approach of how the solution has been obtained.

5.1 OBJECTIVES

By the end of this laboratory assignment, you should be able to:

1. Compute, display, and interpret frequency spectra of an AM signal.
2. Use your understanding of frequency-domain analysis for modulating and demodulating audio signals.
3. Prevent aliasing by using filters or changing the sampling rate.

5.2 BACKGROUND

In the following sections, we discuss the effect of amplitude modulation and sampling on the signal spectrum using common properties of the Fourier transform. Both topics can be analyzed by understanding the effect of multiplying a signal by a periodic waveform. We use this information to provide an intuitive view of digital Fourier analysis as "sampling" of both the CT signal and its spectrum, and discuss digital filtering, frequency shifting, and aliasing as needed in your AM system analysis and design.

5.2.1 Frequency Shifting

How can we shift the center frequency of a sampled signal? This can be done by multiplying your signal by a sinusoid in the time domain; this process is called modulation.

Recall that the Fourier transform of a sinusoid is a pair of unit impulse functions at positive and negative fundamental frequencies and multiplication in the time domain corresponds to convolution in the frequency domain. Thus, multiplication of a signal by a sinusoid results in the convolution of shifted δ functions with the input signal spectrum. The spectrum of the modulated signal will comprise our original signal spectrum shifted and centered on the positive and negative fundamental frequencies of the modulating sinusoid.

We can also understand the impact of this modulation processing in the time domain by using trigonometric identities. Suppose that your input signal is $\cos(\omega_1 t)$. If we want to shift this sinusoid to the frequency ω_2, we need to multiply by the frequency $\omega_2 - \omega_1$. From trigonometry we know that:

$$2\cos(\omega_a t)\cos(\omega_b t) = \cos((\omega_a + \omega_b)t) + \cos((\omega_a - \omega_b)t)$$

Our multiplication results in the signal $\cos(\omega_2 t) + \cos[(\omega_2 - 2\omega_1)t]$, while we wanted just $\cos(\omega_2 t)$. The additional unwanted sinusoidal component at ω_2 $-2\omega_1$ can be removed by high-pass filtering the modulated signal.

5.2.2 Aliasing

The term aliasing refers to the distortion that occurs when a continuous-time signal has frequency components at frequencies larger than half of the sampling rate. The process of aliasing describes the phenomenon in which power originally at these high frequencies appears as power in the sampled signal spectrum at a lower frequency, a frequency that can be represented using the given sampling rate.

One common example of aliasing is when wheels on a moving vehicle in a video appear to remain motionless or move "backwards," rotating in a direction opposite to that expected given the direction of travel. This effect can be understood using the idea of sampling: video is generated by displaying 30 still images, or frames, per second; this rate of display is high enough to trick the vision system into perceiving motion. Consider a wheel that makes 30 full rotations per second; this wheel will appear to be identical in each video frame. Thus the wheel appears to be motionless! Rather than seeing the high frequency rotation (30 rotations per second) of the wheel, it appears to be still, at a frequency of zero rotations per second.

Consider a CT signal for which the frequency spectrum in shown Figure 12.5.1. Note that the signal is bandlimited to $B/2\pi$ Hz.

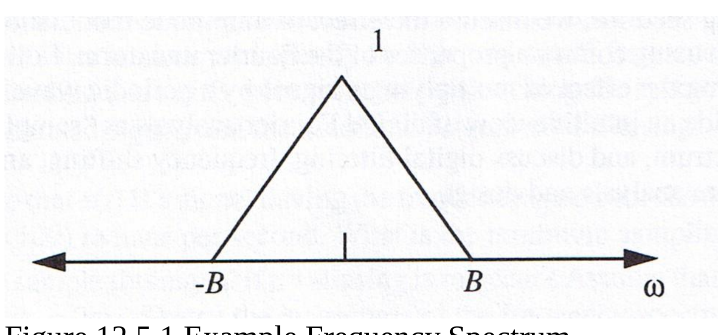

Figure 12.5.1 Example Frequency Spectrum

Sampling a signal can be modeled as multiplication by a train of periodic impulses in the time domain, corresponding to convolution of the signal spectrum with impulses at the harmonic frequencies of the time domain impulse train:

$$x_s(t) = x(t)\sum_{k=-\infty}^{\infty}\delta(t - nT_s) \Leftrightarrow X(\omega)\frac{1}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi k}{T_s}\right)$$

If we sample this signal so that the sampling rate is less than 2B—the Nyquist rate— then the frequency spectrum shown in Figure 12.5.2 results.

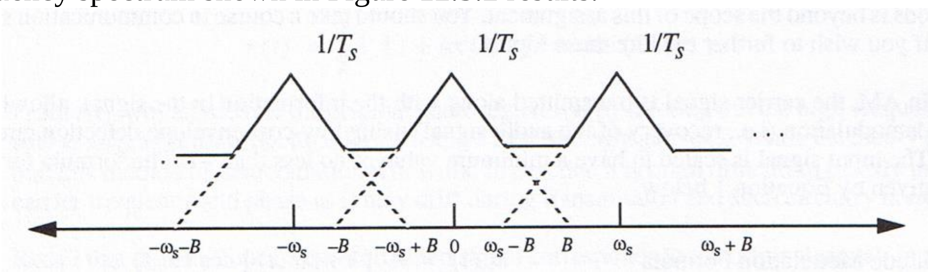

Figure 12.5.2 Aliased Spectrum

The overlapping sections add together, producing a distorted spectrum. Note the shape of the baseband (i.e. centered at zero) spectrum of bandwidth B; it is no longer perfectly triangular in shape, but is now "flat" in the band ω_s - B to B. The original signal spectrum cannot be recovered by using a lowpass filter, causing non-recoverable distortion in the sampled signal.

Two methods are commonly used to prevent aliasing:

1. Sampling at a sufficiently high rate, that is $\omega_s \geq 2B$

2. Low pass filtering the signal prior to sampling so that the signal to be sampled is band-limited to $\omega_s/2$

5.2.3 Amplitude Modulation

Modulation is important because it allows us to transmit signals at different frequencies. Consider amplitude modulation radio systems: all audio signals are baseband signals comprising frequencies in the audible spectrum. Using AM, the audio signals being transmitted by different stations are each shifted to different frequency bands, each centered about the frequency indicated on your radio dial. In this way, signals from different stations do not interfere with each other, which would cause distortion in the received signal. If modulation were not used, then all radio signals would be transmitted at the same time. At best, you would only be able to understand the station for which the signal received at your house is "loudest." At worst, the signals from all stations would simply add together, creating an incredible cacophony of dissonant sounds - which you would be unable to turn off.

AM is a subset of the group of modulation techniques that are said to be linear. Such techniques include double sideband modulation, upper sideband modulation, lower sideband modulation, and AM. All of these linear modulation techniques are similar, in that they shift the input signal up to a transmittable frequency by modulating the original signal using a sinusoid; differences in the bandwidth and frequency bands over which these methods are used result in important differences in implementation and application.

In AM, the carrier signal is transmitted along with the information in the signal, allowing for demodulation (i.e., recovery of the audio signal) using low-cost envelope detection circuitry. The input signal is scaled to have a minimum value of no less than —1. The formula for AM is given by

$$x_c(t) = A_c[1 + ax(t)]\cos(\omega_c t)$$

where A_c is the amplitude of the carrier signal, a is the modulation index - which scales the signal to make sure that its minimum value is approximately -1, and ω_c is the frequency of the carrier signal.

Consider the modulation of a simple waveform 2 $\cos(4\pi t)$ using a 100-Hz carrier wave. A modulation index of 0.5 provides the appropriate scaling. If we let the carrier amplitude be 1, then the modulated output signal is as shown in Figure 12.5.3.

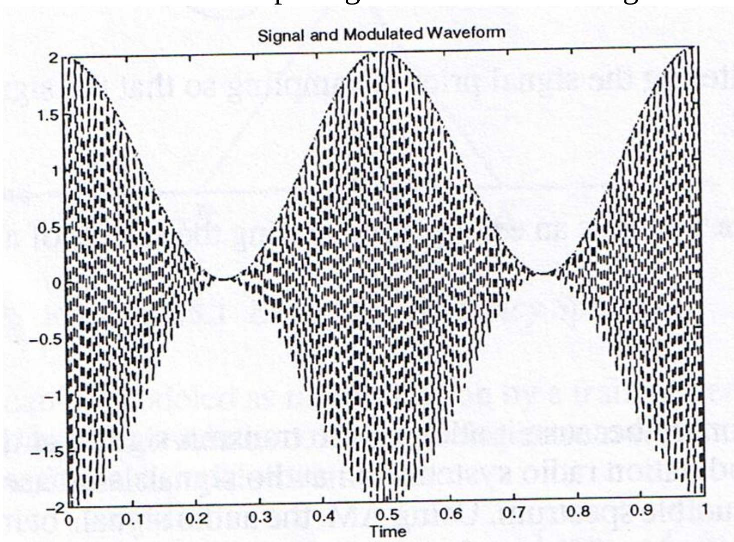

Figure 12.5.3 Shifted Signal and Modulated Output

You can see both the carrier and modulating waveforms clearly. Notice that the envelope of the modulated signal is the shape of the original signal, scaled by the modulation index with a constant shift of 1 on the vertical axis.

We can demodulate this signal by multiplying by a second sinusoid at the carrier frequency, $\cos(\omega_0 t)$, which results in a signal $r(t) = x_c(t)\cos(\omega_0 t)$. When expressed in terms of the original audio signal, it can be seen that $r(t)$ includes a $\cos^2(\omega_0 t)$ term. Substituting the trigonometric identity $\cos^2(\omega_0 t) = [1 + \cos(2\omega_0 t)]/2$ into this expression for $r(t)$ results in

$$r(t) = \frac{1}{2} A_c [1 + ax(t)][1 + \cos(2\omega_0 t)]$$

From *r(t)*, we can recover the original audio signal x(t) by filtering out the high-frequency sinusoid at $2\omega_0$, assuming that ω_0 is large) using a LPF having a gain of 2. While the theory suggests that this method of demodulation will work, in practice it is often difficult to exactly match the carrier frequency and phase as it may drift during transmission and such circuitry is expensive.

In practice, a simple, low-cost envelope detection circuit, like that shown in Figure 12.5.4, is often used to recover the original signal waveform from the modulated signal.

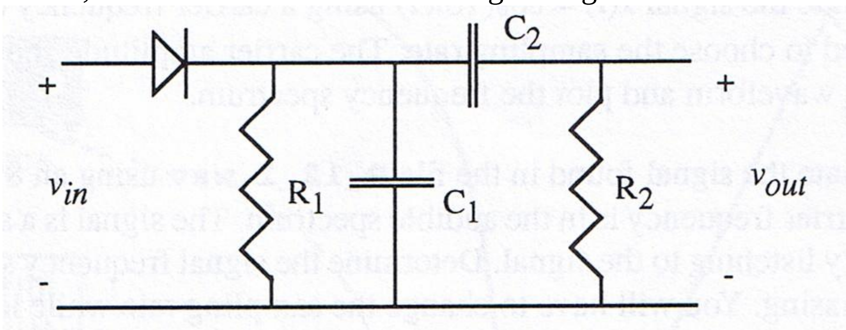

Figure 12.5.4 Passive Envelope Detection Circuit

The diode, R_1, and C_1 act as an envelope detector by allowing the peaks of the waveform to charge the capacitor, which does not completely discharge before the next peak occurs. It operates in a similar manner to a half-wave rectifier, except that the presence of R_1 allows the capacitor to discharge and thus follow a changing waveform. R_2 and C_2 form a high-pass filter to remove the DC component $l/2A_c$, which is a result of transmitting the carrier with the modulated signal.

The envelope detection circuit can operate as long as the carrier frequency is much higher than the highest frequency present in the signal. If the lowpass filter formed by R_1 and $C_1$is set to cut out signals higher than the highest signal frequency, then the carrier tone (which is significantly higher) should disappear. Similarly, the highpass filter formed by R_2 and C_2 should cut out the DC component of the signal.

Since the diode is a nonlinear device, it's difficult to simulate this circuit with standard linear operations. In MATLAB, the easiest way to perform this operation is to simulate the operation of the diode on the R/C circuit: if v_{in} is larger than the capacitor voltage, the capacitor voltage is charged to v_{in}. If v_{in} is smaller, the capacitor voltage discharges through R_1. This is easily accomplished using **for** loops and an **if** statement.

EXPERIMENT

You are to do two things:

1. Successfully simulate AM in MATLAB without having any aliasing occur;
2. Demodulate an AM signal successfully in MATLAB, again without any aliasing.

These are nontrivial problems. You will need to know the frequency spectrum of the input signal, understand how modulation affects the signal, and be able to fix any problems that develop. Beware of aliasing. Remember that long signals require a great deal of computation, so be patient with your computer - it's thinking as fast as it can.

Problem 1. Modulate the signal $x(t) = cos(10\pi t)$ using a carrier frequency of 100 Hz. You are allowed to choose the sampling rate. The carrier amplitude should be 1. Listen to the output waveform and plot the frequency spectrum. You may want to use **fftshift** command to plot the frequency spectrum. You may also need to use **fft** command to calculate fourier coefficients. Set modulation index so that signal's minimum value is -1.

Problem 2. Modulate the signal found in the file `P_12_1.wav` using an 8 kHz carrier; note that this carrier frequency is in the audible spectrum. The signal is a sample of a person talking.

1. Load the .wav file, Listen to the waveform
2. Plot the Fourier coefficients (frequency spectrum)
3. Modulate the signal and set the carrier frequency to 8 kHz
4. Our waveform x currently has higher bandwidth fs/2 = 11025 Hz. We need to down-sample the signal so that the new sampling frequency will be fs2 = 8000 Hz = fc;
5. Set down-sampling frequency to 8000 Hz; Resample our signal at fs2
6. Calculate new FT coefficients and Plot it
7. Our waveform xd now has maximum bandwidth fs2/2 = 4000 Hz. If we modulate at fc = 8000 Hz, we will have a maximum frequency of fc+fs2/2 = 12000 Hz. Therefore, we need to upsample our signal xd so that once we modulate it, we will be able to detect all the relevant frequencies in the modulation. The new sampling frequency is thus fs3=2*(fc+fs2/2) = 2*fc+fs2.
8. Upsample our signal at fs3, Then modulate it and plot the frequency spectrums by calculating FT coefficients

Problem 3. Demodulate the signal you created in **Problem 1**. Make sure the output matches the input (except for amplitude). You may emulate the mathematical/filter technique. Low-pass filter used to clean up the demodulated signal and bring it back to its original form. The input X is the coefficients of the demodulated signal to filter, fs is the sampling frequency, and **fcut** is the cutoff frequency.

Set the DC component to 0. We need to cutoff everything after **fcut**. Get difference in frequency between two consecutive points. Do **fftshift**() so that the coefficients are in order

Calculate the indices of the left cutoff and the right cutoff point. Note that **floor**((fs/2)/df)+1 represents the index of the 0 Hz coefficient, while floor(**fcut*df**) represents the distance in indexes between the 0 Hz coefficient and the cutoff frequency coefficient.

Problem 4. Demodulate the signal found in the file `P_12_2.wav`. You should determine the carrier frequency and bandwidth of the signal before applying any demodulation techniques to it. Listening to the demodulated output should help you to know when you have achieved the correct results.