EECS 3451 Lab 4

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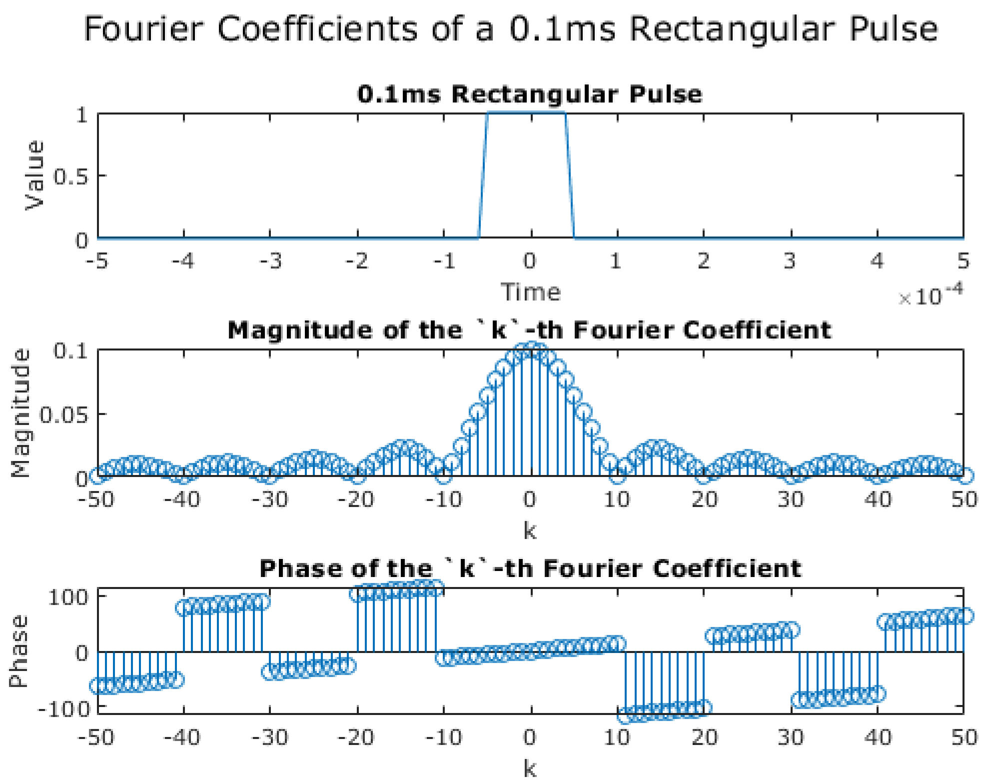
AUTHORS

- Jonathan Baldwin (212095691)
- Mark Savin (212921128)
- Sarwat Shaheen (214677322)

Q1 - Fourier Coefficients of a Rectangular Pulse

```
Fs = 100e3;             % 100 kHz sampling rate
Ts = 1/Fs;              % sampling period

T=1e-3;

t=-T/2:Ts:T/2;

d=-1:T:1;                % 1ms period
w=0.1e-3;                % .1 ms pulse width

x = rectpuls(t,w);

N = length(x);
k=-(N-1)/2:(N-1)/2;
C=zeros(1,length(k));
n=t/Ts;

for i1=1:length(k)
    for i2=1:length(x)
        C(i1)=C(i1)+1/N*x(i2)*exp(-1i*2*pi*k(i1)*n(i2)/N);
    end
end

figure;
sgtitle('Fourier Coefficients of a 0.1ms Rectangular Pulse');
```

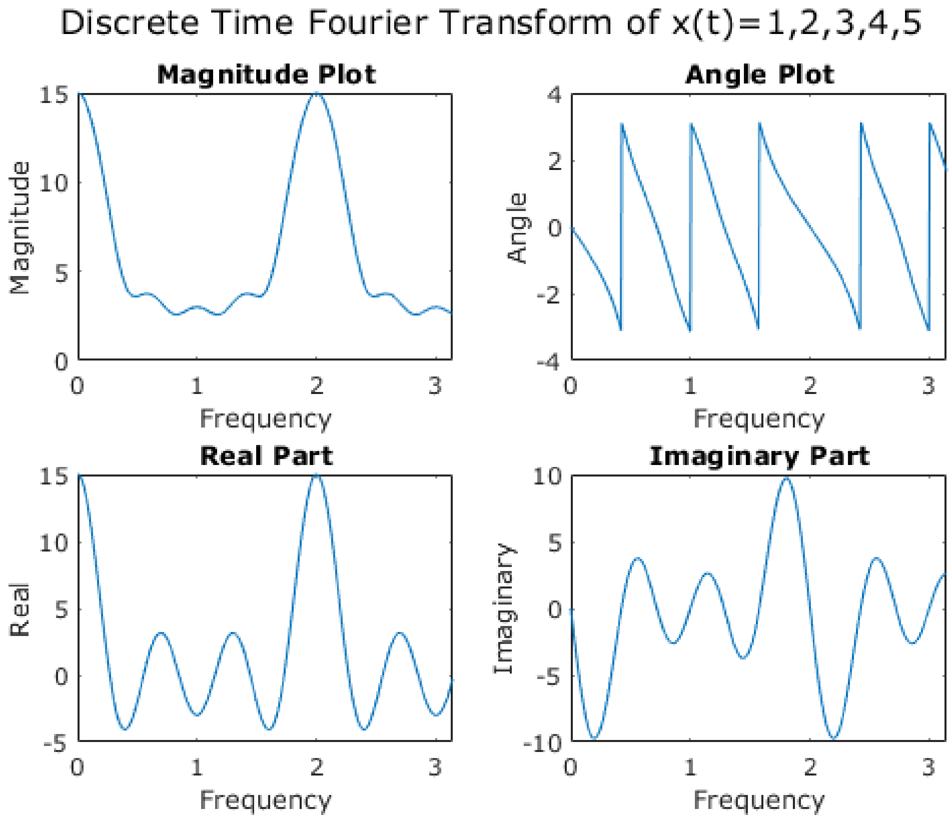
```
plot1 = subplot(3,1,1);
plot(t,x);
title('0.1ms Rectangular Pulse');
xlabel('Time');
ylabel('Value');

plot2 = subplot(3,1,2);
stem(k, abs(C));
title('Magnitude of the `k`-th Fourier Coefficient');
xlabel('k');
ylabel('Magnitude');

plot3 = subplot(3,1,3);
stem(k, angle(C)*130/pi);
title('Phase of the `k`-th Fourier Coefficient');
xlabel('k');
ylabel('Phase');
```



Q2 - DTFT of x(n)={1,2,3,4,5} from 0 to \pi

```
n = -1:3;
x = 1:5;
k = 0:500;
w = (pi/500)*k;
```

```
W = exp(-1i * pi * n' * w);

X = x * W ;

figure;
sgtitle('Discrete Time Fourier Transform of x(t)={1,2,3,4,5}');

subplot(2, 2, 1);
plot(w,abs(X));
title('Magnitude Plot');
xlabel('Frequency');
ylabel('Magnitude');

subplot(2, 2, 2);
plot(w,angle(X));
title('Angle Plot');
xlabel('Frequency');
ylabel('Angle');

subplot(2, 2, 3);
plot(w,real(X));
title('Real Part');
xlabel('Frequency');
ylabel('Real');

subplot(2, 2, 4);
plot(w,imag(X));
title('Imaginary Part');
xlabel('Frequency');
ylabel('Imaginary');
```

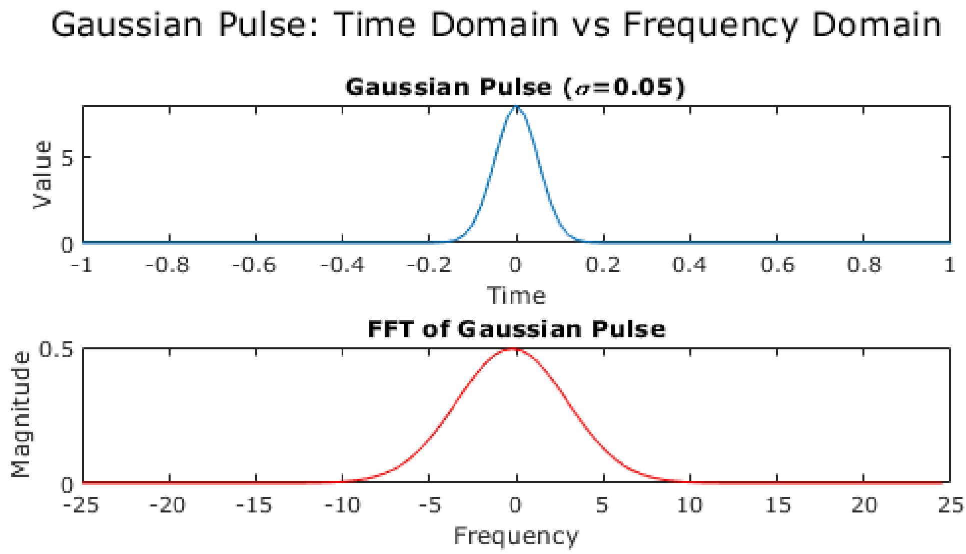


Q3 Gaussian Pulse

When we take the zero-centered FFT of a gaussian pulse, the magnitude of the FFT is shaped like a gaussian pulse.

```
Fs = 50;
Ts = 1/Fs;
sigma = 0.05;

t=-1:Ts:1;

x = 1/sqrt(2*pi*sigma^2)*exp(-t.^2/(2*sigma^2));

N = length(x);
NFFT = N;

F = Fs*(-NFFT/2:NFFT/2-1)/NFFT;
Y = fftshift(fft(x, NFFT));     % Use fftshift to zero-center the fft

figure;
sgtitle("Gaussian Pulse: Time Domain vs Frequency Domain");

plot1 = subplot(3,1,1);
plot(t,x);
title("Gaussian Pulse (\sigma=" + num2str(sigma) + ")");
```

```
xlabel("Time");
ylabel("Value");

plot2 = subplot(3,1,2);
plot(F, abs(Y)/N, 'r');
title("FFT of Gaussian Pulse")
xlabel("Frequency");
ylabel("Magnitude");
```

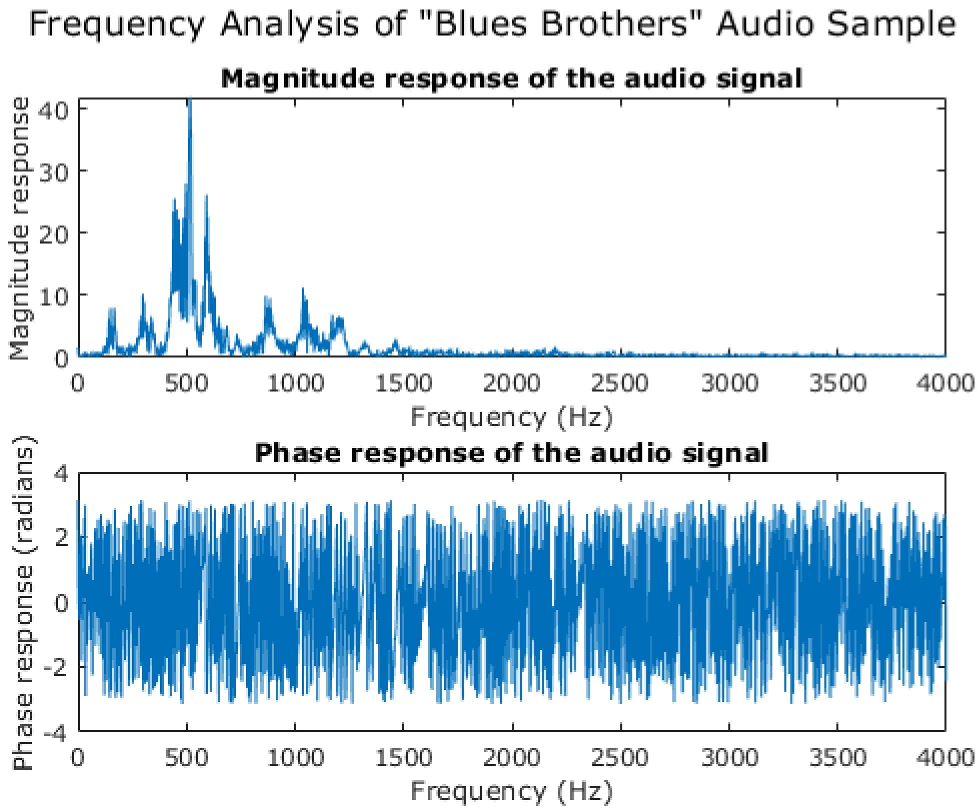


Problem - 4

Here we do frequency analysis on the Blues Brothers clip from Lab 2. When we look at the magnitude response of the audio signal, we can see the largest peaks in the 400-600 Hz range.

```
[x, Fs] = audioread('audio.wav');

N = length(x);
NFFT = 2^12;

X = fft(x, NFFT);
F = Fs*(0:NFFT-1)/NFFT;

figure;
sgtitle('Frequency Analysis of "Blues Brothers" Audio Sample');

plot1 = subplot(2,1,1);
```

```
plot(F(1:NFFT/2),abs(X(1:NFFT/2)));
title('Magnitude response of the audio signal');
xlabel('Frequency (Hz)');
ylabel('Magnitude response');

plot2 = subplot(2,1,2);
plot(F(1:NFFT/2),angle(X(1:NFFT/2)));
title('Phase response of the audio signal');
xlabel('Frequency (Hz)');
ylabel('Phase response (radians)');
```

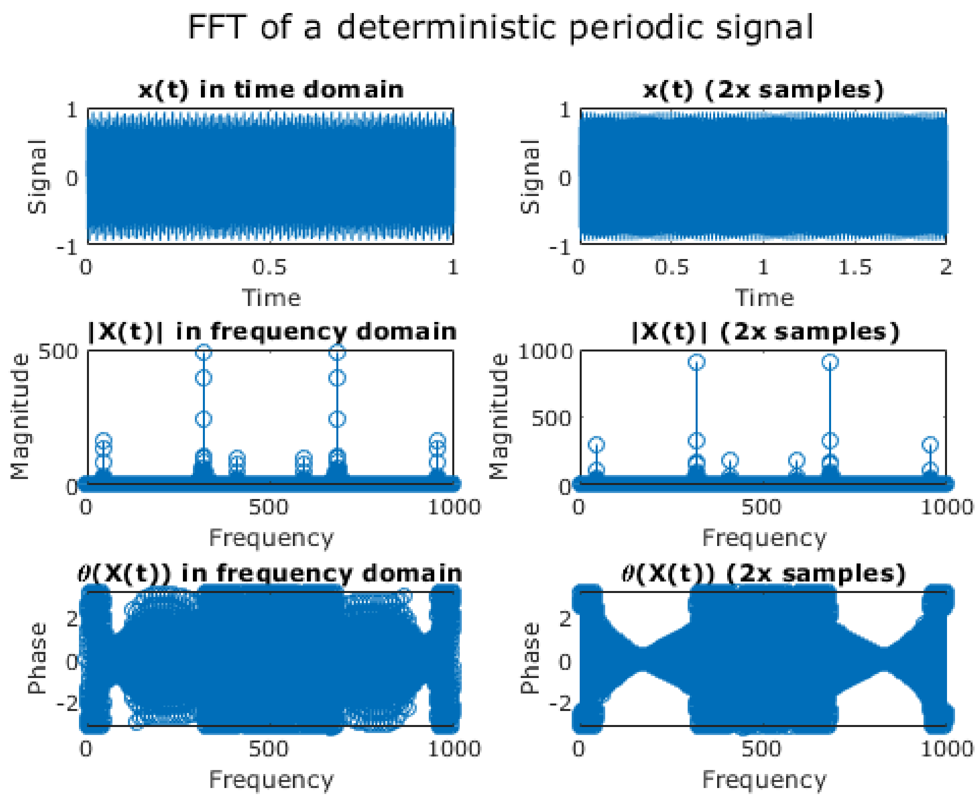


Problem - 5

To increase the frequency resolution, we can either increase the number of samples while keeping sample rate constant (which has the downside of needing to capture and process more samples), or decrease the sampling rate while keeping number of samples constant (which has the downside of lowering the maximum frequency we can resolve.) The former is preferable here.

```
Ts = 0.999e-3;
Fs = 1/Ts;
t = 0:Ts:1;
f1 = 1000;
x = sin(2*f1*t) + (1/3)*sin(2*3*f1*t) + (1/5)*sin(2*5*f1*t);

N = length(X);
NFFT = N;
```

```
F = Fs*(0:NFFT-1)/NFFT;
X = fft(x, NFFT);

figure;
sgtitle("FFT of a deterministic periodic signal");

subplot(3,2,1);
plot(t, x);
xlabel("Time");
ylabel("Signal");
title("x(t) in time domain");

subplot(3,2,3);
stem(F,abs(X));
xlabel("Frequency");
ylabel("Magnitude");
title("|X(t)| in frequency domain");

subplot(3,2,5);
stem(F,angle(X));
xlabel("Frequency");
ylabel("Phase");
title("\theta(X(t)) in frequency domain");

Ts = 0.999e-3;
Fs = 1/Ts;
t = 0:Ts:2;
f1 = 1000;
x = sin(2*f1*t) + (1/3)*sin(2*3*f1*t) + (1/5)*sin(2*5*f1*t);

N = length(X);
NFFT = N;
F = Fs*(0:NFFT-1)/NFFT;
X = fft(x, NFFT);

subplot(3,2,2);
plot(t, x);
xlabel("Time");
ylabel("Signal");
title("x(t) (2x samples)");

subplot(3,2,4);
stem(F,abs(X));
xlabel("Frequency");
ylabel("Magnitude");
title("|X(t)| (2x samples)");

subplot(3,2,6);
stem(F,angle(X));
xlabel("Frequency");
ylabel("Phase");
title("\theta(X(t)) (2x samples)");
```

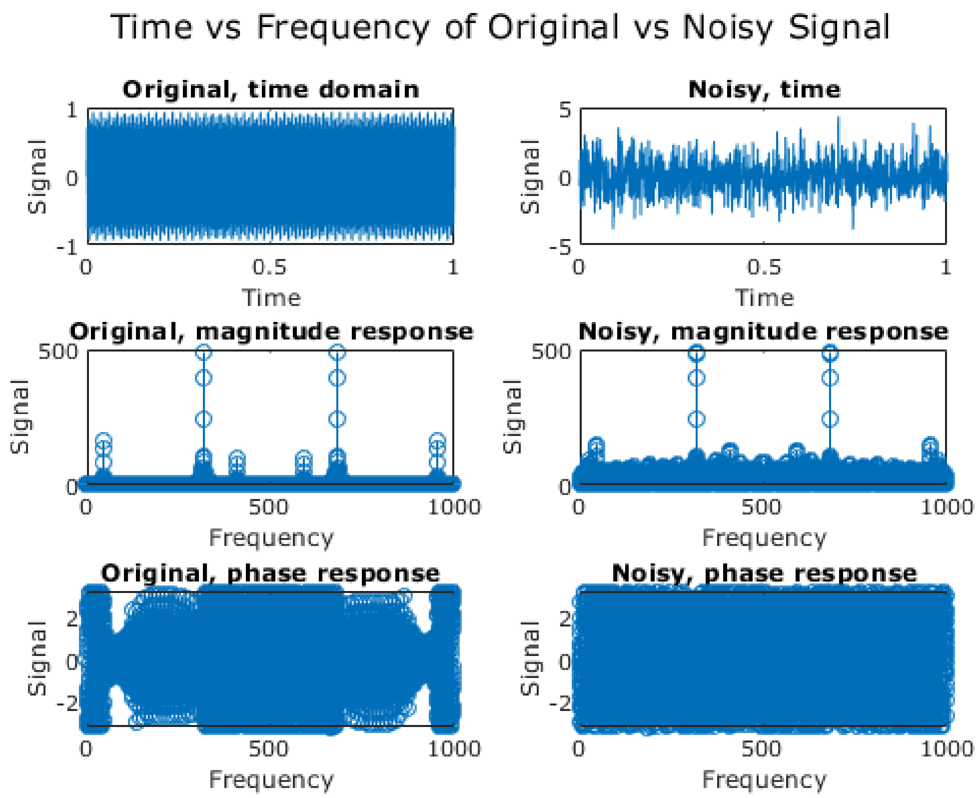


Problem - 6

When we plot the noisy signal against the original, we can see that the noise is a lot more noticeable in the time domain, compared to the frequency domain.

```
Ts = 0.999e-3;
Fs = 1/Ts;
t = 0:Ts:1;
f1 = 1000;
x = sin(2*f1*t) + (1/3)*sin(2*3*f1*t) + (1/5)*sin(2*5*f1*t);
xNoisy = x + randn(size(x));

N = length(X);
NFFT = N;
F = Fs*(0:NFFT-1)/NFFT;
X = fft(x, NFFT);
XNoisy = fft(xNoisy, NFFT);

figure;
sgtitle("Time vs Frequency of Original vs Noisy Signal");

subplot(3,2,1);
plot(t,x);
xlabel("Time");
ylabel("Signal");
```

```
title("Original, time domain");

subplot(3,2,3);
stem(F,abs(X));
xlabel("Frequency")
ylabel("Signal")
title("Original, magnitude response");

subplot(3,2,5);
stem(F,(angle(X)));
xlabel("Frequency")
ylabel("Signal")
title("Original, phase response");

subplot(3,2,2);
plot(t,xNoisy);
xlabel("Time");
ylabel("Signal");
title("Noisy, time");

subplot(3,2,4);
stem(F,abs(XNoisy));
xlabel("Frequency")
ylabel("Signal")
title("Noisy, magnitude response");

subplot(3,2,6);
stem(F,(angle(XNoisy)));
xlabel("Frequency")
ylabel("Signal")
title("Noisy, phase response");
```



What we learned

In this lab we learned how to calculate Fourier coefficients in MATLAB, how to use the Fast-Fourier Transform to convert from the time domain to the frequency domain, how to increase the frequency resolution of the FFT, and how noise is less perceptible in the frequency domain.

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