US Real GDP Growth Prediction from Lagged GDP Growth & Spread Between 10-year Treasury Constant Maturity and 3-month Treasury Constant Maturity

John Balzani

11/14/2019

Abstract

Can the spread between long-term and short-term interest rates predict future economic growth? The purpose of this research is to explore whether or not the difference between the 10-year Treasury Constant Maturity Yield and the 3-month Treasury Constant Maturity Yield (10-year 3-month term spread) is significant in predicting US real GDP growth in the following 4 quarters, above the past values of real GDP growth itself. In particular, one of the purposes of this research is to explore if these predictions can be made accurately for the 2008 recession and current post-2008 recession business cycle. Different lagged values of the term spread and real GDP growth are tested in an Autoregressive Distributed Lag (ADL) model, and this is compared to an Autoregressive (AR) model with 4 lags of real GDP growth. Data from 1983 - April 2008 are used to estimate the model, and data after that are used to evaluate the model performance. It is found that the 1-quarter and 2-quarter lagged values of real GDP growth and the 2-quarter lagged value of the 10-year 3-month term spread are significant in predicting US real GDP growth from 1983-2008. However, these do not accurately predict US real GDP growth from July 2008 - July 2019, with a RMSE of 2.41% annualized quarterly real growth and a MAE of 1.61%. The model predicts the recession 2 quarters too late in late 2008, and does not fully capture the quarterly variability in annualized real GDP growth. During times of more stable economic growth, such as post-2015, the model is more accurate, with a RMSE and MAE of 1.19% and 0.89% respectively for this time period.

Introduction:

As yields are an indication of the market's projections of future economic growth, it would make sense that yields may predict future economic growth. The difference between long term and short term yields may also predict future GDP growth.Past research has shown a link between the term spread between long term and short term yields and future GDP growth.

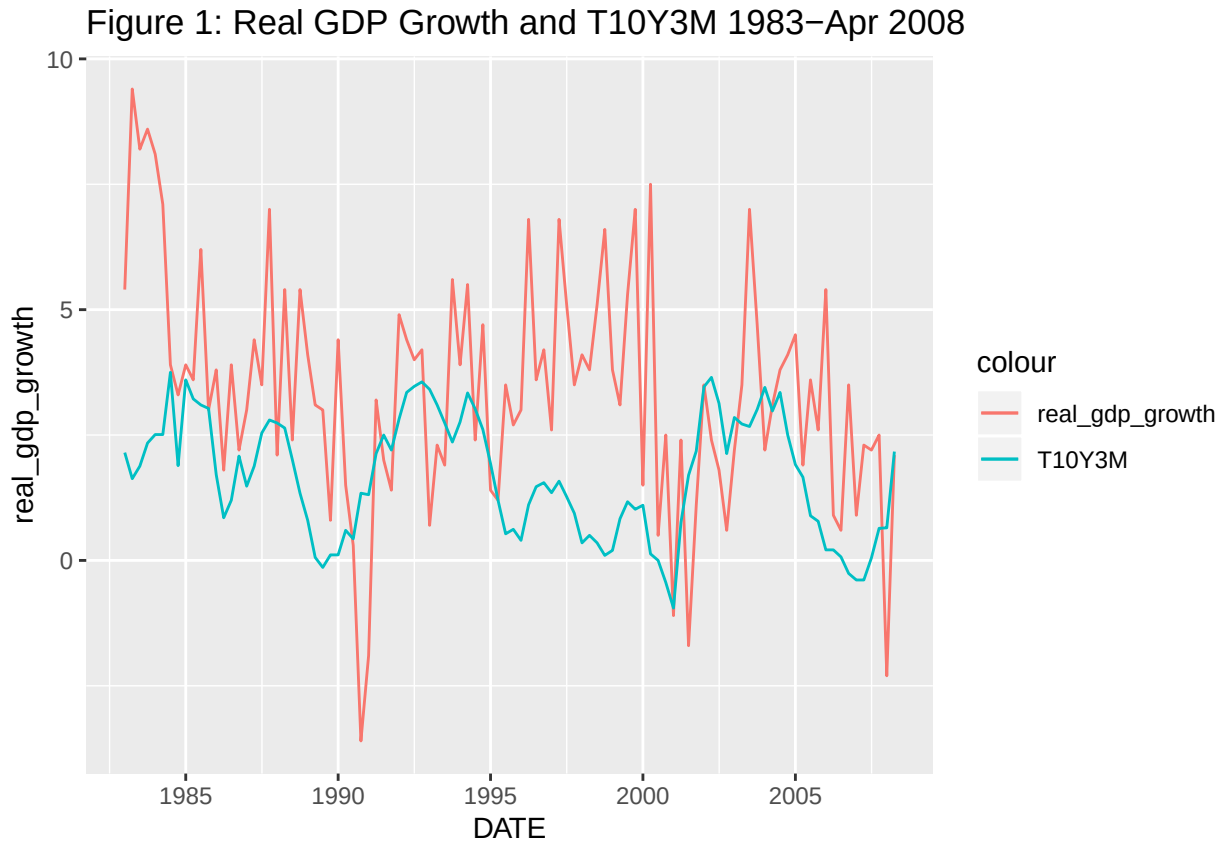
Methodology:

Below is a short description of the data used in this report.
Data:
US Real GDP Growth, seasonally adjusted annual rate
Frequency: Quarterly
Source: https://fred.stlouisfed.org/series/A191RL1Q225SBEA

T10Y3M:
Frequency: Daily
Source: https://fred.stlouisfed.org/series/T10Y3M
Note: T10Y3M data is confined to the data on first day of the month in which data is available. Only quarterly data matching the data on real GDP growth is used.

Note: April 2008 was chosen to separate the estimation and evaluation samples because this was the latest data point before the serious recession of 2008.

Plot of Yield Difference and Real GDP Growth:



For the unrestricted model, the 4 most recent lagged values of T10Y3M and the 4 most recent lagged values of real GDP growth are tested. A 5% level of significance is used for all tests throughout the study.

Unrestricted Model:

real_gdp_growth = alpha + beta1*T10Y3M_lag + beta2*T10Y3M_lag2 + beta3*T10Y3M_lag3 + beta4*T10Y3M_lag4 + gamma1*real_gdp_growth_lag + gamma2*real_gdp_growth_lag2 + gamma3*real_gdp_growth_lag3 + gamma4*real_gdp_growth_lag4 + epsilon, where epsilon is the error term

Test for Stationarity - Augmented Dickey-Fuller (ADF) Test for Real GDP Growth:

First an ADF test is performed to test whether or not real GDP growth is stationary.

Model for ADF test: delta_real_gdp_growth = alpha_adf_1 + rho*real_gdp_growth_lag + gamma_adf_1*delta_real_gdp_ + gamma_adf_2*delta_real_gdp_growth_lag2 + gamma_adf_3*delta_real_gdp_growth_lag3 + gamma_adf_4*delta_real_gdp_growth_lag4 + epsilon_adf_1

Note: Starting with ADF test for all 4 lags of delta_real_gdp_growth.

ADF test for lags 1-4 of delta_real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 2, End = 102
##
## Call:
## dynlm(formula = delta_real_gdp_growth ~ real_gdp_growth_lag +
##     delta_real_gdp_growth_lag + delta_real_gdp_growth_lag2 +
##     delta_real_gdp_growth_lag3 + delta_real_gdp_growth_lag4,
##     data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.9956 -1.3521 -0.1175  1.2718  4.5747
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  1.76956    0.50070   3.534 0.000634 ***
## real_gdp_growth_lag         -0.53862    0.13594  -3.962 0.000144 ***
## delta_real_gdp_growth_lag   -0.12814    0.13184  -0.972 0.333549
## delta_real_gdp_growth_lag2   0.16013    0.12258   1.306 0.194588
## delta_real_gdp_growth_lag3   0.02429    0.11821   0.205 0.837651
## delta_real_gdp_growth_lag4   0.13147    0.09355   1.405 0.163168
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.055 on 95 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.3914, Adjusted R-squared:  0.3594
## F-statistic: 12.22 on 5 and 95 DF,  p-value: 3.752e-09
```

Conclusion: ADF test should be repeated with lag length 3, as the absolute value of the t statistic of the last lagged value is less than 1.6. Rule of Thumb: Set a maximum value for the lag length, and estimate the test regression with that lag length. If the the absolute value of the last lagged value in the test regression is less than 1.6, then reduce the lag length by one and retest (Ng and Perron "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power," ECTA, 2001.).

ADF test with lags 1-3 of delta_real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = delta_real_gdp_growth ~ real_gdp_growth_lag +
##     delta_real_gdp_growth_lag + delta_real_gdp_growth_lag2 +
##     delta_real_gdp_growth_lag3, data = combined_data_filtered_ts)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.357 -1.235 -0.223  1.413  5.569
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  1.89679    0.46810   4.052 0.000102 ***
## real_gdp_growth_lag         -0.56110    0.12572  -4.463 2.18e-05 ***
## delta_real_gdp_growth_lag   -0.12893    0.12471  -1.034 0.303802
## delta_real_gdp_growth_lag2   0.16280    0.11993   1.357 0.177777
## delta_real_gdp_growth_lag3   0.01288    0.09569   0.135 0.893184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.105 on 97 degrees of freedom
## Multiple R-squared:  0.374,  Adjusted R-squared:  0.3482
## F-statistic: 14.49 on 4 and 97 DF,  p-value: 2.611e-09
```

Conclusion: The ADF test should be repeated with lag length 2, as the absolute value of the t statistic of the last lagged value is less than 1.6.

ADF test with lags 1 and 2 of delta_real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = delta_real_gdp_growth ~ real_gdp_growth_lag +
##     delta_real_gdp_growth_lag + delta_real_gdp_growth_lag2, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.3245 -1.2418 -0.2145  1.3985  5.5503
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                   1.8783     0.4453   4.218 5.49e-05 ***
## real_gdp_growth_lag          -0.5553     0.1174  -4.729 7.57e-06 ***
## delta_real_gdp_growth_lag    -0.1331     0.1202  -1.108    0.271
## delta_real_gdp_growth_lag2    0.1541     0.1006   1.532    0.129
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.094 on 98 degrees of freedom
## Multiple R-squared:  0.3738, Adjusted R-squared:  0.3547
## F-statistic:  19.5 on 3 and 98 DF,  p-value: 5.39e-10
```

Conclusion: The ADF test should be repeated with lag length 1, as the absolute value of the t statistic of the last lagged value is less than 1.6.

ADF test with lag 1 of delta_real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = delta_real_gdp_growth ~ real_gdp_growth_lag +
##     delta_real_gdp_growth_lag, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.6824 -1.2279 -0.0702  1.3521  6.2245
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 1.63892    0.41977   3.904 0.000173 ***
## real_gdp_growth_lag        -0.48381    0.10849  -4.460 2.17e-05 ***
## delta_real_gdp_growth_lag -0.24055    0.09824  -2.448 0.016105 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.108 on 99 degrees of freedom
## Multiple R-squared:  0.3589, Adjusted R-squared:  0.3459
## F-statistic: 27.71 on 2 and 99 DF,  p-value: 2.782e-10
```

Conclusion: The t value of real_gdp_growth_lag is `-4.46`, which is below the critical value of -2.9, so we reject the null hypothesis of non-stationarity of real GDP growth. Real GDP growth is stationary.

Test for Stationarity - Augmented Dickey-Fuller Test for T10Y3M:

Model for ADF test: delta_T10Y3M = alpha_adf_2 + rho1 *T10Y3M_lag + beta_adf_1* delta_T10Y3M_lag + beta_adf_2*delta_T10Y3M_lag_2 + beta_adf_3* delta_T10Y3M_lag3 + beta_adf_4*delta_T10Y3M_lag4 + epsilon_adf_2

Note: Starting with ADF test for all 4 lags of delta_T10Y3M.

ADF test for lags 1-4 of delta_T10Y3M:

```
##
## Time series regression with "ts" data:
## Start = 2, End = 102
##
## Call:
## dynlm(formula = delta_T10Y3M ~ T10Y3M_lag + delta_T10Y3M_lag +
##     delta_T10Y3M_lag2 + delta_T10Y3M_lag3 + delta_T10Y3M_lag4,
##     data = combined_data_filtered_ts)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.66178 -0.41618 -0.02724  0.28322  1.75307
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        0.35148    0.11334   3.101 0.002539 **
## T10Y3M_lag        -0.20703    0.05772  -3.587 0.000532 ***
## delta_T10Y3M_lag   0.14638    0.09766   1.499 0.137222
## delta_T10Y3M_lag2  0.21112    0.09171   2.302 0.023515 *
## delta_T10Y3M_lag3  0.15100    0.09182   1.644 0.103394
## delta_T10Y3M_lag4  0.04238    0.08862   0.478 0.633607
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5767 on 95 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.1417, Adjusted R-squared:  0.09656
## F-statistic: 3.138 on 5 and 95 DF,  p-value: 0.01151
```

Conclusion: ADF test should be repeated with lag length 3, as the absolute value of the t statistic of the last lagged value is less than 1.6.

ADF test for lags 1-3 of delta_T10Y3M:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = delta_T10Y3M ~ T10Y3M_lag + delta_T10Y3M_lag +
##     delta_T10Y3M_lag2 + delta_T10Y3M_lag3, data = combined_data_filtered_ts)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.52041 -0.42185 -0.03459  0.35251  1.66610
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        0.34795    0.10937   3.182 0.001968 **
## T10Y3M_lag        -0.21430    0.05460  -3.925 0.000162 ***
## delta_T10Y3M_lag   0.06469    0.09327   0.694 0.489607
## delta_T10Y3M_lag2  0.17846    0.09152   1.950 0.054054 .
## delta_T10Y3M_lag3  0.21087    0.08768   2.405 0.018064 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5885 on 97 degrees of freedom
## Multiple R-squared:  0.1576, Adjusted R-squared:  0.1229
## F-statistic: 4.538 on 4 and 97 DF,  p-value: 0.002106
```

Conclusion: The t value of T10Y3M_lag is `-3.925`, which is below the critical value of -2.9, so we reject the null hypothesis of non-stationarity for T10Y3M. T10Y3M is stationary.

Cointegration: As the series are both stationary, no need to test for cointegration.

Granger Causality Test for T10Y3M on Real GDP Growth:

Next a test for Granger causality is performed to see if T10Y3M Granger-causes real GDP growth.

Unrestricted model: real_gdp_growth = alpha + beta1*T10Y3M_lag + beta2*T10Y3M_lag2 + beta3*T10Y3M_lag3 + beta4*T10Y3M_lag4 + gamma1*real_gdp_growth_lag + gamma2*real_gdp_growth_lag2 + gamma3*real_gdp_growth_lag3 + gamma4*real_gdp_growth_lag4 + epsilon

Restricted model: real_gdp_growth = alpha_r + gamma_r1*real_gdp_growth_lag + gamma_r2*real_gdp_growth_lag2 + gamma_r3*real_gdp_growth_lag3 + gamma_r4*real_gdp_growth_lag4 + epsilon_r

p value of F statistic:

```
## [1] 0.0898
```

This is above 0.05, so we cannot reject the null hypothesis that T10Y3M does not Granger cause real GDP growth at a 5% level of significance. However, since much other literature indicates that T10Y3M can predict real GDP growth, I will explore its use as a predictor.

Models:

Unrestricted Model with lags 1-4 of T10Y3M and real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ T10Y3M_lag + T10Y3M_lag2 +
##     T10Y3M_lag3 + T10Y3M_lag4 + real_gdp_growth_lag + real_gdp_growth_lag2 +
##     real_gdp_growth_lag3 + real_gdp_growth_lag4, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.0169 -1.2366 -0.2082  1.2693  4.5872
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           1.33289    0.53312   2.500  0.01416 *
## T10Y3M_lag           -0.04601    0.33231  -0.138  0.89018
## T10Y3M_lag2           0.77436    0.43573   1.777  0.07881 .
## T10Y3M_lag3          -0.85303    0.42990  -1.984  0.05018 .
## T10Y3M_lag4           0.48985    0.31612   1.550  0.12464
## real_gdp_growth_lag   0.31486    0.10226   3.079  0.00273 **
## real_gdp_growth_lag2  0.27017    0.10722   2.520  0.01345 *
## real_gdp_growth_lag3 -0.19135    0.10582  -1.808  0.07379 .
## real_gdp_growth_lag4  0.02670    0.09803   0.272  0.78594
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.059 on 93 degrees of freedom
## Multiple R-squared:  0.2709, Adjusted R-squared:  0.2082
## F-statistic:  4.32 on 8 and 93 DF,  p-value: 0.000187
```

Conclusion: Lags 1-4 of T10Y3M and lags 3 and 4 of real GDP growth are not significant in predicting US real GDP growth in this model. As lag 1 of T10Y3M has the highest p value, it is eliminated and the model is re-estimated.

Model with lags 2-4 of T10Y3M and lags 1-4 of real GDP growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ T10Y3M_lag2 + T10Y3M_lag3 +
##     T10Y3M_lag4 + real_gdp_growth_lag + real_gdp_growth_lag2 +
##     real_gdp_growth_lag3 + real_gdp_growth_lag4, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.9960 -1.2301 -0.2093  1.2780  4.5927
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           1.31205    0.50874   2.579  0.01146 *
## T10Y3M_lag2           0.73393    0.32172   2.281  0.02479 *
## T10Y3M_lag3          -0.85219    0.42761  -1.993  0.04917 *
## T10Y3M_lag4           0.49030    0.31445   1.559  0.12230
## real_gdp_growth_lag   0.31531    0.10167   3.101  0.00254 **
## real_gdp_growth_lag2  0.26874    0.10616   2.531  0.01302 *
## real_gdp_growth_lag3 -0.19046    0.10507  -1.813  0.07306 .
## real_gdp_growth_lag4  0.02969    0.09512   0.312  0.75562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.049 on 94 degrees of freedom
## Multiple R-squared:  0.2708, Adjusted R-squared:  0.2165
## F-statistic: 4.986 on 7 and 94 DF,  p-value: 7.865e-05
```

Conclusion: Lag 4 of T10Y3M and lags 3 and 4 of real GDP growth are not significant in predicting US real GDP growth in this model. As lag 4 of real GDP growth has the highest p value, it is eliminated and the model is re-estimated.

Model with lags 2-4 of T10Y3M and lags 1-3 of real GDP growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ T10Y3M_lag2 + T10Y3M_lag3 +
##     T10Y3M_lag4 + real_gdp_growth_lag + real_gdp_growth_lag2 +
##     real_gdp_growth_lag3, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.0683 -1.2295 -0.2002  1.2648  4.5167
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.35846    0.48421   2.806  0.00609 **
## T10Y3M_lag2            0.71710    0.31566   2.272  0.02536 *
## T10Y3M_lag3           -0.82573    0.41713  -1.980  0.05065 .
## T10Y3M_lag4            0.48030    0.31132   1.543  0.12621
## real_gdp_growth_lag    0.31066    0.10010   3.104  0.00252 **
## real_gdp_growth_lag2   0.27869    0.10078   2.765  0.00683 **
## real_gdp_growth_lag3  -0.18037    0.09949  -1.813  0.07301 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.039 on 95 degrees of freedom
## Multiple R-squared:   0.27,  Adjusted R-squared:  0.2239
## F-statistic: 5.857 on 6 and 95 DF,  p-value: 3.145e-05
```

Conclusion: Lags 3 and 4 of T10Y3M and lag 3 of real GDP growth are not significant in predicting US real GDP growth in this model. As lag 4 of T10Y3M has the highest p value, it is eliminated and the model is re-estimated.

Model with lags 2 and 3 of T10Y3M and lags 1-3 of real GDP growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ T10Y3M_lag2 + T10Y3M_lag3 +
##     real_gdp_growth_lag + real_gdp_growth_lag2 + real_gdp_growth_lag3,
##     data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.2823 -1.4706 -0.1217  1.2864  4.1659
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.46410    0.48278   3.033  0.00312 **
## T10Y3M_lag2            0.70049    0.31773   2.205  0.02987 *
## T10Y3M_lag3           -0.41585    0.32389  -1.284  0.20225
## real_gdp_growth_lag    0.28910    0.09983   2.896  0.00468 **
## real_gdp_growth_lag2   0.28907    0.10128   2.854  0.00529 **
## real_gdp_growth_lag3  -0.15379    0.09869  -1.558  0.12247
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.053 on 96 degrees of freedom
## Multiple R-squared:  0.2517, Adjusted R-squared:  0.2128
## F-statistic: 6.459 on 5 and 96 DF,  p-value: 3.236e-05
```

Conclusion: Lag 3 T10Y3M and lag 3 of real GDP growth are not significant in predicting US real GDP growth in this model. As lag 3 of T10Y3M has the highest p value, it is eliminated and the model is re-estimated.

Model with lag 2 of T10Y3M and lags 1-3 of real GDP growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ T10Y3M_lag2 + real_gdp_growth_lag +
##     real_gdp_growth_lag2 + real_gdp_growth_lag3, data = combined_data_filtered_ts)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.068 -1.341 -0.168  1.230  4.766
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.45284    0.48431   3.000  0.00343 **
## T10Y3M_lag2            0.35880    0.17416   2.060  0.04206 *
## real_gdp_growth_lag    0.27545    0.09959   2.766  0.00680 **
## real_gdp_growth_lag2   0.27282    0.10082   2.706  0.00805 **
## real_gdp_growth_lag3 -0.15735    0.09898  -1.590  0.11517
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.06 on 97 degrees of freedom
## Multiple R-squared:  0.2389, Adjusted R-squared:  0.2075
## F-statistic: 7.611 on 4 and 97 DF,  p-value: 2.24e-05
```

Conclusion: Lag 3 of real GDP growth is not significant in predicting US real GDP growth in this model. It is eliminated and the model is re-estimated.

Model with lag 2 of T10Y3M and lags 1 and 2 of real GDP growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ T10Y3M_lag2 + real_gdp_growth_lag +
##     real_gdp_growth_lag2, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.4361 -1.3297 -0.0296  1.1921  5.4639
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           1.21370    0.46393   2.616   0.0103 *
## T10Y3M_lag2           0.35441    0.17549   2.020   0.0462 *
## real_gdp_growth_lag   0.23918    0.09769   2.448   0.0161 *
## real_gdp_growth_lag2  0.22535    0.09704   2.322   0.0223 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.076 on 98 degrees of freedom
## Multiple R-squared:  0.2191, Adjusted R-squared:  0.1951
## F-statistic: 9.163 on 3 and 98 DF,  p-value: 2.11e-05
```

Conclusion: The twice lagged values of T10Y3M and once and twice lagged values of real GDP growth are jointly significant in predicting US real GDP growth at the 5% level.

Autoregressive Model (AR Model): The ADL model is compared to an AR model of only GDP growth, in order to further explore if T10Y3M has predictive values above the past values of the real_gdp_growth time series. An AR(4) model consisting of the 4 last lagged values of real_gdp_growth is first tested. This model is equivalent to the restricted model tested earlier in the Granger causality test.

real_gdp_growth = alpha_ar + gamma_ar1*real_gdp_growth_lag + gamma_ar2*real_gdp_growth_lag2 + gamma_ar3*real_gdp_growth_lag3 + gamma_ar4*real_gdp_growth_lag4 + epsilon_ar

AR Model with lags 1-4 of real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ real_gdp_growth_lag + real_gdp_growth_lag2 +
##     real_gdp_growth_lag3 + real_gdp_growth_lag4, data = combined_data_filtered_ts)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.357 -1.235 -0.223  1.413  5.569
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.89679    0.46810   4.052 0.000102 ***
## real_gdp_growth_lag    0.30997    0.10089   3.072 0.002755 **
## real_gdp_growth_lag2   0.29173    0.10804   2.700 0.008176 **
## real_gdp_growth_lag3 -0.14992    0.10580  -1.417 0.159703
## real_gdp_growth_lag4 -0.01288    0.09569  -0.135 0.893184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.105 on 97 degrees of freedom
## Multiple R-squared:  0.2057, Adjusted R-squared:  0.173
## F-statistic: 6.281 on 4 and 97 DF,  p-value: 0.0001545
```

Conclusion: The 3 and 4 times lagged values of real GDP growth are not significant in predicting US real GDP growth in this model. As lag 4 has the highest p value, it is eliminated and the model is re-estimated.

AR Model with lags 1-3 of real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ real_gdp_growth_lag + real_gdp_growth_lag2 +
##     real_gdp_growth_lag3, data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.3245 -1.2418 -0.2145  1.3985  5.5503
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.87830    0.44526   4.218 5.49e-05 ***
## real_gdp_growth_lag    0.31162    0.09964   3.127  0.00232 **
## real_gdp_growth_lag2   0.28723    0.10223   2.810  0.00599 **
## real_gdp_growth_lag3  -0.15411    0.10059  -1.532  0.12874
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.094 on 98 degrees of freedom
## Multiple R-squared:  0.2056, Adjusted R-squared:  0.1813
## F-statistic: 8.454 on 3 and 98 DF,  p-value: 4.739e-05
```

Conclusion: The 3 times lagged values of real GDP growth is not significant in predicting US real GDP growth in this model. It is eliminated and the model is re-estimated.

AR Model with lags 1-2 of real_gdp_growth:

```
##
## Time series regression with "ts" data:
## Start = 1, End = 102
##
## Call:
## dynlm(formula = real_gdp_growth ~ real_gdp_growth_lag + real_gdp_growth_lag2,
##     data = combined_data_filtered_ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.6824 -1.2279 -0.0702  1.3521  6.2245
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.63892    0.41977   3.904 0.000173 ***
## real_gdp_growth_lag    0.27564    0.09749   2.827 0.005679 **
## real_gdp_growth_lag2   0.24055    0.09824   2.448 0.016105 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.108 on 99 degrees of freedom
## Multiple R-squared:  0.1866, Adjusted R-squared:  0.1701
## F-statistic: 11.35 on 2 and 99 DF,  p-value: 3.641e-05
```

Conclusion: Lags 1 and 2 of real GDP growth are significant in predicting real GDP growth at the 5% level of significance.

Model Comparison:

Below is a table showing the AIC and BIC values for the ADL model and the AR model.

```
## # A tibble: 3 x 3
##   Criteria  ADL.Model AR.Model
##   <fct>         <dbl>    <dbl>
## 1 R Squared     0.219    0.187
## 2 AIC           1.52     1.53
## 3 BIC           1.60     1.58
```

Optimal Model: Model real_gdp_growth = alpha_final + beta_final *T10Y3M_lag2 + gamma_final_1* real_gdp_growth_lag + gamma_final_2*real_gdp_growth_lag2 + epsilon_final is the optimal model as the information criteria values are roughly equal overall and the R squared value is higher for the ADL model.

A summary of the optimal model is shown again below.

```
##
## ================================================
##                           Dependent variable:
##                       ---------------------------
##                             real_gdp_growth
## ------------------------------------------------
## T10Y3M_lag2                     0.354**
##                                 (0.175)
##
## real_gdp_growth_lag             0.239**
##                                 (0.098)
##
## real_gdp_growth_lag2            0.225**
##                                 (0.097)
##
## Constant                        1.214**
##                                 (0.464)
##
## ------------------------------------------------
## Observations                      102
## R2                               0.219
## Adjusted R2                      0.195
## Residual Std. Error         2.076 (df = 98)
## F Statistic              9.163*** (df = 3; 98)
## ================================================
## Note:                *p<0.1; **p<0.05; ***p<0.01
```

Model Evaluation:

Below is a table showing the test statistic and p value of the RESET test (p=1) and the Jarque-Bera test applied to the optimal model.

```
## # A tibble: 2 x 3
##   Test        Test.Statistic p.value
##   <fct>                <dbl>   <dbl>
## 1 RESET (p=1)          1.98    0.122
## 2 Jarque-Bera          0.255   0.12
```

The additional parameter from the RESET test is not significant, indicating that the assumption that a linear model is correct cannot be rejected. The Jarque-Bera test indicates that the null hypothesis that the distribution of normality of the residuals cannot be rejected. These test results do not signal misspecification of the model.

Note: Chow break test and Chow forecast test are not done, as a plot of the data shows no evidence of a break in the data, and the test is not appropriate in this situation.

Breusch-Godfrey (BG) Test for serial correlation in residuals:

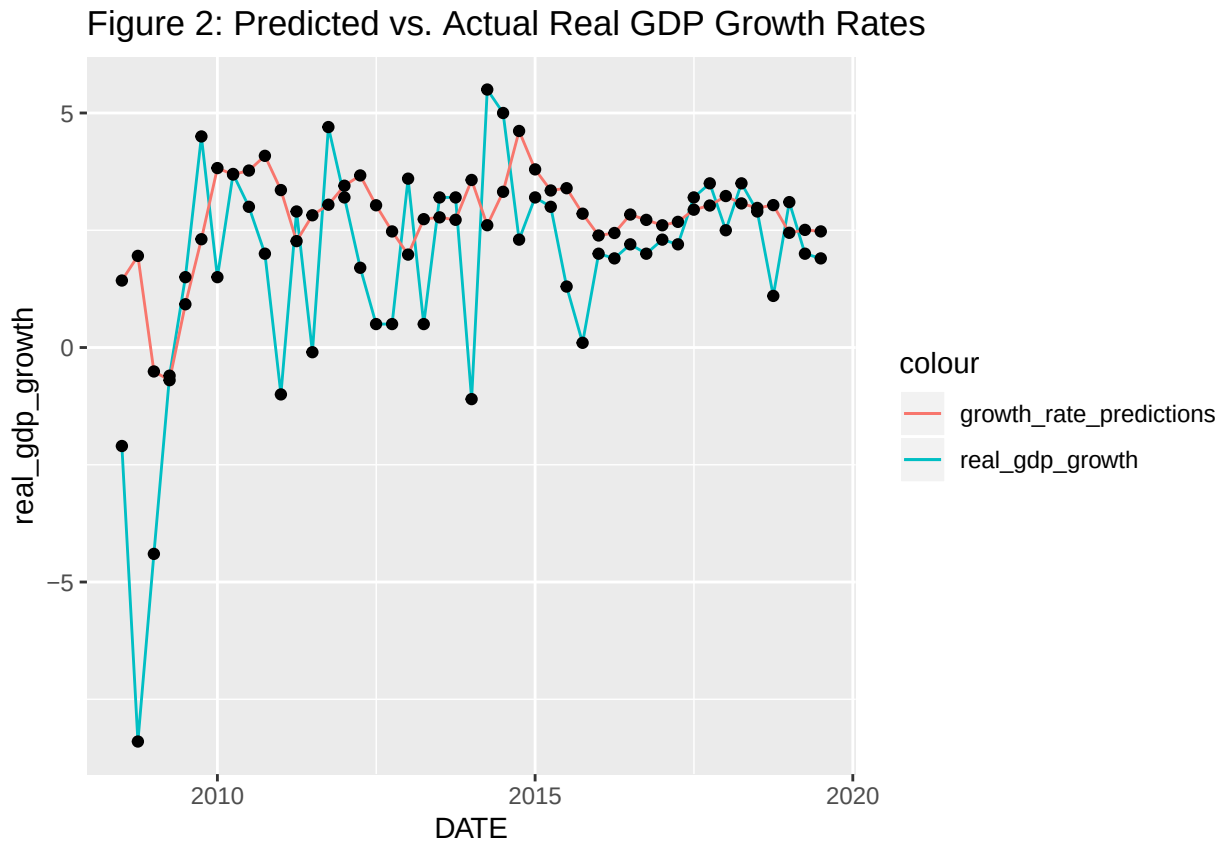
```
##
##  Breusch-Godfrey test for serial correlation of order up to 2
##
## data:  reg_lag_2_12
## LM test = 5.7033, df = 2, p-value = 0.05775
```

As the p value is above 0.05, the null hypothesis of no serial correlation is not rejected. This test result does not signal any misspecfication of the model.

US Real GDP Growth Quarterly Predictions, Annualized Rate, for Quarters 1 - 45:
Evaluation Sample: 2008-07-01 to 2019-07-01:

```
##  [1]  1.43  1.95 -0.51 -0.70  0.92  2.31  3.83  3.69  3.77  4.09  3.36
## [12]  2.27  2.82  3.04  3.45  3.67  3.03  2.47  1.98  2.74  2.78  2.72
## [23]  3.57  2.61  3.32  4.62  3.80  3.35  3.40  2.85  2.39  2.44  2.83
## [34]  2.72  2.61  2.68  2.94  3.03  3.23  3.07  2.98  3.04  2.45  2.51
## [45]  2.48
```

Plot of Predicted vs. Actual Growth Rates July 2008 - July 2019:



Root Mean Square Error (RMSE) of Predictions:

```
## [1] 2.41
```

Mean Absolute Error (MAE) of Predictions:

```
## [1] 1.62
```

RMSE of Predictions post-2015:

```
## [1] 1.19
```

MAE of Predictions post-2015:

```
## [1] 0.89
```

Conclusion:

The RMSE indicates a forecasting error of about `2.41`% on an annualized quarterly growth basis, and the MAE indicates a forecasting error of about `1.62`%. These are high, so the model using 1983-2008 data does not accurately predict real GDP growth from July 2008 - July 2019. The model predicts a recession in late 2008, after the recession was already under way. The model does not fully capture the variability in real GDP growth, but is more accurate in times of more stable economic growth. After 2015, when real GDP growth is more stable, the model becomes more accurate, with a RMSE of `1.19` and a MAE of `0.89`.