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STA 3032

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Project 2.2

3.38

X /Y	0	1	2	3
0	0	1/30	2/30	3/30
1	1/30	2/30	3/30	4/30
2	2/30	3/30	4/30	5/30

a) $P(x \le 2, y = 1) = (1+2+3)/30 = 6/30 =$ **1/5**
b) $P(x>2, y \le 1) = (3+4)/30 =$ **7/30**
c) $P(x>y) = (1+2+3+3+4+5)/30 = 18/30 =$ **3/5**
d) $P(x+y=4) = (4+4)/30 = 8/30 =$ **4/15**

3.76

a) The marginal distribution is $fx_1(x_1) = \int 6x_2 \, dx_2 = 3x_1^2$ when $0 < x_1 < 1$. This is a valid density function because $fx_1(x_1) \ge 0$ for all x_1 and $\int_0^1 3x_1^2 \, dx_1 = 1$.
b) $P(x_2 < 0.5 \mid x_1 = 0.7) = (2/0.7^2) \int_0^{0.5} x_2 \, dx_2 = 25/49 =$ **0.5102**
c) The 65th percentile of the marginal distribution is $\int_0^{0.65} 3x_1^2 \, dx_1 = x_1^3$ evaluated from 0.65 to 0 = **0.2746**

3.78

P(System Works) = (0.95)(0.99)(0.92) = **0.86526**

4.62

$Z = -2X + 4Y - 3$

$\sigma_z = 4\sigma_x + 16\sigma_y = 4(5) + 16(3) =$ **68**

4.67

$E[g(X,Y)] = E(X/Y^3 + X^2Y) = E(X/Y^3) + E(X^2Y)$

$E(X/Y^3) = \int_1^2 \int_0^1 2x(x+2y)/7y^3 \, dxdy = 2/7 = \int_1^2 (1/3y^3 + 1/y^2)dy = 15/84$

$E(X^2Y) = \int_1^2 \int_0^1 (2x^2y(x+2y))/7 \, dxdy = 2/7 = \int_1^2 y(1/4 + 2y/3)dy = 139/252$

$E[g(X,Y)] = 15/84 + 139/252 =$ **46/63**

X and Y are independent if $f(x,y) = f_x(x)f_y(y)$

$f_x(x) = \int_0^1 2/7(x+2y)dx = (1+4y)/7$

$f_y(y) = \int_1^2 2/7(x+2y)dy = 2(x+3)/7$

$f(x,y) = \int_1^2 \int_0^1 2/7(x+2y)dxdy = 7/2$

Since $f(x,y)$ does not equal $f_x(x)f_y(y)$ X and Y are **not independent**.

4.88

Given that $\mu = 900$ and $\sigma = 900$, Chebyshev's theorem says that for $k = 2$, $P(-900 < X < 2700) >= 0.75$. According to 4.85, $P(-900 < X < 2700) = 1-e^{-3} = 0.9502$, so the theorem holds.

For $k = 3$, Chebyshev's theorem says $P(-1800 < X < 3600) >= 0.8889$. According to 4.85, $P(-1800 < X < 3600) = 1-e^{-4} = 0.9817$, so the theorem holds.

4.98

a) The marginal density of x is

x	0	1	2
G(x)	0.2	0.32	0.48

The marginal density of y is

Y	0	1	2
h(y)	0.26	0.35	0.39

The conditional density of x given y =2

x	0	1	2
$F_{X\|Y=2}(x,y)$	4/39	5/39	30/39

b) $E(X) = 0(0.12 + 0.04 + 0.04) + 1(0.08 + 0.09 + 0.05) + 2(0.06 + 0.12 + 0.30) =$ **1.28**

$E(X^2) = 0^2(0.2) + 1^2(0.32) + 2^2(0.48) = 2.24$

$Var(X) = E(X^2) - (E(X))^2 = 2.24 - 1.28^2 =$ **0.6016**

c) $E(X|Y = 2) = 1(5/39) + 2(30/39) =$ **65/39**

$E(X^2|Y = 2) = 1^2(5/39) + 2^2(30/39) =$ **125/39**

$Var(X|Y =2) = E(X^2|Y = 2) - (E(X|Y = 2))^2 = 125/39 - (65/39)^2 =$ **50/117 = 0.4274**