Universidad Politécnica de Madrid

Escuela Técnica Superior de Ingenieros Industriales

Máster en Automática y Robótica

Human Robot Interaction

Bilateral Control with Wave Variables

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November 9, 2023

Interacción Humano Robot

Study of bilateral control with delays using wave variables

Given a bilateral master/slave control system with the following dynamics:

$$M_m \ddot{x}_m(t) = f_{op}(t) - B_m \dot{x}_m(t) - G_f f_e(t)$$
$$M_s \ddot{x}_s(t) = f_s(t) - B_s \dot{x}_s(t) - K_e x_s(t)$$

Where:

- $\mathbf{M_{m,s}} = \mathbf{1}$ Kg, $\mathbf{B_{m,s}} = \mathbf{1}$, are the mass and friction of master and slave respectively.
- $\boldsymbol{K_e} = \mathbf{1}$ N/m, is the impedance of the environment which is modeled as the damping constant of a spring.
- $\boldsymbol{G_f} = \mathbf{1}$, in the force reflection gain.

It is intended to study the behavior of such a system for different values of its parameters.

First, given the above equations, the following block diagram can be constructed (Fig. 1).

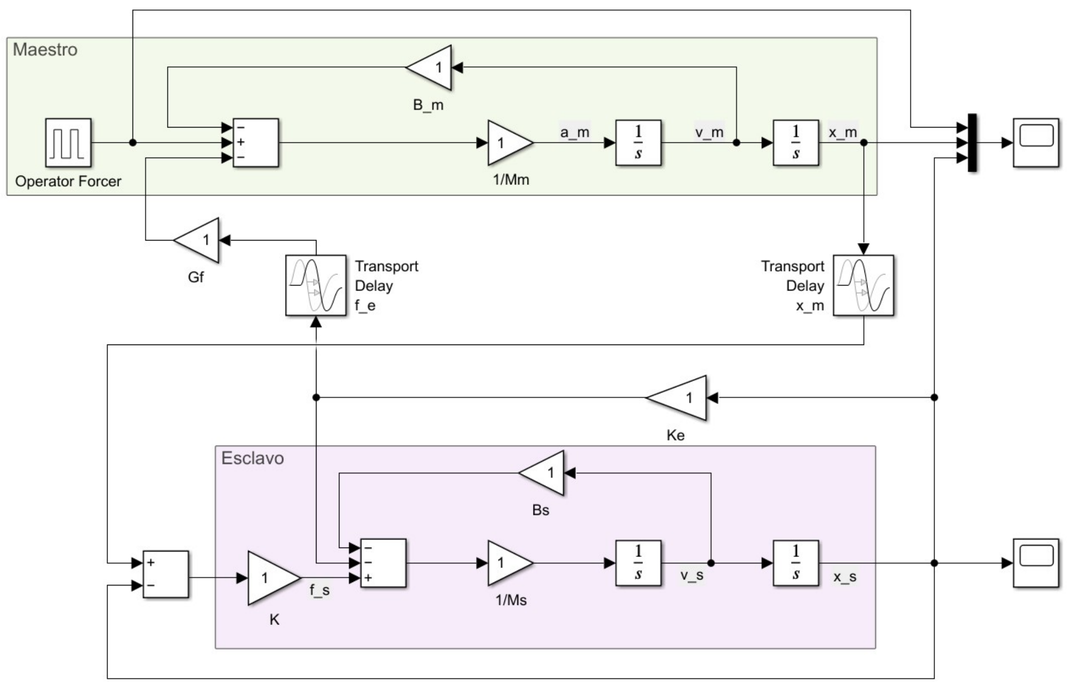


Figure 1: *Block diagram of bilateral master/slave control with communication delays. K or $\mathrm{K_s}$ is the gain of the slave control loop (it is dimensionless) and we initialize it to 1.*

1.1 System characterization and threshold value of delay for stability

We start by using the proposed values (all parameters with unit value) and in the case of no delays ($T = 0$). In Fig. 2 can be seen the response of the system versus time: the blue signal represents the action of the human operator (in this case a pulse train), the intense orange signal corresponds to the position indicated by the master device (by integrating twice the acceleration that the master prints on the device) and the yellow signal represents the position followed by the slave device. Therefore, we see how there is no delay between the two signals but there is an error in the positioning. We now include a delay in the communication and look for

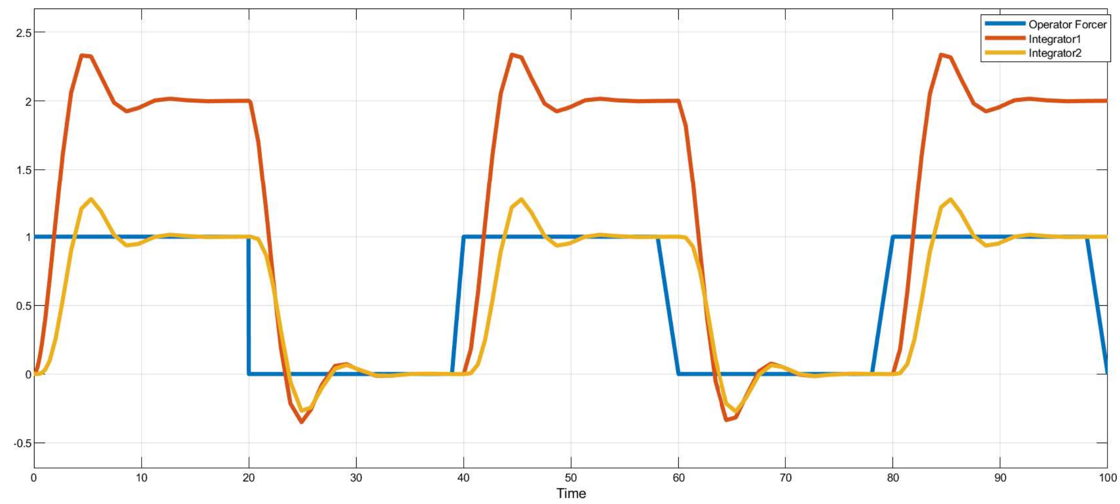


Figure 2: *System response with all its unitary parameters and without delay in communications.*

the value of the delay (T) that makes the system *unstable*. After trying different values we have determined that for a value of $T = 0.9$ seconds the system becomes unstable, as can be seen in Fig. 3.

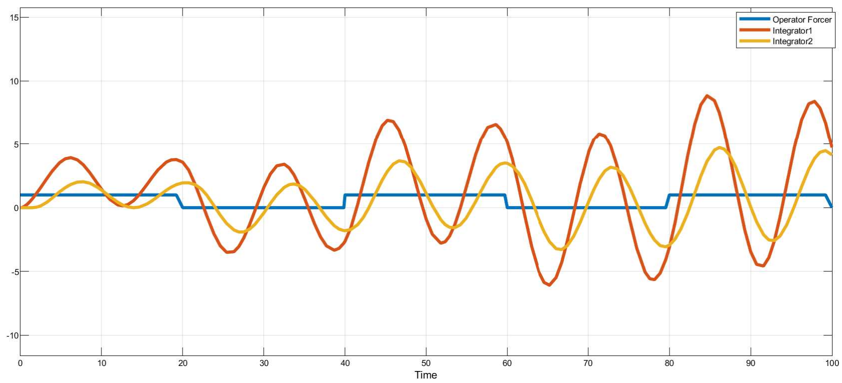


Figure 3: *Unstable behavior of the system for a delay value of $T = 0.9$ s.*

1.2 Parametric adjustment and sensitivity study

We continue with the characterization of the parameters that define the system. These are $\mathrm{M_{m,s}}$, $\mathrm{B_{m,s}}$, $\mathrm{K_e}$, K, $\mathrm{G_f}$. We are asked to find the values of these parameters that cause the bilateral system to be unstable with a delay between 30-50 ms.

Before starting it is worth noting some assumptions that have been made: on the one hand, we assume that the behavior of both delay blocks are identical (i.e., the same delay exists in both directions of the communication channel), on the other hand, we take a constant value of the delay of 50 ms throughout the parametric study, finally, master and slave are considered identical (same mass, same friction coefficients).

First, a physical analysis of the system and the expected values of some of the parameters is necessary. In this way, by means of an intelligent search, we can shorten what would have been a huge exploration.

- $\mathbf{G_f = 1}$: it makes sense because we want the reflection of forces to be transparent. In later analysis we will see that increasing it will mean destabilizing the system (it would be the case of applications such as telesurgery where small movements of the slave we want to be strongly reflected to the master).
- $\mathbf{K \gg 10}$: large values ($\boldsymbol{\sim 50 - 100}$) of the slave control loop gain stabilize the system.
- $\mathbf{K_{\boldsymbol{e}}}$: represents the modeling of the environment as if it were a spring, since it relates position and force. High values of $\mathbf{K_{\boldsymbol{e}}}$ represent rigid contact between the slave and the environment. We will start with a value of **1** N/m being this situation as if the slave was in contact with a soft environment.
- $\mathbf{M_m = M_s}$: we choose for them the same mass and of a small value, starting with **0.25** Kg.
- $\mathbf{B_{\boldsymbol{s}} \gg 1}$: it models the own frictions of the motor. Being a dissipative phenomenon, it stabilizes the system. We are interested in values greater than 1.
- $\mathbf{B_{\boldsymbol{m}} \gg 1}$: as before, the master can also have frictions that dissipate energy, the same criterion is followed.

The system for the values proposed above and a delay of 50 ms behaves as shown in Fig. 4. This behavior will be the starting point in our parametric analysis (see center column in Tab. 1).

The results obtained are gathered in Tab. 1. The classification criterion responds to the behavior before the 40 ms pulse train with a 50% duty cycle and 4 N intensity, although sometimes it has been necessary to extend the simulation times in order to adapt to the slowness of the system and to be able to correctly characterize its behavior. The outputs of the system that showed an oscillatory behavior with increasing amplitude have been classified as unstable, when there is no oscillation they have been classified as *overdamped*, on the contrary when the oscillation was stable but pronounced they have been established as *underdamped* behavior. About the case of K_e it should be noted that its increase or decrease does not affect the

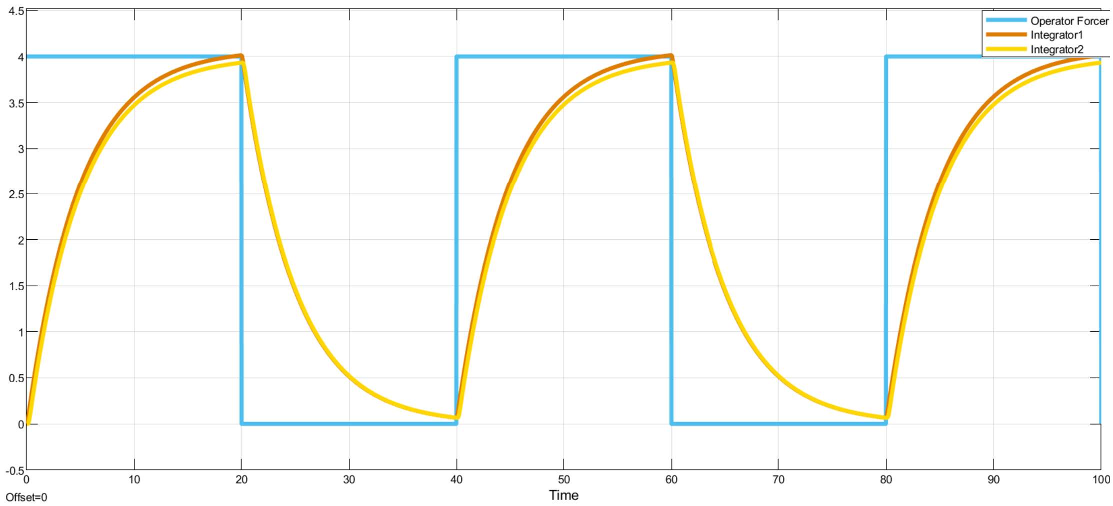


Figure 4: *Behavior of the master-slave system with the reference values (INIT column in* Tab. 1*). The slave follows the master very closely and the realized force matches the displacement.*

stability, in all cases it is stable. If we think about the physical sense of this variable we see that it models the relationship between the motion of the *slave* and its interaction with the environment. Very high values of K_e mean that the slave is in contact with a very rigid solid then in the graph it will be observed that the position of the slave does not reach that of the master. For unit values of K_e the solid in contact allows itself to be deformed but opposes in the same proportion as the deformation it undergoes. Finally, values less than unity imply very deformable solids, where just a slight force command by the master can mean large motion for both master and slave as nothing opposes it. We have marked with an asterisk (*) the case of very high values of K_e because short oscillatory transients appear which are then damped. We believe that they could represent the oscillation that the slave would undergo when colliding with a very rigid object.

	$\downarrow\downarrow$	$\downarrow$	**INIT**	$\uparrow$	$\uparrow\uparrow$
$\mathbf{G_f}$	overdamped (∼ 0.01)	overdamped (∼ 0.1)	1	underdamped (∼ 20)	unstable (∼ 100)
$\mathbf{B_{m,s}}$	unstable (∼ 0.05)	underdamped (∼ 0.5)	5	overdamped (∼ 50)	overdamped (∼ 100)
$\frac{\mathbf{1}}{\mathbf{M_{m,s}}}$ (kg^{-1})	underdamped (∼ 0.04)	underdamped (∼ 0.4)	4	underdamped (∼ 40)	underdamped (∼ 400)
$\mathbf{K}$	stable *very big error*	stable *big error*	50	stable *small error*	stable *no error*
$\mathbf{K_e}$ (N/m)	stable (∼ 0.1)	stable (∼ 0.5)	1	overdamped (∼ 10)	overdamped (∼ 100)*

Tabla 1: *Parametric analysis of the system. The values have been modified one by one, keeping all other parameters constant. A color code ranging from red to green is used to characterize the different behaviors of the system for easy visualization.*

Regarding the unstable cases, their occurrence is as expected. For example, in the case of very small values of friction it is expected that the system becomes unstable because the dissipative character is reduced and the accumulation of energy in the communication channel destabilizes the control. On the other hand, as already mentioned, large values of G_f make sense in some applications but we see that they imply instability in the system, while if the movements of the slave are very weakly reflected in the master they are stable (overdamped) but we need very long simulation periods.

It is also requested to find the values of the parameters that cause the system to be unstable when the delay is bounded between 30 and 50 ms. In the Fig. 5 can be seen the unstable system, this has been achieved by reducing the value of the friction in both devices. It has also been necessary to reduce the value of the slave control loop gain and the other parameters have been varied looking for those that destabilize the system. The only one that has not been changed has been the force reflection gain (G_f) which has been kept at 1 to maintain transparency.

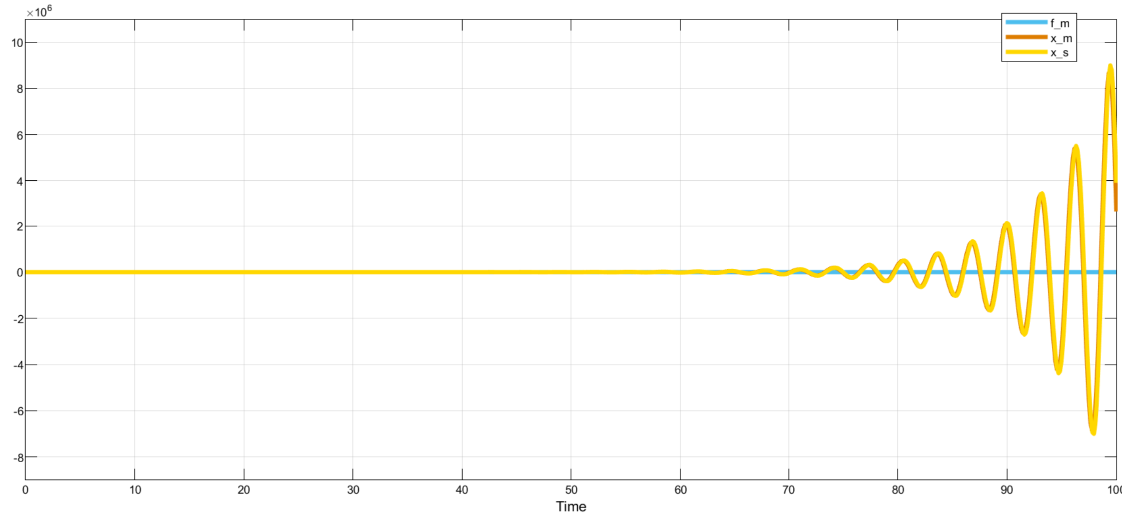


Figure 5: *Unstable system with a delay of* 50 *ms and the following parameter values:* $\mathrm{M_{m,s}} = 0.5$ *Kg,* $\mathrm{B_{m,s}} = 0.1$ *N s/m,* $\mathrm{K_e} = 2$ *N/m,* $\mathrm{K} = 10$*,* $\mathrm{G_f} = 1$.

1.3 Bilateral control scheme using wave variables

To guarantee the stability of the bilateral system we use the wave variables. This is a control technique based on *passivity*. We proceed to perform a transformation of the speed and force signals of the master and slave before entering and leaving the delay block, i.e. the communication channel. The equations that model this transformation are as follows:

$$u_m = \frac{1}{\sqrt{2b}}(f_{hd} + bv_m) \tag{1}$$
$$v_{ed} = -\frac{1}{b}(f_e - \sqrt{2b}\,u_s) \tag{2}$$
$$w_s = \frac{1}{\sqrt{2b}}(f_e - bv_{ed}) \tag{3}$$
$$f_{hd} = bv_m + \sqrt{2b}\,w_m \tag{4}$$

where b (in kg/s or N/m) is the *characteristic impedance* of the line, $f_h d$ is the force reflected by the slave (not yet affected by the gain), v_m is the velocity of the master, f_e is the force of the environment, $v_e d$ is the force of the environment. The new wave variables are: u_m, u_s, w_s, w_m.

These equations and wave variables correspond to the schematic in the FIG. 6.

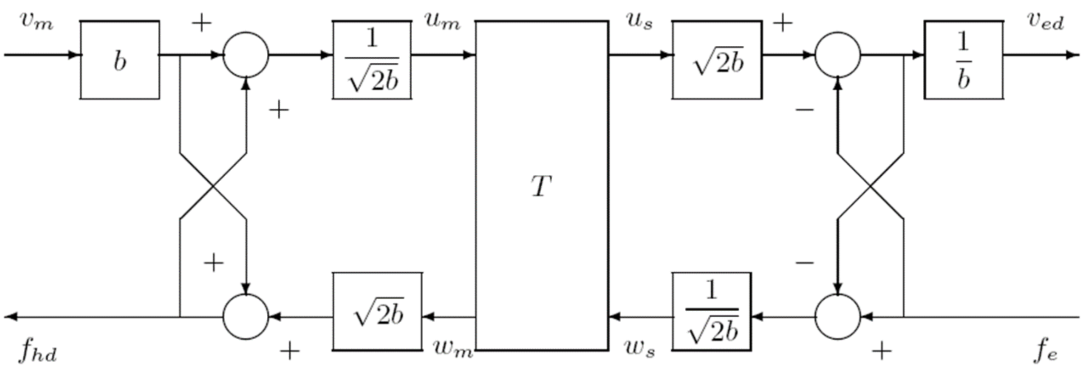


Figure 6: *Passivity-based transmission line. Source [1].*

This new scheme has been implemented in Matlab resulting in the new architecture of the FIG. 7. The blocks, boxed in different colors in this figure, perform the transformation into wave variables and correspond to the ecuaciones (1)–(4) as follows:

- Orange block → equation 1
- Blue block → equation 2
- Red block → equation 3
- Green block → equation 4

In the FIG. 8 you can see the content of the *orange* block, which implements the equation 1.

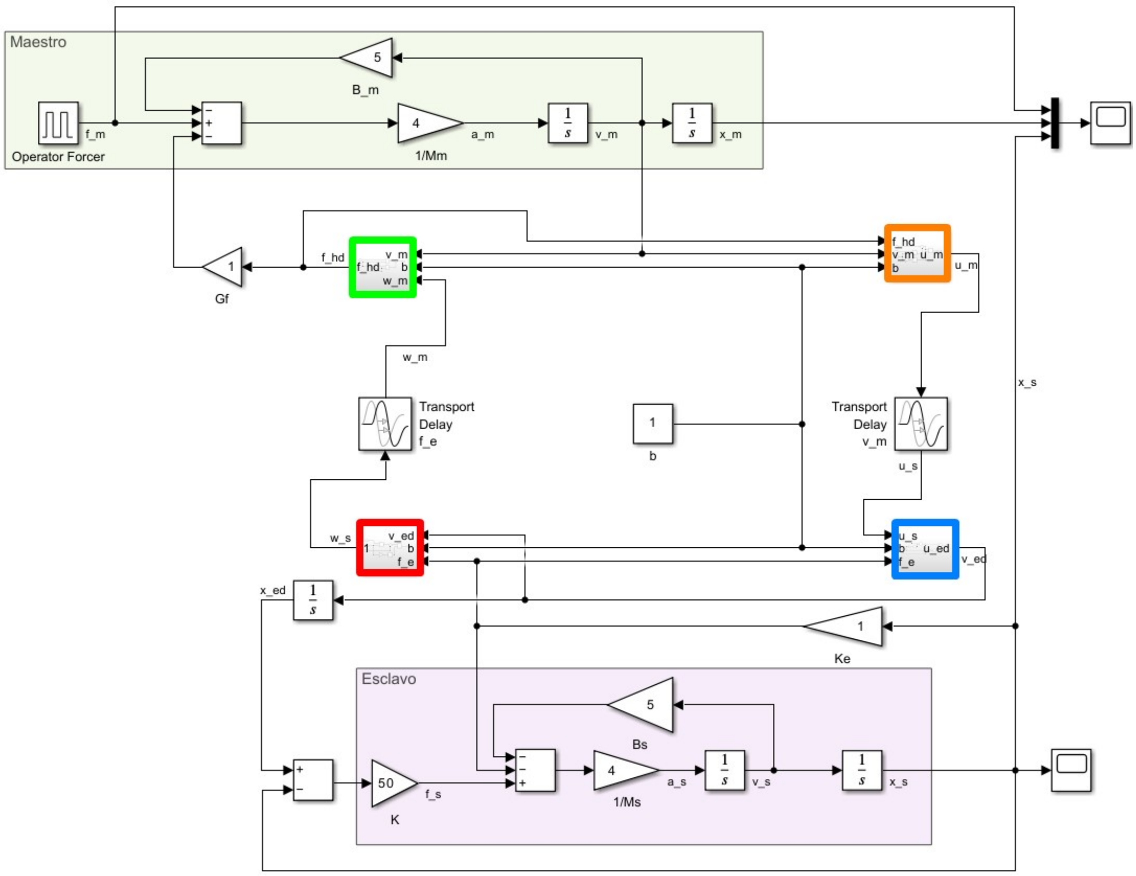


Figure 7: *Block diagram of bilateral master/slave control with communication delays **using wave variables**. The colored blocks model the transformation from system variables to wave variables.*

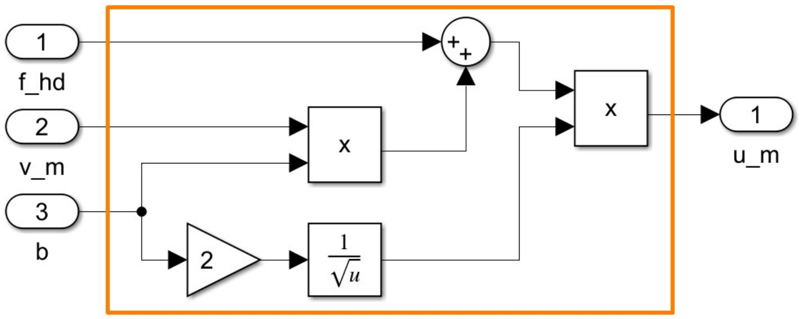


Figure 8: *Orange block that implements the transformation from v_m to the wave variable u_m thanks to the characteristic impedance of the line (b) and the force reflected by the slave ($f_h d$) (see* Ec. *1).*

On the other hand, it can be seen how after applying this change of variables, the stability of the system is highly benefited. For example, in the Fig. 9 it can be seen how the previously unstable system (Fig. 5) now, thanks to the use of wave variables, becomes stable.

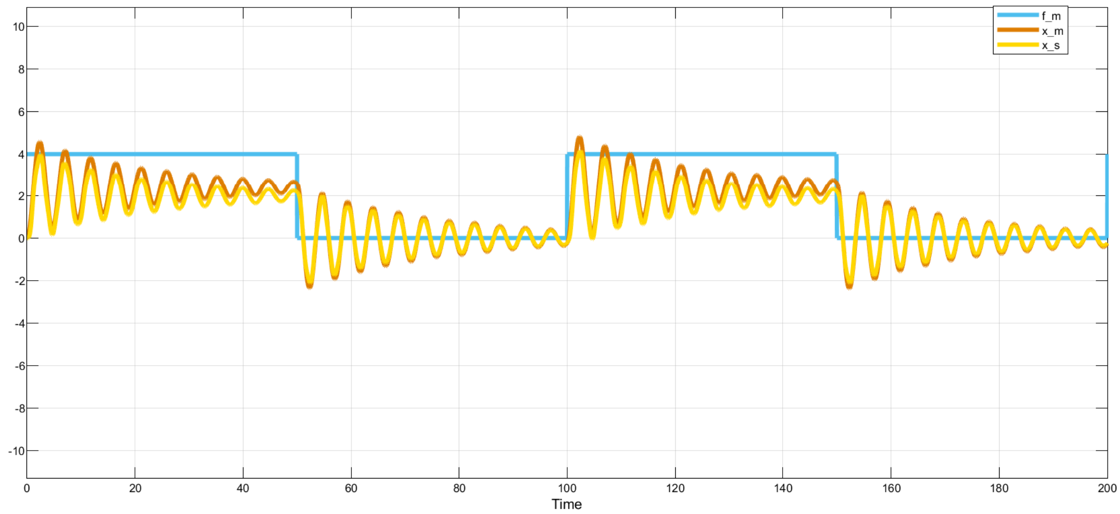


Figure 9: ***Stable*** *system with a delay of* 50 *ms and the same parameter values as in the* FIG. 5*, but this time employing the scheme of the* FIG. 7 *(use of wave variables).*

1.4 Parametric study using wave variables

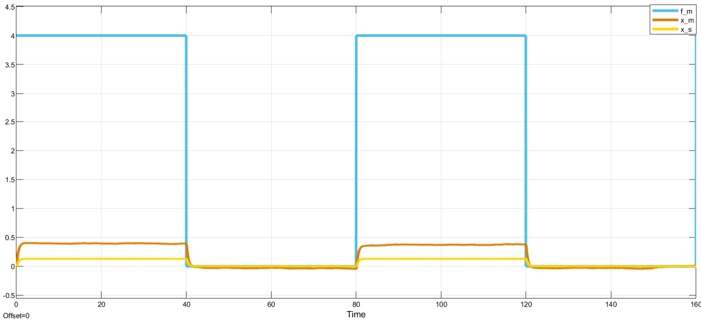
It remains to analyze again the behavior of the system but this time analyzing how it affects the increase and decrease of the delays (T) and also the change in the type of environment (K_e). A table similar to TAB. 1 is constructed.

For this analysis we will again start from the same parameters *INIT* of the TAB. 1 but using wave variables (see appendices for the FIG. 11) and with a value of the line impedance (b) of $1, kg/s$. The initial communication delay (T) has been set at 50 ms. The results of the study can be seen in the TAB. 2.

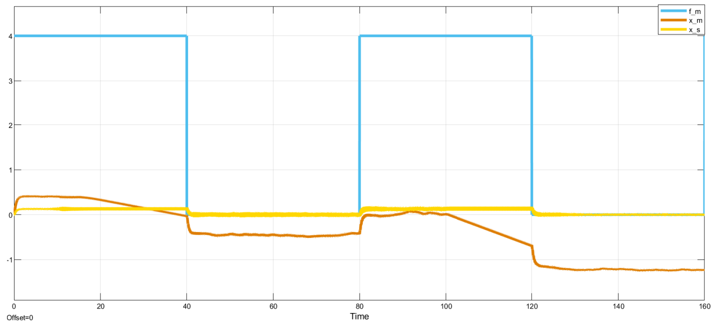
	⇊	↓	**INIT**	↑	⇈
$\mathbf{K_e}$ (N/m)	stable (∼ 0.01)	stable (∼ 0.5)	1	stable (∼ 10)	unstable (∼ 100)
T (ms)	stable (∼ 0)	unstable (∼ 5)	50	stable (∼ 500)	stable (∼ 1000)

Tabla 2: *Parametric analysis of the system when using wave variables. Different environments are studied, modeled by the constant K_e and the communication delay T. This second table only considers the stability or non-stability resulting from the study in the behavior of the system.*

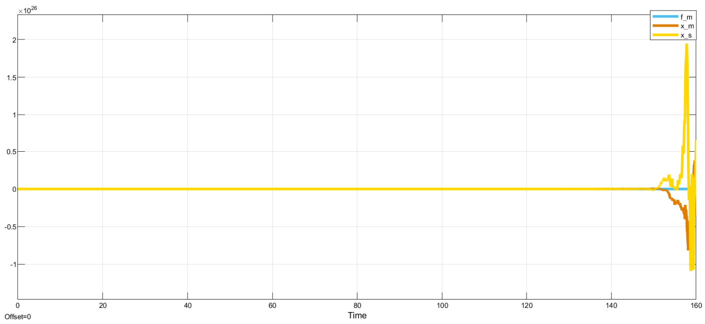
As can be seen in the table above, very soft environments do not destabilize the system. As K_e (stiffness of the environment) increases, however, it happens that up to a value of $K_e \approx 30$ the system has a stable behavior (FIG. 10a) and when taking a value close to $K_e = 31.5$ the system becomes completely unstable (FIG. 10b). Regarding the delay, if the delay does not exist ($T = 0$) the system is stable, but very small values of the delay, $T \approx 5$ ms, contrary to what one would expect, do result in an unstable system (FIG. 10c). On the other hand, delays of half a second ($T \approx 500$ ms) or even one second ($T \approx 1000$ ms) do result in stable but noisy slave response (affected by marked oscillation, see FIG. 10d).



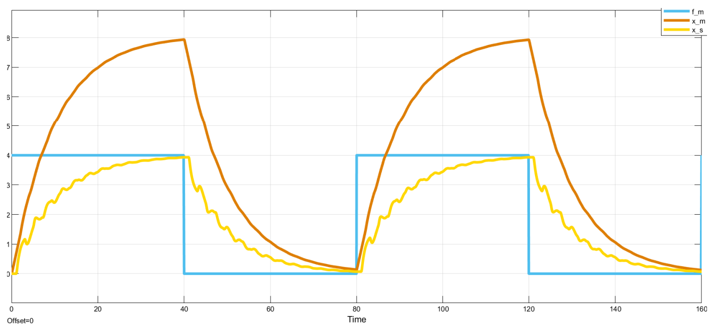
(a) $\boldsymbol{K_e = 31}$ *(N/m)*



(b) $\boldsymbol{K_e = 31.5}$ *(N/m)*



(c) $\boldsymbol{T = 5}$ *(ms)*



(d) $\boldsymbol{T = 1000}$ *(ms)*

Figure 10: *System behaviors with wave variables under different values of K_e y T.*

1.5 Discussion and conclusions

The stability analysis carried out has allowed first of all to get in touch with a bilateral control architecture. In addition, through the parametric study, a deeper knowledge of the behavior of the system in the face of disturbances or different scenarios has been obtained. Secondly, the use of passivity control by means of the wave variables has shown the benefit for the stability of the implemented control when there are delays in the communication.

It is worth highlighting some of the results obtained. Regarding the system *without* wave variables, we have verified that the instabilities are mainly the result of a decrease of the friction phenomena and, on the other hand, of the excessive increase of the force reflection gain. With respect to the system *with* wave variables, it is observed that excessively large values of the stiffness constant of the environment (above approximately $K_e \approx 30$ N/m) destabilize the system, and that this transition from stability to non-stability is drastic. Regarding the delay (T), values up to 1 second are stable for the system, but with some oscillation at the output (slave position). Finally, very small delay values (but not zero, $T \approx 5$ ms) result in an unstable system, and again, with a very abrupt transition to instability.

This study is of great relevance for the improvement of bilateral control systems. We believe that a possible improvement would result from a more extensive analysis, with joint variation of up to two parameters. On the other hand, a study based on real devices of the "master" and "slave" could allow to narrow down the analysis and thus relax some of the assumptions made. Finally, it would remain to address the study for communication delay values that were not equal, something frequent in real systems, for example in IoT environments or in public spaces (due to bandwidth sharing).

References

[1] S. Hirche, M. Ferre, J. Barrio, C. Melchiorri, and M. Buss. "Bilateral Control Architectures for Telerobotics". In: *Advances in Telerobotics*. Ed. by M. Ferre, M. Buss, R. Aracil, C. Melchiorri, and C. Balaguer. Berlin, Heidelberg: Springer Berlin Heidelberg, 2007, pp. 163–176. ISBN: 978-3-540-71364-7. DOI: 10.1007/978-3-540-71364-7_11. URL: https://doi.org/10.1007/978-3-540-71364-7_11.

Annexes

In figure Fig. 11 you can see the behavior of the reference system (INIT) with a line impedance value of $b = 1kg/s$.

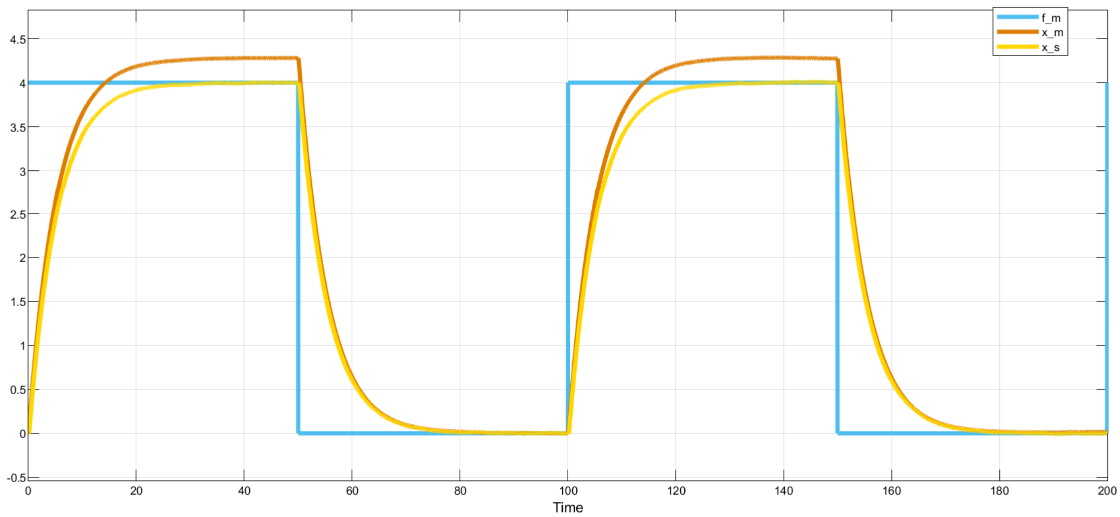


Figure 11: *System behavior for INIT parameters of tables 1 and 2.*