ECE 310

Homework 3:

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Problem 1:

$y[n] = ay[n-1] + x[n]$

$y[-1] = c$

$x[n] = b\delta[n]$

$\rightarrow y[n] = a^{n+1}c + a^n b u[n]$

a) $\delta[n] \rightarrow \boxed{S} \rightarrow h[n] = y[n]$

$n=0 \rightarrow y[0] = ac + \delta[0] = ac+1$

$n=1 \rightarrow y[1] = a y[0] + \delta[1] = a(ac+1) = a^2c + a$

$n=2 \rightarrow y[2] = ay[1] + \delta[2] = a(ac+1) = a^3c + a^2$

$\rightarrow h[n] = a^n(ac+1)u[n]$

b) As the system depends only on past values $(y[n-1])$ or present value $(x[n])$ and no future value, the system is causal

c) $y[n] = ?$

$x[n] = 2b\delta[n]$

$\rightarrow y[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k]$ ~~and ...~~

$(a^n(ac+1)u[n]) * 2b\delta[n] = 2b\, a^n(ac+1)u[n]$

$= 2b(a^{n+1}cu[n] + a^n u[n])$

No, because ~~...~~

$2b(a^{n+1}cu[n] + a^n u[n]) \neq 2b(a^{n+1}c + a^n u[n]) \rightarrow$ not linear

d) $y[-1] = c = 0$

(a) $\rightarrow n=0 \rightarrow y[0] = a\cdot 0 + \delta[0] = 1$

$n=1 \rightarrow y[1] = a\cdot 1 + \delta[1] = a$

$n=2 \rightarrow y[2] = a\cdot a + \delta[2] = a^2$

$\rightarrow h[n] =$ ~~...~~ $a^n u[n]$

(b) the system is still causal (doesn't change anything)

(c) $y[n] = h[k] * x[n-k] = a^n u[n] * 2b\delta[n] = 2ba^n u[n]$

$= 2\cdot y_1[n] = 2a^n b u[n]$

$\rightarrow$ now the system ~~...~~ seems linear, but we would have to check for the superposition

e) $y[n] = ?$

$x[n] = b\delta[n-1]$

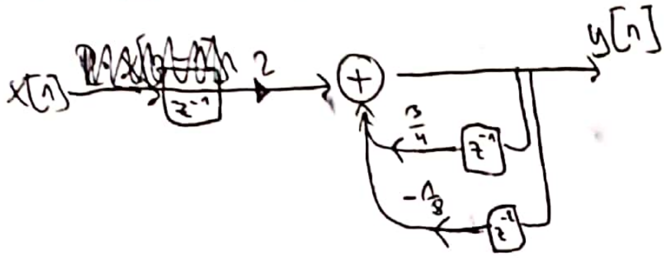
$\rightarrow$ ~~...~~ $y[n] = b\delta[n-1] * a^n u[n] = a^n b u[n-1]$

$y[n-n_0] = a^{(n-n_0)} b u[n-n_0]$ if $n_0 = 1$

$\rightarrow y[n-1] = a^{n-1} b u[n-1] \rightarrow$ time-varying system

Problem 2:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1]$$



Problem 3:

$z^{-1} = 2e^{-j\frac{2\pi}{3}}$

$z^{1} = 2e^{j\frac{2\pi}{3}}$

$x[n] = bz^{n}$

$$H(z) = \sum_n h[n]z^{-n} = z + z^{-1}$$

$y[n] = ?$

$$x[n] = 2\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)^n = 2e^{j\frac{2\pi}{3}n}$$

a) $y[n] = H(z)\,x[n]$

$$= \left(z + \frac{1}{z}\right)2\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)^n = \left(z + \frac{1}{z}\right)2e^{j\frac{2\pi}{3}n}$$

$$= 2e^{j\frac{2\pi}{3}n}\left(e^{j\frac{2\pi}{3}} + e^{-j\frac{2\pi}{3}}\right)$$

$$= 2e^{j\frac{2\pi}{3}(n+1)} + 2e^{j\frac{2\pi}{3}(n-1)}$$

b) $$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

$$= 2e^{j\frac{2\pi}{3}n} * \left(\delta[n+1] + \delta[n-1]\right) = 2e^{j\frac{2\pi}{3}(n+1)} + 2e^{j\frac{2\pi}{3}(n-1)}$$

with

$$H(z) = \sum_n h[n]z^{-n} = z + z^{-1}$$

$$\rightarrow h[n] = \delta[n+1] + \delta[n-1]$$

Problem 4:

a) $x[n] = \left(\frac{1}{3}\right)^n u[-n] \rightarrow X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n u[-n]\, z^{-n}$

$\mathcal{Z}\{x[-n]\} = X(z^{-1})$

$a^n u[n] \overset{z}{\leftrightarrow} \frac{1}{1-az^{-1}} \rightarrow X(z) = \frac{1}{1-3z}$, ROC $= |z| < \frac{1}{3}$

b) $x[n] = \left(\frac{1}{3}\right)^n\left(u[n] - u[n-5]\right) \rightarrow X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n\left(\delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)\right)z^{-n}$

$$= 1 + \frac{1}{3}z^{-1} + \frac{1}{9}z^{-2} + \frac{1}{27}z^{-3} + \frac{1}{81}z^{-4}, \quad \text{ROC}: z \neq 0$$

c) ~~$x[n] = \alpha^{|n|}$ with~~

$x[n] = \left(\frac{1}{2}\right)^{n-3}u[n-2] = 2\cdot\left(\frac{1}{2}\right)^{n-2}u[n-2] \rightarrow X(z) = 2z^{-2}\frac{1}{1-\frac{1}{2}z^{-1}} = \frac{2z^{-2}}{1-\frac{1}{2}z^{-1}}$, ROC $: |z| > \frac{1}{2}$

d) $x[n] = \alpha^{|n|}$, $0 < |\alpha| < 1$

$\rightarrow$ if $n < 0 \rightarrow x[n] = \alpha^{-n} \rightarrow X(z) = \frac{1}{1-\alpha z^{-1}}$

if $n > 0 \rightarrow x[n] = \alpha^{n} \rightarrow X(z) = \frac{1}{1-\alpha z^{-1}}$

$\rightarrow X(z) = \frac{1}{1-\alpha^{|n|}z^{-1}}$, ROC: $|z| > \alpha^{|n|}$

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Problem 5:

$x[n] = a^n u[n] \leftrightarrow X(z) = \frac{z}{z-a}$, ROC: $|z| > |a|$

a) $Y(z) = \frac{a^2 z}{1 - az} = \frac{a^2 z}{\frac{a}{a} - az} = \frac{(a^2) z}{(-a)z - (-\frac{a}{a})} = \frac{a^2 z}{(-a)z - (-\frac{a}{a})} \cdot \left(\frac{a^{-1}}{a^{-1}}\right) = \frac{az}{-z - (-a^{-1})}$

$\rightarrow y[n] = -a^{(1-n)} u[1-n]$

b) $Y(z) = \frac{z-1}{z + \frac{3}{4}}$ and $n > 0$:

$Y(z) = \frac{z}{z + \frac{3}{4}} - \frac{1}{z + \frac{3}{4}} \Leftrightarrow \left(-\frac{3}{4}\right)^n u[n] - \left(-\frac{3}{4}\right)^{n-1} u[n-1]$

$\rightarrow y[n] = \left(-\frac{3}{4}\right)^n u[n] - \left(-\frac{3}{4}\right)^{n-1} u[n-1]$

c) $Y(z) = \log\left(\frac{z + \frac{1}{2}}{z}\right) \rightarrow \left(\log\left(\frac{z + \frac{1}{2}}{z}\right)\right)' = \frac{\frac{z + \frac{1}{2} - z}{z^2}}{\frac{z + \frac{1}{2}}{z}} = \frac{\frac{1}{2}}{z(z + \frac{1}{2})} = \frac{1}{2} \frac{z^{-1}}{z + \frac{1}{2}}$ $\quad n_1 = az^{-1}$

$= \frac{1}{2} z^{-2} \cdot \frac{z}{z + \frac{1}{2}}$

$\rightarrow y[n] = \frac{1}{2}\left(-\frac{1}{2}\right)^{n-2} u[n-2] \rightarrow y[n] = \frac{1}{2}\left(-\frac{1}{2}\right)^{n-2} \cdot n\, u[n-2] \rightarrow y[n] = \frac{1}{2}\left(-\frac{1}{2}\right)^{n-2} n \cdot u[n-2]$

d) $Y(z) = \frac{2z}{z + \frac{3}{\sqrt{2}}(1+j)} = 2 \cdot \frac{z}{z + \frac{3}{\sqrt{2}}(1+j)} \leftrightarrow y[n] = 2 \cdot \left(-\frac{3}{\sqrt{2}}(1+j)\right)^n u[n]$

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