

Name _____

UIN _____

Section: 9:00 AM 12:00 PM 3:00 PM

Score _____

Problem	Pts.	Score
1	24	
2	16	
3	20	
4	18	
5	14	
6	8	
Total		

Instructions

- This is an open-book exam. You are allowed to use all course notes, and notes that have been prepared by you (digital notes are allowed). However use of calculator or any other computational software is not permitted. You are also not permitted to use any other material including HW solutions, past exam solutions, and on-line resources.
 - Students are expected to follow the UI Student Honor Code. Any evidence of communication or collaboration or use of prohibited material and resources will result in an automatic zero in the exam followed by appropriate action per FAIR.
 - **Show all your work to receive full credit for your answers.**
 - When you are asked to “calculate”, “determine”, or “find”, this means providing closed-form expressions (i.e., without summation or integration signs).
 - Begin your answer to each problem on a new page. Sub-parts of a problem can be on the same page.
 - Highlight your final answer to each part of the problem.
 - Neatness counts. If we are unable to read your work, we cannot grade it.
 - You can write your answers on a tablet or on paper.
 - You will have 30 minutes to upload your work on gradescope, i.e., the exam must be scanned (if written on paper), and uploaded (emailed to the instructors) by 11:00 PM CT. The email should have a time-stamp no later than 11:00AM CT.
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Problem 1:

An LTI system has the following frequency response:

$$H_d(\omega) = \sin(2\omega) + j \cos(2\omega) + 1 \quad (1)$$

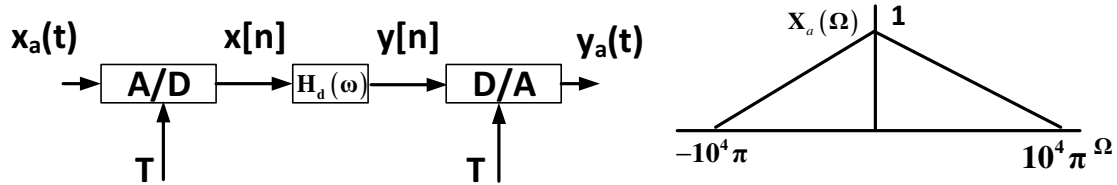
1. Determine whether $h[n]$ is real-valued or not.
2. Determine the response $y[n]$ when $x[n] = \sin(\frac{\pi}{4}n) + 2e^{-j\frac{\pi}{2}n} + 1$.
3. Determine the response $y[n]$ when the input is $2x[n-2]$.

Problem 2:

Consider the sequence $\{x[n]\}_{n=0}^9 = \{1, 0, 0, 0, 0, 0, 4, 0, 2, 0\}$ with DFT coefficients $\{X[k]\}_{k=0}^9$.

1. Express the DFT coefficients $\{X_1[k]\}_{k=0}^4$ of the sequence $\{x_1[n]\}_{n=0}^4 = \{1, 0, 0, 4, 2\}$ in terms of $\{X[k]\}_{k=0}^9$.

2. Express the DFT coefficients $\{X_2[k]\}_{k=0}^4$ of the sequence $\{x_2[n]\}_{n=0}^4 = \{4, 0, 2, 0, 1\}$ in terms of $\{X[k]\}_{k=0}^9$.

Problem 3:

Consider the DSP system shown above. Assume that the A/D and D/A converters are ideal. The input signal $x_a(t)$ is bandlimited to 10000π rad/s as shown. It is desired to filter this signal with a **high-pass** filter that will pass frequencies greater than 3500 Hz by using a digital filter $H_d(\omega)$ as shown in the figure above.

1. Determine T such that there is no aliasing at the output of A/D converter.

2. Determine maximum T such that the overall system comprising A/D, digital filter, and D/A realizes the desired highpass filter.

3. Assume $T = 5 \times 10^{-5}s$. Sketch Fourier transform of $x[n]$, $y[n]$, and $y_a(t)$.

Problem 4:

Consider the two finite-length sequences:

$$x = \underset{\uparrow}{\{-1, 1, 0, -1, 1\}} \text{ and } h = \underset{\uparrow}{\{-1, 4, 1, 4, 3, -2\}}$$

1. Let $Y_d(\omega) = X_d(\omega)H_d(\omega)$ where $X_d(\omega), H_d(\omega)$ denote the DTFT of $x[n], h[n]$. Compute $y[n]$.

2. Let $Y[k] = X[k]H[k]$, where $X[k], H[k]$ denote the 7-point DFT of $x[n], h[n]$. Find $y[n]$.

3. **True or False:** Zero padding both sequences to length $N = 9$ is adequate to guarantee that linear and circular convolutions coincide.

Problem 5:

A continuous-time signal $x_c(t) = \cos(56\pi t)$ is sampled at a rate of 140 Hz for 6 seconds to produce a discrete-time signal $x[n]$ with length $L = 840$.

1. Let $X[k]$ be the L -point DFT of $x[n]$. At what value(s) of k will $X[k]$ have the greatest magnitude?
2. Suppose that $x[n]$ is zero-padded to a total length of $N = 2048$. At what value(s) of k does the N -point DFT have the greatest magnitude?

Problem 6:

Answer **True** or **False** to each of the following statements (-1 for wrong answer, minimum score=0)

1. Let $X[m]$, $(0 \leq m \leq 31)$ and $X_d(\omega)$ respectively be the 32-point DFT and DTFT of a sequence $\{x_n\}_{n=0}^7$ that is zero-padded to length 32. Then $X[24] = X_d\left(\frac{-\pi}{2}\right)$.

True/False

2. The signal $x[n] = \cos\left(\frac{\pi n}{4}\right)$ is obtained by sampling the signal $x_a(t) = \cos(\Omega_0 t)$ at $T = \frac{1}{1000}$ samples/s. The only possible value of Ω_0 is 250π rad/s.

True/False.

3. The DTFT of $x[n] = \sin\left(\frac{\pi n}{4}\right)$, $0 \leq n \leq 64$ in $(-\pi, \pi)$ is
- $$X_d(\omega) = \frac{\pi}{j} \left(\delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right) \right)$$

True/False.

4. The 64-point DFT of the sequence $x[n] = \cos\left(\frac{2\pi n}{28}\right)$, $0 \leq n \leq 31$ will have only two non-zero elements.

True/False.

5. Let $\{X[k]\}_{k=0}^3$, denote the 4-point DFT of $\{x[n]\}_{n=0}^3 = \{1, -1, 1, -1\}$. Then $X[0] = 0$.

True/False