Topic covered in this homework is: Fast Fourier Transform. Homework will be graded for (1) completion and (2) Three randomly picked problems will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

Calculate the DFT of the length-4 sequence $x[n] = [1, -2, 3, -4]$ explicitly "by hand" using the decimation-in-time FFT algorithm.

Solution: Half length sequences are as follows:

$$x_e[n] = [1, 3]$$

$$x_o[n] = [-2, -4]$$

Subsequently,

$$X_e[k] = 1 + 3e^{j\frac{2\pi}{2}k}, \quad k = 0, 1$$

$$X_o[k] = -2 - 4e^{j\frac{2\pi}{2}k}, \quad k = 0, 1$$

Then, decimation-in-time FFT algorithm is as follows:

$$X[k] = X_e[k] + e^{-j\frac{2\pi}{4}k}X_o[k] \quad k = 0, 1$$

$$X[k + N/2] = X_e[k] - e^{-j\frac{2\pi}{4}k}X_o[k] \quad k = 0, 1$$

$$\implies X[0] = -2, X[1] = -2 - 2j, X[2] = 10, X[3] = -2 + 2j$$

Problem 2:

A flowgraph illustrating the decimation-in-time FFT algorithm is shown below. Complete the connections in the output stage, clearly identifying the connection weights. Also determine the values of the indices $a, b, \ldots, h$.

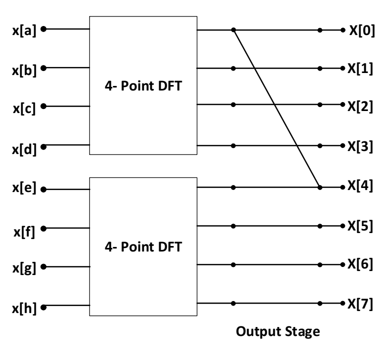

Figure 1: An 8-point decimation-in-time FFT algorithm.

Solution:

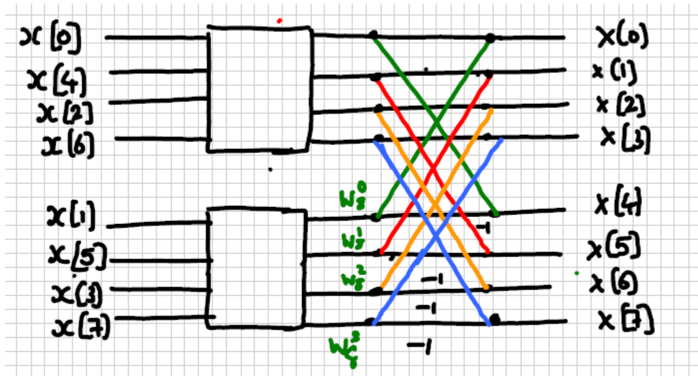

Figure 2: Problem2 Solution

Problem 3:

Assume that a real multiply-add takes one nanosecond and that the amount of time to compute a DFT is determined by the amount of time it takes to perform all of the multiply-adds sequentially, i.e., you have access to only one multiply-add unit.

1. How much time does it take to compute a 1024-point DFT directly?
   Solution: Complexity of N-point DFT is N^2 **complex** multiply-adds. Note that 1 complex MA is 4 real MAs. So, 1024-point DFT needs 4×1024^2 real multiply-add operations. Total time required is $T_{DFT} = 4 \times 1024^2 \times 1ns = 4.2ms$

2. How much time is required if an FFT is used?
   Solution: Complexity of N-point FFT is $Nlog_2N$ **complex** multiply-adds. So, 1024-point FFT needs $4 \times 1024 \times log_2(1024)$ real multiply-add operations. Total time required is $T_{FFT} = 4 \times 1024 \times log_2(1024) \times 1ns = 40.96\mu s$
   Note: $\frac{T_{DFT}}{T_{FFT}} \approx 100$

3. Repeat the above two parts for a 16384-point DFT.

$$T_{DFT} = 4 \times 16384^2 \times 1ns = 1.07s$$

$$T_{FFT} = 4 \times 16384 \times log_2(16384) \times 1ns = 0.916ms$$

   Note: $\frac{T_{DFT}}{T_{FFT}} \approx 1170$

Problem 4:

Two complex sequences of lengths $L = 8192$ and $M = 512$ respectively are to be linearly convolved. Compute the number of complex multiply-adds required if:

1. The convolution is performed directly in the sequence-domain.
   Solution: Direct convolution requires $L \times M$ multiply-add operations.

$$\Longrightarrow = 8192 \times 512 = 4194304$$

2. Using radix-2 FFTs.
   Solution: FFT requires zero padding to next power of 2. In this case $N = 16384$. Then, convolution by FFT requires $3N \times log_2(N) + N$ multiply-add operations.

$$\Longrightarrow = 3 \times 16384 \times log_2(16384) + 16384 = 704512$$