Digital Signal Processing | ECE 310 (Spring 2020)
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Homework 4

Topics covered in this homework are: inverse z-transform, system analysis via the z-transform and system transfer functions.

Problem 1: ROCs and inverse z-transforms

Find all possible ROCs for the following z-transforms and determine the associated inverse z-transform for each case.

(a) $\dfrac{z^2 - 2z}{z^2 + 4z + 3}$

(b) $\dfrac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})}$

Problem 2: z-transform properties and inverse z-transform

Evaluate the convolution of the sequences $h[n] = (1/3)^n u[n]$ and $x[n] = 2^n u[-n]$ using z-transform properties and the inverse z-transform.

Problem 3: z-transform properties and difference equations

Consider the system described as

$$y[n] = \sum_{k=-\infty}^{n} kx[k]$$

(a) Find a difference equation for this system. Is this a constant coefficient difference equation?

(b) Take the z-transform to both sides of the difference equation to express $Y(z)$ in terms of $X(z)$.

(c) Let $y[n] = \sum_{k=0}^{n} k2^{-k}, \ \ n \geq 0$. Use your answer in part (b) to find $Y(z)$ and the corresponding ROC.

Problem 4: System Cascades

Two systems with unit-pulse responses

$$h_1[n] = u[n] + \left(\frac{1}{4}\right)^n u[n], \qquad h_2[n] = \delta[n] - \delta[n-1]$$

are in serial connection.

(a) For each of the individual systems, as well as for the overall system, determine whether they are BIBO stable.

(b) Determine the unit pulse response of the overall system.

(c) Find the difference equation for the overall system.

Problem 5: System Analysis

Consider the system described by the following difference equation (or LCCDE) with zero initial conditions:

$$y[n] = \frac{1}{2}y[n-2] + x[n] - x[n-1], \quad \text{for } n = 0, 1, 2, \ldots$$

(a) Find the transfer function and its ROC.

(b) Find the impulse response of the system.

(c) Determine the output $y[n]$ to the input $x[n] = (1/4)^n u[n]$.

Problem 6: z-transform properties and difference equations

Let $X(z) = e^{1/z}$ for a right-sided signal $x[n]$ starting at $n = 0$ and having initial value $x[0] = 1$.

(a) Take the derivative of $X(z)$ and use z-transform properties to obtain a recursion for $x[n]$.

(b) Find the inverse z-transform by solving the recursion for $x[n]$ with initial condition $x[0] = 1$.

(c) Obtain the inverse z-transform via the power series method.