Digital Signal Processing | ECE 310 (Spring 2020)
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Homework 9

Assigned: 04/01 - Due: 04/08

Topics covered in this homework are: DT response of an analog system, DFT properties, the DFT and its properties. Homework is due at 5:00 PM on Wednesdays. Homework will be graded for (1) completion and (2) one randomly picked problem will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

Consider the communication link shown below: The channel frequency response $H_c(\Omega)$ is shown

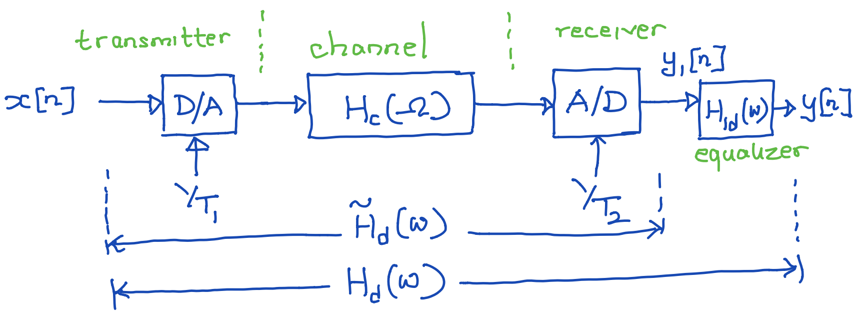

Figure 1: A communication system.

below: We wish to design this link, i.e., select the values of the D/A and A/D sample rates and

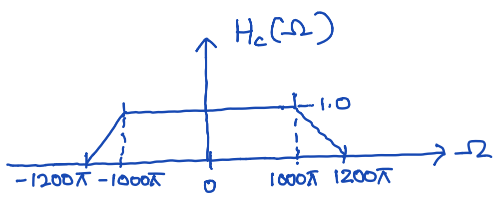

Figure 2: The channel frequency response.

the equalizer frequency response $H_{1d}(\omega)$, such that the end-to-end equivalent discrete-time impulse response $h[n] = \delta[n]$ so that the final output $y[n] = x[n]$. In other words, the receiver is able to recover the transmitted data $x[n]$ in spite of the distortion introduced by the channel due to the roll-off in its frequency response (see Fig. 2). In practice, the channel will also introduce noise, which can be accounted for via the use of material from ECE 313. But, we will ignore noise in this problem to keep things simple. In the following, assume that the D/A is ideal.

(a) Find the minimum values of the D/A and A/D sample rates $\frac{1}{T_1}$ and $\frac{1}{T_2}$, and the equalizer frequency response $H_{1d} = c$ where c is a constant, such that the end-to-end impulse response $h[n] = \delta[n]$.

(b) If the D/A sample rate is fixed at $\frac{1}{T_1} = 1.2\,\text{kS/s}$ so that the entire channel bandwidth is utilized, find an appropriate value of the A/D sample rate $\frac{1}{T_2}$ and the equalizer frequency response $H_{1d} = c$ where c is a constant, such that the end-to-end impulse response $h[n] = \delta[n]$.

(c) If $\frac{1}{T_1} = \frac{1}{T_2} = 1.2\,\mathrm{kS/s}$, which is a typical scenario, find the equalizer frequency response $H_{1d}(\omega)$ needed to obtain an end-to-end impulse response $h[n] = \delta[n]$.

Problem 2:

Consider the length-32 sequence $\{x[n]\}_{n=0}^{31}$. A sequence of length-64 $\{y[n]\}_{n=0}^{63}$ is generated by setting $y[n] = x[2m]$ ($m = 0, \ldots, 31$), i.e., for even values of n, and $y[n] = 0$ for odd values of n. Determine $\{Y[k]\}_{k=0}^{63}$ in terms of $\{X[k]\}_{k=0}^{31}$.

Problem 3:

Given two sequences $\{x_1[n]\}_{n=0}^{31}$ and $\{x_2[n]\}_{n=0}^{31}$, a new sequence $\{y[n]\}_{n=0}^{63}$ is composed by interleaving $x_1[n]$ and $x_2[n]$ as follows: $y[n] = x_1[m]$ when $n = 2m$ i.e., n is even, otherwise $y[n] = x_2[m]$ when $n = 2m + 1$, i.e., n is odd, where $m = 0, \ldots, 31$. If $\{X_1[k]\}_{k=0}^{31}$ is the DFT of $\{x_1[n]\}_{n=0}^{31}$ and $\{X_2[k]\}_{k=0}^{31}$ is the DFT of $\{x_2[n]\}_{n=0}^{31}$, express the DFT $\{Y[k]\}_{k=0}^{63}$ of $\{y[n]\}_{n=0}^{63}$ in terms of the $\{X_1[k]\}_{k=0}^{31}$ and $\{X_2[k]\}_{k=0}^{31}$.

Problem 4:

Given the sequence $\{x[n]\}_{n=0}^{3} = \{2, 0, 6, 4\}$ with DFT $\{X[k]\}_{k=0}^{3} = \{X[0], X[1], X[2], X[3]\}$:

(a) Express the DFT $\{\hat{X}[k]\}_{k=0}^{3}$ of the sequence $\{\hat{x}[n]\}_{n=0}^{3} = \{-2, 1, 0, 3\}$ in terms of $\{X[k]\}_{k=0}^{3}$.

(b) validate the relationship derived in part (a) by computing $\{X[k]\}_{k=0}^{3}$ and $\{\hat{X}[k]\}_{k=0}^{3}$ directly from their corresponding time-domain sequences.

Problem 5:

Consider two length-4 sequences $\{x[n]\}_{n=0}^{3} = \{1, 0, -1, 0\}$ and $h[n]_{n=0}^{3} = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$.

(a) Compute the DFTs $\{X[k]\}_{k=0}^{3}$ and $\{H[k]\}_{k=0}^{3}$.

(b) Compute the circular convolution $y[n] = x[n] \circledast h[n]$ directly in the sequence domain.

(c) Compute the circular convolution $y[n] = x[n] \circledast h[n]$ in the frequency domain using DFT and IDFT. Compare your answer with the one in part (b).

Problem 6:

Consider the two length-3 sequences $\{x[n]\}_{n=0}^{2} = \{1, -1, 1\}$ and $h[n]_{n=0}^{1} = \{-1, 1\}$.

(a) Compute the linear convolution $y[n] = x[n] * h[n]$ directly in the sequence domain.

(b) Compute the linear convolution $y[n] = x[n] * h[n]$ in the frequency domain using DFT and IDFT after appropriately zero-padding the length-4 sequences. Compare your answer with the answer in part (a).