Topics covered in this homework are: Sampling, Analog Frequency response of DSP system. Homework is due at 5:00 PM on Wednesdays. Homework will be graded for (1) completion and (2) three randomly picked problems will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

Consider the discrete time system shown below. The CTFT $X_a(\Omega)$ of $x_a(t)$ and the frequency response $H_d(\omega)$ are also shown. Given that $T_1 = T_2 = \dfrac{1}{8 \times 10^3}$ s.

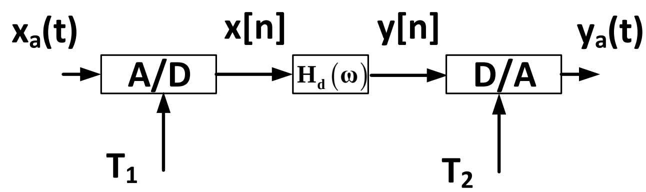
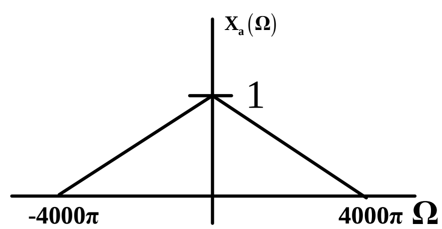
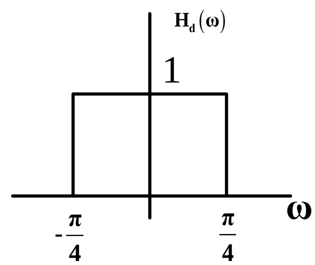


Figure 1: DSP system for Problem 1





1. Sketch $X_d(\omega)$ and $Y_d(\omega)$ over $|\omega| \leq \pi$.
2. Assume the D/A is ideal. Sketch the spectrum $Y_a(\Omega)$

Solution:

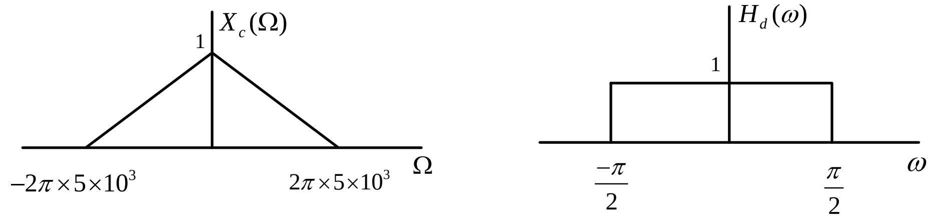
Problem 2:

Consider the DSP system shown in fig. 1. The output $y[n]$ of the filter $H_d(\omega)$ is described by $y[n] = x[n] - \sqrt{2}x[n-1] + x[n-2]$. Given $x(t) = cos(500\pi t) + cos(1000\pi t)$. Compute $y_a(t)$ for:

1. $T_1 = T_2 = \dfrac{1}{1500}$s
2. $T_1 = T_2 = \dfrac{1}{750}$s.

Problem 3:

For the system shown in Fig. (1), the CTFT of $x_a(t)$ and the frequency response $H_d(\omega)$ are shown below,



Sketch and label the CTFT of $y_a(t)$ for each of the following cases:

(a) $1/T_1 = 1/T_2 = 10^4$

(b) $1/T_1 = 1/T_2 = 2 \times 10^4$

(c) $1/T_1 = 2 \times 10^4, 1/T_2 = 10^4$

(d) $1/T_1 = 10^4, 1/T_2 = 2 \times 10^4$

Problem 4:

In communication systems a message signal, $x(t)$, needs to be transmitted over long distance. This is achieved by taking help of a high frequency carrier signal, $x_c(t)$. This process is called modulation and is given by:

$$\begin{aligned} x_a(t) &= 2cos\left(\Omega_c t\right) x(t) \\ &= \left(e^{j\Omega_c t} + e^{-j\Omega_c t}\right) x(t) \end{aligned}$$

and is illustrated in Fig. 2 below.

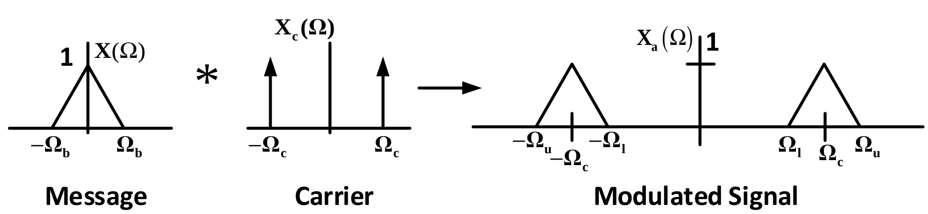


Figure 2: DSP system

At the receiver, the incoming signal must be demodulated. The demodulation can be implemented digitally as shown in Fig. 3. Generally the carrier frequency is large compared to signal bandwidth. This requires the sampling frequency to be large. In this problem we will look at implementing digital demodulation by sampling at lower than Nyquist frequency. Consider the system in Fig. 3. Assume $\Omega_C = 2\pi 100000$ rad/s, $\Omega_b = 2\pi 5000$ rad/s.

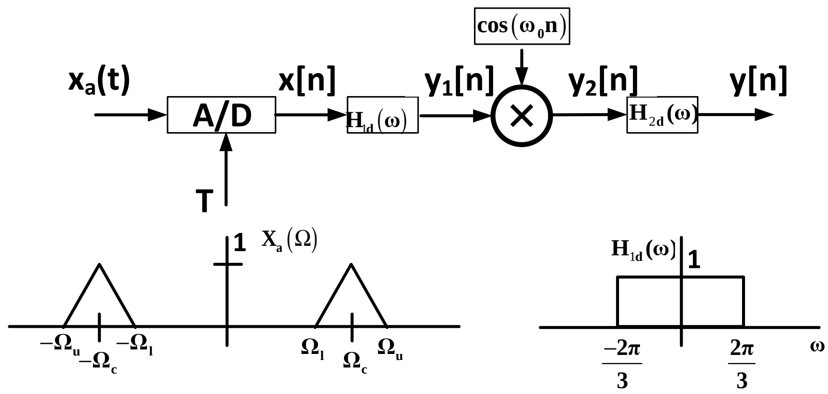


Figure 3: DSP system

1. What is the sampling period T such that no aliasing occurs at the output of the A/D converter.
2. Assume $T = \dfrac{1}{150000}$s. Sketch $X_d(\omega), Y_{1d}(\omega)$ in $(-2\pi, 2\pi)$. Please note: H_{1d} is periodic with period 2π.
3. Let $\omega_0 = \dfrac{2\pi}{3}$. Sketch $Y_{2d}(\omega)$ in $(-2\pi, 2\pi)$.
4. Sketch the frequency response of $H_{2d}(\omega)$ so that the spectrum of $y[n]$ is equivalent to the signal obtained by demodulating $x_a(t)$.

Problem 5:

A speech signal $x_a(t)$ is assumed to be bandlimited to 15kHz. It is desired to filter this signal with a bandpass filter that will pass the frequencies between 500Hz and 5kHz by using a digital filter $H_d(\omega)$ shown in Fig. 1.

1. Determine the Nyquist sampling rate for the input signal.
2. Sketch the frequency response $H_{d,1}(\omega)$ for the necessary discrete-time filter, when sampling at the Nyquist rate.
3. Find the largest sampling period T for which the overall system comprising A/D, digital filter response ($H_{d,2}(\omega)$), and D/A realize the desired band pass filter.
4. For the system using T from part (3), sketch the necessary $H_d(\omega)$.