ECE 310

Homework 2:

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Problem 1:

1. $y[n-2] = x^2[n] + 3y[n]$ $\quad z[n] = y[n-2] - 3y[n] = x^2[n]$

linearity: $(ax_1[n] + bx_2[n])^2 \neq ax_1^2[n] + bx_2^2[n] = az_1[n] + bz_2[n]$

$\tilde{z}[n] = \tilde{x}^2[n]$ → not linear

causality: $z[n]$ depends only on $x^2[n]$ → causal

shift-variance: $z[n-n_0] = x^2[n-n_0] = \tilde{x}^2[n] = \tilde{z}[n] = z[n-n_0]$ → shift-invarient

2. $y[n] = y[n-1] + \sum_{m=-\infty}^{n} x[m]$ $\quad z[n] = y[n] - y[n-1] = \sum_{m=-\infty}^{n} x[m]$

linearity: $\tilde{z}[n] = \sum_{m=-\infty}^{n} \tilde{x}[m] = \sum_{m=-\infty}^{n}(ax_1[m] + bx_2[m]) = a\sum_{m=-\infty}^{n} x_1[m] + b\sum_{m=-\infty}^{n} x_2[m] = a\tilde{z}_1[m] + b\tilde{z}_2[m]$ → linear

causality: $z[n]$ depends only on $\sum_{m=+\infty}^{n} x[m]$ → causal

shift-variance: $z[n-n_0] = \sum_{m=-\infty}^{+\infty} x[n-n_0]$ → time-invarient

3. $y[n] = x[-n]$

linearity: $a_1y_1[n] + a_2y_2[n] = a_1x_1[-n] + a_2x_2[-n] = y_3[n] = y(a_1x_1[n] + a_2x_2[n])$ → linear

causality: $y[n]$ depends only on $x[-n]$ → $x[-4]$ will depend on $x[4]$ → non-causal

time-varying: $y[n-n_0] = x[-(n-n_0)] = x[-n+n_0]$ → time-varient

4. $y[n] = x[n^2]$

linearity: $a_1x_1[n^2] + a_2x_2[n^2] = a_1y_1[n] + a_2y_2[n] = y_3[n]$ → linear

causality: $y[n]$ depends only on $x[n^2]$ → non-causal $\quad |y[2] = x[4]|$

time-varying: $y[n-n_0] = x[(n-n_0)^2] \neq x[n^2-n_0^2]$ → time-invarient

5. $y[n] = x[n] + v[n]$

linearity: $\tilde{y}[n] = ax_1[n] + bx_2[n] + av_1[n] + bv_2[n] \neq a(x_1[n] + v_1[n]) + b(x_2[n] + v_2[n]) = ay_1[n] + by_2[n]$ → nonlinear

causality: $y[n]$ depends only on $x[n] + v[n]$ → causal

time-varying: $y[n-n_0] = x[n-n_0] + v[n]$ → time-varient

Problem 2:

$x_1[n] = [-4 \ \underset{\uparrow}{2} \ -4]$ $\quad$ $y_1[n] = [-1 \ \underset{\uparrow}{5} \ 5 \ 0 \ 2]$

$x_2[n] = [-4 \ \underset{\uparrow}{2} \ 0] \rightarrow \underset{\text{linear}}{\boxed{T}} \rightarrow$ $y_2[n] = [1 \ 1 \ -2 \ 0 \ 1]$

$x_3[n] = [0 \ \underset{\uparrow}{2} \ 2]$ $\quad$ $y_3[n] = [1 \ 1 \ 0 \ 0 \ 2]$

1. $x[n] = \delta[n] = \{0, 1, 0\} = \frac{1}{4}\{x_1[n] - x_2[n] - 2x_3[n]\} \rightarrow \boxed{T} \rightarrow -\frac{1}{4}(y_1[n] - y_2[n] - 2y_3[n])$ as T is a linear system

$\rightarrow \delta[n] \rightarrow \boxed{T} \rightarrow -\frac{1}{4}\{-4; \underset{\uparrow}{2}; 7; 0; -3\}$

$= \{1; -\frac{1}{2}; -\frac{7}{4}; 0; \frac{3}{4}\} = y[n]$

2. $\delta[n-1] = \{0; 0; 1\} = \frac{1}{4}\{x_2[n] - x_1[n]\} \rightarrow T \rightarrow \frac{1}{4}\{y_2[n] - y_1[n]\} = \{\frac{1}{2}; \underset{\uparrow}{-1}; -\frac{7}{4}; 0; -\frac{1}{4}\}$

So we can see that the system is shift-varient because the output is not the same as it should be.

Problem 3:

$x[n] \rightarrow \boxed{T_1} \xrightarrow[z[n]]{} \boxed{T_2} \rightarrow y[n]$

1) ~~[illegible]~~ Yes, a combination of linear systems is linear by commutativity

2) $z[n] = e^{x[n]}$ ~~[illegible] not linear~~ [illegible]

$y[n] = \log(z[n]) \rightarrow$ $\quad x[n] \rightarrow T_1 \rightarrow e^{x[n]} \rightarrow T_2 \rightarrow \log(z[n]) = x[n]$

log and exp $\rightarrow S_1$ and S_2 being non-linear

Output is the input $\rightarrow$ overall system is linear

3) $z[n] = x[n]e^{j\frac{\pi n}{4}}$ $\quad z[n-n_0] = x[n-n_0]e^{j\frac{\pi}{4}[n-n_0]} \neq x[n-n_0]e^{j\frac{\pi}{4}n} = x[n-n_0] = \tilde{x}[n]$

$y[n] = x[n]e^{-j\frac{\pi n}{4}}$ $\quad y[n-n_0]$ idem $\quad S_1 + S_2 \rightarrow$ not shift-invariant

The overall system is shift-invariant as $y[n] = x[n]e^{-jn\frac{\pi}{4}}e^{jn\frac{\pi}{4}} = x[n]$

$\rightarrow$ same output as input

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Problem 4:

1. $h[n] = \left(\frac{1}{2}\right)^n u[n]$ → causal because $u[n] = 0$ if $n < 0$
   $\sum_{-\infty}^{+\infty} |h[n]| = \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n = 2 < +\infty$ → stable

2. $h[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1]$
   $= \left(\frac{1}{2}\right)^n u[n] + 2^n u[\underbrace{-(n+1)}]$ → start at $-(n+1) \Rightarrow n = -1$
   → not causal because $u[-(n+1)] = 0$ for $n < -1$
   $\sum_{n=-\infty}^{+\infty} |h[n]| = +\infty$ → not stable

3. $h[n] = \left(\frac{1}{4}\right)^n u[n+5] - u[n-5]$ → non-causal because $u[n+5] = 0$ for $n < -5$ only
   and stable as $\sum_{n=-\infty}^{+\infty} |h[n]| < +\infty$

4. $h[n] = \left(\frac{1}{4}\right)^{|n|}$ ⇒ not causal because $h[n] \neq 0$ if $n < 0$ and stable as $\sum_{n=-\infty}^{+\infty} |h[n]| < +\infty$

5. $h[n] = \sin(\pi n) u[n]$ → causal because $u[n] = 0$ if $n < 0$
   and unstable as $\sin(\pi n)$ is divergent

Problem 5:

1. $x[n] = (\delta[n] - \delta[n-1])$
   $h[n] = (0,5)^n u[n]$

   $x[n] * h[n] = h[n] * (\delta[n] - \delta[n-1])$
   $= h[n] - h[n-1]$
   $= 0,5^n [u[n] - 2u[n-1]]$

2. $x[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$
   $h[n] = \{1, \underset{\uparrow}{3}, 3\}$

   1 2 3
   1 1 1 1 1
   1 2 3
   1 2 3
   1 2 3

   → $y[n] = x[n] * h[n] = \{1, \underset{\uparrow}{3}, 6, 6, 6, 5, 3\}$

   ~~$h[k-n]$ if $k < -2 \to 0$~~

   $y[n] = 0$ for $n > 5$

   $h[k-n]$ → if $n < -2 \to 0$
   $n = -1 \to y[-1] = 3$, $y[0] = 5$, $y[1] = 6$, $y[2] = y[3] = 6$, $y[4] = 3$,
   $y[5] = 1$,

3. $x[n] = u[n-10]$

$h[n] = \cos(n)u[n]$

$y[n] = x[n] * h[n]$

$= \sum_{k=-\infty}^{+\infty} x[n-k]\, h[k] = \sum_{k=0}^{n-10} \cos k\, u[n-10]$

$\rightarrow k > 0$

$n-10-k \geq 0$

$k \leq n-10$

$$\sum_{k=0}^{n-10} (\cos k) = \frac{1}{2}\left(\sum_{k=0}^{n-10} e^{jk} + \sum_{k=0}^{n-1} e^{-jk}\right) = \frac{1}{2}\left(\frac{1-e^{j(n-9)}}{1-e^{j}} + \frac{1-e^{-j(n+9)}}{1-e^{-j}}\right)$$

$$\rightarrow x[n] * h[n] = \frac{1}{2}\left(\frac{1-e^{j(n-9)}}{1-e^{j}} + \frac{1-e^{-j(n+9)}}{1-e^{-j}}\right)u[n-10]$$

Problem 6:

$x[n] = \left(\frac{1}{3}\right)^n u[n]$

$h[n] = \left(\frac{2}{3}\right)^n u[n]$

$w[n] = u[n] - u[n-10]$

1. $y_1[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{+\infty} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{n-k} u[n-k]u[k] \quad n-k \geq 0$$

$\rightarrow k \leq n$

$$= \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{n-k} = \left(\frac{2}{3}\right)^n \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{-k}$$

$$= \left(\frac{2}{3}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \quad (\text{for } n \geq 0) = \left(\frac{2}{3}\right)^n \cdot \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = \left(\frac{2}{3}\right)^n \cdot \left(2-2\left(\frac{1}{2}\right)^{n+1}\right)$$

$$= \left(\frac{2}{3}\right)^n \cdot 2\left(1-\left(\frac{1}{2}\right)^{n+1}\right) \quad n \geq 0$$

$\frac{\left(\frac{1}{3}\right)^k}{\left(\frac{2}{3}\right)^k} = \frac{1}{2}$

2. $$y_2[n] = x[n] * w[n] = \sum_{k=-\infty}^{+\infty} \left(\left(\frac{1}{3}\right)^{n-k} u[n-k]\right)\left(u[k] - u[k-10]\right)$$

$\rightarrow k \geq 0$

$k-10 < 0$

$\rightarrow k \leq 10$

$$= \sum_{k=0}^{10} \left(\frac{1}{3}\right)^{n-k} u[n-k]$$

$\rightarrow n-k \geq 0$

$k \leq n \quad , n \leq 10$

$$= \sum_{k=0}^{n} \left(\frac{1}{3}\right)^{n-k} = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} \left(\frac{1}{3}\right)^{-k}$$

$$= \left(\frac{1}{3}\right)^n \cdot \frac{1-\left(\frac{1}{3}\right)^{n+1}}{1-\frac{1}{3}} = \left(\frac{1}{3}\right)^n \cdot \frac{1-\left(\frac{1}{3}\right)^{n+1}}{\frac{2}{3}} = \left(\frac{1}{3}\right)^n \cdot \left(\frac{3}{2}\right)\left[1-\left(\frac{1}{3}\right)^{n+1}\right]$$

$n \leq 10$

3. $$y_3[n] = x[n] * \left(\frac{1}{5}h[n-1] + \frac{1}{10}w[n+9]\right)$$

$$= \frac{1}{5}y_1[n-1] + \frac{1}{10}y_2[n+9]$$

$$= \frac{1}{5}\left[\left(\frac{2}{3}\right)^{n-1} \cdot 2\left(1-\left(\frac{1}{2}\right)^{n}\right)\right] + \frac{1}{10}\left[\left(\frac{1}{3}\right)^{n+9} \cdot \left(\frac{3}{2}\right)\left[1-\left(\frac{1}{3}\right)^{n+10}\right]\right]$$

$$= \frac{1}{5}\left[\left(\frac{2}{3}\right)^n \cdot 3\left(1-\left(\frac{1}{2}\right)^n\right)\right] + \frac{1}{10}\left[\frac{59049}{2}\left(\frac{1}{3}\right)^n\left[1-\left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^{10}\right]\right]$$

$$\approx \frac{3}{5}\left(\frac{2}{3}\right)^n\left(1-\left(\frac{1}{2}\right)^n\right) + \frac{59049}{20}\left(\frac{1}{3}\right)^n\left[1-\left(\frac{1}{3}\right)^n\right]$$