Topics covered in this homework are: complex numbers, discrete-time (DT) sequences, and properties of DT systems. Homework will be graded for (1) completion and (2) one randomly picked problem will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1: Complex Numbers

Consider the following complex numbers:

(a) $2e^{-j\pi/6} + 3e^{j\pi/3}$

(b) $\dfrac{(\sqrt{2} - j\sqrt{2})^{2n}}{(\sqrt{8} + j\sqrt{8})^{n}}$

Evaluate and represent these numbers in both Cartesian and polar forms.

Problem 2: Complex Numbers

Determine all roots of $z^4 = 1$ on the complex plane.

Problem 3: Complex Signals

Suppose that $x[n]$ is a sequence of complex numbers (complex signal). The *conjugate symmetric part* of $x[n]$ is defined as $x_{\text{cs}}[n] = \frac{1}{2}(x[n] + x^*[-n])$, where $x^*[n]$ is the complex conjugate of $x[n]$. Let $x^r_{\text{cs}}[n] = \text{Re}\{x_{\text{cs}}[n]\}$ and $x^i_{\text{cs}}[n] = \text{Im}(x_{\text{cs}}[n])$. Argue that $x^r_{\text{cs}}[n] = x^r_{\text{cs}}[-n]$ and $x^i_{\text{cs}}[n] = -x^i_{\text{cs}}[-n]$, i.e., $x^r_{\text{cs}}[n]$ has *even symmetry* and $x^i_{\text{cs}}[n]$ has *odd symmetry.*

Problem 4: Basic Discrete-Time Signals

Sketch the following signals:

1. $n^2(u[n+2] - u[n-4])$
2. $u[-n+2]u[n+6]$
3. $\sin\left(\frac{\pi}{4}n\right)\delta[n-1] + \cos(\frac{\pi}{7}(2n-1))\delta[n-4]$
4. $\cos(3n), n = 0, 1, \ldots, 10$

where $u[n]$ is the unit step signal and $\delta[n]$ is the unit sample signal.

Problem 5: Finite-Length Signals

Express the signal $\{x[n]\} = \{\cdots, 0, 2, 0, -3, 1, \underset{\uparrow}{4}, 0, 3, 1, -2, 0, 4, 0 \cdots\}$ in terms of the unit sample (impulse) $\delta[n]$. Here, $\cdots$ denotes zeros.

Problem 6: **Properties of Discrete-Time Systems**

Consider the following discrete-time systems:

(a) $y[n] = x[n] + 2x[n-1] + 3x[n-3]$

(b) $y[n] = \cos(x[n])$

(c) $y[n] = x[n]e^{j\frac{\pi}{4}n}$

(d) $y[n] = x[0]x[n] + x[1]x[n+1]$

Determine if the systems are: (a) linear (b) time-invariant.