Homework 10

Problem 1: DFT and DTFT

Let $\{X[k]\}_{k=0}^{50}$ and $X_d(\omega)$ respectively be the 51-point DFT and DTFT of a *real-valued* sequence $\{x[n]\}_{n=0}^{17}$ that is zero-padded to length 51. Determine all the correct relationships in the following and justify your answer.

1. $X[49] = X_d(-\frac{4\pi}{51})$.
2. $X[2] = X_d^*(-\frac{4\pi}{51})$
3. $X[1] = X_d(\frac{104\pi}{51})$
4. $X[25] = X_d(\pi)$

Problem 2: DFT of a Cosine

A continuous-time signal $x_c(t) = \cos(24\pi t)$ is sampled at a rate of 120 Hz for 5 seconds to produce a discrete-time signal $x[n]$ with length $L = 600$.

1. Let $X[k]$ be the L-point DFT of $x[n]$. At what value(s) of k will $X[k]$ have the greatest magnitude?
2. Suppose that $x[n]$ is zero-padded to a total length of $N = 1024$. At what value(s) of k does the N-point DFT have the greatest magnitude?

Problem 3: Circular and Linear Convolution

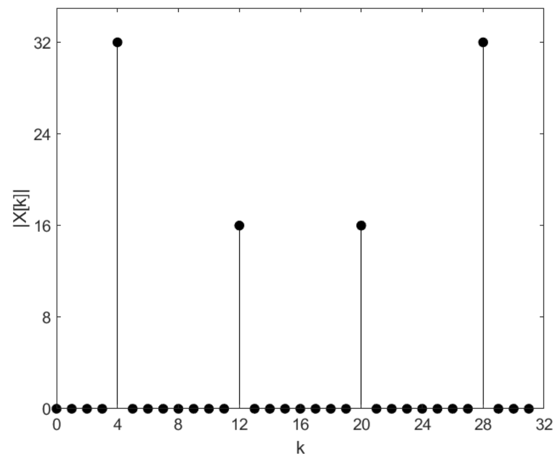
Consider the two finite-length sequences:

$$x = \{\underset{\uparrow}{-1}, 2, -3, 4, -5\} \text{ and } h = \{\underset{\uparrow}{1}, 1, 1\}$$

1. Compute the linear convolution $x * h$.
2. Compute the circular convolution $x \circledast_5 h$.
3. What is the smallest value of N so that the N-point circular convolution is equal to the linear convolution?

Problem 4: DFT for the Sum of Cosines

Suppose that the signal $x_c(t) = A_0 \cos(\Omega_0 t) + A_1 \cos(\Omega_1 t)$ is sampled at a rate of 64 kHz for 1/2 msec. The DFT of the obtained signal is provided by the plot.



1. Assume that Ω_0 and Ω_1 are both less than the Nyquist frequency. Find A_0, A_1, Ω_0, and Ω_1.
2. Suppose that $x_c(t)$ was instead sampled at 128 kHz for 1/2 msec. Sketch the new DFT magnitude plot and clearly label all nonzero values.