Homework 7

Problem 1: Special Filters

Consider a stable causal LTI system given by

$$H(z) = \frac{z^{-1} - 0.3}{1 - 0.3z^{-1}}.$$

Compute and sketch the magnitude response of the system. What kind of filter is this?

Problem 2: Sampling a Cosine

The sequence $x[n] = \cos\left(\frac{\pi}{4}n\right), \ -\infty < n < \infty$ was obtained by sampling the continuous-time signal $x_a(t) = \cos(\Omega_0 t), \ -\infty < t < \infty$ at a sampling rate of 2000 samples/sec. Find two possible values of Ω_0 that could have resulted in the sequence $x[n]$.

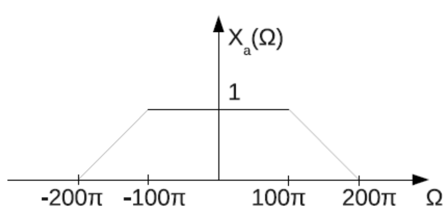
Problem 3: Sampling a Cosine (again)

The continuous-time signal $x_a(t) = \cos(150\pi t)$ is sampled with sampling period T_s to obtain a discrete-time signal $x[n] = x_a(nT_s)$.

1. Compute and sketch the magnitude of the continuous-time Fourier transform of $x_a(t)$ and the discrete-time Fourier Transform of $x[n]$ for $T_s = 1$ ms and $T_s = 2$ ms.
2. What is the maximum sampling period $T_{s,\max}$ such that no aliasing occurs in the sampling process?

Problem 4: Sampling

The continuous-time signal $x_a(t)$ has the continuous-time Fourier transform shown in the figure below. The signal $x_a(t)$ is sampled with sampling interval T_s to get the discrete-time signal $x[n] = x_a(nT_s)$. Sketch $X_d(\omega)$ (the DTFT of $x[n]$) for the sampling intervals $T_s = 1/50, 1/200, 1/400$ sec.



Problem 5: Energy Relationship

Let $x_a(t)$ be a bandlimited signal to $\Omega_{\max}$, i.e., $X_a(\Omega) = 0, |\Omega| > \Omega_{\max}$. Suppose that the signal is sampled with sampling frequency $F_s \geq 2F_{\max}$ (frequencies in Hz). Denote by E_d the energy of $x[n] = x_a(nT_s)$, i.e., $E_d = \sum_{n=-\infty}^{\infty} |x[n]|^2$ and by E_a the energy of $x_a(t)$, i.e., $E_a = \int_{-\infty}^{\infty} |x_a(t)|^2 dt$. Show that

$$E_d = \frac{E_a}{T_s}.$$

Homework 7

Verify the energy relationship for the bandlimited signal $x_a(t)$ with $X_a(\Omega) = 2$ for $\Omega \in [-1000\pi, 1000\pi]$ and 0 otherwise. Assume that the signal is sampled at the Nyquist rate.