Topics covered in this homework are: Continuous-Time Fourier TRansform(CTFT) and Discrete-Time Fourier Transform(DTFT). Homework is due at 5:00 PM on Wednesdays. Homework will be graded for (1) completion and (2) one randomly picked problem will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

Determine if the following systems are BIBO Stable. Give brief justification. For each case in which the system is determined to be unstable, find a bounded real-valued input that will produce an unbounded output. Does DTFT exist for parts (3) - (6) ?

1. System with input-output relationship, $y[n] = x[n]sin^2(x[n])$
2. System with input-output relationship, $y[n] = nx[n]$
3. Causal LSI system with transfer function $H(z) = \dfrac{z-7}{z^2+\dfrac{1}{9}}$
4. Anticausal LSI system with transfer function $H(z) = \dfrac{z-7}{z^2+\dfrac{1}{9}}$
5. $H(z) = \dfrac{z}{(z-0.7)(z^2+z+1)}$, $h[n]$ is LSI and two-sided
6. Causal LSI system with transfer function $H(z) = \dfrac{z^2}{z^2-\sqrt{2}z+1}$

Problem 2:

The input $x[n] = 2^n\left(u[n] - 3u[n-1]\right)$ to an unknown LSI system produces output $y[n] = (3^n - 2^n)u[n]$.

1. Compute the z-transform of $x[n]$ and $y[n]$. Identify the region of convergence of $X(z)$ and $Y(z)$.
2. Compute the transfer function $H(z)$. Identify the region of convergence
3. Is the impulse response $h[n]$ unique? Can $h[n]$ represent a stable system? Can it represent a causal system?

Problem 3:

Compute the CTFT of the following sequences

1. $x(t) = \delta(5t-2)$
2. $x(t) = e^{-3t}u(t)$
3. $x(t) = e^{-3t} * \delta(5t-2)$

4. $x(t) = 4sin(2000\pi t)$

Problem 4:

Determine the DTFT of the sequences given below. For parts 1 and 2 plot the magnitude and phase of DTFT.

1. $x[n] = u[n+3] - u[n-5]$
2. $x[n] = 2\delta[n+1] - 2\delta[n]$
3. $x[n] = \left(\frac{1}{3}\right)^n u[n-4]$
4. $x[n] = (1-n)\left(\frac{1}{3}\right)^n u[n]$
5. $x[n] = (1-n)3^n u[n]$
6. $x[n] = \left(\frac{1}{3}\right)^n cos\left(\frac{\pi(n-5)}{2}\right) u[n]$

Problem 5:

Let $x[n]$ be an arbitrary signal not necessarily real-valued with DTFT $X_d(\omega)$. Express the DTFT of the following signals in terms of $X_d(\omega)$.

1. $y[n] = x^*[-n+4]$
2. $y[n] = x[n-2]cos\left(\frac{\pi n}{4}\right)$

Problem 6:

Determine the signal $x[n]$ corresponding to each of the following cases.

1. $X_d(\omega) = 2 + 3e^{-j\omega} + 2e^{-3j\omega} - e^{-5j\omega}$
2. Assume $x[n]$ is real-valued.

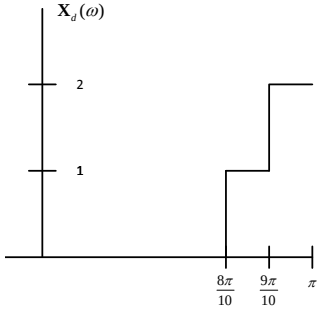

Figure 1: Figure for Problem 6-2