Topics covered in this homework are: System Properties, Convolution. Homework is due at 5:00 PM on Wednesdays. Homework will be graded for (1) completion and (2) one randomly picked problem will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

Determine if the following systems are **linear** or **non-linear**, **causal** or **non-causal**, **shift-invariant** or **shift-varying**.

1. $y[n-2] = x^2[n] + 3y[n]$
2. $y[n] = y[n-1] + \sum_{m=-\infty}^{n} x[m]$
3. $y[n] = x[-n]$
4. $y[n] = x[n^2]$
5. $y[n] = x[n] + u[n]$

Problem 2:

Consider the **linear** system T shown below. The outputs $y_k[n], k = 1, 2, 3$ of the system for three inputs $x_k[n], k = 1, 2, 3$ are shown below.

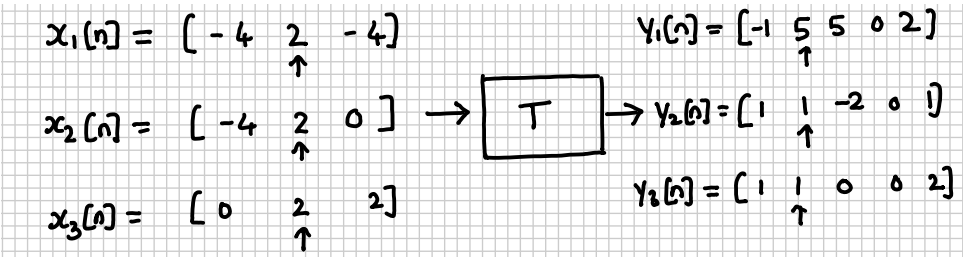

Figure 1: Figure for Problem 2

1. Find the output of the system when the input $x[n] = \delta[n]$.
2. Can the system be Shift-Invariant?

Problem 3:

Consider two systems in cascade as shown below.

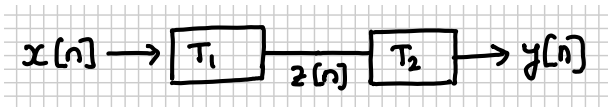

Figure 2: Figure for Problem 3

1. Assume S_1 and S_2 are linear. Will the overall system be linear?
2. Assume $z[n] = e^{x[n]}$ and $y[n] = log(z[n])$. Are S_1 and S_2 linear? What can you say about the overall system?
3. Assume $z[n] = x[n]e^{j\frac{\pi n}{4}}$ and $y[n] = x[n]e^{-j\frac{\pi n}{4}}$. Are S_1 and S_2 Shift Invariant? What can you say about the overall system?

Problem 4:

Determine if the following systems with given impulse response are causal and(stable?).

1. $h[n] = \left(\frac{1}{2}\right)^n u[n]$
2. $h[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1]$
3. $h[n] = u[n+5] - u[n-5]$
4. $h[n] = \left(\frac{1}{4}\right)^{|n|}$
5. $h[n] = sin(\pi n)u[n]$

Problem 5:

Compute the convolution $x[n] * h[n]$ for $x[n]$ and $h[n]$ shown below.

1. $x[n] = (\delta[n] - \delta[n-1]), h[n] = (0.5)^n u[n]$
2. $x[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}, h[n] = \{1, \underset{\uparrow}{2}, 3\}$
3. $x[n] = u[n-10], h[n] = cos(n)u[n]$

Problem 6:

Let $x[n] = \left(\frac{1}{3}\right)^n u[n], \quad h[n] = \left(\frac{2}{3}\right)^n u[n], \quad w[n] = u[n] - u[n-10]$. Compute:

1. $y_1[n] = x[n] * h[n]$
2. $y_2[n] = x[n] * w[n]$
3. Use results obtained in parts (1) and (2) to compute $y_3[n] = x[n] * \left(\frac{1}{5}h[n-1] + \frac{1}{10}w[n+9]\right)$