Topics covered in this homework are: DTFT properties, Fourier analysis of LTI systems and frequency response (magnitude and phase). Homework is due at 5:00 PM on Wednesdays. Homework will be graded for (1) completion and (2) one randomly picked problem will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

It is known that DTFT $X_d(\omega)$ is a real function of ω. Now answer the following:

(a) What property does $x[n]$ need to satisfy? Provide reasons.

(b) Given that $x[n] = 0, \forall |n| > 3$, $x[-3] = -j$, $x[-2] = 1 + \frac{1}{2}j$, $x[0] = 1$, $x[1] = 2 + j$, determine the remaining values of $x[n]$.

(c) If $x[n]$ is a real-valued sequence, determine the values of $X_d(\frac{3\pi}{4})$, $X_d(-\frac{\pi}{4})$, $X_d(-\frac{5\pi}{4})$ and $X_d(\frac{7\pi}{4})$ if $X_d(-\frac{3\pi}{4}) = -2$ and $X_d(\frac{\pi}{4}) = 3.5$.

Problem 2:

Consider a causal LTI system described by the following difference equation:

$$y[n] + \frac{1}{2}y[n-4] = x[n] - x[n-2] \tag{1}$$

(a) Determine the frequency response $H_d(\omega)$.

(b) Will the impulse response $h[n]$ be real-valued or complex? Provide reasons.

(c) Determine $y[n]$ when the input is $x[n] = \sin(\frac{\pi}{4}n)$.

Problem 3:

Consider the LTI system with frequency response:

$$H_d(\omega) = 1 + \sin(2\omega) - j\cos(2\omega) \tag{2}$$

Determine the response $y[n]$ when:

(a) $x[n] = 3 + 2\sin(\frac{\pi}{4}n) + 4e^{-j\frac{\pi}{2}n}$.

(b) $x[n] = 1, \forall n$.

(c) the input is a 2-sample delayed version of $x[n]$ in part (a).

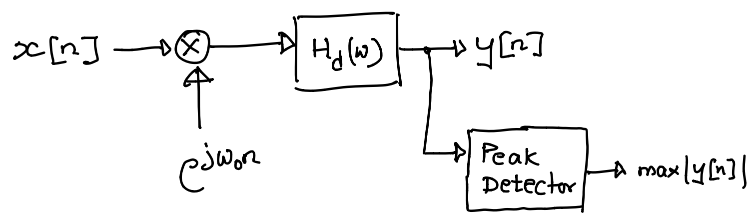

Figure 1: System set-up for Problem 4.

Problem 4:

The magnitude $H_d(\omega)|$ of an LTI system is given by:

$$|H_d(\omega)| = |2cos(2\omega)| \tag{3}$$

A complex sinusoid $x[n] = ae^{j\omega_1 n}$ $(a > 0)$is fed at the input to the LTI system after multiplying it by $e^{j\omega_o n}$ where the modulation frequency $-\pi < \omega_o < \pi$ is under user control. A peak detector at the output estimates the magnitude of the output $y[n]$. As ω_o is swept from 0 to π, it is observed that the magnitude of the output nulls out when $\omega_o = \frac{\pi}{8}$ and $\omega_o = \frac{5\pi}{8}$ and its peak magnitude is 6.

(a) What are the possible values of ω_1 and the amplitude a of $x[n]$?

(b) If $x[n] = 4e^{j\pi n}$ and ω_o is swept from 0 to $-\pi$, at what values of ω_o will the output magnitude peak and what will be its peak magnitude?

(c) If $x[n] = 4e^{j\pi n}$ and ω_o is swept from 0 to $-\pi$, at what values of ω_o will the output magnitude null out?

Problem 5:

Which of the following sequences are eigenfunctions of stable LTI systems:

(a) $x[n] = e^{j\frac{2\pi}{3}n}$.

(b) $x[n] = 3^n$ and the system is also causal.

(c) $x[n] = 3^n u[n]$.

(d) $x[n] = cos(\omega_o n)$.

Problem 6:

Solve Problem 3 from HWK5.