Topics covered in this homework are: Difference equations, z-transforms, and z-transform properties. Homework is due at 5:00 PM on Wednesdays. Homework will be graded for (1) completion and (2) one randomly picked problem will be graded. Submissions will be using gradescope. Please solve problems on your own in order to maximally benefit from this homework.

Problem 1:

The following difference equation:

$$y[n] = ay[n-1] + x[n] \tag{1}$$

with initial conditions $y[-1] = c$ and $x[n] = b\delta[n]$ has the solution:

$$y[n] = a^{n+1}c + a^n bu[n] \tag{2}$$

Now answer the following. Provide reasons.

(a) What is the impulse response $h[n]$ of the system in (1)?

(b) Is this a causal system?

(c) Obtain an expression for the response $y[n]$ when $x[n] = 2b\delta[n]$. Based on your answer, what can you say about the linearity of the system?

(d) Repeat Part (a)-(c) if the system is at *initial rest*.

(e) Obtain an expression for the response $y[n]$ when $x[n] = b\delta[n-1]$ under initial rest conditions. What can you say about the time-invariance of this system?

Problem 2: DE vs. Block-diagram

Sketch a block diagram of the system described by the difference equation:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1] \tag{3}$$

Assume the input $x[n]$ is real-valued.

Problem 3:

An LTI system is described by its system transfer function $H(z) = \sum_n h[n]z^{-n} = z + z^{-1}$. Find its output $y[n]$ when the input is $x[n] = 2(-\frac{1}{2} + j\frac{\sqrt{3}}{2})^n$ using:

(a) the eigenfunction property of LTI systems.

(b) the convolution sum.

Problem 4:

Determine the z-transform $X(z)$ of the following sequences and sketch the pole-zero plot:

(a) $x[n] = (\frac{1}{3})^n u[-n]$

(b) $x[n] = (\frac{1}{3})^n (u[n] - u[n-5])$

(c) $x[n] = (\frac{1}{2})^{n-3} u[n-2]$

(d) $x[n] = \alpha^{|n|}$ with $0 < |\alpha| < 1$

Problem 5:

Given the z-transform pair

$$x[n] = a^n u[n] \Longleftrightarrow X(z) = \frac{z}{z-a}, \quad \text{with ROC:} |z| > |a| \tag{4}$$

employ only the z-transform properties to obtain $y[n]$ from $Y(z)$ and its ROC:

(a) $Y(z) = \frac{a^2 z}{1-az}$ ROC:$|z| < |a^{-1}|$

(b) if $Y(z) = \frac{z-1}{z+\frac{3}{4}}$ and $y[n]$ is a causal sequence

(c) $Y(z) = \log(\frac{z+\frac{1}{2}}{z})$ ROC:$|z| > |\frac{1}{2}|$

(d) $Y(z) = \frac{2z}{z+\frac{3}{\sqrt{2}}(1+j)}$ ROC:$|z| > |3|$