CME 211 Project: Part 1

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Description

The conjugate gradient (CG) method for solving the linear system $Ax = b$ requires a few matrix and vector operations that can be generalized into a few operations. The most important function is a General Matrix Multiply (GEMM) that takes the form $c = \alpha Ax + \beta b$ where c, x, and b are vectors, α and β are scalars, and A is a CSR matrix. We can use this function for all of the matrix multiplication operations required in the CG method. Additionally, we also implement a weighted vector sum called daxby which is the expression $d = \alpha x + \beta y$. Finally, we have simple operations where we calculate the dot product and L_2 norm. These form the basis for all of the basic operations required to perform the CG method.

Algorithm

Data: Matrix A and vector b to solve $Ax = b$
Result: Solution for vector x in $Ax = b$
$u_n = \mathbf{1}$;
$r_n = b - Au_n$;
$l_{2,0} = \text{norm}_2(r_n)$;
$p_n = r_n$;
$n = 0$;
while $n < n_{max}$ **do**
    n++;
    $\alpha = r_n^T r_n / \left(p_n^T A p_n\right)$;
    $u_{n+1} = u_n + \alpha p_n$;
    $r_{n+1} = r_n - \alpha A p_n$;
    $l_{2,r} = \text{norm}_2\left(r_{n+1}\right)$;
    if $l_{2,r}/l_{2,0} < \mathtt{tol}$ **then**
        break;
    end
    $\beta = r_{n+1}^T r_{n+1} / (r_n^T r_n)$;
    $p_n = r_{n+1} + \beta p_n$;
    $u_n = u_{n+1}$;
    $r_n = r_{n+1}$;
end

Algorithm 1: Conjugate Gradient Method for solving $Ax = b$