8. Wastewater Math Fundamentals

8.1 Units

To measure any quantity or compare two physical quantities we need a universally accepted standard called Unit. The most common measurements involve measuring - length, weight and time. International System of Units (SI), the modern form of the metric system is the globally accepted standard. In the United States, it is customary to measure the physical quantities in English Engineering Units.

Fundamental Units		
Dimension	**English Engineering Units**	**SI**
time	second (s)	second (s)
length	foot (ft)	meter (m)
mass	pound mass (lb)	kilogram

The measurement of any physical quantity is expressed in terms of a number - which is the quantity and a specific unit. Thus, a measurement of 5000 ft is basically 5000 of the of length as measured in ft.

Using the fundamental physical measurements, mathematical calculations can be made to measure other physical quantities such as area (ft2), volume (ft3), velocity (ft/s), flow (ft3/s), density (lbs/ft3).

Depending on the what is being measured or quantified, there are appropriate and customary units of measure - for example - miles and inches for length, gallons and acre-ft for volume and milligrams and tons for mass.

8.1.1 Unit Conversions

Unit conversion is a process for changing the units of a measured quantity without changing its value. It involves utilizing a conversion factor which expresses the relationship between units that is used to change the units of a measured quantity without changing the value. Examples of conversion factors include:

Fundamental Units	
Dimension	**Conversion Factor**
time	$\frac{60\ sec}{min}, \frac{1,440\ sec}{day}$
length	$\frac{12\ in}{ft}, \frac{5,280\ ft}{mile}$
mass	$\frac{2,000\ lbs}{ton}, \frac{1000\ gm}{mg}$
Derived Units	
Dimension	**Conversion Factor**
area	$\frac{43,560\ ft^2}{acre}, \frac{60\ sec}{min}$
volume	$\frac{27\ ft^3}{yd}, \frac{7.48\ gal}{ft^3}$

The numerator and the denominator of any conversion factor always equals one, they have the same value expressed in different units.

For converting one measurement unit to another.

Step 1: Make sure the original unit is for the same measurement as the conversion unit. So if the original unit is for area, say ft^2 the conversion unit can be another area unit such as in^2 or acre but it cannot be gallons as gallon is a unit of volume.

Step 2: Write down the conversion formula as:

$$Quantity\ in\ converted\ unit = Quantity\ (\cancel{Original\ Unit}) * Conversion\ Factor\ \frac{Conversion\ unit}{\cancel{Original\ unit}}$$

Unit conversions may involve single factor where the original unit value is multiplied by the conversion factor to obtain the measured parameter in the converted (desired) unit.
For example:
Converting 1000 ft^3 to cu. yards:

$$1000\cancel{ft^3} * \frac{cu.yards}{27\cancel{ft^3}} = 37 cu.yards$$

Other unit conversions may require multiplying by known constants along with conversion factors.
For example:

1. Converting 3.5 ft^3/sec to MGD:
$$\frac{3.5\ \cancel{ft^3}}{\cancel{sec}} * \frac{7.48\ \cancel{gal}}{\cancel{ft^3}} * \frac{MG}{10^6 \cancel{gal}} * \frac{1440 * 60\ \cancel{sec}}{day} = 2.3\ MGD$$
2. Converting 1,000 L water to lbs:
$$1000\ \cancel{L} * \frac{\cancel{gal}}{3.785\ \cancel{L}} * \frac{8.34\ lbs}{\cancel{gal}} = 2,203\ lbs$$
$(Note: 8.34\ lbs/gal\ is\ density\ of\ water - a\ constant)$

8.2 Area & Volume

Perimeter (P)/Circumference (C)

Rectangle: P [ft] = 2L [ft] + 2W [ft]
where L = length and W = width

Circle: C [ft] = π × D [ft]
where π = constant = 3.1415; and D = diameter

Area (A)

Rectangle: A [ft²] = L [ft] × W [ft]
where L = Length and W = Width

Circle: where π = constant = 3.1415; D = diameter

$$A\left[ft^2\right] = \frac{1}{4} \times \pi \times D^2 \left[ft^2\right]$$

Volume (V)

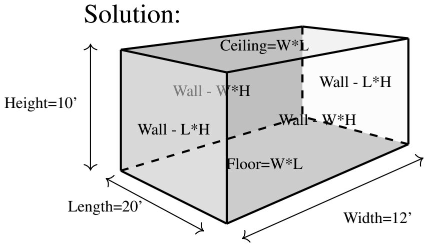
Regular Prism: V [ft³] = A $_{base}$ [ft²] × H [ft]

where A_{base} is the area of the base; and H is the height or depth of the tank

Cone: $V\left[ft^3\right] = \frac{1}{3} A_{base}\left[ft^2\right] \times H\left[ft\right]$

8.2.1 Example Problems

1. The floor of a rectangular building is 20 feet long by 12 feet wide and the inside walls are 10 feet high. Find the total surface area of the inside walls of this building
Solution:



2 Walls W*H + 2 Walls L*H= $2*12*10ft^2 + 2*20*10ft^2$
$= 240 + 400 = \boxed{640ft^2}$
2 Walls W*H + 2 Walls L*H + Floor + Ceiling= $2*12*10ft^2 + 2*20*10ft^2 + 2*12*20ft^2$
$= 240 + 400 + 480 = \boxed{1,120ft^2}$

8.3 Concentration

Concentration is typically expressed as mg/l which is the weight of the constituent (mg) in 1 l (liter) of solution (wastewater). As 1 l of water weighs 1 million mg, a concentration of 1 mg/l implies 1 mg of constituent per 1 million mg of water or one part per million (ppm). **Thus, mg/l and ppm are synonymous.** Sometimes the constituent concentration is expressed in terms of percentage.

For example: sludge containing 5% solids or a 12.5% chlorine concentration solution.

As one liter of water weighs 1,000,000 mg, one percent of that weight is 10,000 mg. So 1% solids implies 10,000 mg of solids per liter or 10,000 mg/l or 10,000 ppm.

$$1\%concentration = 10,000 \;\; ppm \;\; or \;\; \frac{mg}{l}$$

$$0.1\%concentration = 1,000 \;\; ppm \;\; or \;\; \frac{mg}{l}$$

$$0.01\% concentration = 100 \;\; ppm \;\; or \;\; \frac{mg}{l}$$

$$10\% concentration = 100,000 \;\; ppm \;\; or \;\; \frac{mg}{l}$$

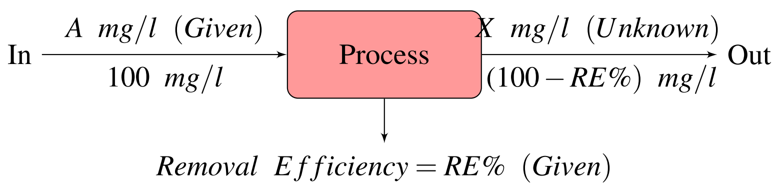
$$5\% concentration = 50,000 \;\; ppm \;\; or \;\; \frac{mg}{l}$$

$$A \;\; 12.5\% \;\; bleach \;\; solution \;\; contains \;\; 12.5\% \;\; or \;\; 125,000 \;\; \frac{mg}{l} \;\; of \;\;\; active \;\; chlorine$$

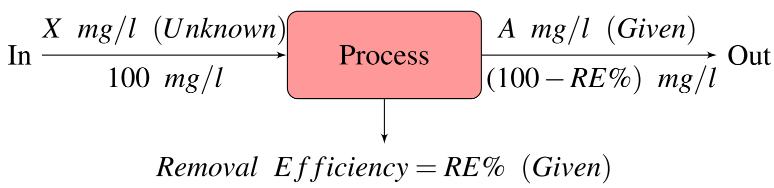
8.4 Process Removal Efficiency

- Process removal rate or removal efficiency is the percentage of the inlet concentration removed.
- It is used for quantifying the pollutant removal during wastewater treatment and is established based upon the amount of a particular wastewater constituent entering and leaving a treatment process.
- $Process \;\; Removal \;\; Rate \;\; (\%) = \frac{Pollutant \;\; In - Pollutant \;\; Out}{Pollutant \;\; In} * 100$
- If 10 units of a pollutant are entering a process and 8 units of pollutant are leaving (process removes 2 units), then the process removal rate for that pollutant is (10-8)/10*100=20%. In this example the process is 20% efficient in removing that particular pollutant.
- The amount of pollutant can be measured in terms of concentration (mg/l) or in terms of mass loading (lbs). The pounds formula is used for calculating the mass loadings.

The above example is for calculating the removal efficiency using the inlet and outlet concentrations or mass loading.
The methods below can be used for calculating either the inlet or outlet pollutant concentrations, if the removal efficiency and the corresponding inlet or outlet concentrations are given.
Case 1: Calculating outlet conc. (X) given the inlet conc. and removal efficiency (RE%):



Using the fact that if the inlet concentration was 100 mg/l, the outlet concentration would be 100 minus the removal efficiency.
Setup the equation as: $\frac{Out}{In} : \frac{X \;\; mg/l}{A \;\; mg/l} = \frac{100 - RE\%}{100}$
Calculate X using cross multiplication - if $\frac{A}{B} = \frac{C}{D} \implies A = B * \frac{C}{D}$:
$X \;\; mg/l = A \;\; mg/l * \frac{100 - RE\%}{100}$
Case 2: Calculating inlet conc. (X) given the outlet conc. and removal efficiency (RE%):



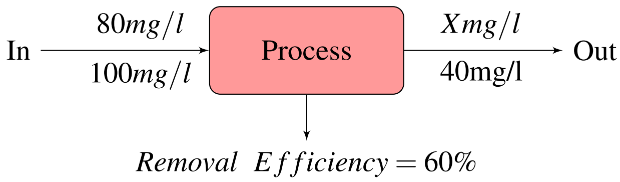
Using the fact that if the inlet concentration was 100 mg/l, the outlet concentration would be 100 minus the removal efficiency.
Setup the equation as: $\frac{In}{Out} : \frac{X \;\; mg/l}{A \;\; mg/l} = \frac{100}{100 - RE\%}$

Calculate X using cross multiplication - if $\frac{A}{B} = \frac{C}{D} \implies A = B * \frac{C}{D}$:

$X\ \ mg/l = A\ \ mg/l * \dfrac{100}{100 - RE\%}$

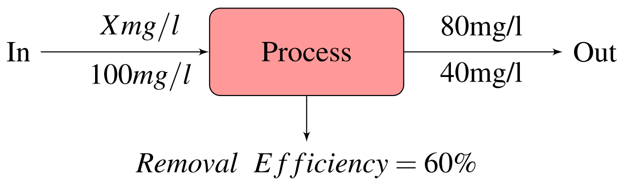
8.4.1 Example Problems

1. What is the % removal efficiency if the influent concentration is 10 mg/L and the effluent concentration is 2.5 mg/L?
   $Removal\ \ Rate(\%) = \dfrac{In - Out}{In} * 100 \implies \dfrac{10 - 2.5}{10} * 100 = \boxed{75\%}$
2. Calculate the outlet concentration if the inlet concentration is 80 mg/l and the process removal efficiency is 60%
   Solution:



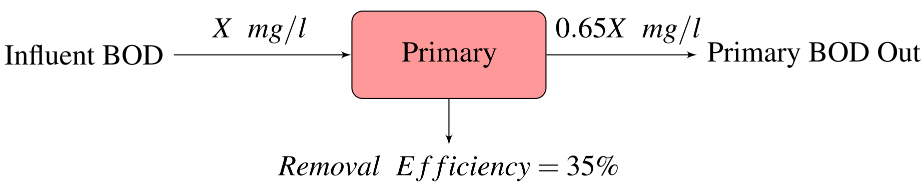
$\dfrac{Out}{In}\ :\ \dfrac{Actual\ \ Outlet(X)}{80} = \dfrac{100 - 60}{100}$
$\implies \dfrac{Actual\ \ Outlet(X)}{80} = 0.4$
$\implies Actual\ \ Outlet(X) = 0.4 * 80 = \boxed{32mg/l}$

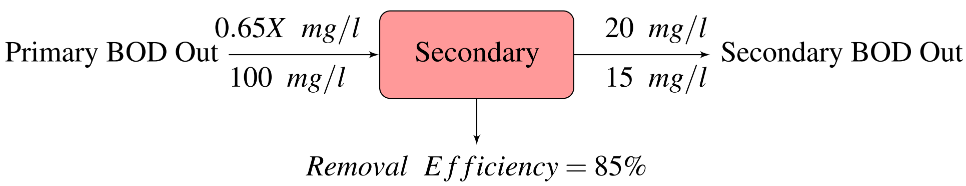
3. Calculate the inlet concentration if the outlet concentration is 80 mg/l and the process removal efficiency is 60%



$\dfrac{In}{Out}\ :\ \dfrac{Actual\ \ inlet\ \ (X)}{80} = \dfrac{100}{100 - 60} \implies \dfrac{Actual\ \ inlet\ \ (X)}{80} = 2.5$
Rearranging the equation: $Actual\ \ inlet(X) = 2.5 * 80 = \boxed{200mg/l}$

4. If a plant removes 35% of the influent BOD in the primary treatment and 85% of the remaining BOD in the secondary system, what is the BOD of the raw wastewater if the BOD of the final effluent is 20mg/l
   Solution:





For the Secondary process:

$$\frac{In}{Out} : \frac{0.65X}{20} = \frac{100}{15} \implies X \ mg/l = \frac{100*20}{15*0.65} = \boxed{205 \ mg/l}$$

Alternate Solution #1

$$X\frac{mg}{l} \xrightarrow{\text{Influent BOD}} \boxed{Primary} \xrightarrow[\text{X-0.35X=X*(1-0.35)=0.65X}\frac{mg}{l}]{\text{Primary Effluent BOD}} \boxed{Secondary} \xrightarrow[\text{0.65X-0.5525X=(0.65-0.5525)X=0.0975X}]{\text{Secondary Effluent BOD}}$$

$\downarrow$ (0.35X)BOD Removed $\qquad$ $\downarrow$ (0.65*0.85)X = 0.5525X BOD Removed

$$\implies 0.0975X = 20 \implies X = \frac{20}{0.0975} = \boxed{205\frac{mg}{l}}$$

Alternate Solution #2:

$$X\frac{mg}{l} \xrightarrow{\text{Influent BOD}} \boxed{Primary} \xrightarrow[\text{0.65X}]{\text{Primary Effluent BOD}} \boxed{Secondary} \xrightarrow[\text{(0.65*0.15)X}]{\text{Secondary Effluent BOD}}$$

$\downarrow$ (0.35X)BOD Removed $\qquad$ $\downarrow$ (0.65X*0.85)BOD Removed

Primary Effluent BOD = Influent BOD * (1-Primary BOD Removal), and
Secondary Effluent BOD=[Primary Effluent BOD]*(1-Secondary BOD Removal)
Secondary Eff. BOD=[Influent BOD * (1-Primary BOD Removal)]*(1-Secondary BOD Removal)
Therefore, 20 = [X*(1-0.35)] * (1-0.85)= X*0.65*0.15

$$\implies 20 \ \frac{mg}{l} = 0.0975X \implies X = \frac{20}{0.0975} = \boxed{205 \ \frac{mg}{l}}$$

8.5 Pumping

For Grades I & II, pumping rate problems include the following:

8.5.1 Calculating volume pumped given the pump flow rate

Method:

Step 1. Multiply the pump flow rate by the time interval

Make sure:

- The time units - in the given time interval and in the pump flow rate match

8.5.2 Calculating time to pump a certain volume

Method: Step 1. Calculate the total volume pumped

Step 2. Divide the total volume by the pump flow rate

Make sure:

- The volume units - in the volume that needs to be pumped and in the pump flow rate match
- The time unit in the pump flow rate needs to be converted to the time unit that you need the answer in

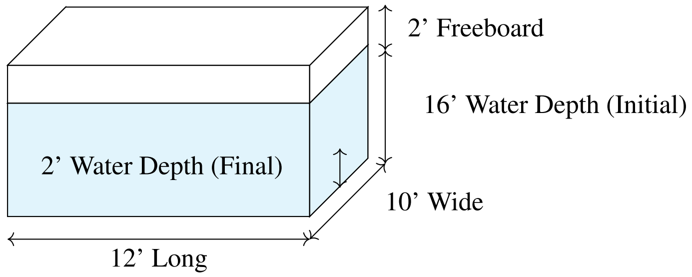
8.6 Example Problems

1. A sludge pump is set to pump 5 minutes each hour. It pumps at the rate of 35 gpm. How many gallons of sludge are pumped each day?
   Solution:
   $$\frac{35 \ gal \ sludge}{\cancel{min}} * \frac{5 \ \cancel{min}}{\cancel{hr}} * \frac{24 \ \cancel{hr}}{day} = \boxed{\frac{4,200 \ gallons}{day}}$$

2. A sludge pump operates 5 minutes each 15 minute interval. If the pump capacity is 60 gpm, how many gallons of sludge are pumped daily?
   $$\frac{60 \ gal \ sludge}{\cancel{min}} * \frac{5 \ \cancel{min}}{15 \ \cancel{min}} * 1440\frac{\cancel{min}}{day} = \boxed{\frac{28,800 \ gal \ sludge}{day}}$$

3. Given the tank is 10ft wide, 12 ft long and 18 ft deep tank including 2 ft of freeboard when filled to capacity. How much time (minutes) will be required to pump down this tank to a depth of 2 ft when the tank is at maximum capacity using a 600 GPM pump

Solution:



Volume to be pumped=$12\ ft * 10\ ft * (16-2)\ ft = 1,680 ft^3$

$$\implies \frac{1,680 \cancel{ft^3} * 7.48 \frac{\cancel{gal}}{\cancel{ft^3}}}{600 \frac{\cancel{gal}}{min}} = \boxed{21 min}$$