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TO: Distribution

FROM: R. D. Reasenberg

REFERENCE: 1) Memo dated 24 January 1974, Distribution/R. D. Reasenberg (74-1) Convergence Indicator in PEP

2) Memo dated 28 July 1975, Distribution/R. D. Reasenberg (75-2) The PEP A Priori Facility

SUBJECT: The RMS Predicted Residual Calculated in the Analyze Link of PEP

In the early part of 1973, R. D. White added the a priori facility to PEP. The memo describing that facility is dated 28 July 1975. A memo dated 24 January 1974, which describes a means of calculating the rms of the predicted postfit residuals without calculating those residuals, does not consider the effect of the a priori facility. These two features are now used with sufficient frequency that it would be useful to remove the conflict between them. This memorandum documents the equations required to remove that conflict.

We consider a least-squares adjustment, Δ, to a parameter vector, x. The a priori estimate, x_o, has a covariance B_o. Then from Equation 9 of 75-2

$$\Delta \equiv \hat{x} - \bar{x} = (B + B_o)^{-1}(u + v) \qquad (1)$$

where $B = A^{\dagger}R^{-1}A$ is the new information matrix, $u = A^{\dagger}R^{-1}r$ is the "right hand side" of the normal equations before the addition of the a priori information, R is the noise covariance, A is the sensitivity matrix (in PEP, the DERIV vector), and $\underline{r}$ is the prefit residual vector. In Equation 1, $v = B_o(x_o - \bar{x})$ is the contribution of the a priori parameter estimates to the "right hand side" of the normal equations, and $\bar{x}$ is the nominal (or starting) value of x for the current iteration. The predicted postfit residual is given by Equation 6 of 74-1:

$$r_p = r - A\Delta \qquad (2)$$

By combining Equations 1 and 2 we obtain

$$\chi_p^2 = r_p^{\dagger} R^{-1} r_p = \chi_o^2 - N^2 \tag{3}$$

$$N^2 = H + H^{\dagger} - I \tag{4}$$

where

$$\chi_o^2 = r^{\dagger} R^{-1} r \tag{5}$$

$$H = r^{\dagger} R^{-1} A (B + B_o)^{-1} (u + v) = (u + v)^{\dagger} \Delta - v^{\dagger} \Delta \tag{6}$$

$$I = (u + v)^{\dagger} (B + B_o)^{-\dagger} B (B + B_o)^{-1} (u + v)$$

$$= \Delta^{\dagger} (u + v) - \Delta^{\dagger} B_o \Delta \tag{7}$$

By combining terms, we find

$$N^2 = N_o^2 - 2v^{\dagger} \Delta + \Delta^{\dagger} B_o \Delta \tag{8}$$

where N_o was defined (as N) in Equation 5 of 74-1

$$N_o^2 = \Delta^{\dagger} u \tag{9}$$

Implementation

Equation 8 gives the two terms that must be added to the code. The first of these is zero when there are no *a priori* estimates. Both B_o and v are saved on the *a priori* data set, IBUF2. Keeping these in MAIN does not seem useful. B_o can be read and multiplied by Δ one row at a time to generate the vector $B_o\Delta$.

Distribution

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