Chandler

PLANETARY EPHEMERIS PROGRAM

Contract # NOOO14-89-C-2432

Final Report
For the period 29 September 1989 through 28 October 1990

Principal Investigator

John F. Chandler

November 1990

Prepared for
Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5000

Smithsonian Institution
Astrophysical Observatory
Cambridge, MA 02138

The Smithsonian Astrophysical Observatory
is a member of the
Harvard-Smithsonian Center for Astrophysics

The United States Naval Observatory Technical Officer for this grant is Dr. P. Kenneth Seidelmann, Nautical Almanac Office, 34th Street & Mass. Avenue, NW., Washington, DC 20390.

PLANETARY EPHEMERIS PROGRAM

Contract # NOOO14-89-C-2432

Final Report
For the period 29 September 1989 through 28 October 1990

Principal Investigator

John F. Chandler

BODFN

SBFN1

November 1990

Prepared for
Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5000

Smithsonian Institution
Astrophysical Observatory
Cambridge, MA 02138

The United States Naval Observatory Technical Officer for this grant is Dr. P. Kenneth Seidelmann, Nautical Almanac Office, 34th Street & Mass. Avenue, NW., Washington, DC 20390.

Harvard-Smithsonian Center for Astrophysics

60 Garden Street, Cambridge, MA 02138

(617) 495-7000
Telex # 921428

1990 September 26

Dr. P. Kenneth Seidelmann
United States Naval Observatory
Nautical Almanac Office
34th St. & Mass. Ave., NW.
Washington, DC 20390

Dear Ken,

This letter and its enclosure constitute the report on our activities under contract N00014-89-C-2432 (effective dates 1989 September 29 through 1990 October 28). The focus for this year has been the numerical integration code in PEP, and the enclosure includes documentation of both the logic flow and underlying physical models in the principal subroutines of the n-body and planet integrators. The models as documented are purely descriptive, *i.e.*, they embody what the PEP code actually does. Not surprisingly, we have verified that the documentation included in this report agrees with standard physics. However, a small surprise turned up in the preliminary comparison: a bug in a section of code that had not been used. Partly as an exercise for Mark Murison and partly as the first step in the code upgrade, this bug was fixed.

This report illustrates the basic properties of the eventual "comprehensive" PEP documentation that will result from our project. There are varying degrees of detail in the descriptions, ranging from thorough exposition of the physics to simple display of the subroutine calling structure. These correspond to the relative importance of the subroutines and to the amount of physics they contain.

Not yet available, but expected soon, is a report from Ken Nordtvedt describing his development of a more general framework for tests of general relativity. Bob Reasenberg spoke to Ken at the Fairbank meeting in Rome. Ken has apparently been making good progress, and Bob and I look forward to the completion of his work, which will be the basis for the most significant changes we expect to make.

For future work, we have several more pieces of the integration code to document (for example, the subroutines responsible for the post-Newtonian terms in the equations of motion), and the sections of code dealing with observations. Our goal is to compare the resulting documentation with Ken Nordtvedt's framework at post-Newtonian order and thereby reorganize PEP to include a more general model and, at the same time, to be more maintainable.

Yours truly,

John

John F. Chandler

enc.
JFC/sas

Contents

Preface

Preface

The following documentation of PEP, the Planetary Ephemeris Program, represents an in-progress snapshot of current work. We have decided that firm understanding of the *physics* embedded in PEP is the highest priority. Later we will concentrate on other goals, including incorporation of a new theoretical parameterized post-Newtonian framework being devised by K.H. Nordtvedt, documentation of the observables sections of PEP, and documentation aimed at the user of PEP.

The first routines subjected to detailed analysis are those involved in the integration of the equations of motion and of the partial derivatives. For each routine, the associated documentation will consist, as needed, of 1) a summary sheet, describing the routine at a glance; 2) a memorandum explaining in detail the physics in the routine; 3) a subroutine call tree; 4) a process structure chart; and 5) a detailed flowchart. An explanation of each of these follows.

The summary sheet for subroutine X contains the call syntax, followed by a brief description of the purpose of the routine. Then each calling argument is explained in enough detail to allow one to use X without having to look at the actual code. Next, under the heading "Calls:", is listed the subprogram dependency; X requires these subprograms in order to run properly. Following this is a list of all the PEP routines which call X. After that is a list of include files required by X. Finally, if X involves any of the global variables in the include files in a fortran EQUIVALENCE, that global variable is listed, as well as the common block it belongs to and the name of the associated include file.

The primary purpose of the memorandum is to document the equations used in the subroutine. This is especially important for the parts of PEP containing physics (as opposed to service routines, numerical analysis routines, *etc.*). For clarity, the order of the presentation does not necessarily follow the sequence of calculation in the subroutine. In general, derivations of the equations are not shown. In addition, not all of the routines require a memorandum.

As part of the overall PEP documentation, subroutine call tree charts are being created. If a routine happens to be the top node of a particular tree, the chart is included with the other documents describing the routine (this is in addition to the presence of the chart in the call tree documentation). In the call tree charts, each routine is contained in one of three box styles. These correspond to position in the tree of the routine: 1) internal node with further branches, 2) "leaf" with no further subprogram dependencies, and 3) internal node which is a top node and is displayed on another chart. In addition, within a "leaf" box style there is a separate notation for program entry points. These styles are illustrated at the top of the call tree chart for PEPMAIN. All PEP routines will be included somewhere in the charts. The top-level nodes that begin each chart were chosen such that each chart fits onto approximately one page.

Each routine that has a summary sheet will also have a process structure chart. This chart is easily recognized by the shaded box at the upper left corner, containing the name of the routine. The structure chart presents a hierarchical overview of the processes that make up the subroutine.

Following the structure chart is a detailed flowchart. The purpose of the flowchart is to document the logic of the routine, which is often complicated in a program as large as PEP. The flowcharts are hierarchical. The high-level charts contain symbols which indicate a further expansion on another (lower-level) chart. Chart expansion is indicated by a large circle containing the label of the expanded chart in capital letters, and often also including some indication of the purpose of the section of code represented by the expanded chart. For example, the flowchart for subroutine PLANET contains an expansion symbol labeled PLNT1, with the indication "N-body integration". The next flowchart is labeled PLNT1 and contains the N-body integration logical flow. For very complicated routines (SBFN1 is a good example), a chart is also included which details the flowchart hierarchy. This chart of flowcharts is a "directory" of the flowcharts for that routine.

PEPMAIN

program PEPMAIN

Description: This is the driver program for PEP.

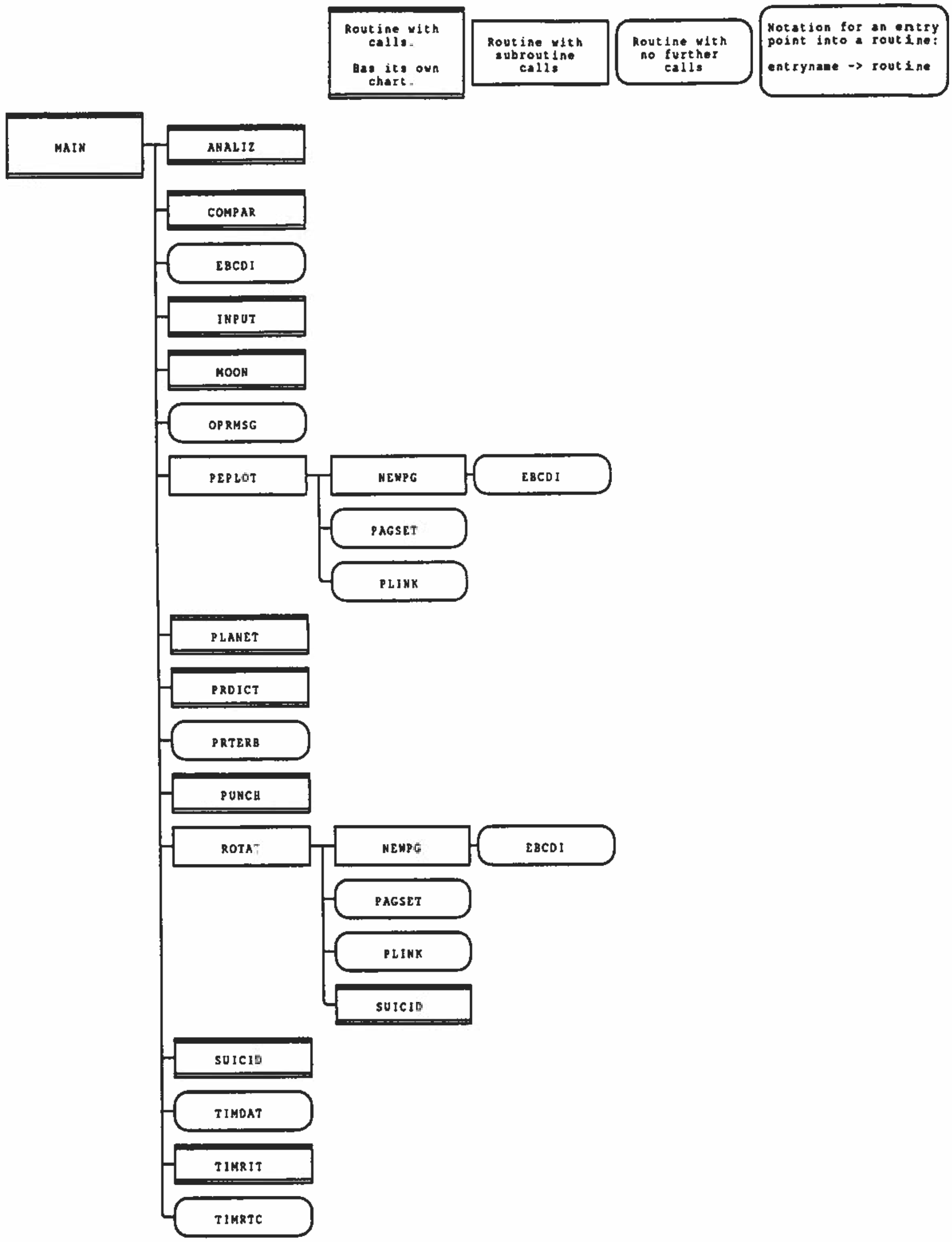
Arguments: none

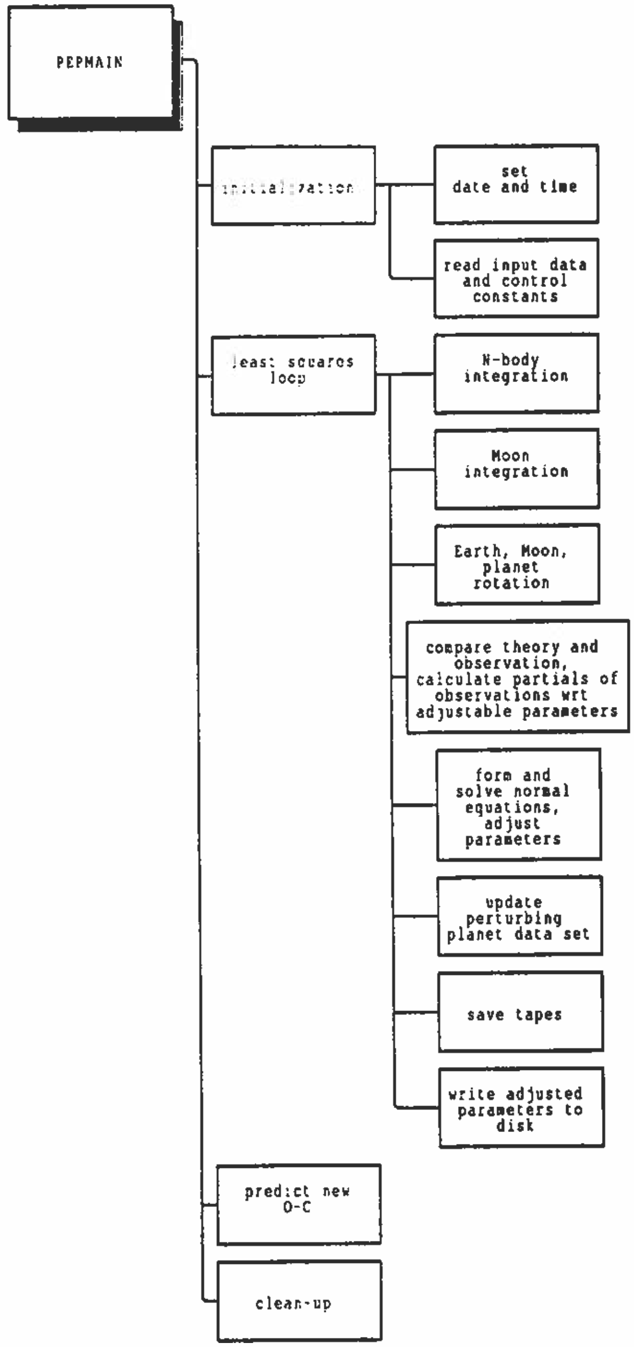
Calls: ANALIZ, COMPAR, EBCDI, INPUT, MOON, OPRMSG, PEPLOT, PLANET, PRDICT, PRTERB, PUNCH, ROTAT, SUICID, TIMDAT, TIMRIT, TIMRTC

Called by: none

Include Files: BDCTRL, CRDBUF, FCNTRL, INODTA, NAMTIM, TIMSTF

Equivalence: none


Routine with calls.
Has its own chart.
Routine with subroutine calls
Routine with no further calls
Notation for an entry point into a routine:
entryname -> routine
MAIN
ANALIZ
COMPAR
EBCDI
INPUT
MOON
OPRMSG
PEPLOT
NEWPG
EBCDI
PAGSET
PLINK
PLANET
PRDICT
PRTERB
PUNCH
ROTAT
NEWPG
EBCDI
PAGSET
PLINK
SUICID
SUICID
TIMDAT
TIMRIT
TIMRTC


PEPMAIN
initialization
set date and time
read input data and control constants
least squares loop
N-body integration
Moon integration
Earth, Moon, planet rotation
compare theory and observation, calculate partials of observations wrt adjustable parameters
form and solve normal equations, adjust parameters
update perturbing planet data set
save tapes
write adjusted parameters to disk
predict new O-C
clean-up

BODFN

real*8 function BODFN(k, j, s)

Description: BODFN is the function called to evaluate the right-hand sides of the N-body equations of motion. Includes asteroid perturbations, time-variable gravitational constant, 2nd harmonic of Sun's gravitational potential, GR motion factors, and ability to treat Earth-Moon system as a single point mass or as two separate bodies for perturbations.

Arguments:

k — Equation number. For a particular body i, the state vector is stored in a one-dimensional array y starting at y(m), where m = 6*(i-1)+1. Thus, for example, k = 15 references the equation for z, and k = 17 references the equation for dy/dt, of body i = 3.

j — Integrator iteration number (generally takes on values from 1 to 3, and is often used as a logic flag).

s — Current time = julian date + 0.5 day.

Calls: BDASTF, BDAST0, PRTCRD

Called by: ADAM, EVAL, NINT, RROAD

Include Files: BDCTRL, BODET, BODSTF, FUNCON, OUTPUT, PARAM, PRTCOD

Equivalence: none

Affected Variables:

link	/bodet/	BODET
mp	/bodet/	BODET

Restrictions: none

BODFN equations:

Upon entry into BODFN, the body currently being integrated will be denoted as body N. Other bodies will be denoted by subscript j, with the exceptions of Earth (subscript E), Moon (subscript M), Earth-Moon barycenter (subscript B), and Sun (subscript S).

Subscripts and their corresponding bodies are

N	Body currently being integrated
S	Sun
E	Earth
M	Moon
B	Earth-moon barycenter
j	bodies other than bodies N, E, M, B

This notation is in keeping with that of the accompanying flowchart.

1. Setup

In the initial setup section, first part, compute the velocity of light in astronomical units per day,

$$c_{AU} \equiv \frac{86400}{AU\ in\ light\ \sec}$$

Define the vector P:

$$\begin{aligned}
P_x &= \sin\Omega_\odot \sin i_\odot \\
P_y &= -\cos i_\odot \sin\varepsilon - \cos\Omega_\odot \sin i_\odot \cos\varepsilon \\
P_z &= \cos i_\odot \cos\varepsilon - \cos\Omega_\odot \sin i_\odot \sin\varepsilon
\end{aligned}$$

where

$\Omega_\odot$ Longitude of the ascending node of Sun's equator on the ecliptic, measured from the mean equinox of 1950.0.

$i_\odot$ Inclination of Sun's equator to the ecliptic.

ε Mean inclination of the ecliptic to the celestial equator of 1950.0 (*i.e.*, the obliquity).

The north pole of the Sun points in the direction of **P**. The x, y, and z components of **P** are the cosines of the angles between the Solar north pole and the axes of the usual right-handed orthogonal coordinate system whose X axis is in the direction of the equinox of 1950.0 and whose Z axis is in the direction of the north celestial pole.

2. Gravitational Constant

Calculate the gravitational constant according to

$$\gamma(t) = k_G^2 \left[1 + \alpha_G (t - t_0) \right]$$

where α_G is the time variation factor for the gravitational constant, called gmvary in BODFN; t is the current time, called s in BODFN; t_0 is the initial time, called ta in BODFN; and k_G is the Gaussian gravitational constant, called gauss in BODFN.

In subroutine BODSET (also in SETUP), the Gaussian constant is set to a value of

$$k_G = 0.01720209895\ (\mathrm{AU})^{3/2}(\text{ephemeris day})^{-1}(M_\odot)^{-1/2}$$

3. Position Vectors involving Earth and Moon

Position vectors are defined

$$\mathbf{R}_{ik} \equiv \mathbf{R}_k - \mathbf{R}_i$$

The Earth and Moon mass factors are

$$\mu_E \equiv \frac{M_E}{M_E + M_M} \qquad \mu_M \equiv \frac{M_M}{M_E + M_M}$$

Loop through all bodies j = 1,...nbody. The following relative position vectors may, depending on the logic flow, be calculated:

$$R_{SE} = R_{SB} - \ _{M}R_{EM}$$
$$R_{SM} = R_{SB} + \ _{E}R_{EM}$$

$$R_{Ej} = R_j - R_{SE}$$
$$R_{Mj} = R_j - R_{SM}$$

$$R_{Bj} = R_j - R_{SB}$$

$$R_{jE} = R_{SE} - R_j$$
$$R_{jM} = R_{SM} - R_j$$

$$R_{Ej} = -R_{jE}$$
$$R_{Mj} = -R_{jM}$$

$$R_{jB} = R_{SB} - R_j$$

$$R_{Bj} = -R_{jB}$$

4. Calculate body-body and Sun-body Distances

For each body j = 1,...nbody, excluding the Earth, calculate

Distance from Sun: R_j and R_j^3

and, for k = 1,...nbody, again excluding Earth, the position vectors

$$R_{jk} = R_k - R_j$$
$$R_{kj} = -R_{jk}$$

5. Acceleration due to Sun and Planets

Acceleration due to Sun:

$$q_\odot = -\gamma(t)\left(1 + \frac{M_N}{M_\odot}\right)\frac{\boldsymbol{R}_N}{R_N^3}$$

Add $\mathbf{q}_\odot$ to BODFN.

Acceleration with respect to Sun due to planets:

For each body j = 1,...nbody, excluding the Earth, sum the vector

$$\boldsymbol{\Omega}_j = M_j\left(\frac{\boldsymbol{R}_{Nj}}{R_{Nj}^3} - \frac{\boldsymbol{R}_j}{R_j^3}\right)$$

Then add $\gamma(t)\sum_j \boldsymbol{\Omega}_j$ to BODFN.

Acceleration due to Earth-Moon system:

If Earth and Moon are treated as a single point mass, add

$$q_{EM} = \gamma(t)\, M_{EM}\left(\frac{\boldsymbol{R}_{NB}}{R_{NB}^3} - \frac{\boldsymbol{R}_B}{R_B^3}\right)$$

to BODFN.

If Earth and Moon are treated as separate masses, add instead the quantity

$$q_{EM} = \gamma(t)\, M_{EM}\left[\mu_E\left(\frac{\boldsymbol{R}_{NE}}{R_{NE}^3} - \frac{\boldsymbol{R}_{SE}}{R_{SE}^3}\right) + \mu_M\left(\frac{\boldsymbol{R}_{NM}}{R_{NM}^3} - \frac{\boldsymbol{R}_{SM}}{R_{SM}^3}\right)\right]$$

to BODFN.

6. Accelerations on Earth and Moon

This section, corresponding to flowchart fragment BODFN4, is executed only if body N is Earth-Moon (npl=3) and Earth and Moon are treated as separate masses (kbdy(3)≥0).

Acceleration on Earth-Moon due to Sun:

If the gravitational constant does not vary in time, the acceleration vector to be added to **BODFN** is

$$\boldsymbol{q} = -\gamma(t)\left(1 + \frac{M_N}{M_\odot}\right)\left(\mu_E \frac{\boldsymbol{R}_{SE}}{R_{SE}^{\;3}} + \mu_M \frac{\boldsymbol{R}_{SM}}{R_{SM}^{\;3}}\right)$$

where $M_N = M_E + M_M$.

Acceleration on Earth-Moon due to planets:

For each body j = 1,...nbody (excluding j=3), add to **BODFN** the acceleration vector

$$\boldsymbol{\Omega}_j = -\gamma(t)\, M_j \left(\mu_E \frac{\boldsymbol{R}_{Ej}}{R_{Ej}^{\;3}} + \mu_M \frac{\boldsymbol{R}_{Mj}}{R_{Mj}^{\;3}} - \frac{\boldsymbol{R}_j}{R_j^3}\right)$$

7. "Extra" Forces

The "extra" forces currently consist of the effects due to general relativity and the Sun's gravitational second harmonic.

If the GR motion factors are to be included in the equations of motion (kbdy(21)≥0), define the quantities

$$\alpha = \boldsymbol{V}_N \cdot \boldsymbol{V}_N \quad \text{and} \quad \beta = \boldsymbol{R}_N \cdot \boldsymbol{V}_N$$

Also, let ζ_N denote the general relativity motion factor for body N (called relftb in BODFN). Add to **BODFN** the vector

$$q_{GR} = \zeta_N \frac{\gamma(t)}{c_{AU}^2 R_N^3} \left[\boldsymbol{R}_N \left(\frac{4\gamma(t)}{R_N^3} - \alpha \right) + 4 \boldsymbol{V}_N \beta \right]$$

Let J_2 be the second harmonic of the gravitational potential of the Sun. If this is to be included in the equations of motion (kbdy(23)≥0), then add to **BODFN** the vector

$$\boldsymbol{h}_{N\odot} = p_N \left(\frac{\boldsymbol{R}_N}{R_N} (5g_N^2 - 1) - 2g_N \boldsymbol{P} \right)$$

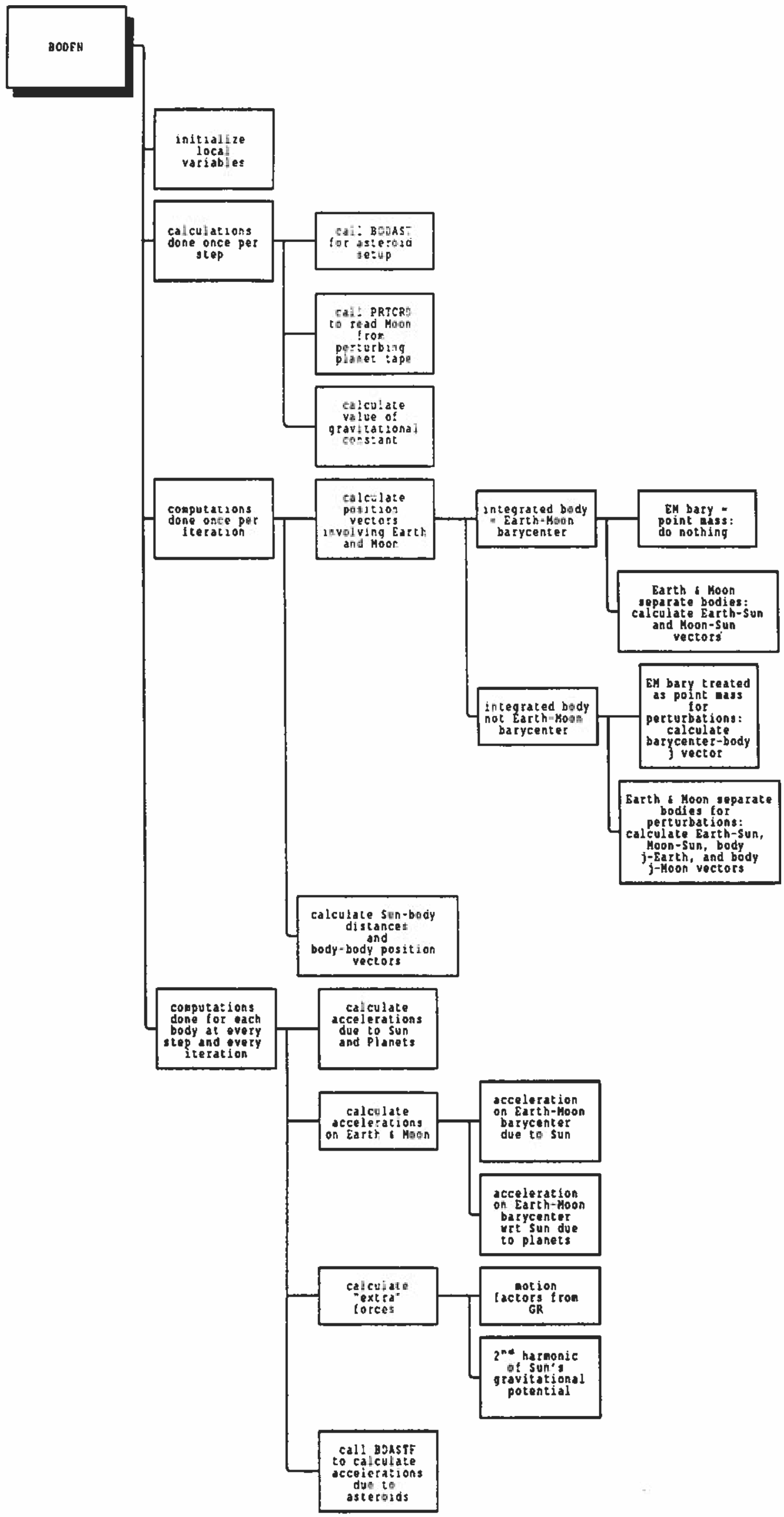
where

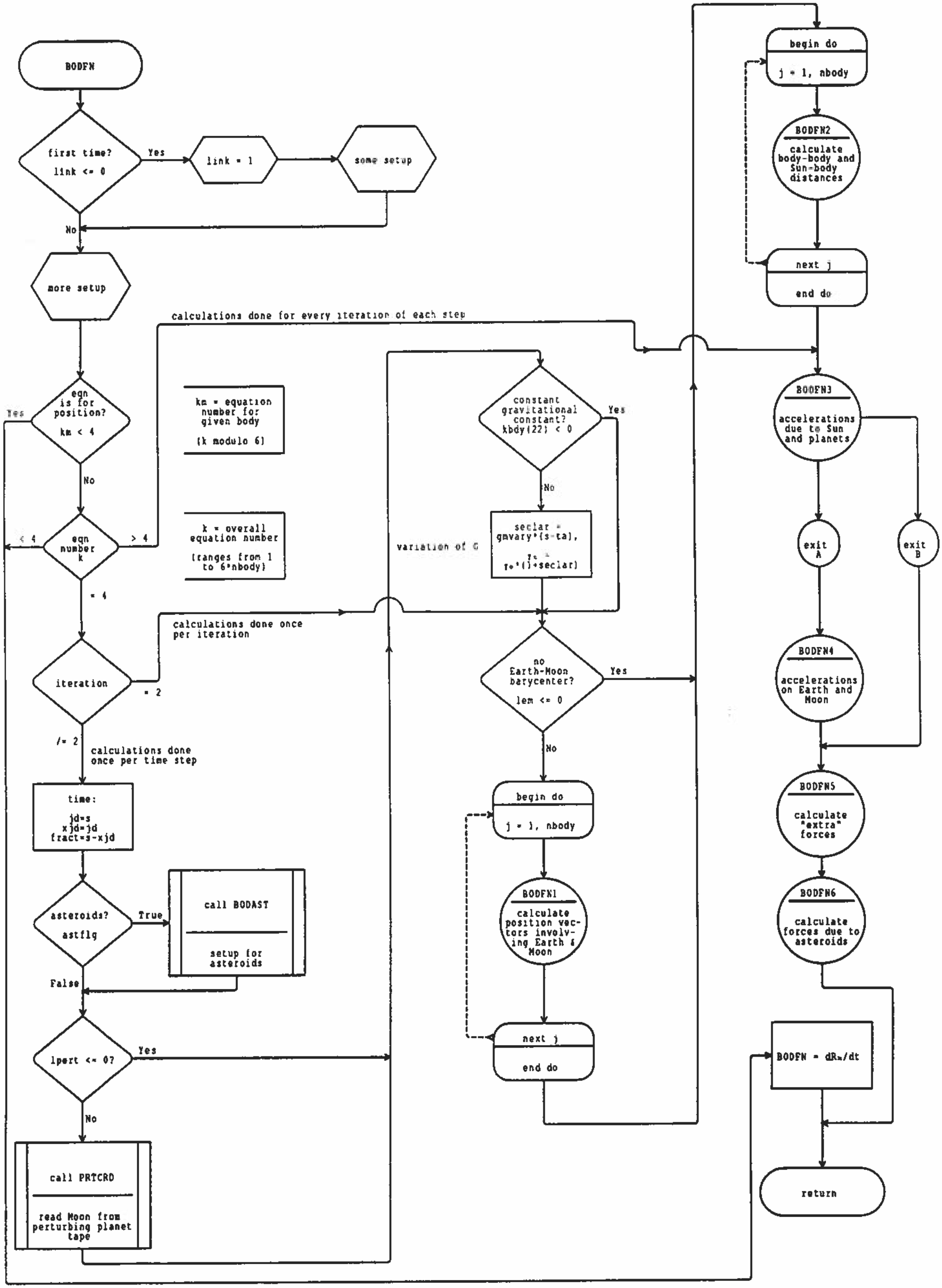
$$g_N \equiv \boldsymbol{P} \cdot \boldsymbol{R}_N$$

$$p_N \equiv 1.5 \left(1 + \frac{M_N}{M_\odot} \right) R_\odot^2 J_2$$

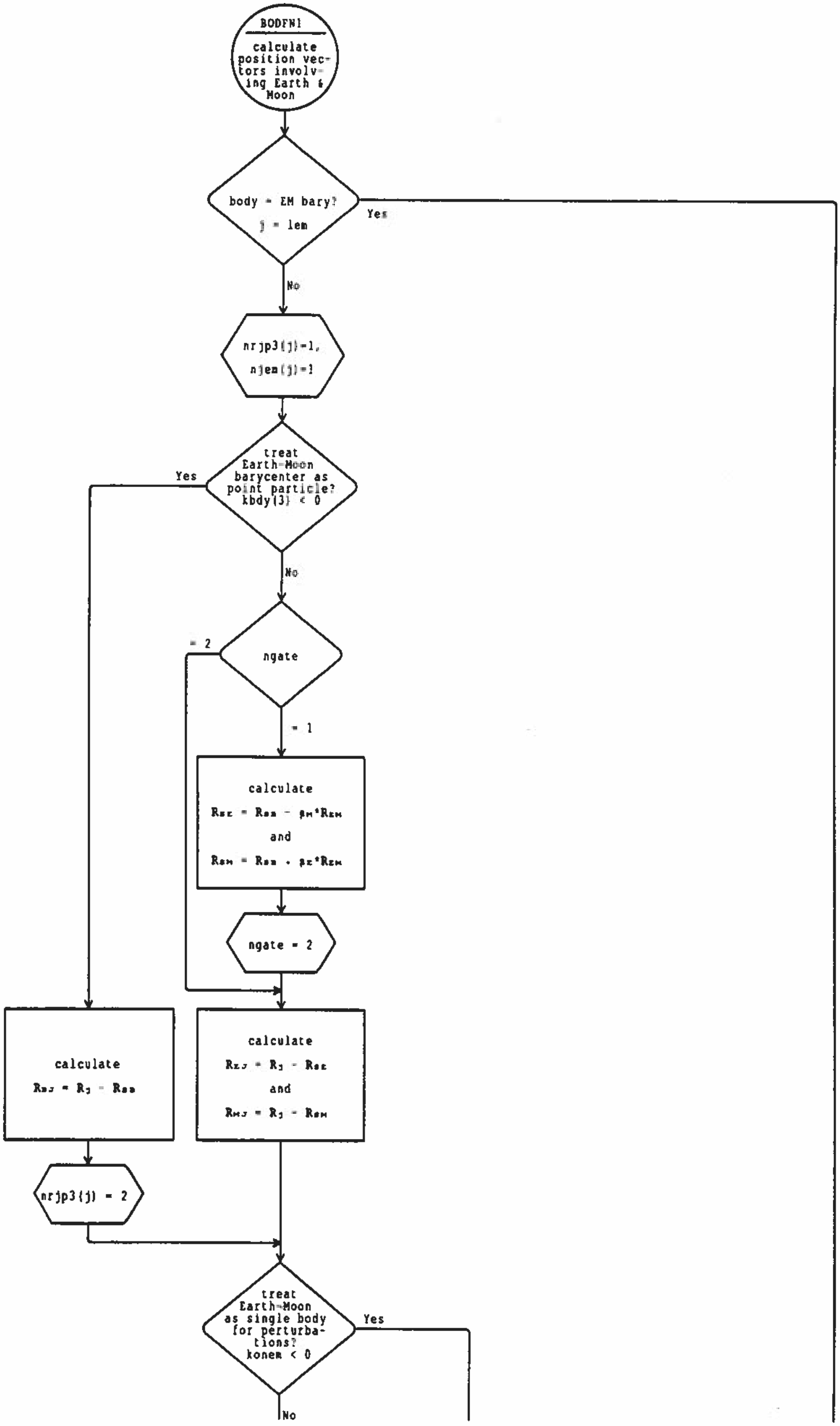
8. Acceleration<u>s</u> due to Asteroids

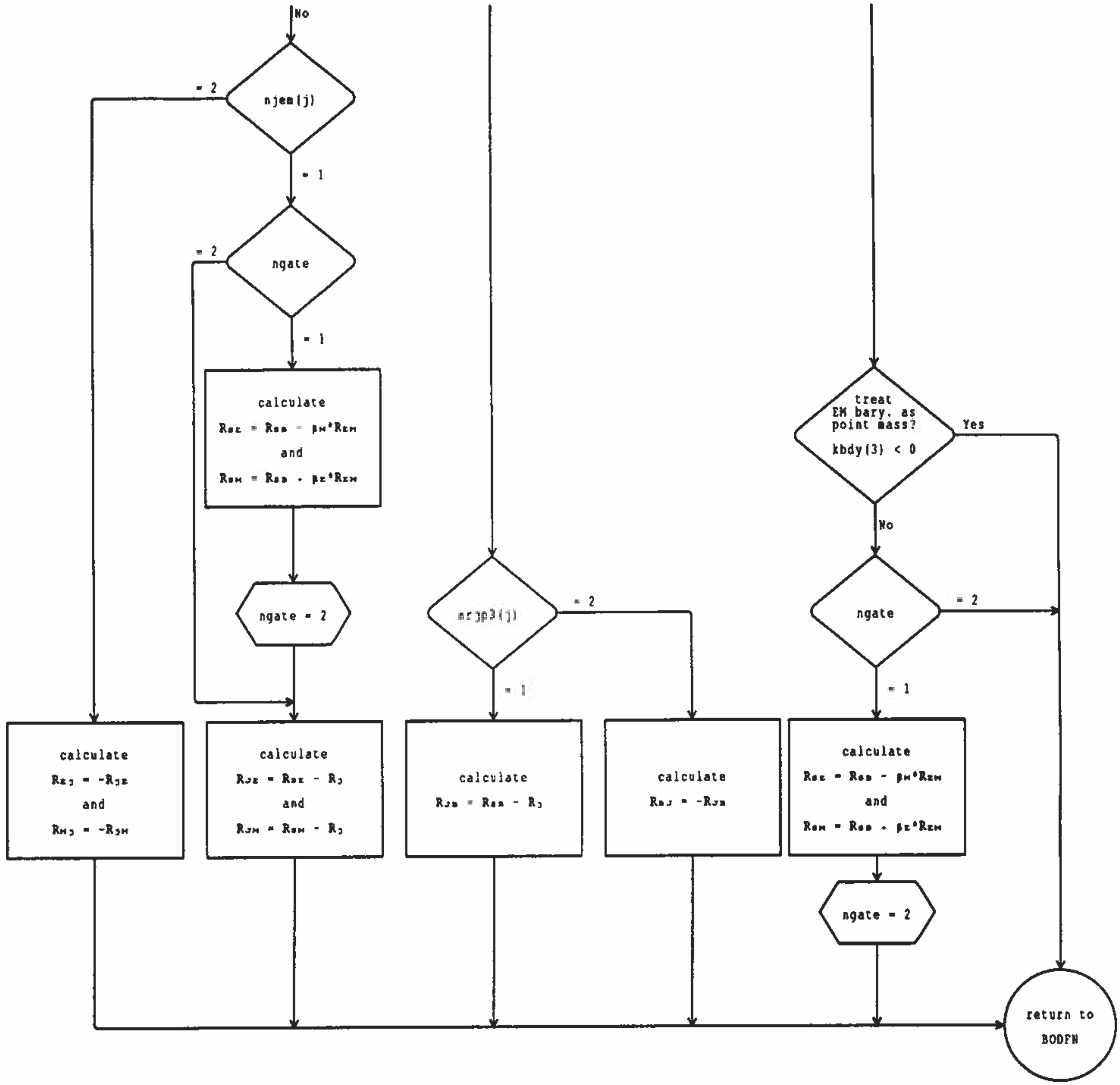
Finally, if asteroids are to be included (astflg=true), the BDASTF routine is called, returning the components of acceleration due to the asteroids. Call this acceleration $\mathbf{q}_{ast}$. Then $\gamma(t)\mathbf{q}_{ast}$ is added to **BODFN**.

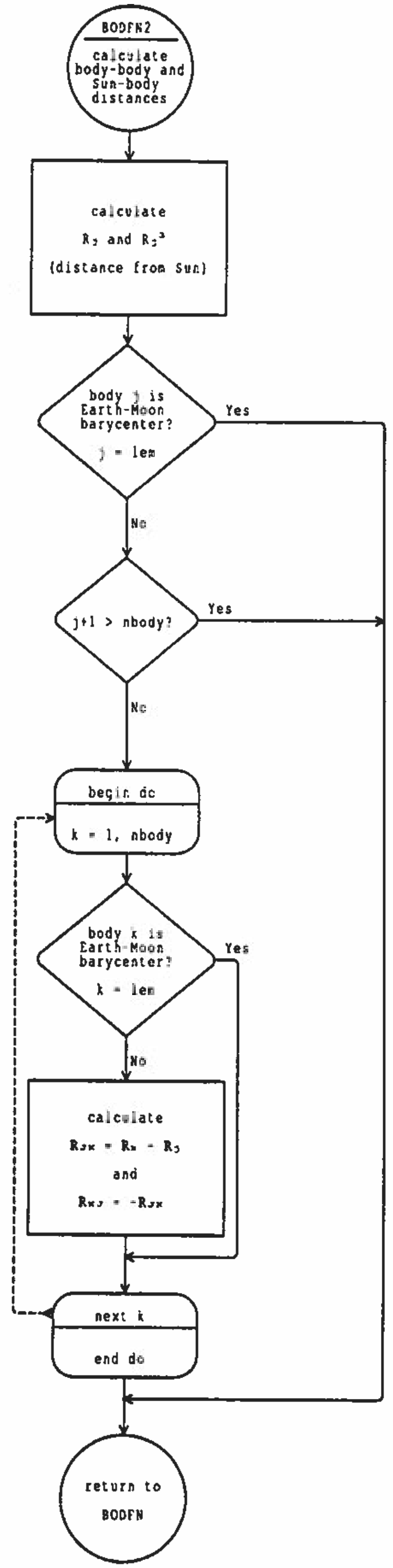

BODFN
initialize local variables
calculations done once per step
call BODAST for asteroid setup
call PRTCRD to read Moon from perturbing planet tape
calculate value of gravitational constant
computations done once per iteration
calculate position vectors involving Earth and Moon
integrated body = Earth-Moon barycenter
EM bary = point mass: do nothing
Earth & Moon separate bodies: calculate Earth-Sun and Moon-Sun vectors
integrated body not Earth-Moon barycenter
EM bary treated as point mass for perturbations: calculate barycenter-body j vector
Earth & Moon separate bodies for perturbations: calculate Earth-Sun, Moon-Sun, body j-Earth, and body j-Moon vectors
calculate Sun-body distances and body-body position vectors
computations done for each body at every step and every iteration
calculate accelerations due to Sun and Planets
calculate accelerations on Earth & Moon
acceleration on Earth-Moon barycenter due to Sun
acceleration on Earth-Moon barycenter wrt Sun due to planets
calculate "extra" forces
motion factors from GR
2nd harmonic of Sun's gravitational potential
call BDASTF to calculate accelerations due to asteroids

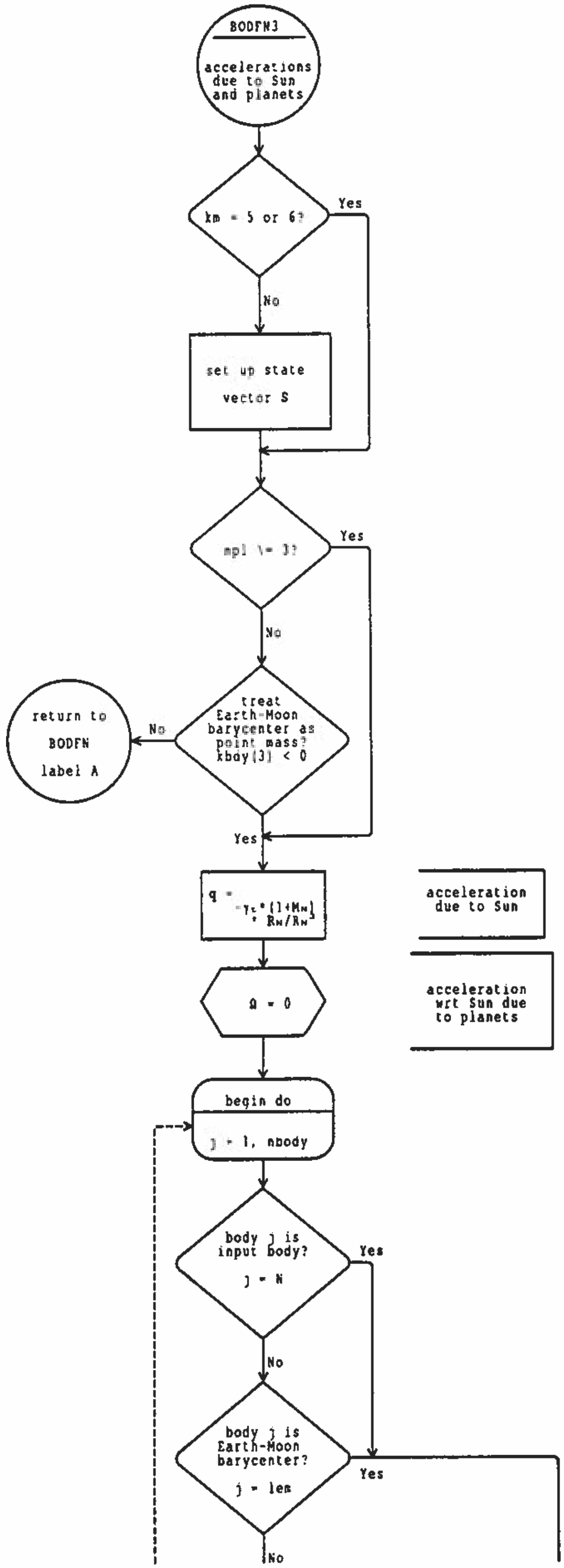


BODFN
first time?
link <= 0
Yes
link = 1
some setup
No
more setup
calculations done for every iteration of each step
eqn
is for
position?
km < 4
Yes
No
km = equation
number for
given body
(k modulo 6)
eqn
number
k
< 4
> 4
= 4
k = overall
equation number
(ranges from 1
to 6*nbody)
iteration
= 2
/= 2
calculations done once
per iteration
calculations done
once per time step
time:
jd=s
xjd=jd
fract=s-xjd
asteroids?
astflg
True
call BODAST
setup for
asteroids
False
lpert <= 0?
Yes
No
call PRTCRD
read Moon from
perturbing planet
tape
constant
gravitational
constant?
kbdy(22) < 0
Yes
No
variation of G
seclar =
gmvary*(s-ta),
seclar)
no
Earth-Moon
barycenter?
lem <= 0
Yes
No
begin do
j = 1, nbody
BODFN1
calculate
position vec-
tors involv-
ing Earth &
Moon
next j
end do
begin do
j = 1, nbody
BODFN2
calculate
body-body and
Sun-body
distances
next j
end do
BODFN3
accelerations
due to Sun
and planets
exit
A
exit
B
BODFN4
accelerations
on Earth and
Moon
BODFN5
calculate
"extra"
forces
BODFN6
calculate
forces due to
asteroids
BODFN = dRₖ/dt
return

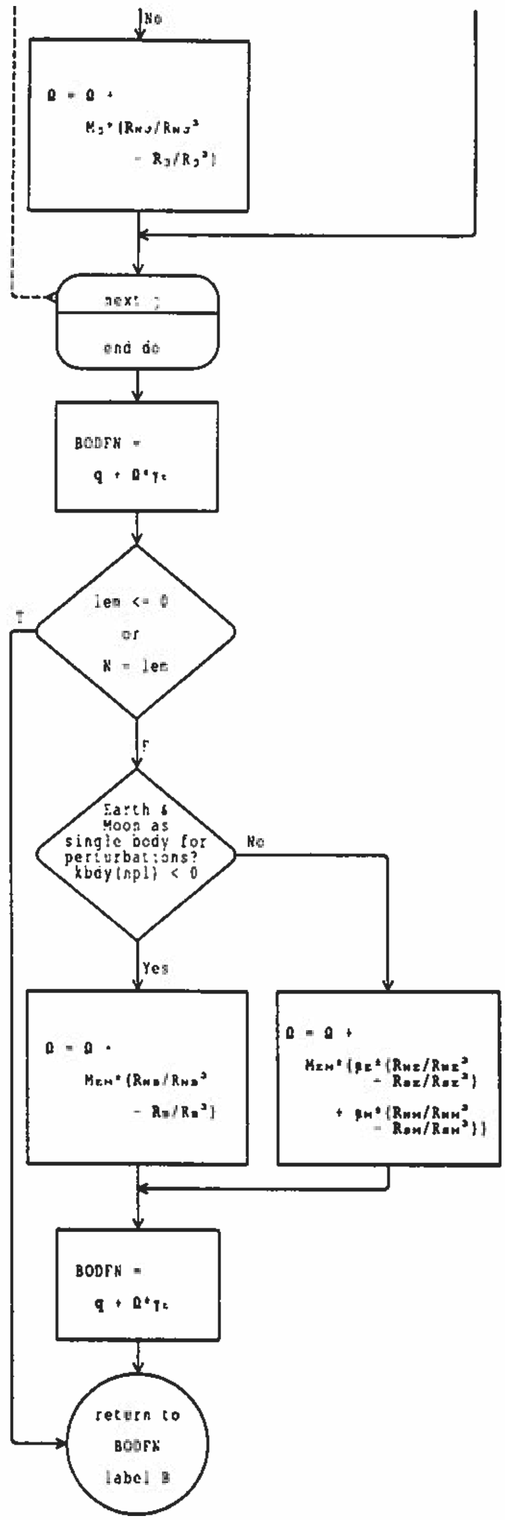

BODFN1
calculate position vectors involving Earth & Moon
body = EM bary?
j = lem
Yes
No
nrjp3(j)=1,
njem(j)=1
treat Earth-Moon barycenter as point particle?
kbdy(3) < 0
Yes
No
ngate
= 2
= 1
calculate
$R_{SE} = R_{SB} - \mu_M * R_{EM}$
and
$R_{SM} = R_{SB} + \mu_E * R_{EM}$
ngate = 2
calculate
$R_{BJ} = R_J - R_{SB}$
calculate
$R_{EJ} = R_J - R_{SE}$
and
$R_{MJ} = R_J - R_{SM}$
nrjp3(j) = 2
treat Earth-Moon as single body for perturbations?
konem < 0
Yes
No


No
njem(j)
= 2
= 1
ngate
= 2
= 1
calculate
$R_{SE} = R_{SB} - \mu_M * R_{EM}$
and
$R_{SM} = R_{SB} + \mu_E * R_{EM}$
ngate = 2
nrjp3(j)
= 2
= 1
treat
EM bary. as
point mass?
kbdy(3) < 0
Yes
No
ngate
= 2
= 1
calculate
$R_{EJ} = -R_{JE}$
and
$R_{MJ} = -R_{JM}$
calculate
$R_{JE} = R_{SE} - R_J$
and
$R_{JM} = R_{SM} - R_J$
calculate
$R_{JB} = R_{SB} - R_J$
calculate
$R_{BJ} = -R_{JB}$
calculate
$R_{SE} = R_{SB} - \mu_M * R_{EM}$
and
$R_{SM} = R_{SB} + \mu_E * R_{EM}$
ngate = 2
return to
BODFN


BODFN2
calculate body-body and Sun-body distances
calculate
R_j and R_j^3
(distance from Sun)
body j is Earth-Moon barycenter?
j = lem
Yes
No
j+1 > nbody?
Yes
No
begin do
k = 1, nbody
body k is Earth-Moon barycenter?
k = lem
Yes
No
calculate
$R_{jk} = R_k - R_j$
and
$R_{kj} = -R_{jk}$
next k
end do
return to
BODFN



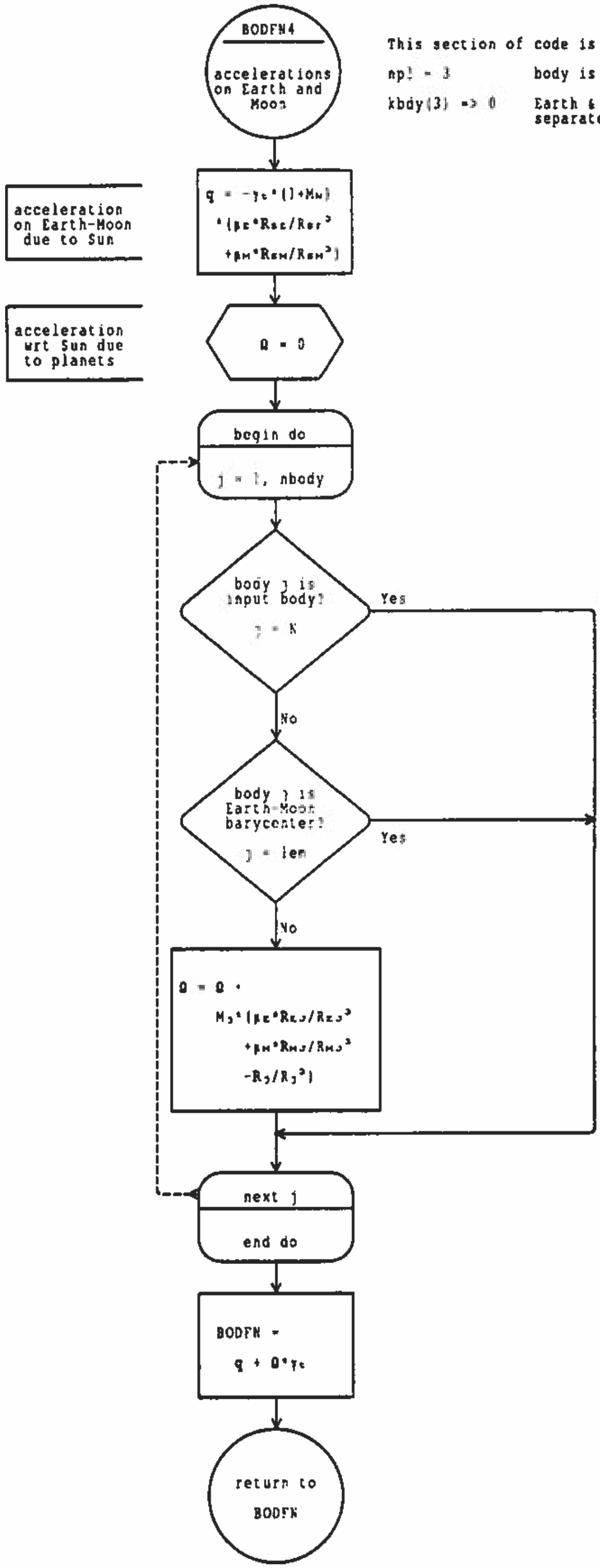
BODFN3
accelerations due to Sun and planets
km = 5 or 6?
Yes
No
set up state vector S
mpl \= 3?
Yes
No
treat Earth-Moon barycenter as point mass? kbdy(3) < 0
No
return to BODFN label A
Yes
q = -γE*(1+MN) * RN/RN³
acceleration due to Sun
Ω = 0
acceleration wrt Sun due to planets
begin do
j = 1, nbody
body j is input body? j = N
Yes
No
body j is Earth-Moon barycenter? j = lem
Yes
No

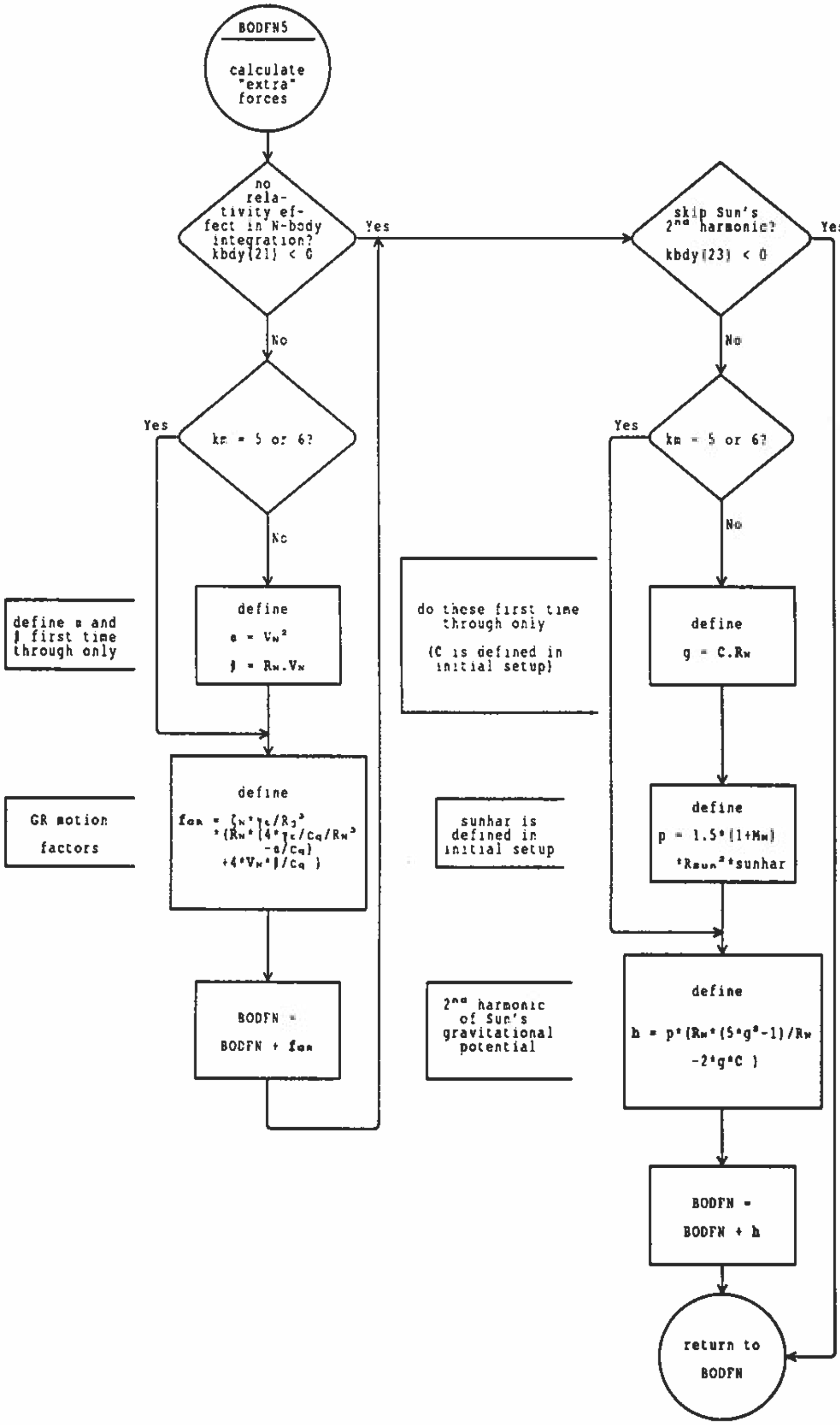


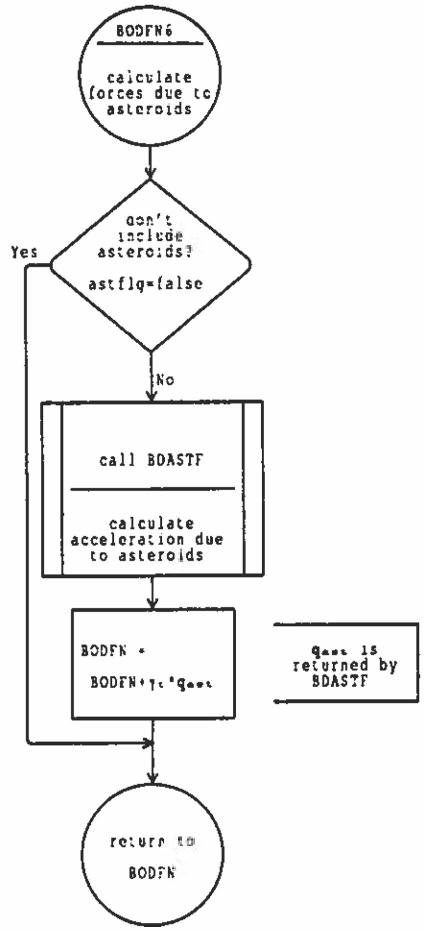
No
Ω = Ω + M_J*(R_NJ/R_NJ^3 - R_J/R_J^3)
next j
end do
BODFN = q + Ω*γ_c
lem <= 0 or N = lem
T
F
Earth & Moon as single body for perturbations? kbdy(npl) < 0
No
Yes
Ω = Ω + M_EM*(R_NB/R_NB^3 - R_B/R_B^3)
Ω = Ω + M_EM*(μ_E*(R_NE/R_NE^3 - R_BE/R_BE^3) + μ_M*(R_NM/R_NM^3 - R_BM/R_BM^3))
BODFN = q + Ω*γ_c
return to BODFN label B

This section of code is executed only if

npl = 3	body is Earth-Moon
kbdy(3) => 0	Earth & Moon are treated as separate masses




BODFN5
calculate "extra" forces
no relativity effect in N-body integration? kbdy(21) < 0
Yes
No
km = 5 or 6?
Yes
No
define
define α and β first time through only
GR motion factors
define
BODFN = BODFN + f
skip Sun's 2nd harmonic? kbdy(23) < 0
Yes
No
km = 5 or 6?
Yes
No
do these first time through only
(C is defined in initial setup)
define
g = C.R
define
sunhar is defined in initial setup
2nd harmonic of Sun's gravitational potential
define
BODFN = BODFN + h
return to BODFN

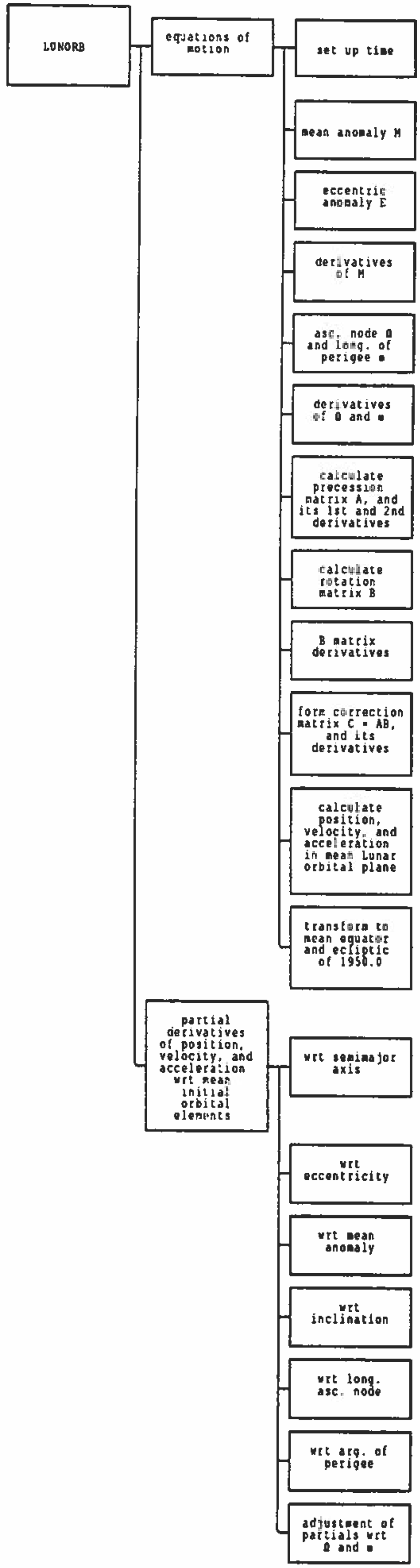

BODFN6
calculate forces due to asteroids
don't include asteroids?
astflg=false
Yes
No
call BDASTF
calculate acceleration due to asteroids
BODFN ←
BODFN+γ_c*q_{ast}
q_{ast} is returned by BDASTF
return to
BODFN

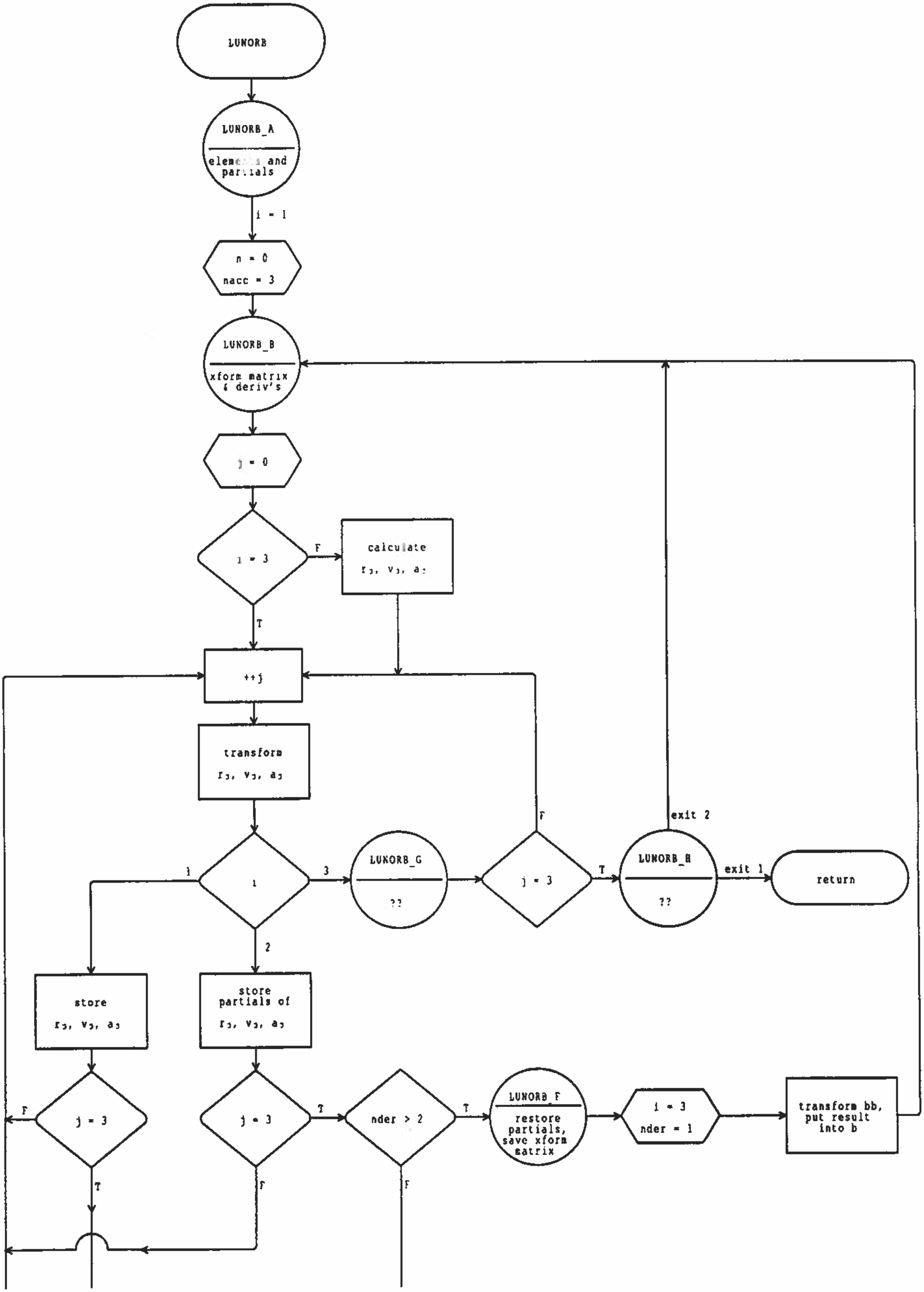
LUNORB

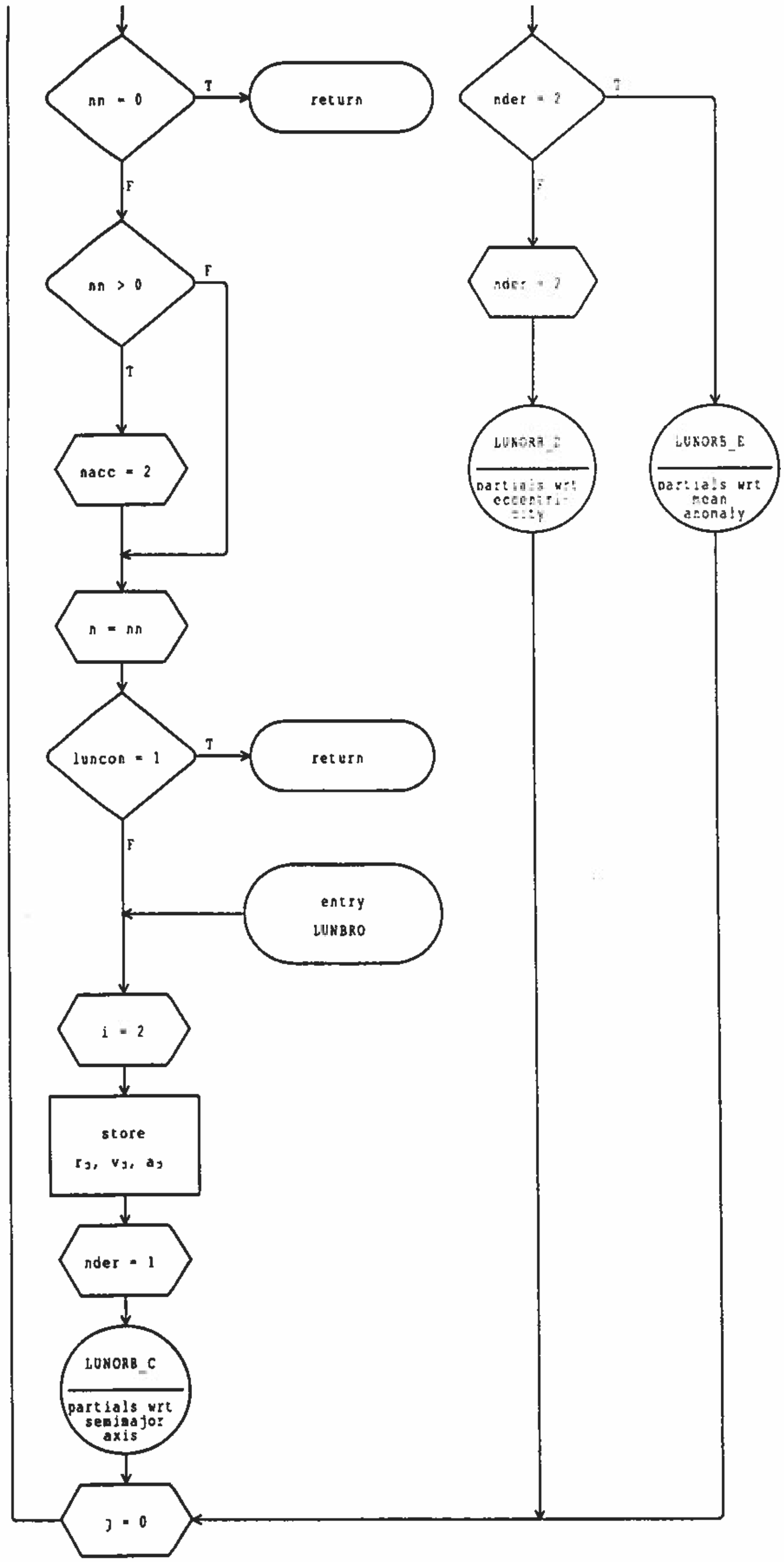
subroutine **LUNORB (jd, fract, nn)**

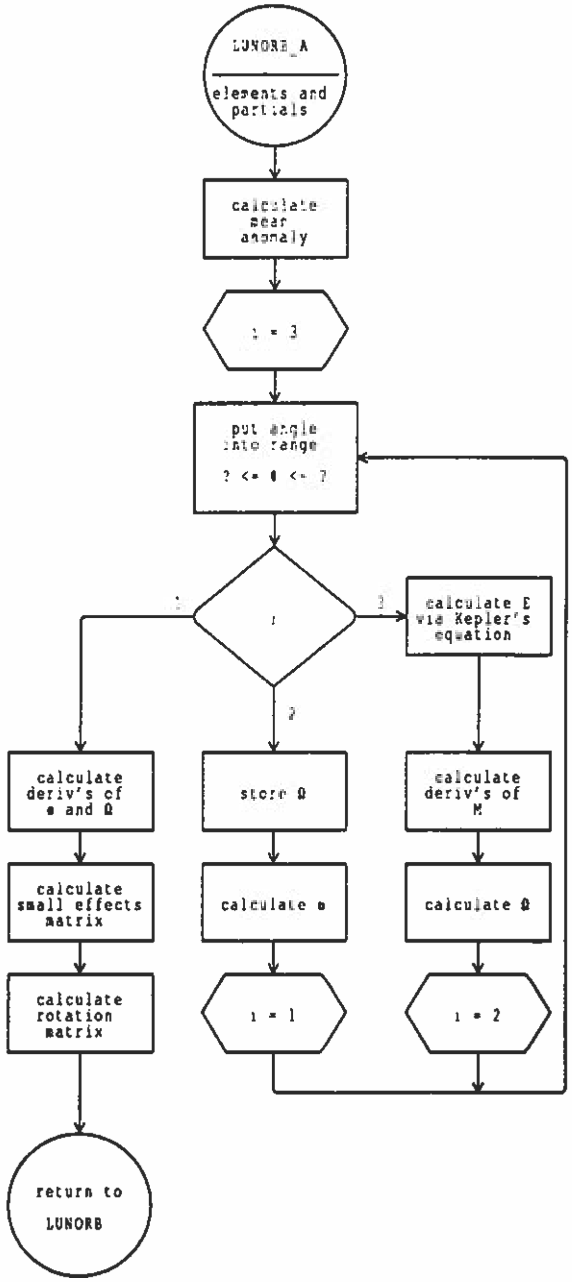
Description:	LUNORB calculates the position, velocity, and acceleration of the mean lunar orbit, using Brown's theory. LUNORB also calculates the partial derivatives of these quantities with respect to the initial mean orbital elements.	
Arguments:	jd	Julian Day Number
	fract	fraction of day from midnight
	nn	control flag:
		0 calculate position, velocity, acceleration
		+1 also calculate derivatives of position, velocity, acceleration
		-1 also calculate derivatives of position and velocity

Calls:	none		
Called by:	LUNSET, MORFN, MOROUT, MORSET, PRTCRD, SBFN		
Entry Points:	LUNORB, LUNBRO		
Includes:	AACOFF, FUNCON, ORBLUN, ORBSTF, PRECMN		
Equivalence:	ww(7)	/orbstf/	ORBSTF

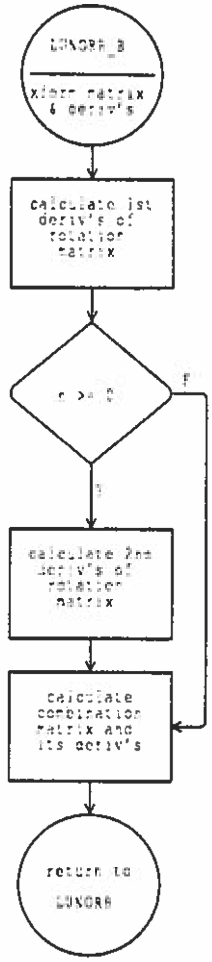

LUNORB
equations of motion
set up time
mean anomaly M
eccentric anomaly E
derivatives of M
asc. node Ω and long. of perigee ω
derivatives of Ω and ω
calculate precession matrix A, and its 1st and 2nd derivatives
calculate rotation matrix B
B matrix derivatives
form correction matrix C = AB, and its derivatives
calculate position, velocity, and acceleration in mean Lunar orbital plane
transform to mean equator and ecliptic of 1950.0
partial derivatives of position, velocity, and acceleration wrt mean initial orbital elements
wrt semimajor axis
wrt eccentricity
wrt mean anomaly
wrt inclination
wrt long. asc. node
wrt arg. of perigee
adjustment of partials wrt Ω and ω

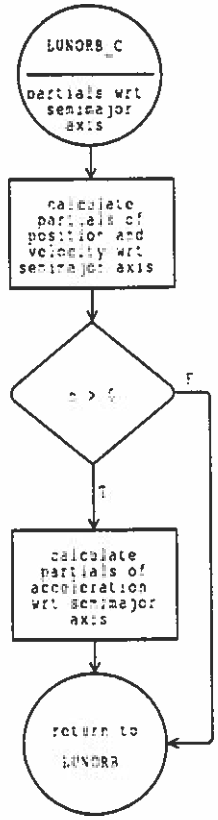

LUNORB
LUNORB_A
elements and partials
i = 1
n = 0
nacc = 3
LUNORB_B
xform matrix & deriv's
j = 0
i = 3
F
calculate
rj, vj, aj
T
++j
transform
rj, vj, aj
F
exit 2
1
i
3
LUNORB_G
??
j = 3
T
LUNORB_H
??
exit 1
return
2
store
rj, vj, aj
store partials of
rj, vj, aj
F
j = 3
j = 3
T
nder > 2
T
LUNORB_F
restore partials, save xform matrix
i = 3
nder = 1
transform bb, put result into b
T
F
F

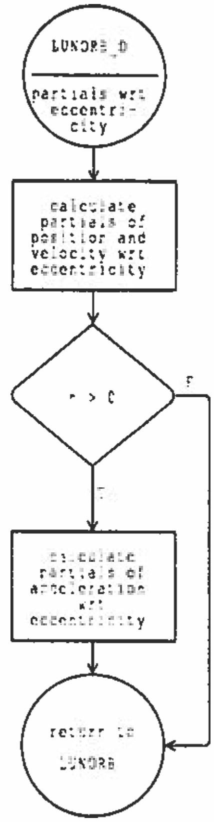

nn = 0
T
return
F
nn > 0
F
T
nacc = 2
n = nn
luncon = 1
T
return
F
entry
LUNBRO
i = 2
store
r3, v3, a3
nder = 1
LUNORB_C
partials wrt
semimajor
axis
j = 0
nder = 2
partials wrt
LUNORB_E
partials wrt
mean
anomaly



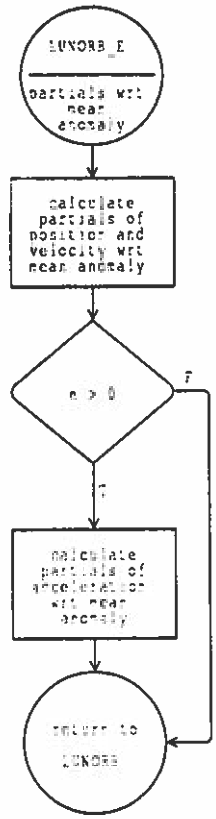
LUNORB_A
elements and partials
calculate mean anomaly
i = 3
put angle into range
i
1
2
3
calculate E via Kepler's equation
calculate deriv's of ω and Ω
calculate small effects matrix
calculate rotation matrix
return to LUNORB
store Ω
calculate ω
i = 1
calculate deriv's of M
calculate Ω
i = 2

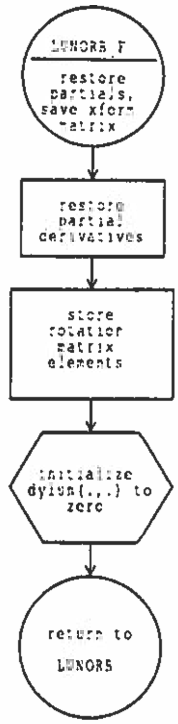

xform matrix
& deriv's
calculate 1st deriv's of rotation matrix
F
calculate 2nd deriv's of rotation matrix
calculate combination matrix and its deriv's
return to
LUNORB

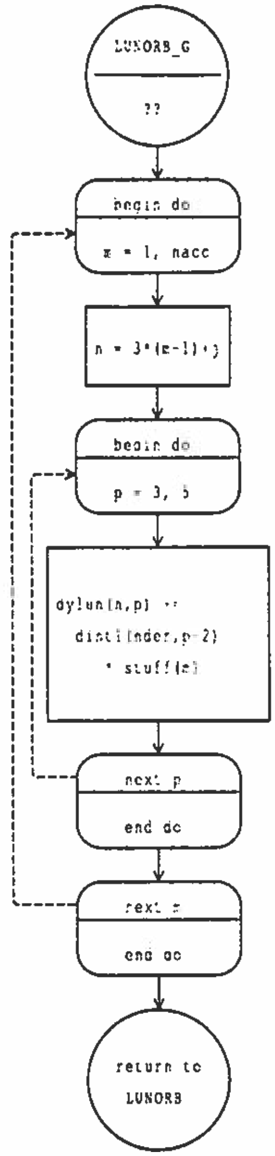

LUNORB_C
partials wrt semimajor axis
calculate partials of position and velocity wrt semimajor axis
F
T
calculate partials of acceleration wrt semimajor axis
return to LUNORB

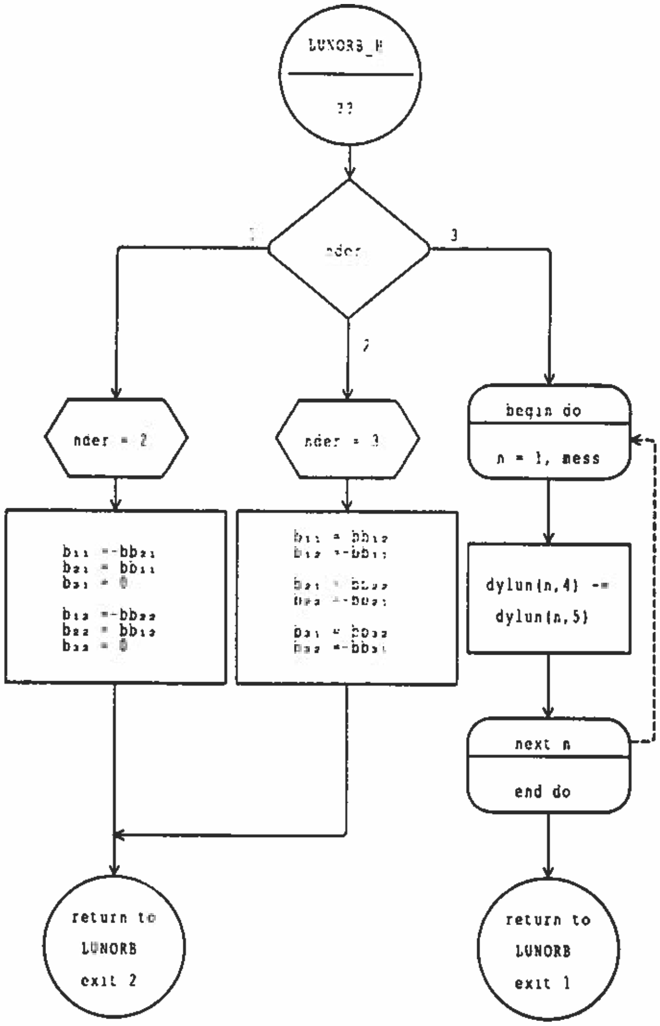


LUNORB_D
partials wrt eccentricity
calculate partials of position and velocity wrt eccentricity
r > 0
F
T
calculate partials of acceleration wrt eccentricity
return to LUNORB


LUNORB_E
partials wrt mean anomaly
calculate partials of position and velocity wrt mean anomaly
e > 0
F
T
calculate partials of acceleration wrt mean anomaly
return to LUNORB


LUNORB_F
restore partials, save xform matrix
restore partial derivatives
store rotation matrix elements
initialize dyisn(.,.) to zero
return to LUNORB


LUNORB_G
??
begin do
m = 1, nacc
begin do
p = 3, 5
next p
end do
next m
end do
return to
LUNORB

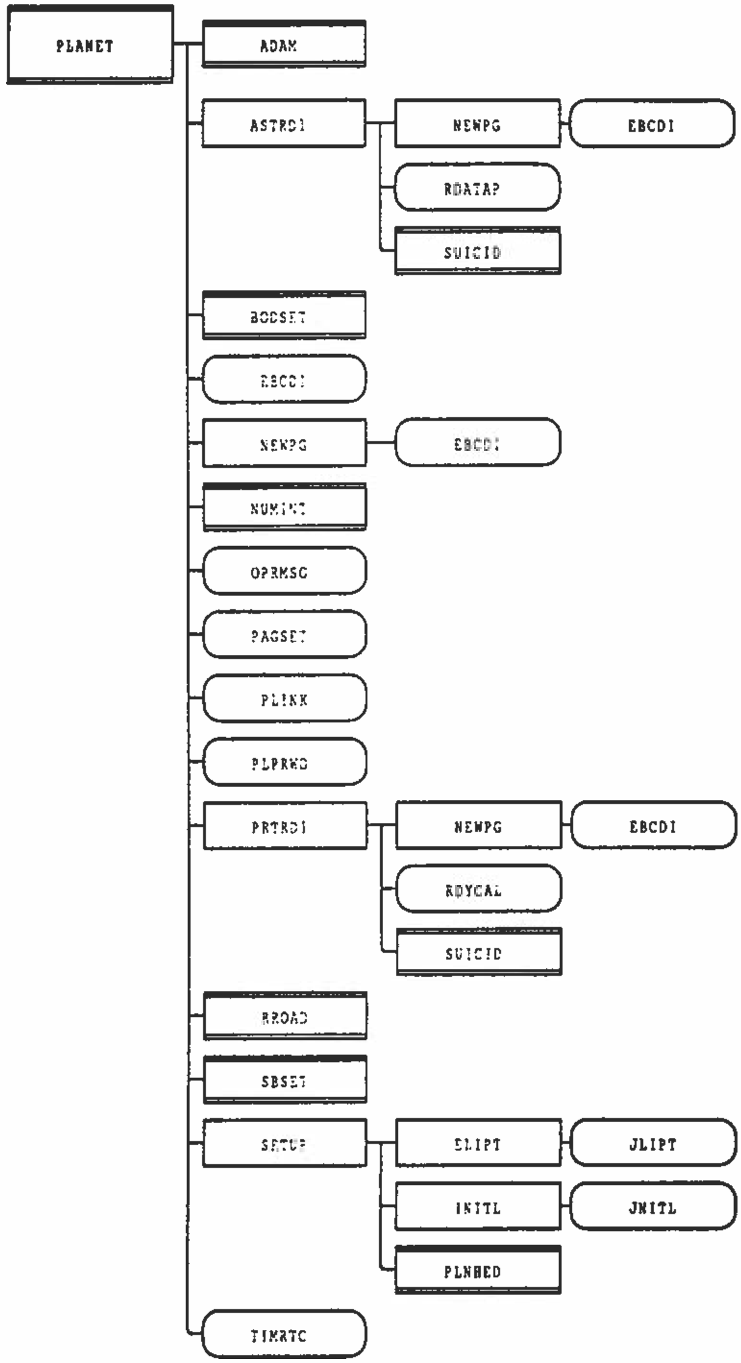


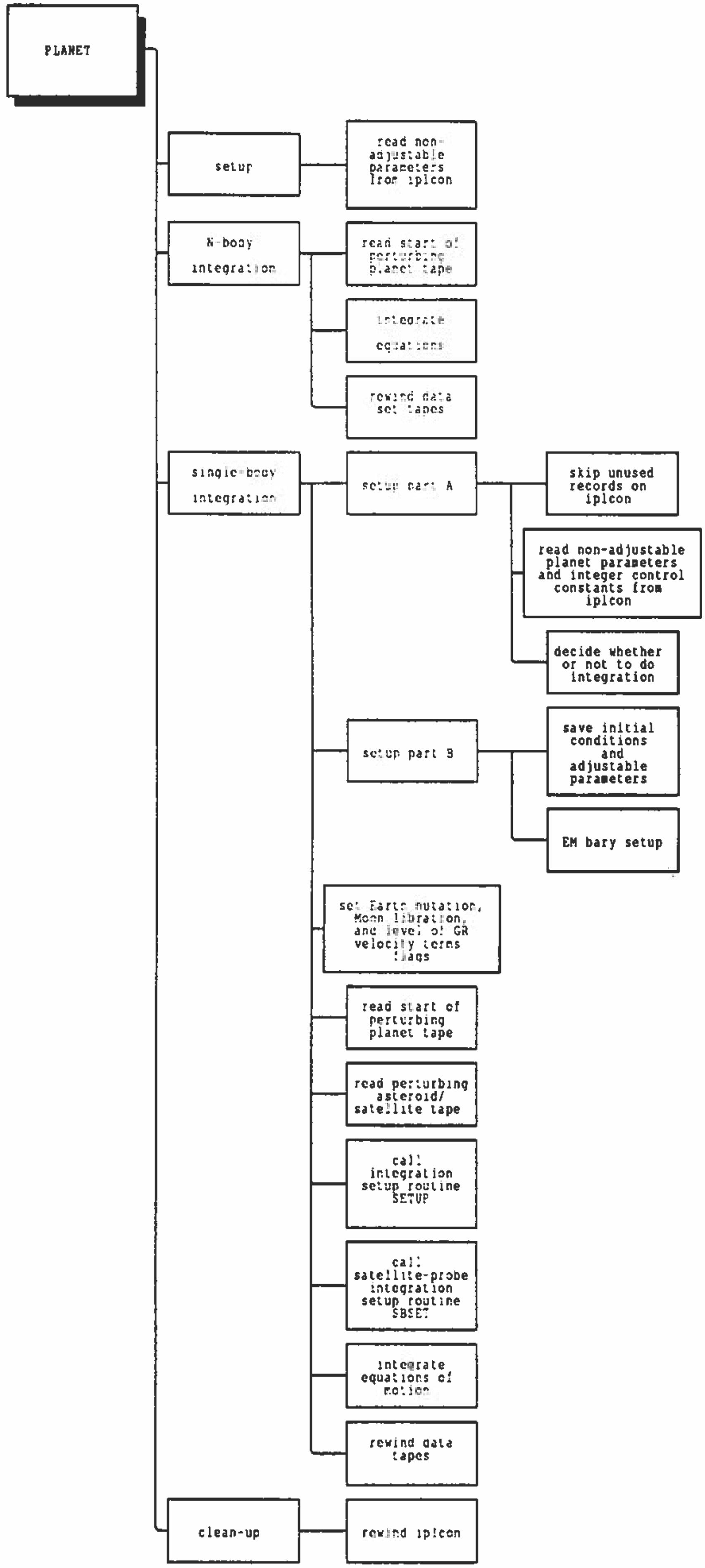
LUNORB_H
nder
3
nder = 2
nder = 3
$b_{11} = -bb_{21}$
$b_{21} = bb_{11}$
$b_{31} = 0$
$b_{12} = -bb_{22}$
$b_{22} = bb_{12}$
$b_{32} = 0$
$b_{11} = bb_{12}$
$b_{12} = -bb_{11}$
$b_{21} = bb_{22}$
$b_{22} = -bb_{21}$
$b_{31} = bb_{32}$
$b_{32} = -bb_{31}$
begin do
n = 1, mess
dylun(n,4) -= dylun(n,5)
next n
end do
return to LUNORB exit 2
return to LUNORB exit 1

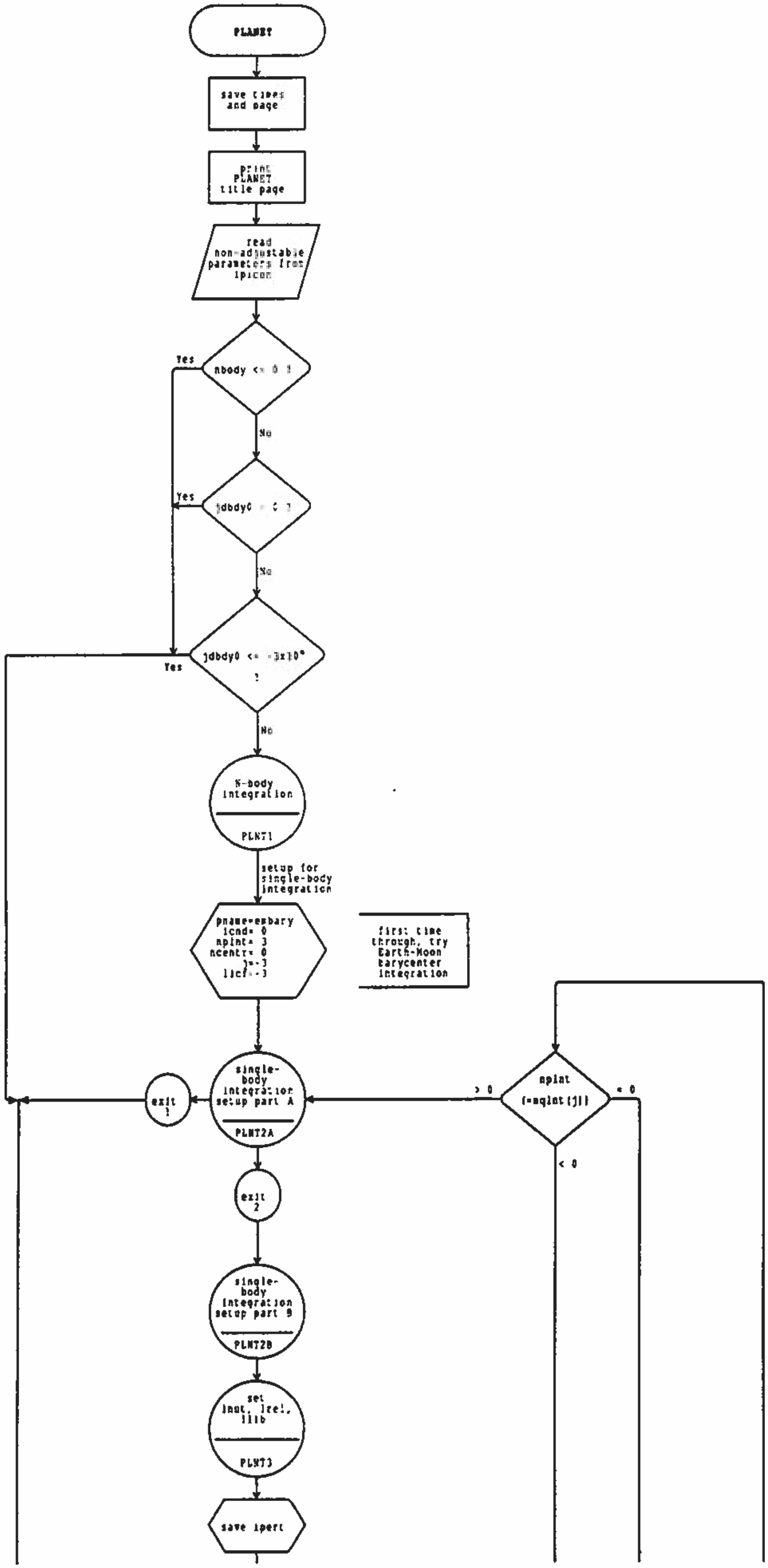
PLANET

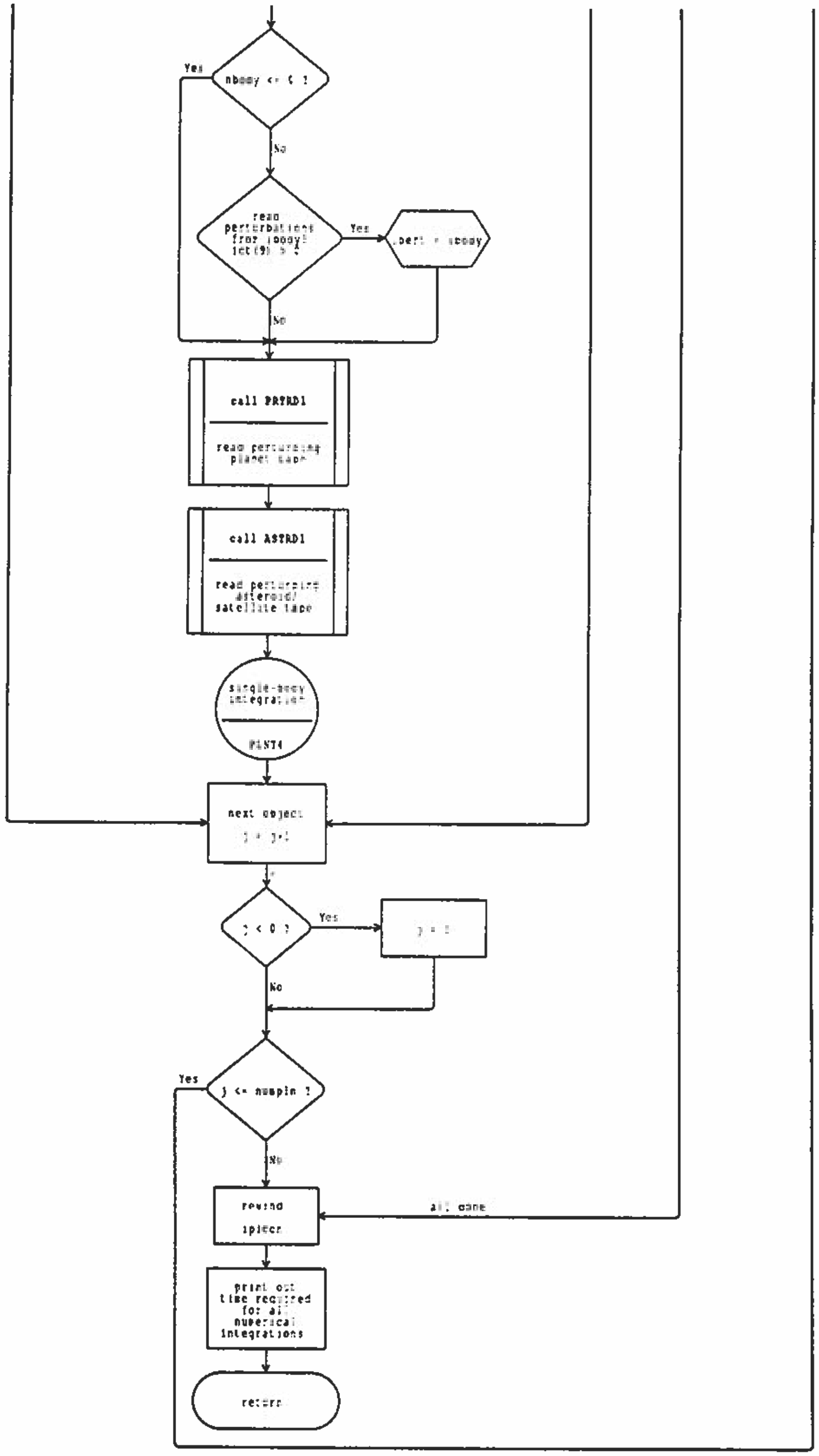
subroutine **PLANET (kfit)**

Description:	PLANET is the driver routine for numerical integration of the equations of motion.
Arguments:	kfit control integer: 0 Integrations are in the midst of orbit fitting iterations. 1 Integrations are after orbit fitting convergence. The integration interval will be extended, but without partials calculations.
Calls:	ADAM, ASTRD1, BODSET, EBCDI, NEWPG, NUMINT, OPRMSG, PAGSET, PLINK, PSPRSD, PRTRD1, RROAD, SBSET, SETUP, TIMRTC
Called by:	MAIN
Include Files:	BDCTRL, EMPCND, FCNTRL, INODTA, NAMTIMQ, PARAM, PETINA, PETUNA, PLNDTA, STBTST, STINT, TIMSTF
Equivalence:	name(2) /petuna/ PETUNA

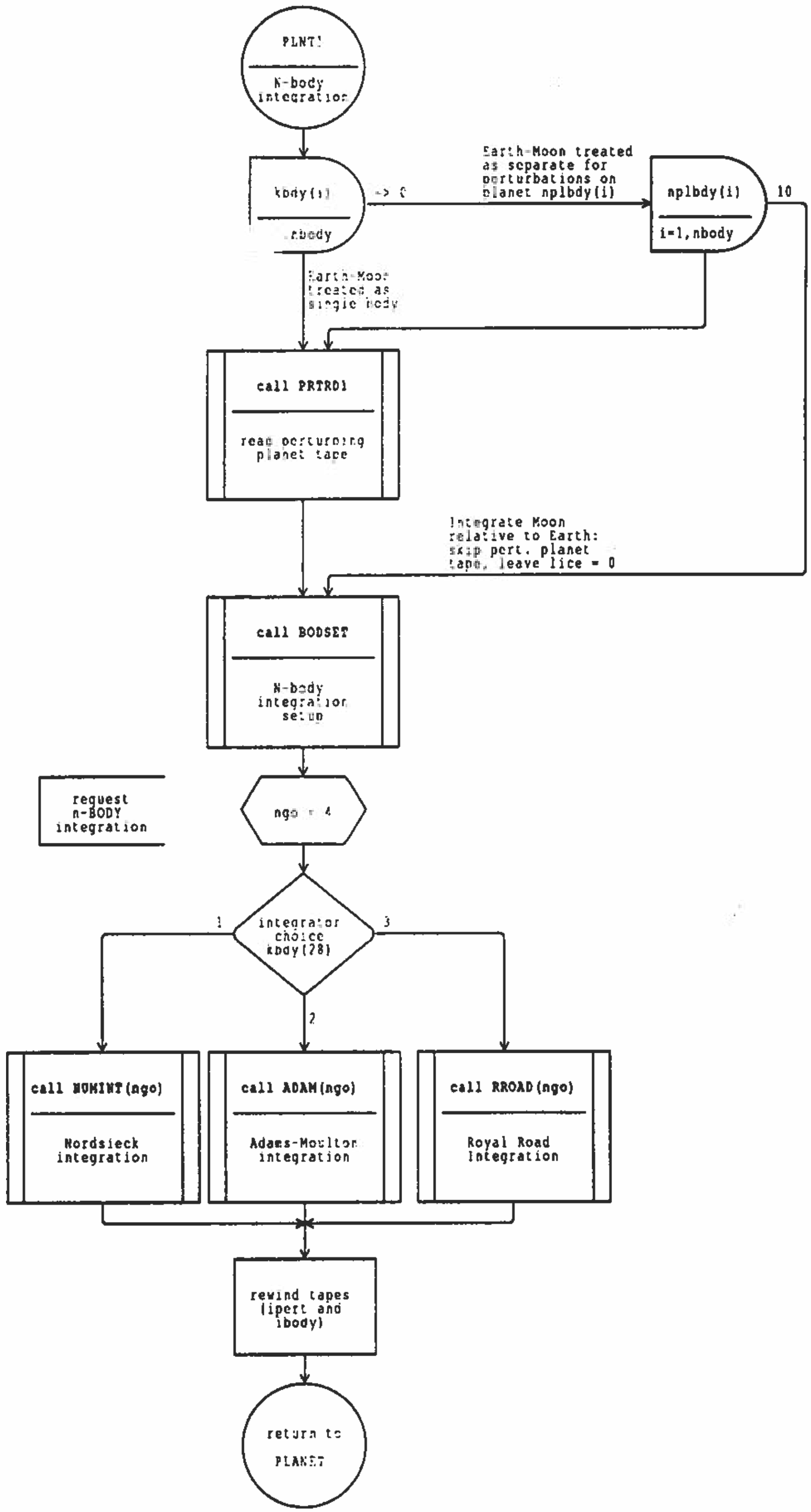

PLANET
ADAM
ASTRDI
NEWPG
EBCDI
RDATAP
SUICID
BODSET
EBCDI
NEWPG
EBCDI
NUMINT
OPRMSG
PAGSET
PLINK
PLPRWD
PRTRDI
NEWPG
EBCDI
RDYCAL
SUICID
RROAD
SBSET
SETUP
ELIPT
JLIPT
INITL
JNITL
PLNHED
TIMRTC

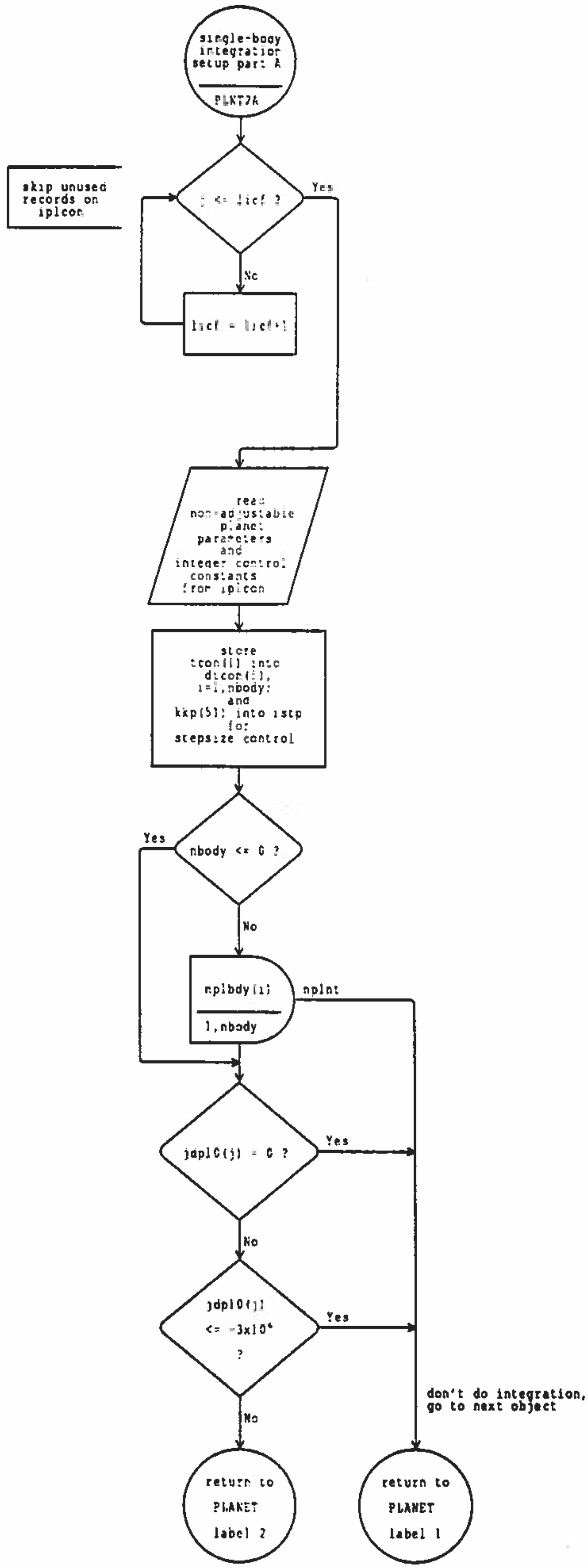

PLANET
setup
read non-adjustable parameters from iplcon
N-body integration
read start of perturbing planet tape
integrate equations
rewind data set tapes
single-body integration
setup part A
skip unused records on iplcon
read non-adjustable planet parameters and integer control constants from iplcon
decide whether or not to do integration
setup part B
save initial conditions and adjustable parameters
EM bary setup
set Earth nutation, Moon libration, and level of GR velocity terms flags
read start of perturbing planet tape
read perturbing asteroid/satellite tape
call integration setup routine SETUP
call satellite-probe integration setup routine SBSET
integrate equations of motion
rewind data tapes
clean-up
rewind iplcon


PLANET
save times and page
print PLANET title page
read non-adjustable parameters from ipicon
nbody <= 0 ?
Yes
No
jdbdy0 = 0 ?
Yes
No
jdbdy0 <= -3x10⁶ ?
Yes
No
N-body integration
PLNT1
setup for single-body integration
pname=embary
icnd= 0
nplnt= 3
ncentr= 0
j=-3
ijcf=-3
first time through, try Earth-Moon barycenter integration
single-body integration setup part A
PLNT2A
exit 1
exit 2
nplnt (=nqlnt(j))
> 0
< 0
< 0
single-body integration setup part B
PLNT2B
set inut, irel, iiib
PLNT3
save ipert

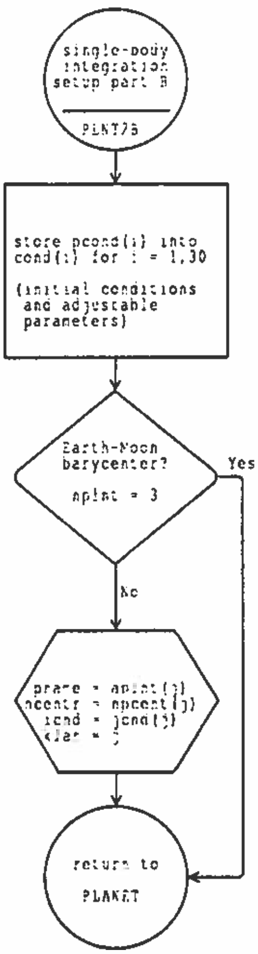


Yes
No
Yes
No
call PRTRD1
call ASTRD1
read perturbing asteroid/ satellite tape
next object
Yes
No
Yes
j <= numpln ?
No
rewind
all done
print out time required for all numerical integrations
return


PLNT1
N-body integration
kbdy(i)
nbody
=> 0
Earth-Moon treated as separate for perturbations on planet nplbdy(i)
nplbdy(i)
i=1,nbody
10
Earth-Moon treated as single body
call PRTRD1
read perturbing planet tape
Integrate Moon relative to Earth: skip pert. planet tape, leave lice = 0
call BODSET
N-body integration setup
request n-BODY integration
ngo = 4
integrator choice kbdy(28)
1
2
3
call NUMINT(ngo)
Nordsieck integration
call ADAM(ngo)
Adams-Moulton integration
call RROAD(ngo)
Royal Road Integration
rewind tapes (ipert and ibody)
return to PLANET



single-body integration setup part A
PLNT2A
skip unused records on iplcon
j <= licf ?
Yes
No
licf = licf+1
read non-adjustable planet parameters and integer control constants from iplcon
store tcon(i) into dtcon(i), i=1,nbody; and kkp(51) into istp for stepsize control
nbody <= 0 ?
Yes
No
nplbdy(i)
1,nbody
nplnt
jdpl0(j) = 0 ?
Yes
No
jdpl0(j) <= -3x10⁶ ?
Yes
No
don't do integration, go to next object
return to PLANET label 2
return to PLANET label 1

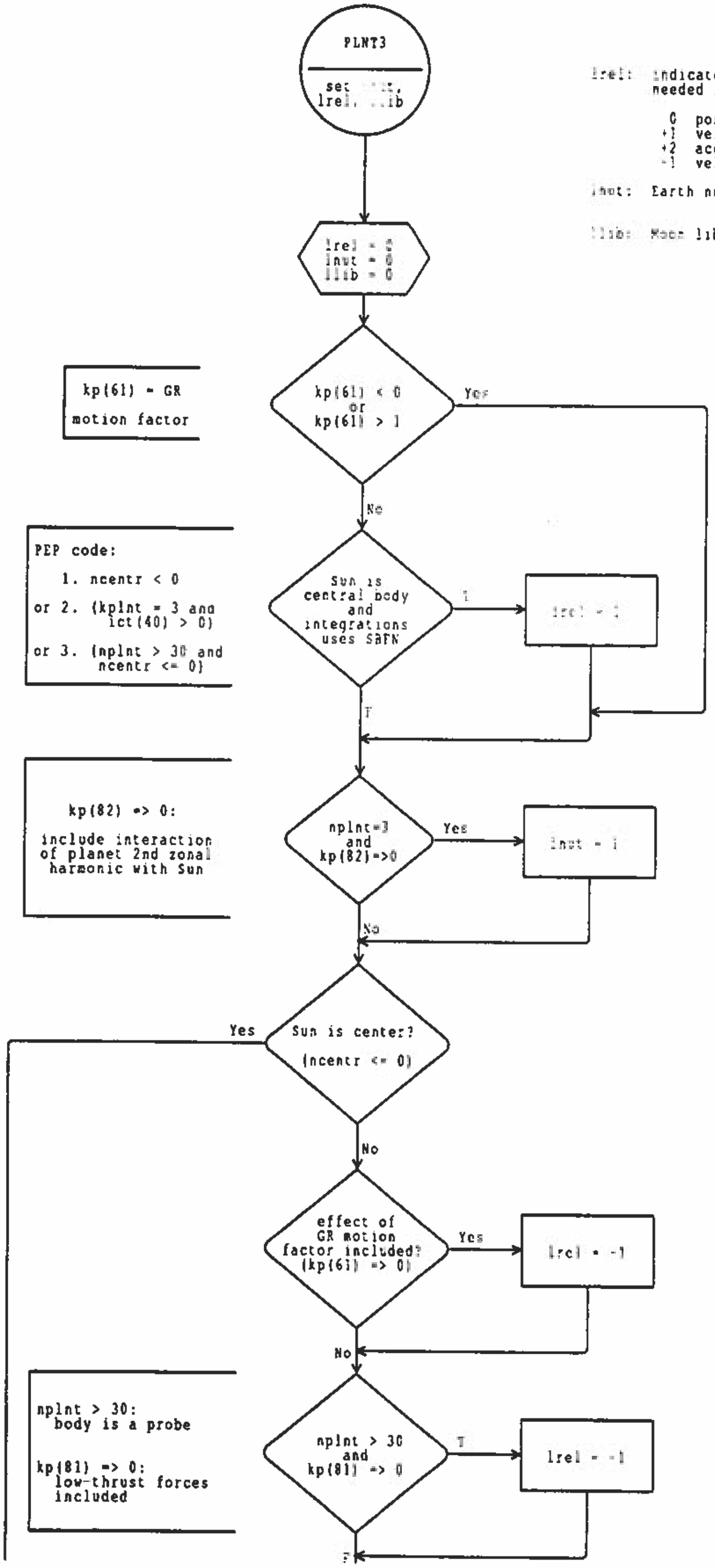

single-body
integration
setup part B
PLNT75
store pcond(i) into
cond(i) for i = 1,30
(initial conditions
and adjustable
parameters)
Earth-Moon
barycenter?
nplnt = 3
Yes
No
ncentr = npcent(j)
icnd = jcnd(j)
return to
PLANET

lrel: indicates number of time derivatives needed for perturbing body coordinates

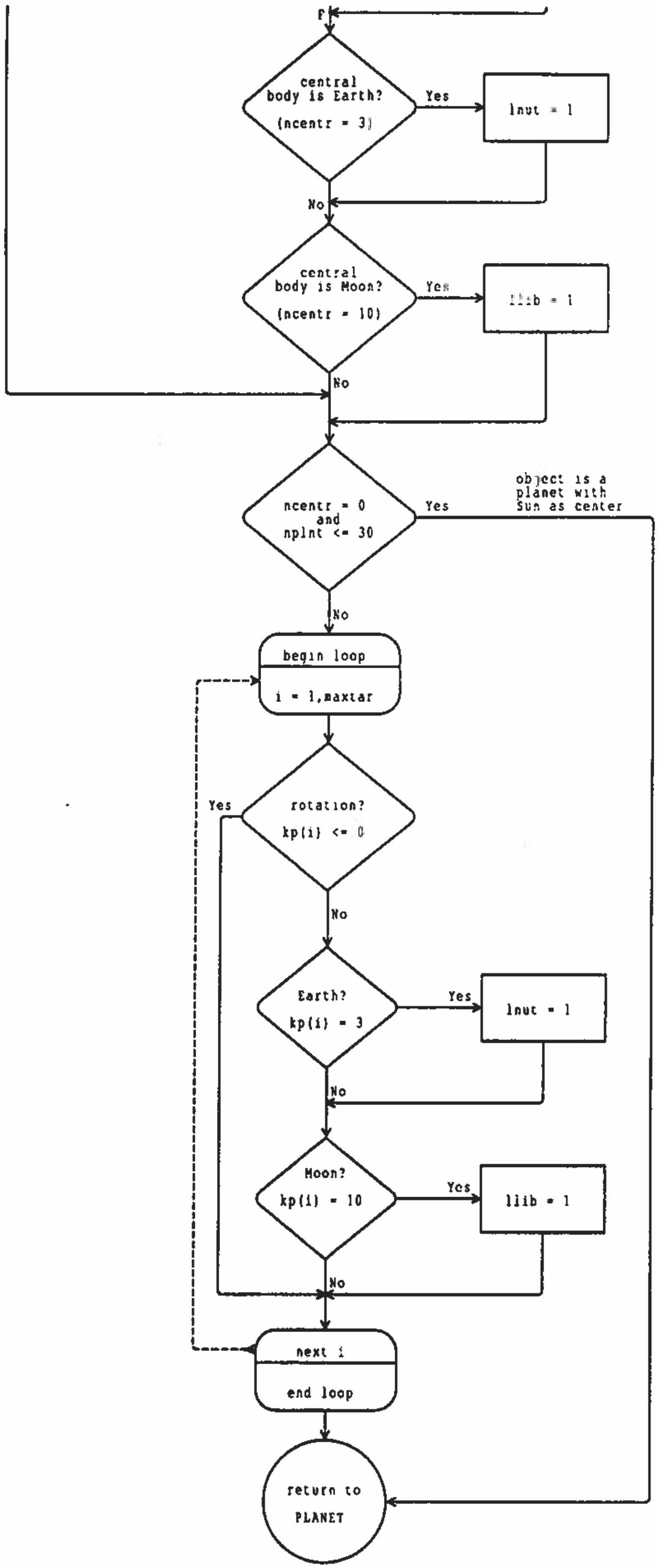
- 0 position only
- +1 velocity
- +2 acceleration
- -1 velocity for central body only

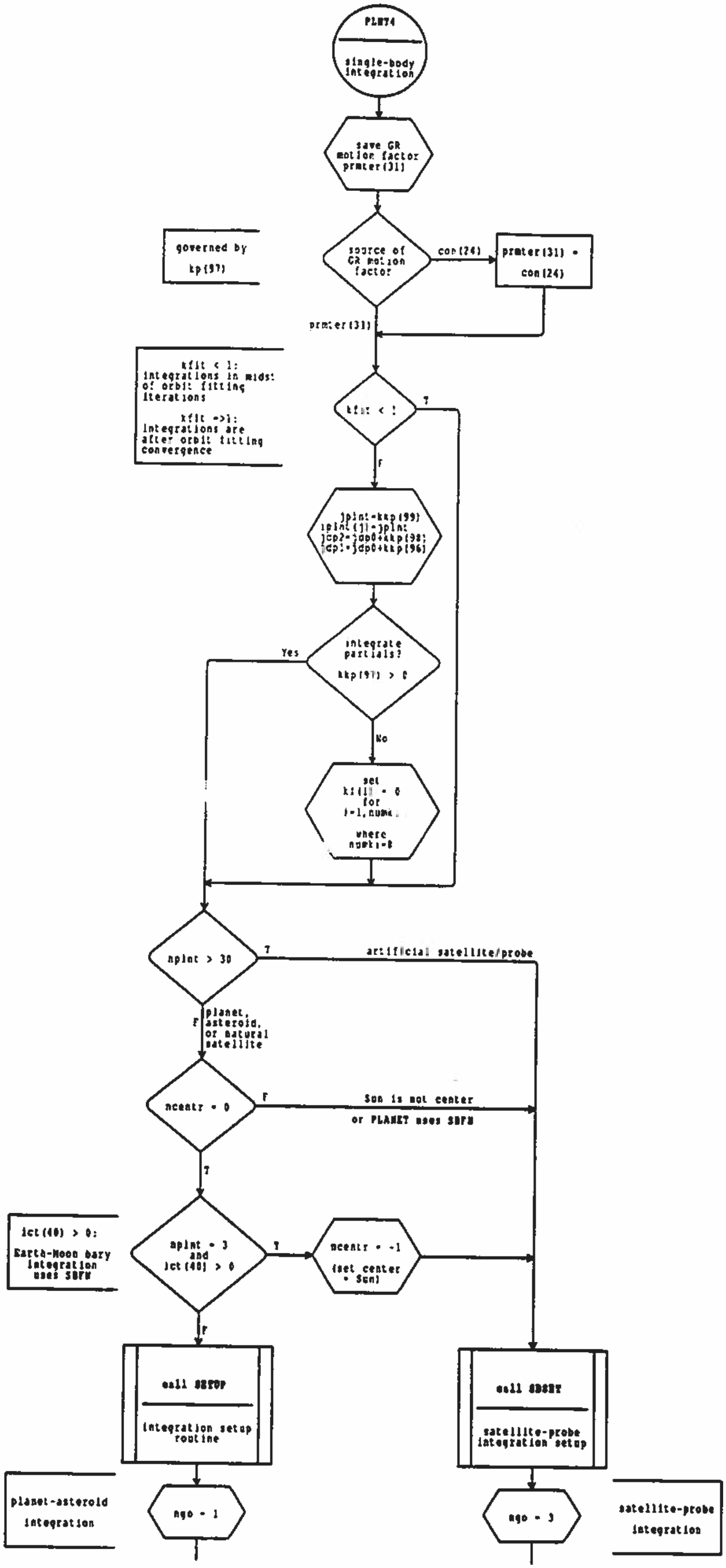
inut: Earth nutation

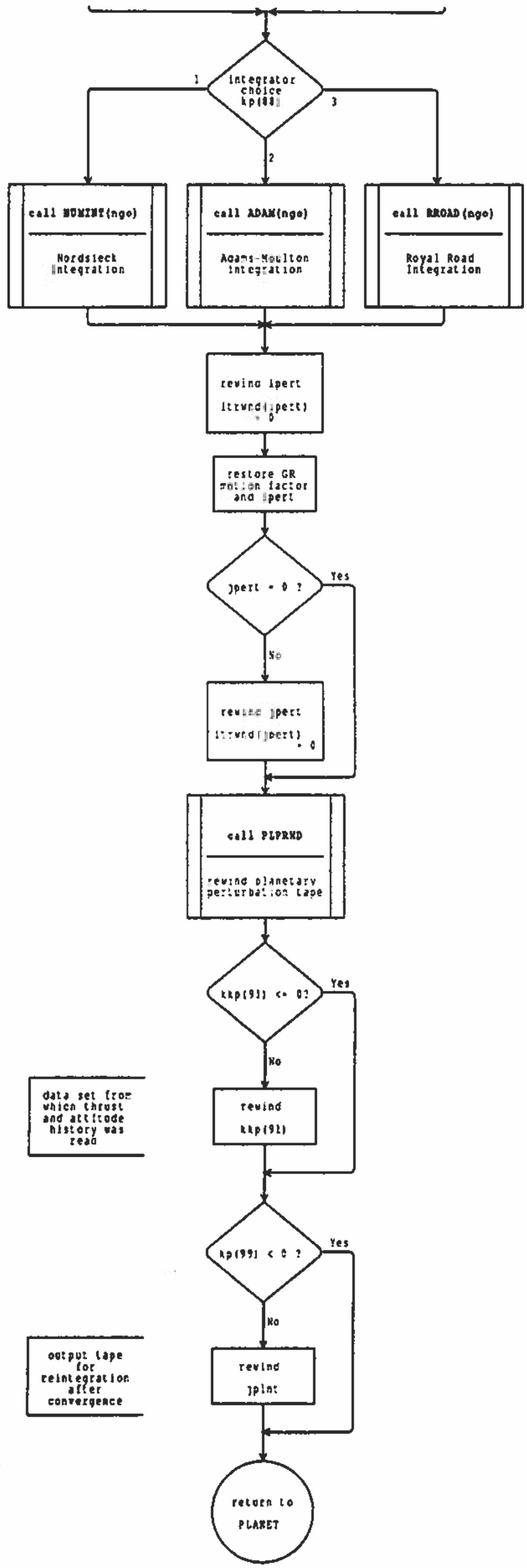
llib: Moon libration



1. Sun is center for satellite/probe integration (i.e., planet integration uses SBFN
2. Earth-Moon bary integration uses SBFN
3. Sun is center & body is artificial satellite or probe (SBFN is used)


F
central body is Earth?
(ncentr = 3)
Yes
lnut = 1
No
central body is Moon?
(ncentr = 10)
Yes
llib = 1
No
ncentr = 0 and nplnt <= 30
Yes
object is a planet with Sun as center
No
begin loop
i = 1,maxtar
Yes
rotation?
kp(i) <= 0
No
Earth?
kp(i) = 3
Yes
lnut = 1
No
Moon?
kp(i) = 10
Yes
llib = 1
No
next i
end loop
return to PLANET


single-body integration
save GR motion factor prmter(31)
source of GR motion factor
con(24)
prmter(31) = con(24)
governed by kp(97)
prmter(31)
kfit < 1: integrations in midst of orbit fitting iterations
kfit =>1: integrations are after orbit fitting convergence
T
F
integrate partials? kkp(97) > 0
Yes
No
where numki=8
npint > 30
artificial satellite/probe
planet, asteroid, or natural satellite
ncentr = 0
Sun is not center
or PLANET uses SBFN
ict(40) > 0: Earth-Moon bary integration uses SBFN
npint = 3 and ict(40) > 0
ncentr = -1 (set center = Sun)
call SETUP
integration setup routine
call SBSET
satellite-probe integration setup
planet-asteroid integration
ngo = 1
ngo = 3
satellite-probe integration



integrator choice kp(88)
1
2
3
call NUMINT(ngo)
Nordsieck Integration
call ADAM(ngo)
Adams-Moulton Integration
call RROAD(ngo)
Royal Road Integration
rewind ipert
itrwnd(ipert) = 0
restore GR motion factor and ipert
jpert = 0 ?
Yes
No
rewind jpert
itrwnd(jpert) = 0
call PLPRWD
rewind planetary perturbation tape
kkp(91) <= 0?
Yes
No
data set from which thrust and attitude history was read
rewind
kkp(91)
kp(99) < 0 ?
Yes
No
output tape for reintegration after convergence
rewind
jplnt
return to PLANET

SBFN

real*8 function SBFN(k, j, s)

Description:	SBFN evaluates (mostly via a call to SBFN1) the equations of motion and partial derivatives for satellites and probes. It does a fair amount of setting up for calculations performed in SBFN1, which contains most of the right hand sides of the differential equations being solved.	

Arguments:

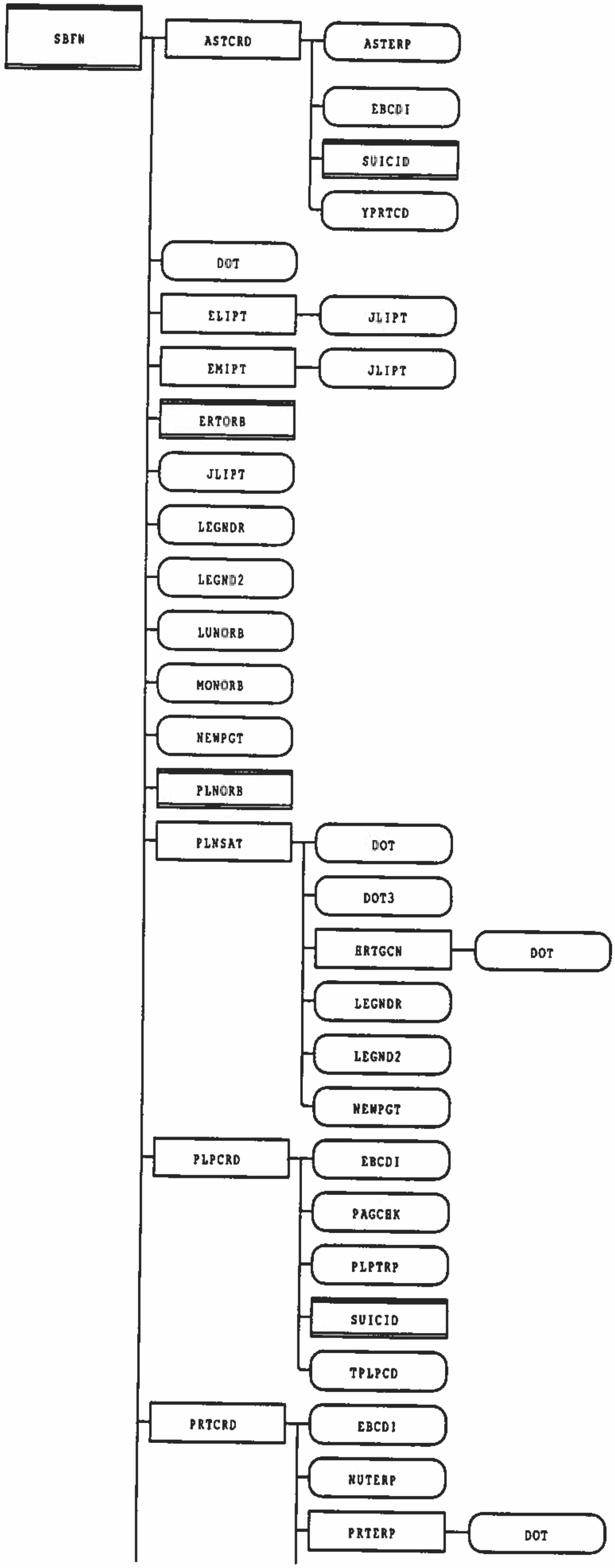
k	Equation number. There are six equations for the motion and six for each partial derivative. A detailed description of k is provided elsewhere.
j	Integrator iteration number (generally takes on values from 1 to 3, and is often used as a logic flag).
s	Current time = julian date + 0.5 day.

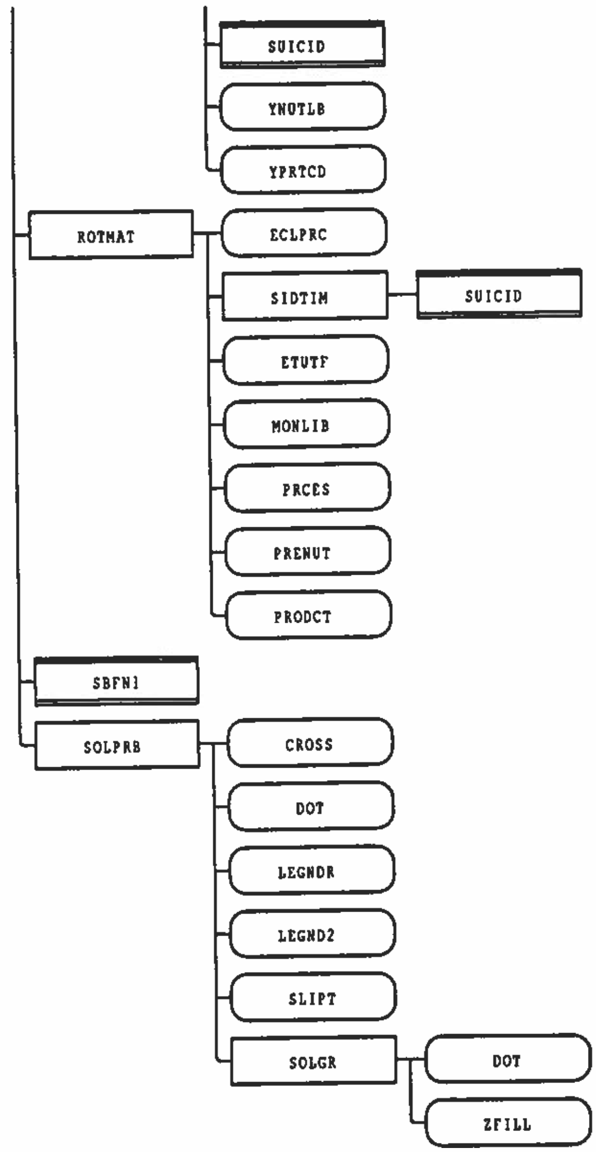
Calls: ASTCRD, DOT, ELLIPT, EMIPT, ERTORB, JLIPT, LEGNDR, LEGND2, LUNORB, MONORB, NEWPGT, PLNORB, PLNSAT, PLPCRD, PRTCRD, ROTMAT, SBFN1, SOLPRB

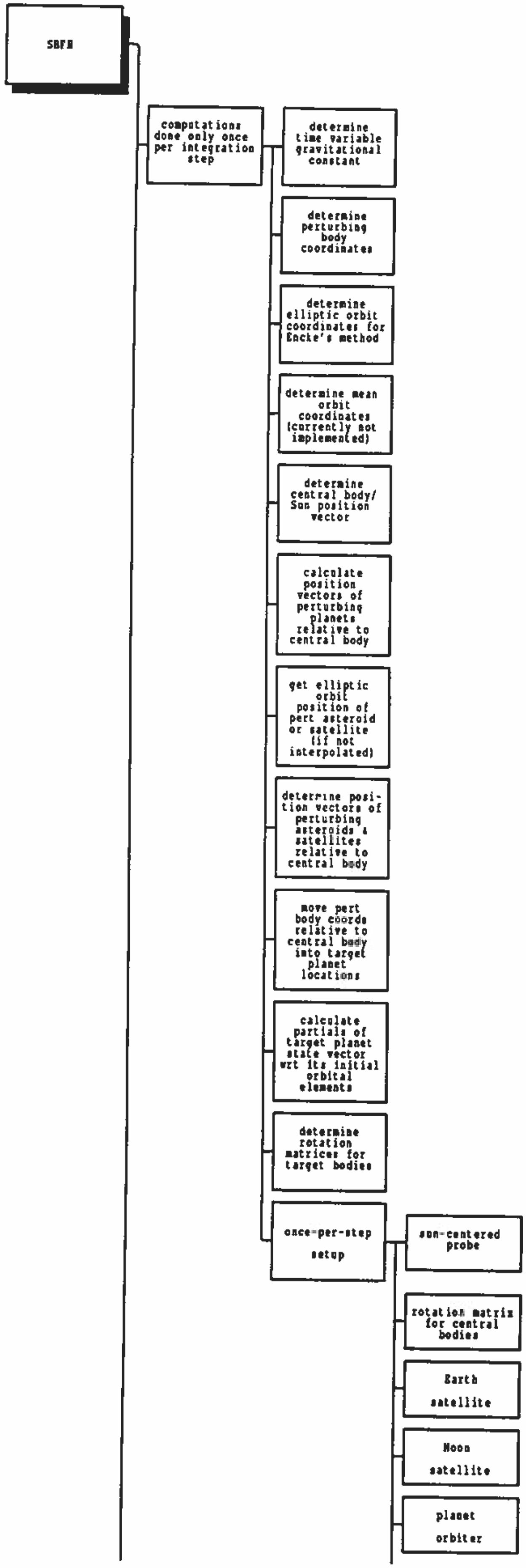
Called by: ADAM, EVAL, NINT, RROAD

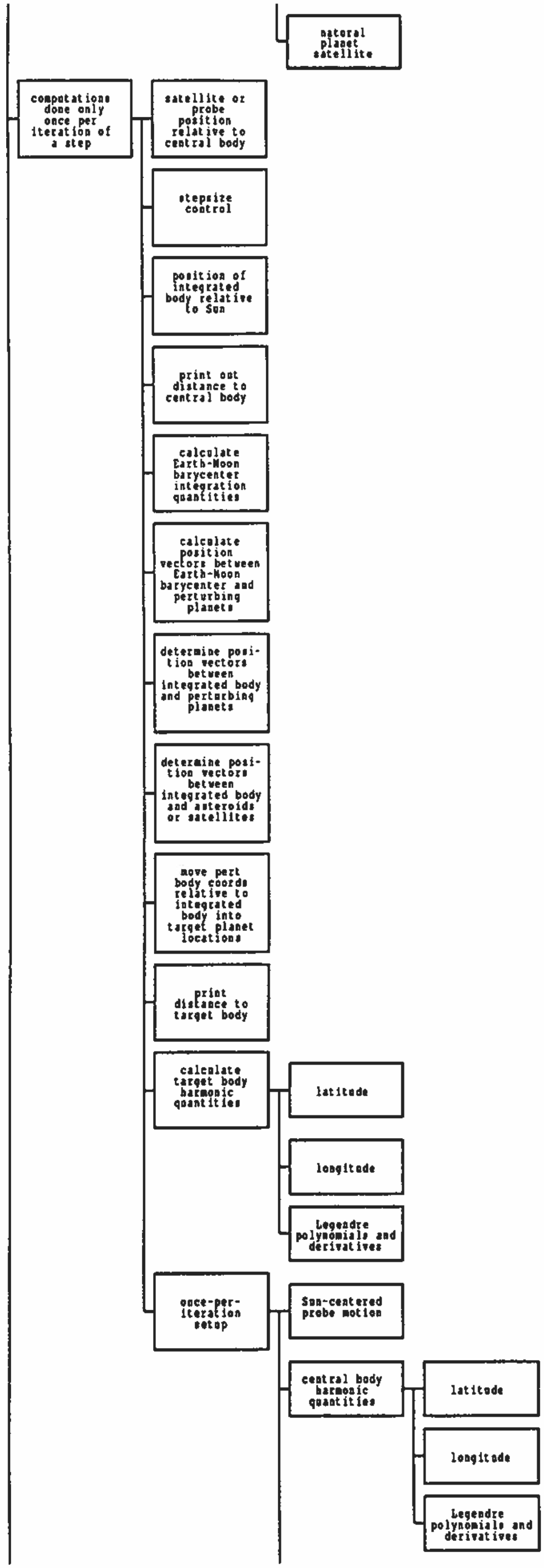
Include Files: CNTHAR, ELLIPS, EMMIPS, FCNTRL, FMMIPS, INODTA, ORBLUN, OUTPUT, PARAM, PETUNA, PRTCOD, SBEMBR, SBROT, SBSTUF, SBTHNG, STINT, TRGHAR

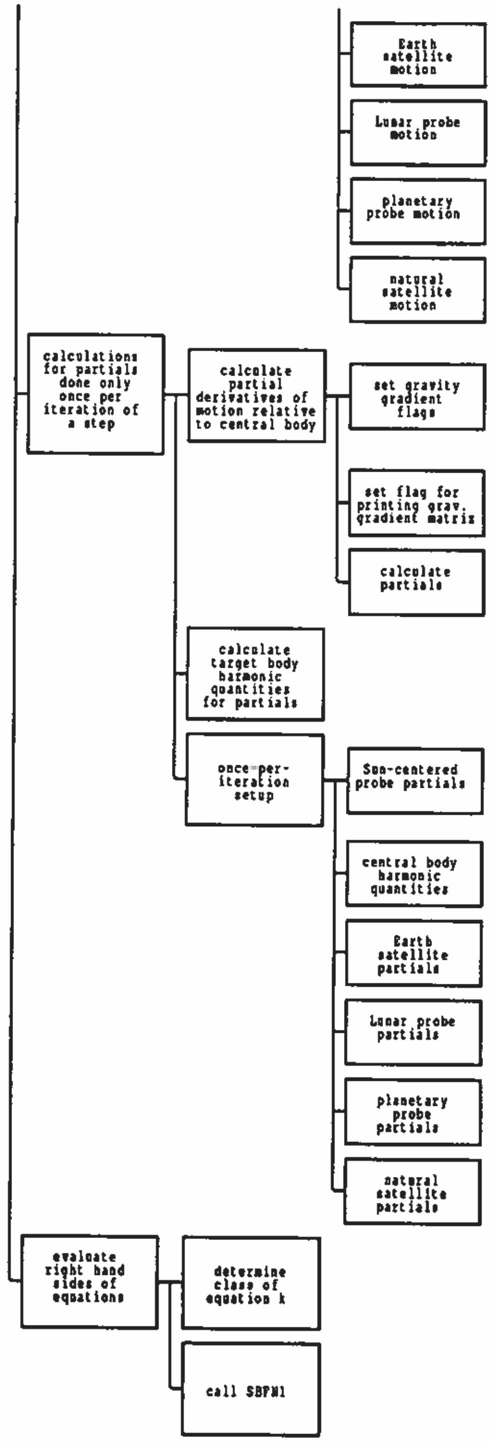
Equivalence: ggfgs(2) /sbstuf/ SBSTUF

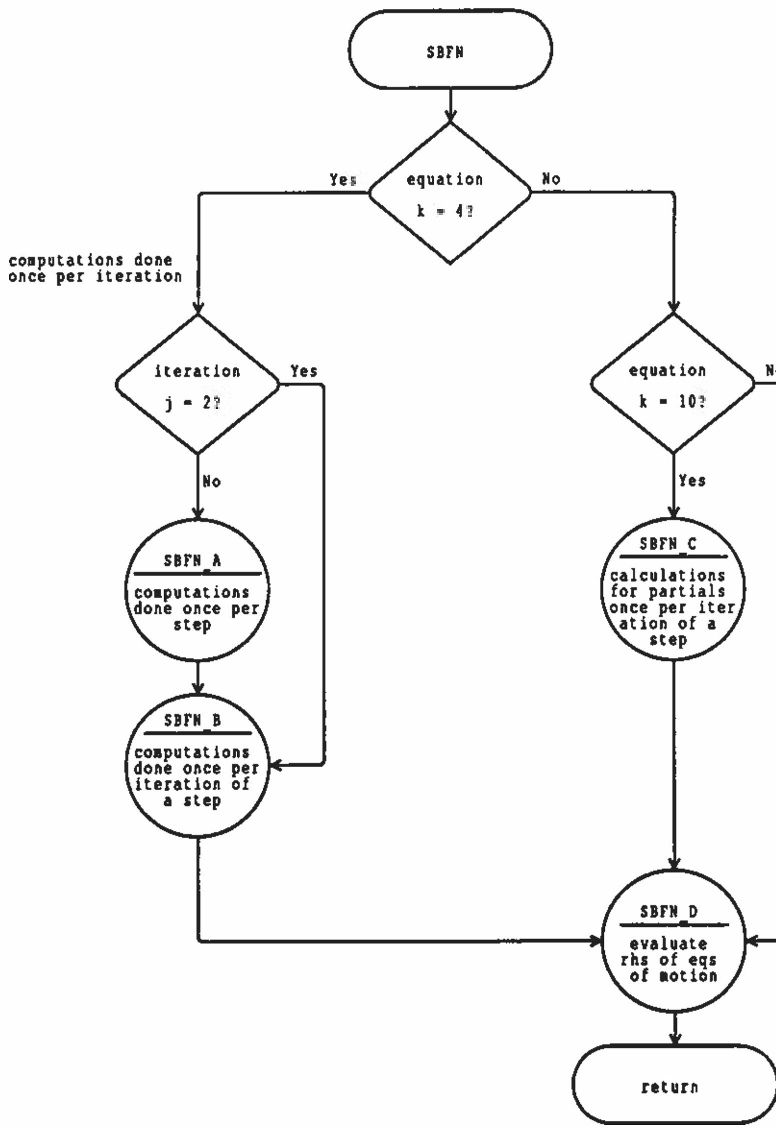

SBFN
ASTCRD
ASTERP
EBCDI
SUICID
YPRTCD
DOT
ELIPT
JLIPT
EMIPT
JLIPT
ERTORB
JLIPT
LEGNDR
LEGND2
LUNORB
MONORB
NEWPGT
PLNORB
PLNSAT
DOT
DOT3
ERTGCN
DOT
LEGNDR
LEGND2
NEWPGT
PLPCRD
EBCDI
PAGCHK
PLPTRP
SUICID
TPLPCD
PRTCRD
EBCDI
NUTERP
PRTERP
DOT

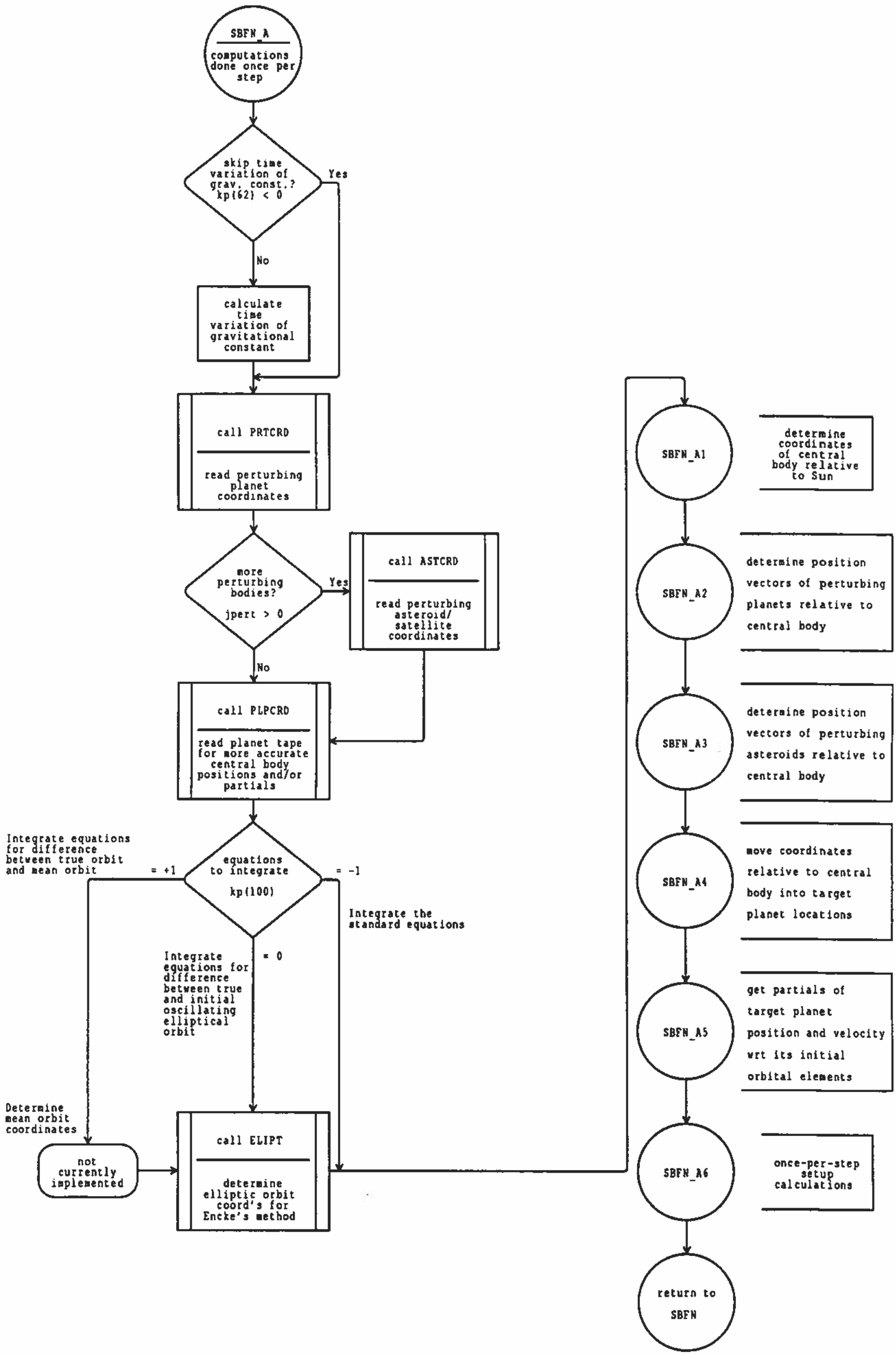

SUICID
YNOTLB
YPRTCD
ROTMAT
ECLPRC
SIDTIM
SUICID
ETUTF
MONLIB
PRCES
PRENUT
PRODCT
SBFN1
SOLPRB
CROSS
DOT
LEGNDR
LEGND2
SLIPT
SOLGR
DOT
ZFILL


SBFN
computations done only once per integration step
determine time variable gravitational constant
determine perturbing body coordinates
determine elliptic orbit coordinates for Encke's method
determine mean orbit coordinates (currently not implemented)
determine central body/ Sun position vector
calculate position vectors of perturbing planets relative to central body
get elliptic orbit position of pert asteroid or satellite (if not interpolated)
determine position vectors of perturbing asteroids & satellites relative to central body
move pert body coords relative to central body into target planet locations
calculate partials of target planet state vector wrt its initial orbital elements
determine rotation matrices for target bodies
once-per-step setup
sun-centered probe
rotation matrix for central bodies
Earth satellite
Moon satellite
planet orbiter

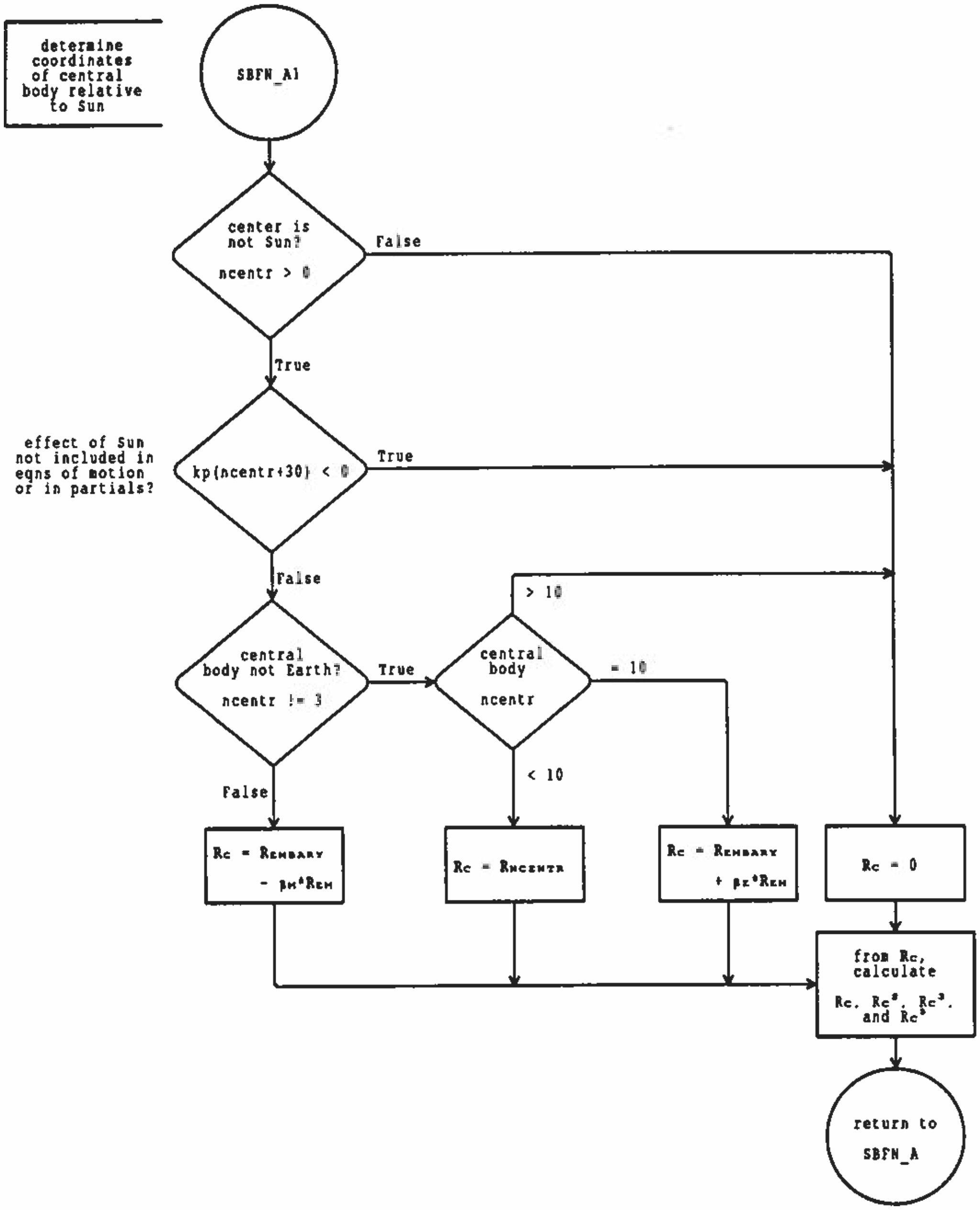

natural planet satellite
computations done only once per iteration of a step
satellite or probe position relative to central body
stepsize control
position of integrated body relative to Sun
print out distance to central body
calculate Earth-Moon barycenter integration quantities
calculate position vectors between Earth-Moon barycenter and perturbing planets
determine position vectors between integrated body and perturbing planets
determine position vectors between integrated body and asteroids or satellites
move pert body coords relative to integrated body into target planet locations
print distance to target body
calculate target body harmonic quantities
latitude
longitude
Legendre polynomials and derivatives
once-per-iteration setup
Sun-centered probe motion
central body harmonic quantities
latitude
longitude
Legendre polynomials and derivatives

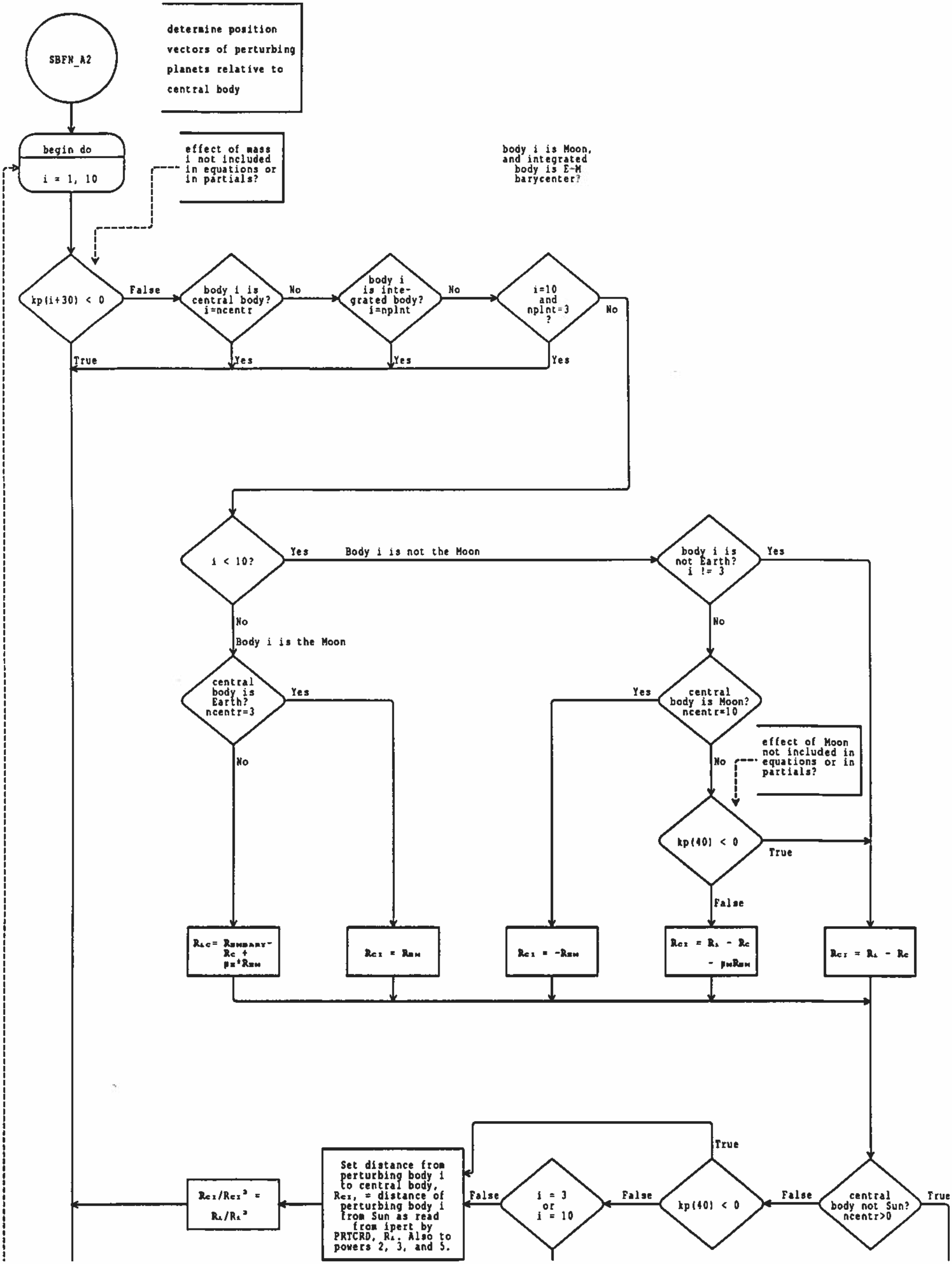

Earth satellite motion
Lunar probe motion
planetary probe motion
natural satellite motion
calculations for partials done only once per iteration of a step
calculate partial derivatives of motion relative to central body
set gravity gradient flags
set flag for printing grav. gradient matrix
calculate partials
calculate target body harmonic quantities for partials
once-per-iteration setup
Sun-centered probe partials
central body harmonic quantities
Earth satellite partials
Lunar probe partials
planetary probe partials
natural satellite partials
evaluate right hand sides of equations
determine class of equation k
call SBPN1

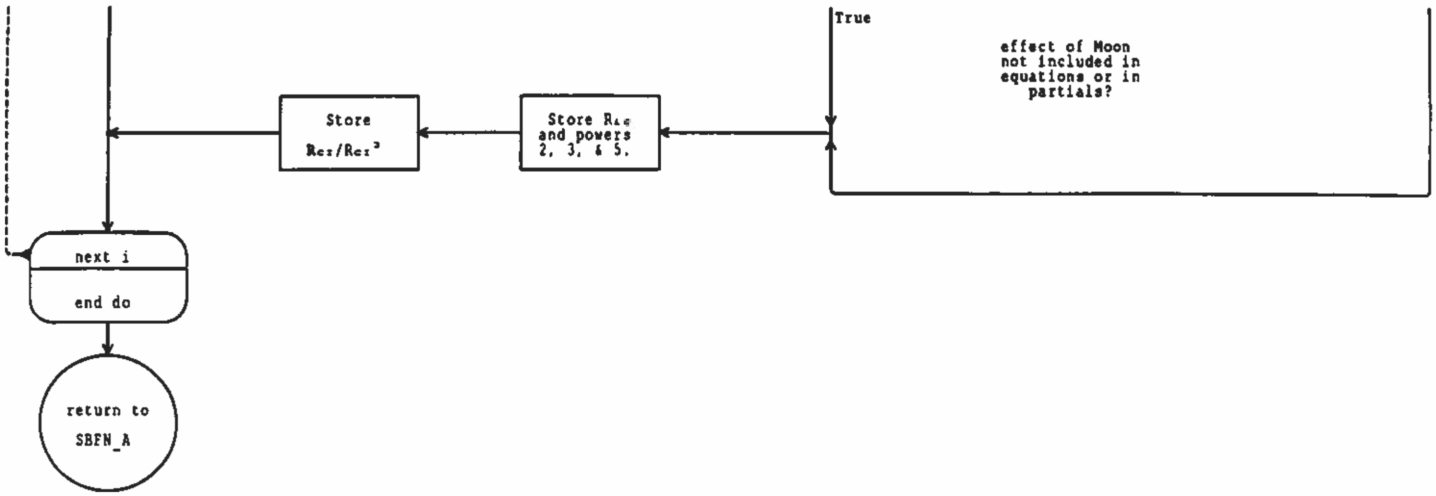

SBFN
equation
k = 4?
Yes
No
computations done
once per iteration
iteration
j = 2?
Yes
No
SBFN_A
computations
done once per
step
SBFN_B
computations
done once per
iteration of
a step
equation
k = 10?
No
Yes
SBFN_C
calculations
for partials
once per iter
ation of a
step
SBFN_D
evaluate
rhs of eqs
of motion
return

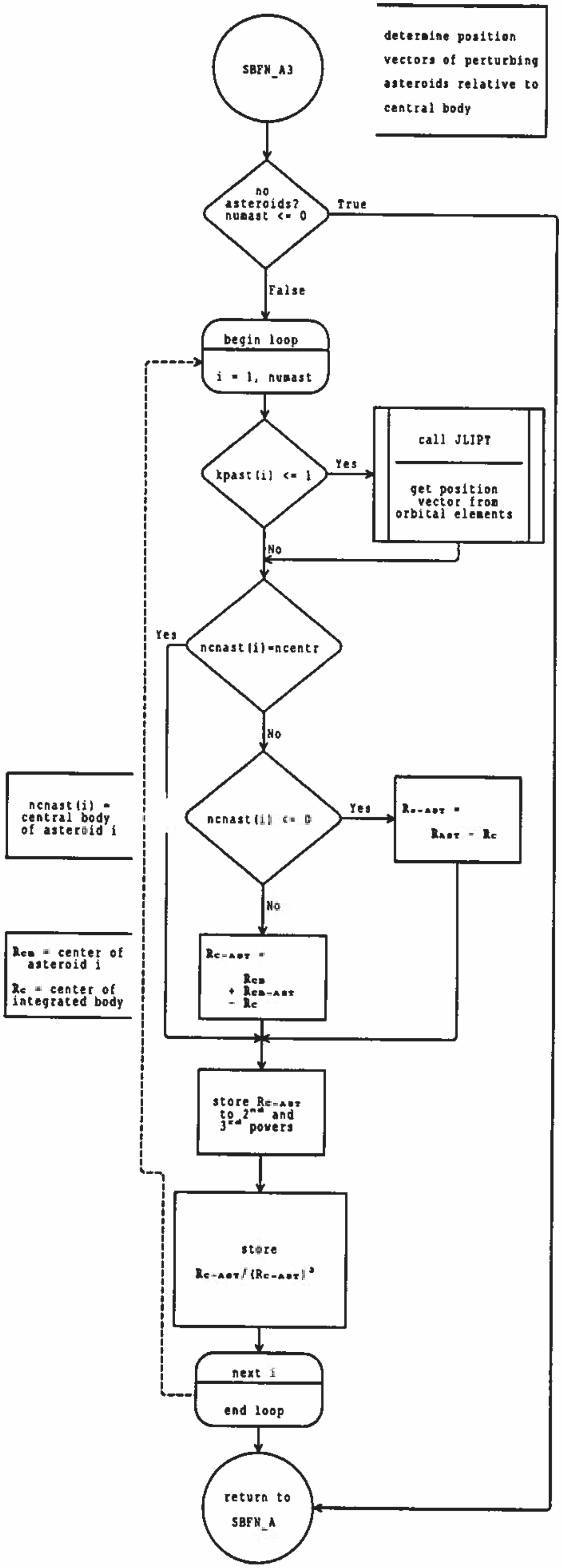


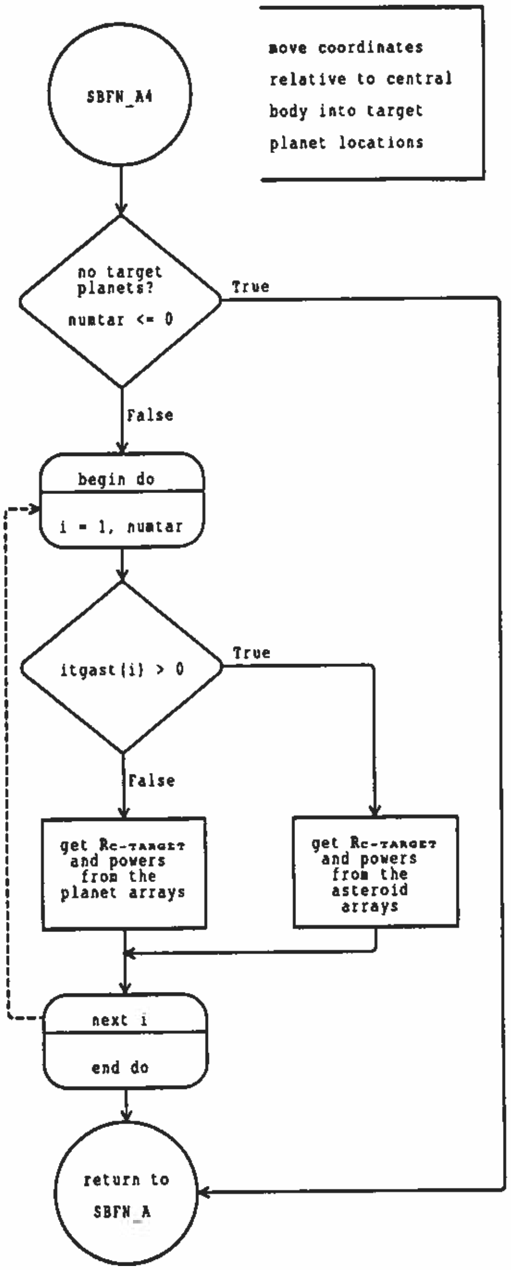
SBFN_A
computations done once per step
skip time variation of grav. const.? kp(62) < 0
Yes
No
calculate time variation of gravitational constant
call PRTCRD
read perturbing planet coordinates
more perturbing bodies?
jpert > 0
Yes
call ASTCRD
read perturbing asteroid/ satellite coordinates
No
call PLPCRD
read planet tape for more accurate central body positions and/or partials
Integrate equations for difference between true orbit and mean orbit
= +1
equations to integrate
kp(100)
= -1
Integrate the standard equations
Integrate equations for difference between true and initial oscillating elliptical orbit
= 0
Determine mean orbit coordinates
not currently implemented
call ELIPT
determine elliptic orbit coord's for Encke's method
SBFN_A1
determine coordinates of central body relative to Sun
SBFN_A2
determine position vectors of perturbing planets relative to central body
SBFN_A3
determine position vectors of perturbing asteroids relative to central body
SBFN_A4
move coordinates relative to central body into target planet locations
SBFN_A5
get partials of target planet position and velocity wrt its initial orbital elements
SBFN_A6
once-per-step setup calculations
return to SBFN

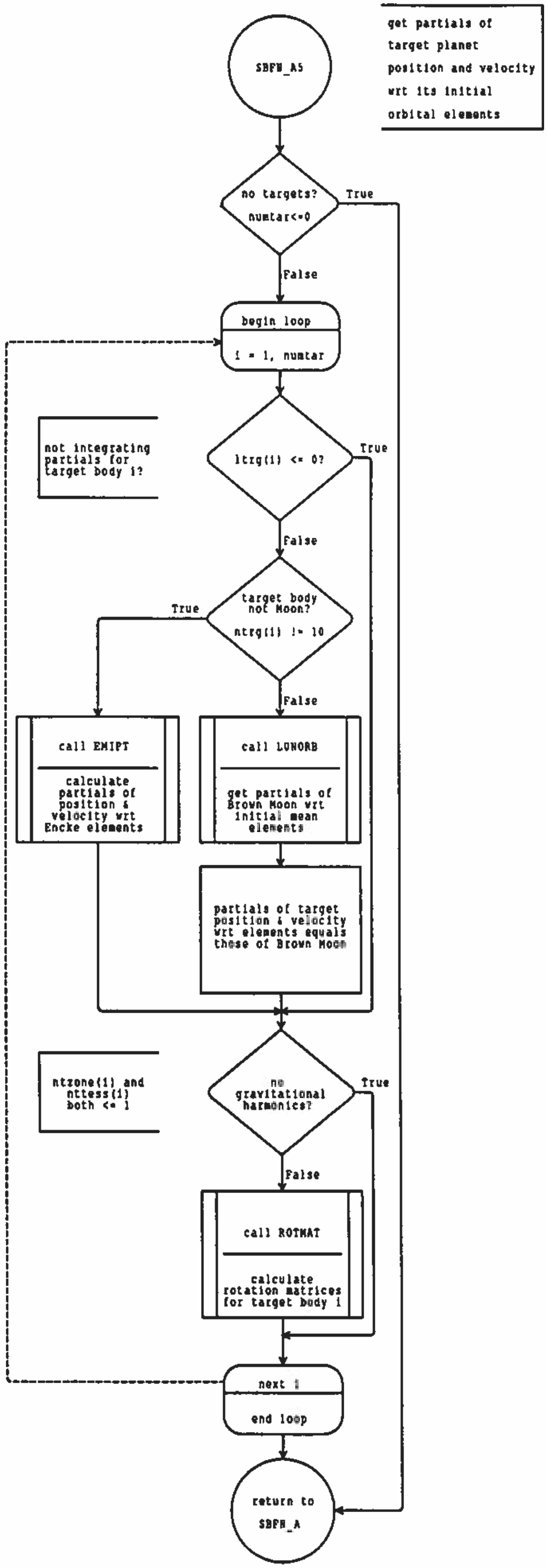

determine coordinates of central body relative to Sun
SBFN_A1
center is not Sun?
ncentr > 0
False
True
effect of Sun not included in eqns of motion or in partials?
kp(ncentr+30) < 0
True
False
central body not Earth?
ncentr != 3
True
central body
ncentr
> 10
= 10
< 10
False
Rc = REMBARY - μM*REM
Rc = RNCENTR
Rc = REMBARY + μE*REM
Rc = 0
from Rc, calculate Rc, Rc², Rc³, and Rc⁵
return to SBFN_A

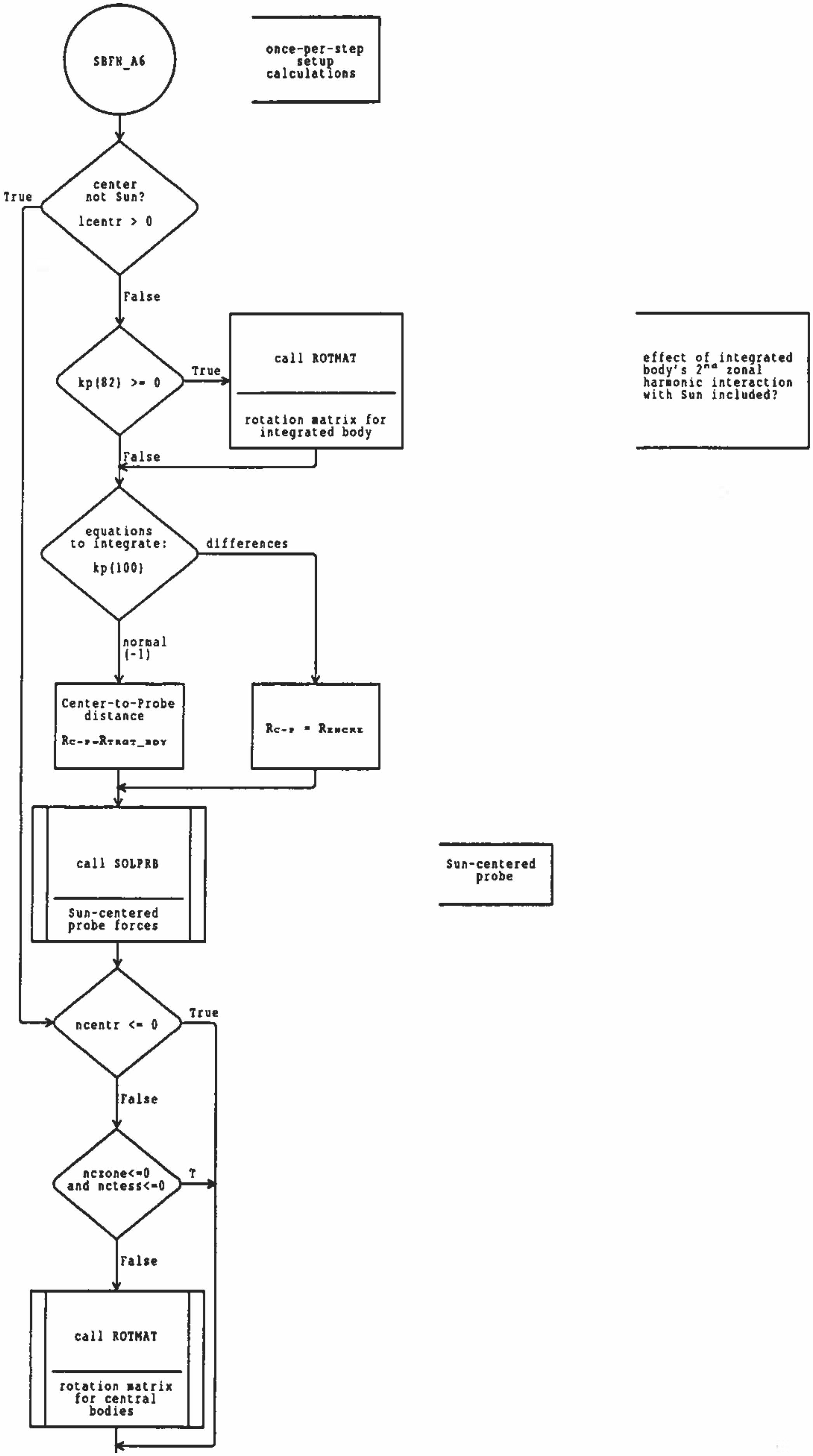

SBFN_A2
determine position vectors of perturbing planets relative to central body
begin do
i = 1, 10
effect of mass i not included in equations or in partials?
body i is Moon, and integrated body is E-M barycenter?
kp(i+30) < 0
False
True
body i is central body? i=ncentr
No
Yes
body i is integrated body? i=nplnt
No
Yes
i=10 and nplnt=3 ?
No
Yes
i < 10?
Yes
Body i is not the Moon
No
Body i is the Moon
body i is not Earth? i != 3
Yes
No
central body is Earth? ncentr=3
Yes
No
central body is Moon? ncentr=10
Yes
No
effect of Moon not included in equations or in partials?
kp(40) < 0
True
False
$R_{iC} = R_{EMBARY} - R_C + \mu_E{}^* R_{EM}$
$R_{CI} = R_{EM}$
$R_{CI} = -R_{EM}$
$R_{CI} = R_i - R_C - \mu_M R_{EM}$
$R_{CI} = R_i - R_C$
central body not Sun? ncentr>0
True
False
kp(40) < 0
True
False
i = 3 or i = 10
False
Set distance from perturbing body i to central body, R_{CI}, = distance of perturbing body i from Sun as read from ipert by PRTCRD, R_i. Also to powers 2, 3, and 5.
$R_{CI}/R_{CI}{}^3 = R_i/R_i{}^3$

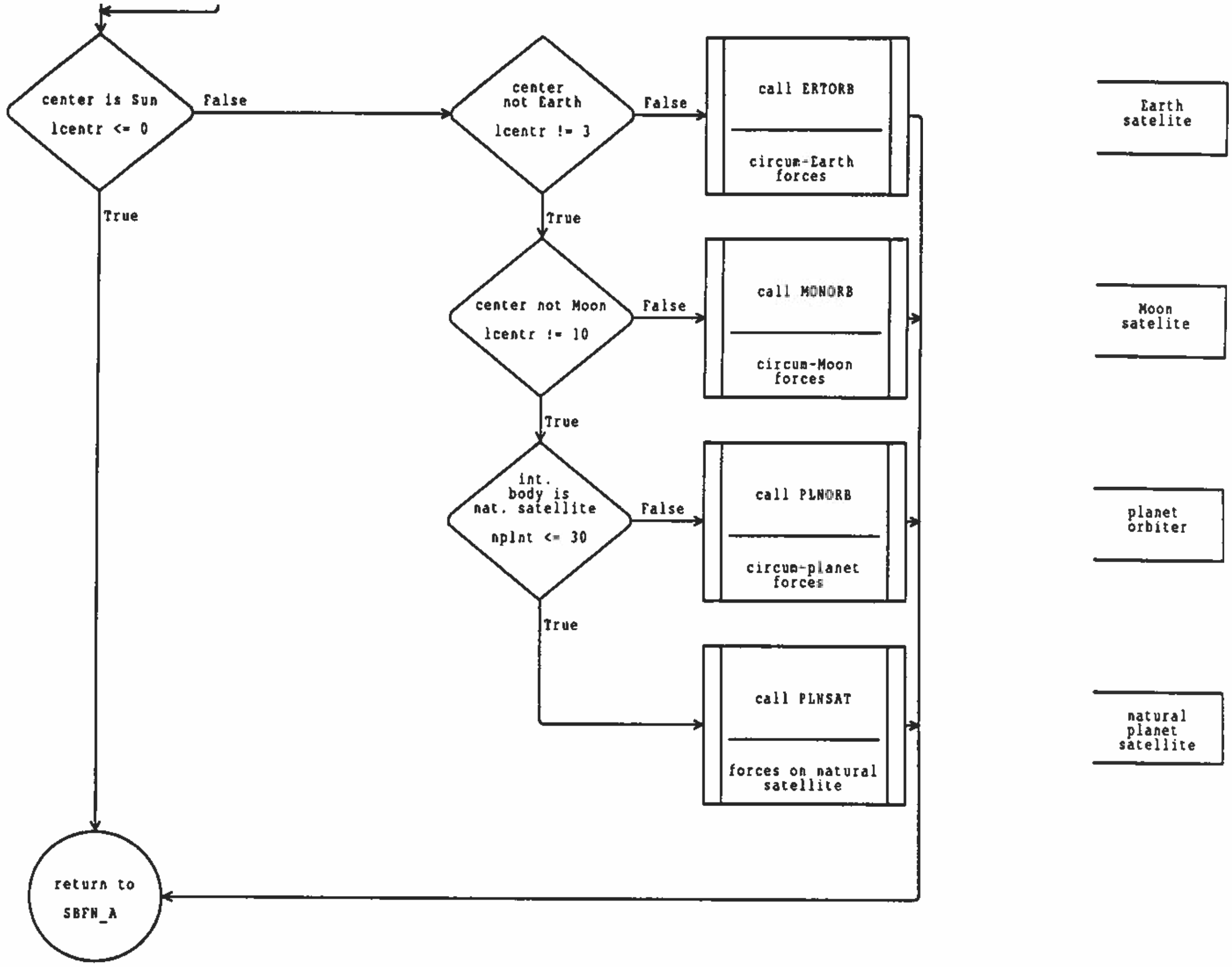

True
effect of Moon not included in equations or in partials?
Store R_{ic} and powers 2, 3, & 5.
Store
R_{ex}/R_{ex}^3
next i
end do
return to SBFN_A


SBFN_A3
determine position vectors of perturbing asteroids relative to central body
no asteroids? numast <= 0
True
False
begin loop
i = 1, numast
kpast(i) <= 1
Yes
call JLIPT
get position vector from orbital elements
No
ncnast(i)=ncentr
Yes
No
ncnast(i) = central body of asteroid i
ncnast(i) <= 0
Yes
Rc-ast = Rast - Rc
No
Rcb = center of asteroid i
Rc = center of integrated body
Rc-ast = Rcb + Rcb-ast - Rc
store Rc-ast to 2nd and 3rd powers
store Rc-ast/(Rc-ast)³
next i
end loop
return to SBFN_A

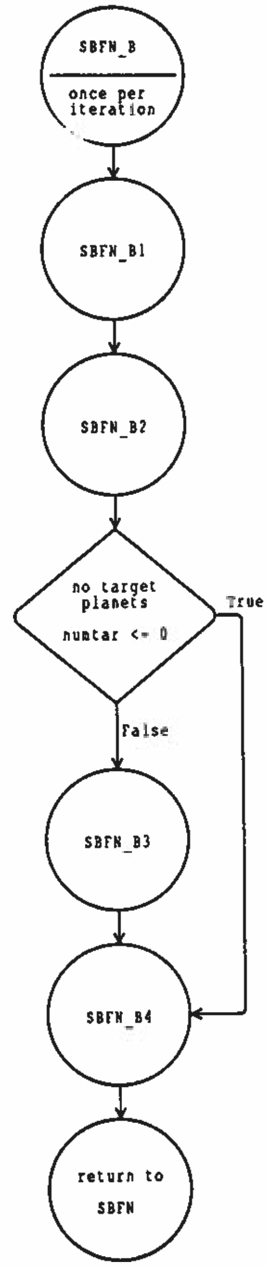

move coordinates
relative to central
body into target
planet locations
SBFN_A4
no target
planets?
numtar <= 0
True
False
begin do
i = 1, numtar
itgast(i) > 0
True
False
get Rc-TARGET
and powers
from the
planet arrays
get Rc-TARGET
and powers
from the
asteroid
arrays
next i
end do
return to
SBFN_A

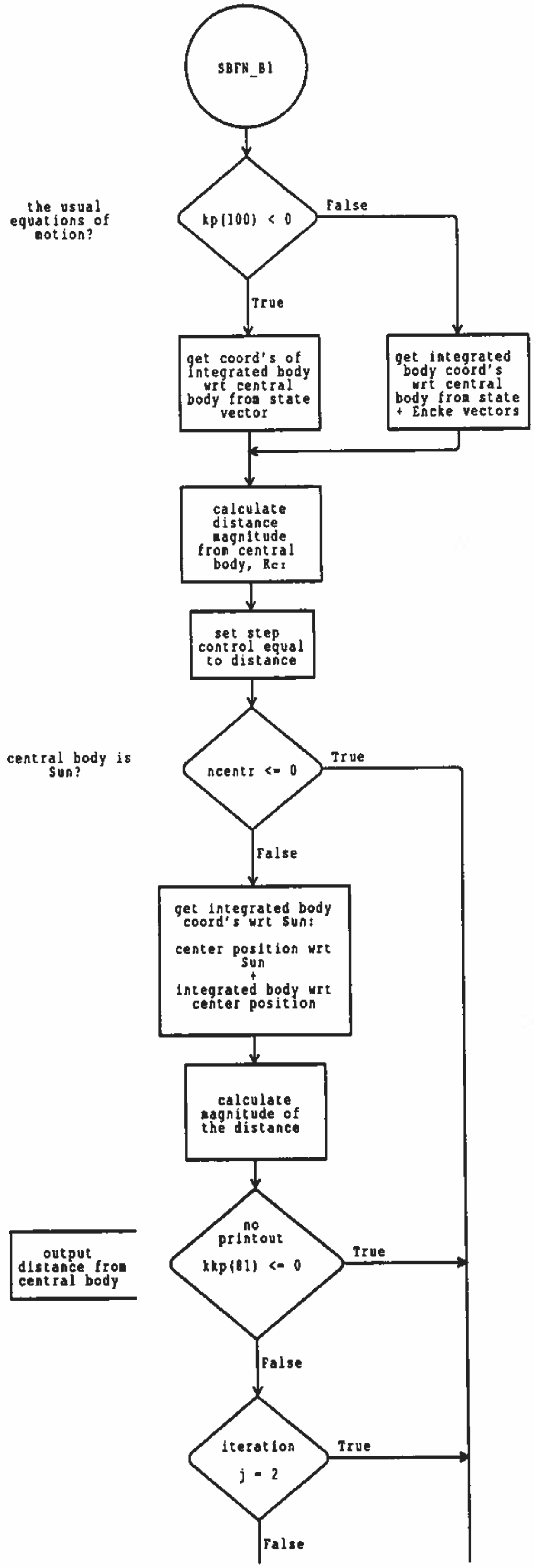

get partials of target planet position and velocity wrt its initial orbital elements
SBFN_A5
no targets? numtar<=0
True
False
begin loop
i = 1, numtar
not integrating partials for target body i?
ltrg(i) <= 0?
True
False
target body not Moon?
ntrg(i) != 10
True
False
call EMIPT
calculate partials of position & velocity wrt Encke elements
call LUNORB
get partials of Brown Moon wrt initial mean elements
partials of target position & velocity wrt elements equals those of Brown Moon
ntzone(i) and nttess(i) both <= 1
no gravitational harmonics?
True
False
call ROTMAT
calculate rotation matrices for target body i
next i
end loop
return to SBFN_A

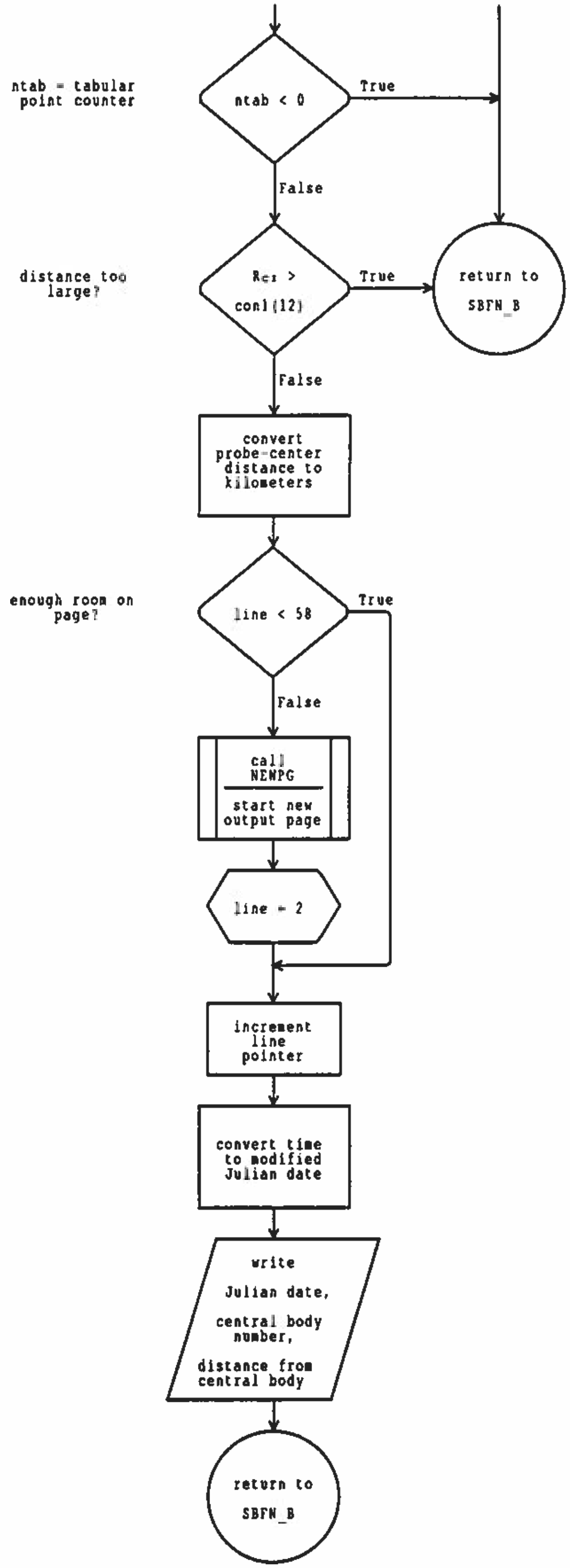

SBFN_A6
once-per-step setup calculations
center not Sun?
lcentr > 0
True
False
kp(82) >= 0
True
call ROTMAT
rotation matrix for integrated body
effect of integrated body's 2nd zonal harmonic interaction with Sun included?
False
equations to integrate:
kp(100)
differences
normal (-1)
Center-to-Probe distance
$R_{c-p}=R_{trgt\_bdy}$
R_{c-p} =
call SOLPRB
Sun-centered probe forces
Sun-centered probe
ncentr <= 0
True
False
nczone<=0 and nctess<=0
T
False
call ROTMAT
rotation matrix for central bodies

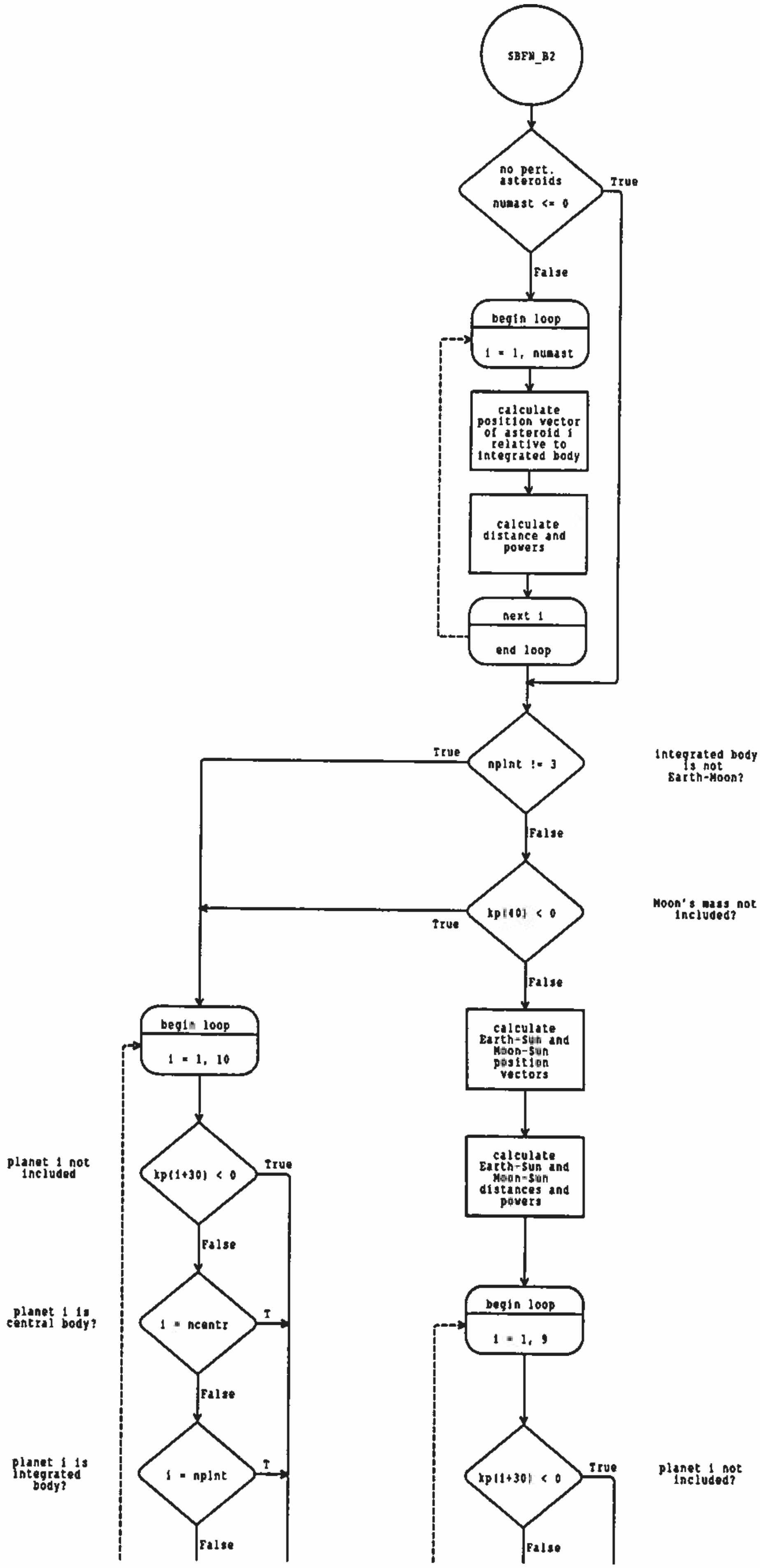


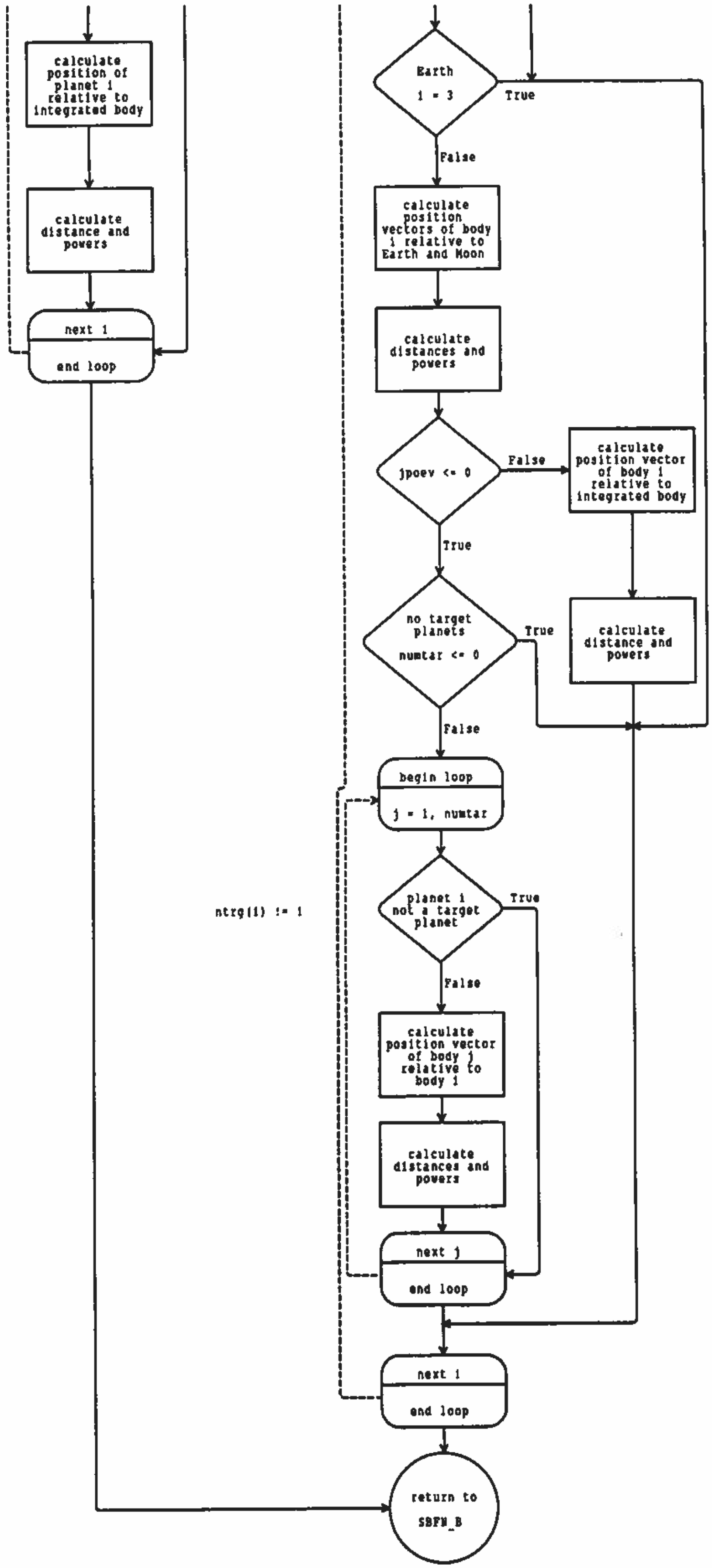
center is Sun
lcentr <= 0
False
True
center not Earth
lcentr != 3
False
True
call ERTORB
circum-Earth forces
center not Moon
lcentr != 10
False
True
call MONORB
circum-Moon forces
int. body is nat. satellite
nplnt <= 30
False
True
call PLNORB
circum-planet forces
call PLNSAT
forces on natural satellite
return to SBFN_A
Earth satelite
Moon satelite
planet orbiter
natural planet satellite

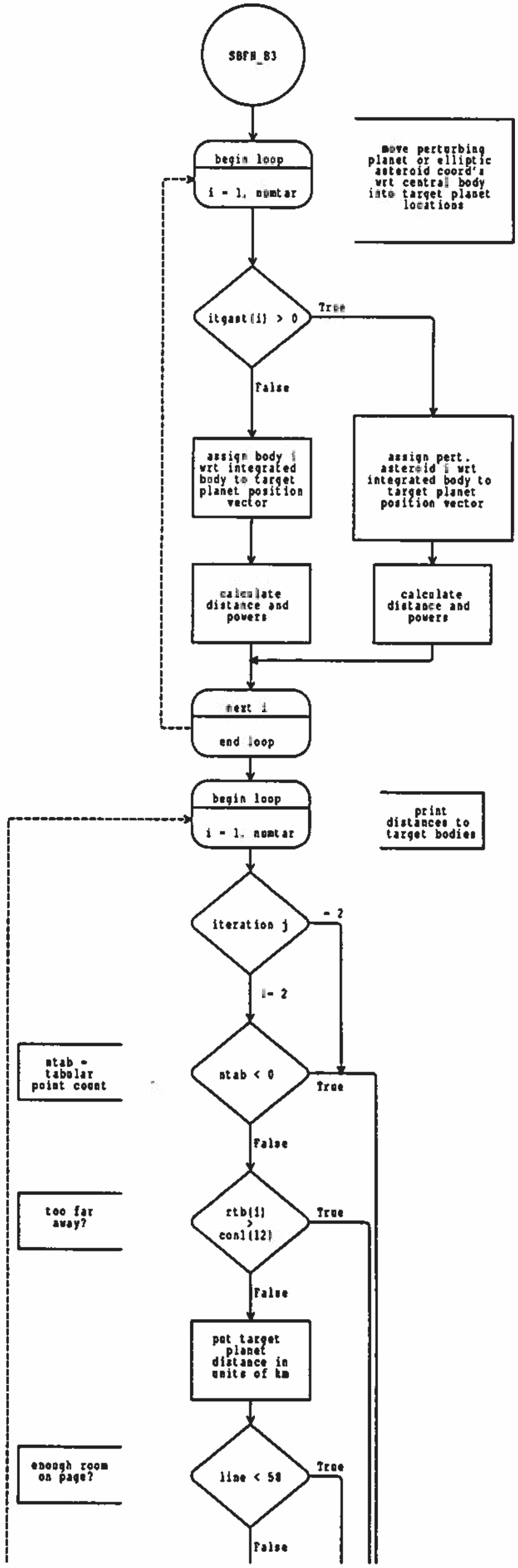

SBFN_B
once per iteration
SBFN_B1
SBFN_B2
no target planets
numtar <= 0
True
False
SBFN_B3
SBFN_B4
return to SBFN


SBFN_B1
the usual equations of motion?
kp(100) < 0
False
True
get coord's of integrated body wrt central body from state vector
get integrated body coord's wrt central body from state + Encke vectors
calculate distance magnitude from central body, Rc1
set step control equal to distance
central body is Sun?
ncentr <= 0
True
False
get integrated body coord's wrt Sun:
center position wrt Sun
+
integrated body wrt center position
calculate magnitude of the distance
output distance from central body
no printout
kkp(81) <= 0
True
False
iteration
j = 2
True
False

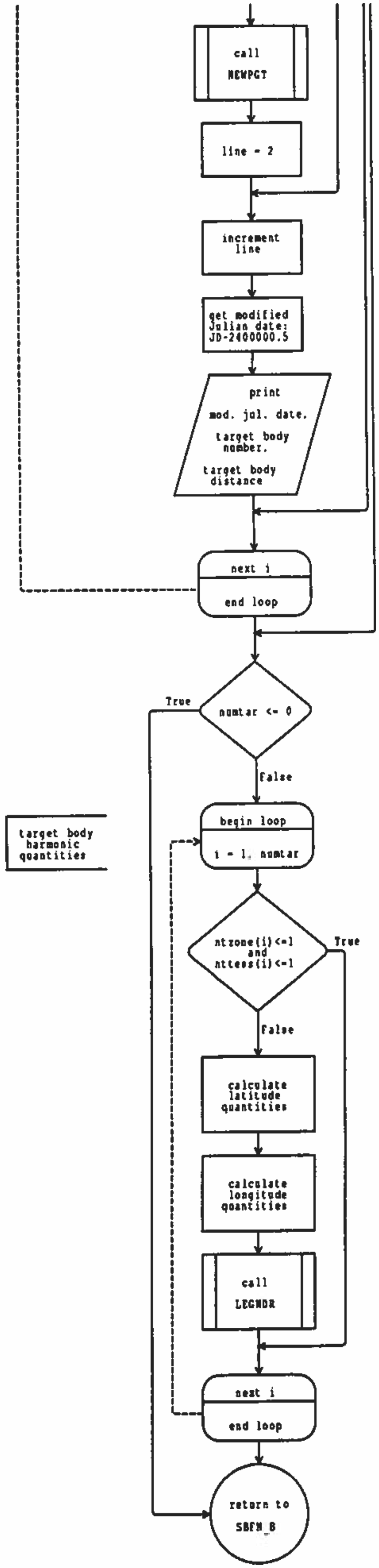

ntab = tabular point counter
ntab < 0
True
False
distance too large?
R_{CI} > conl(12)
True
return to SBFN_B
False
convert probe-center distance to kilometers
enough room on page?
line < 58
True
False
call NEWPG
start new output page
line = 2
increment line pointer
convert time to modified Julian date
write Julian date, central body number, distance from central body
return to SBFN_B


SBFN_B2
no pert. asteroids
numast <= 0
True
False
begin loop
i = 1, numast
calculate position vector of asteroid i relative to integrated body
calculate distance and powers
next i
end loop
nplnt != 3
integrated body is not Earth-Moon?
True
False
kp(40) < 0
Moon's mass not included?
True
False
calculate Earth-Sun and Moon-Sun position vectors
calculate Earth-Sun and Moon-Sun distances and powers
begin loop
i = 1, 10
kp(i+30) < 0
planet i not included
True
False
i = ncentr
planet i is central body?
T
False
i = nplnt
planet i is integrated body?
T
False
begin loop
i = 1, 9
kp(i+30) < 0
planet i not included?
True
False

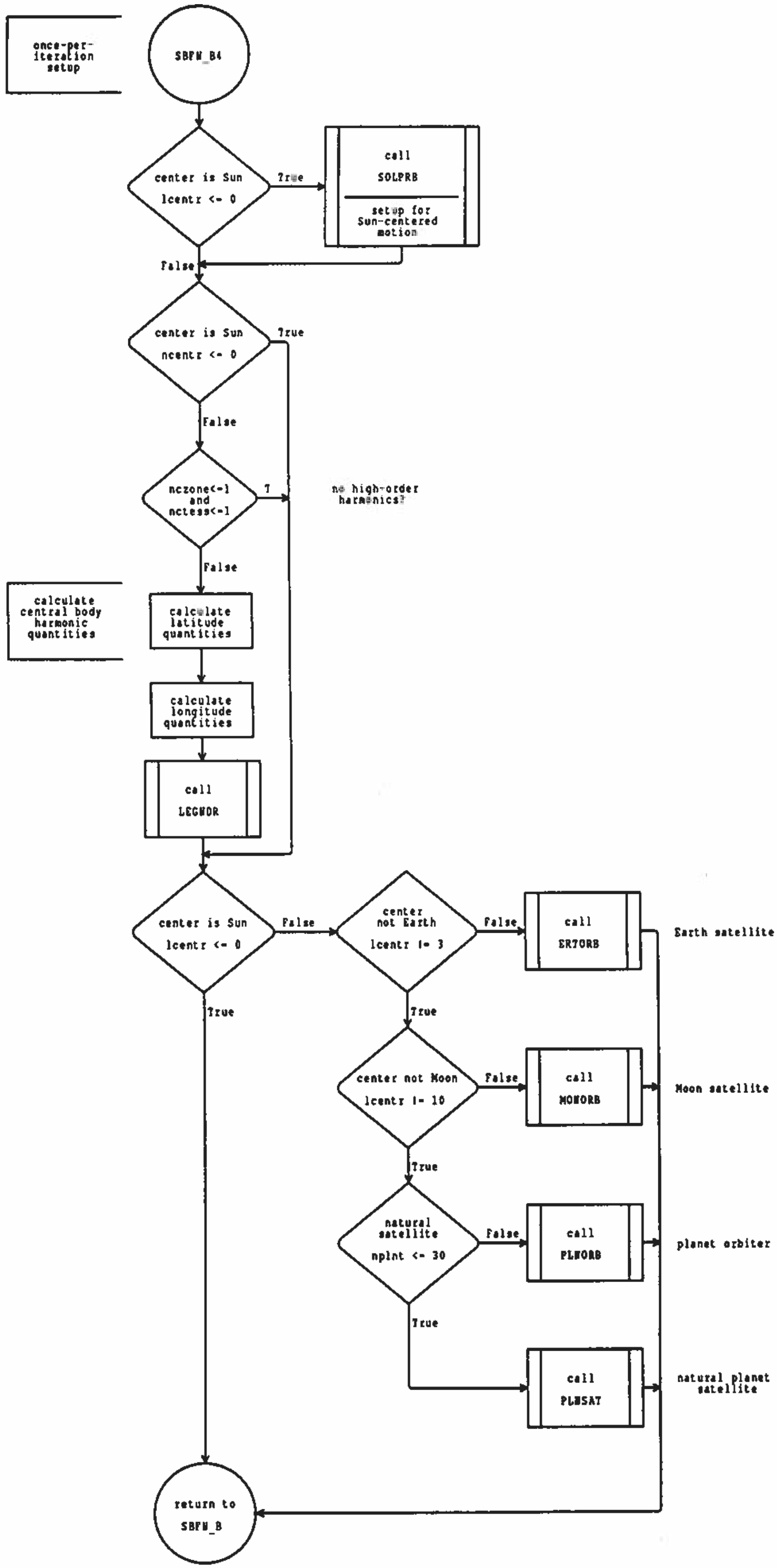

calculate position of planet i relative to integrated body
calculate distance and powers
next i
end loop
Earth
i = 3
True
False
calculate position vectors of body i relative to Earth and Moon
calculate distances and powers
jpoev <= 0
False
calculate position vector of body i relative to integrated body
True
calculate distance and powers
no target planets
numtar <= 0
True
False
begin loop
j = 1, numtar
ntrg(i) != 1
planet i not a target planet
True
False
calculate position vector of body j relative to body i
calculate distances and powers
next j
end loop
next i
end loop
return to
SBFN_B



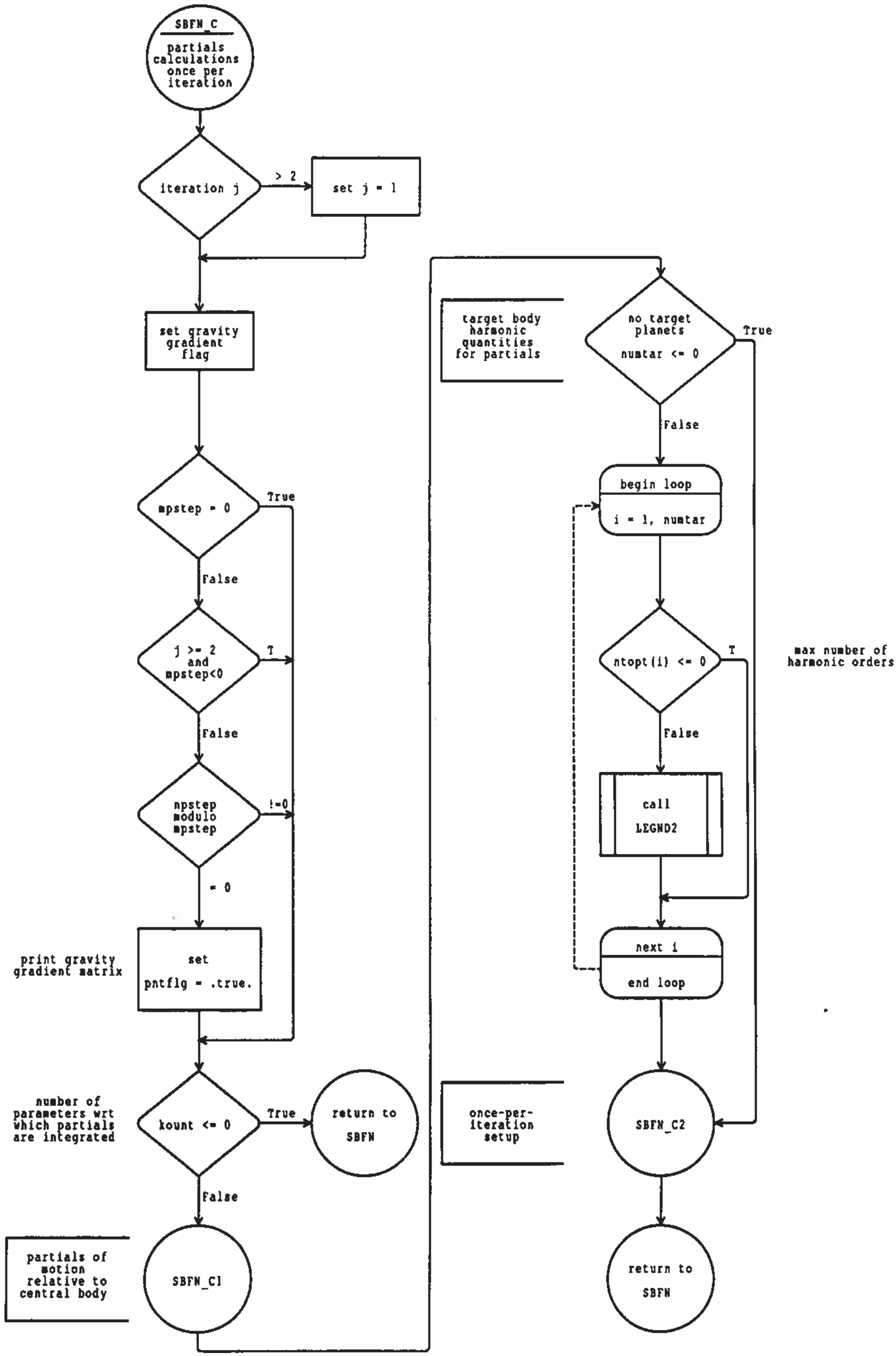
SBFH_B3
begin loop
i = 1, numtar
move perturbing planet or elliptic asteroid coord's wrt central body into target planet locations
itgast(i) > 0
True
False
assign body i wrt integrated body to target planet position vector
assign pert. asteroid i wrt integrated body to target planet position vector
calculate distance and powers
calculate distance and powers
next i
end loop
begin loop
i = 1, numtar
print distances to target bodies
iteration j
= 2
!= 2
ntab = tabular point count
ntab < 0
True
False
too far away?
rtb(i) > con1(12)
True
False
put target planet distance in units of km
enough room on page?
line < 58
True
False

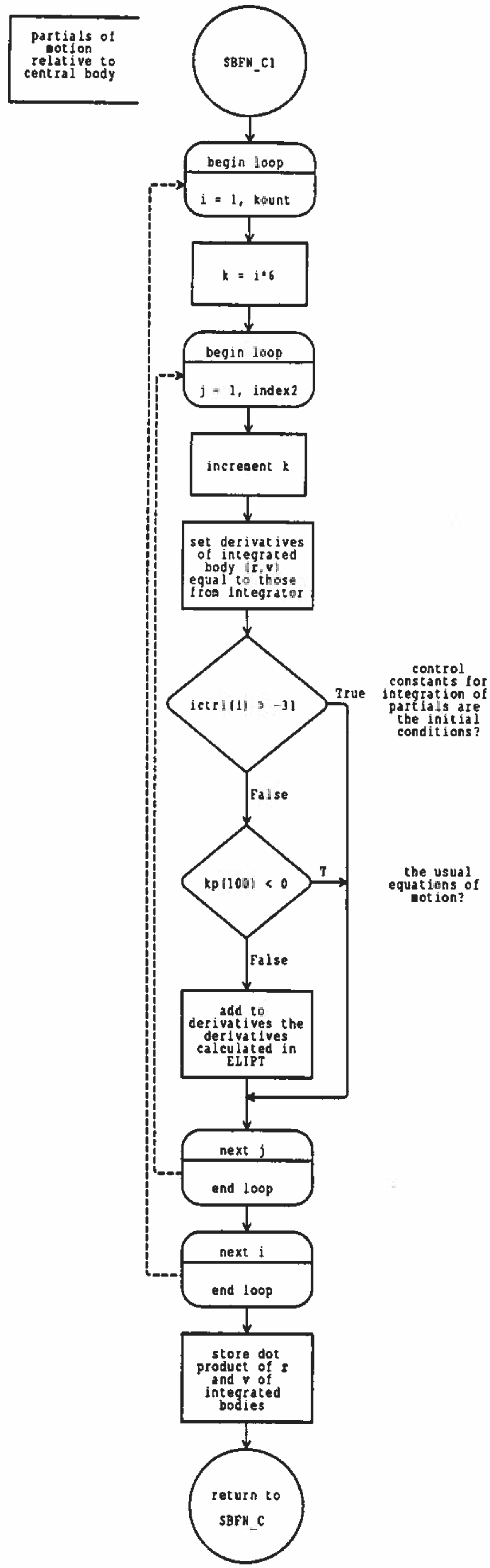


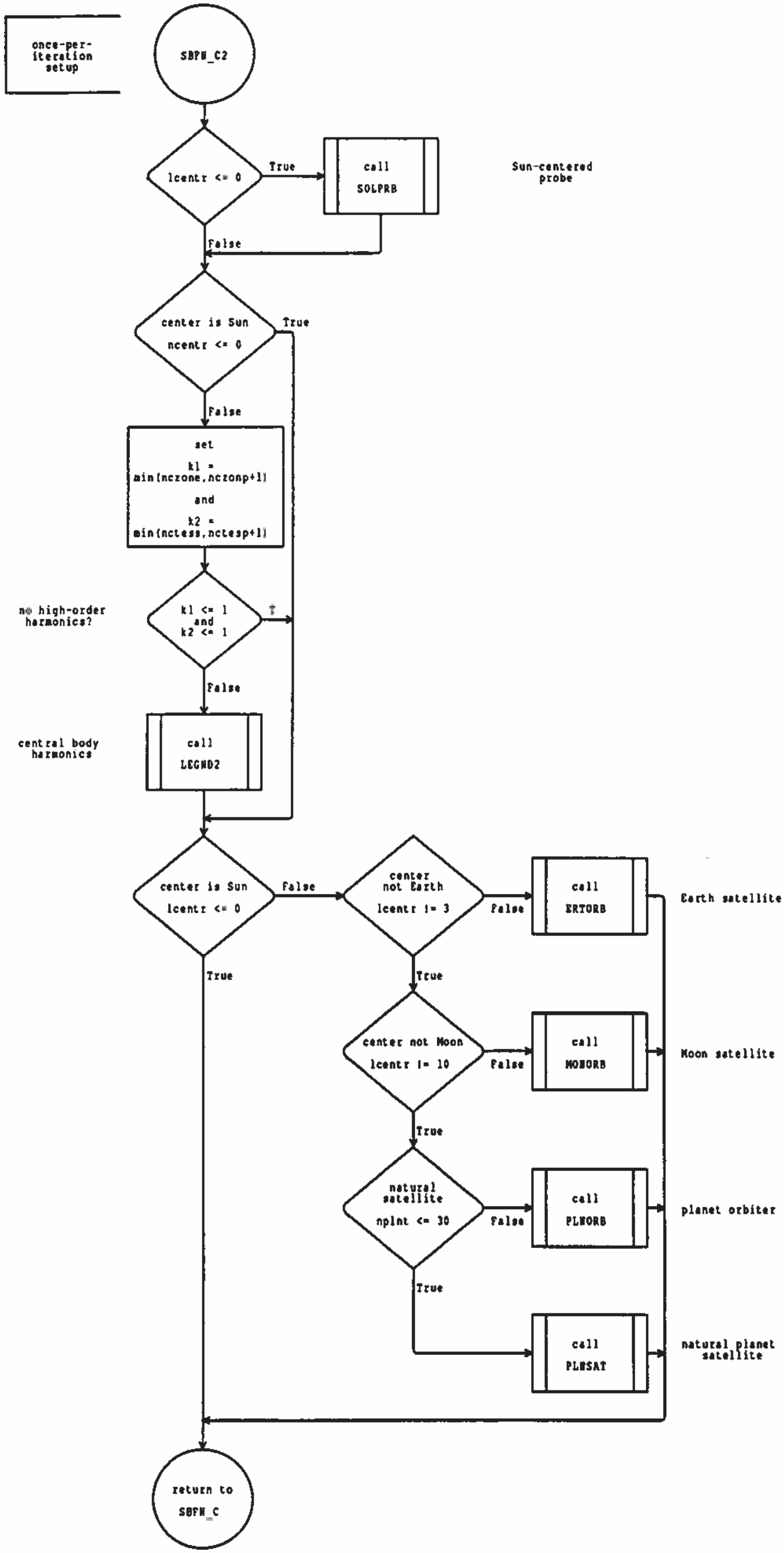
call
NEWPGT
line = 2
increment line
get modified Julian date: JD-2400000.5
print
mod. jul. date.
target body number.
target body distance
next i
end loop
True
numtar <= 0
False
target body harmonic quantities
begin loop
i = 1, numtar
ntzone(i)<=1 and ntteas(i)<=1
True
False
calculate latitude quantities
calculate longitude quantities
call
LEGNDR
next i
end loop
return to
SBFN_B



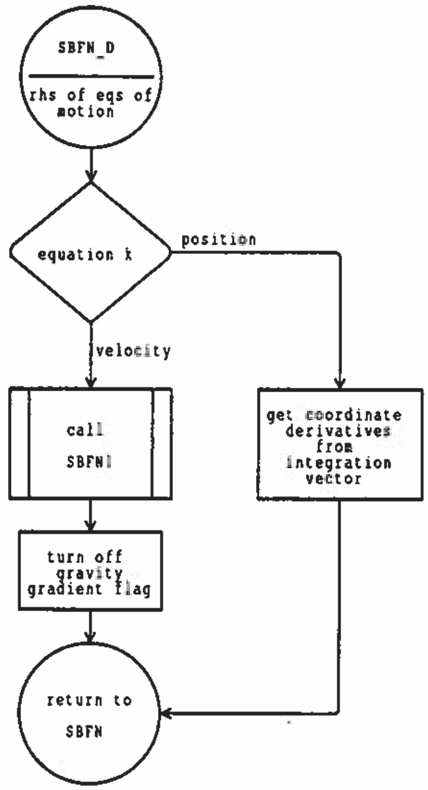
once-per-iteration setup
SBFN_B4
center is Sun
lcentr <= 0
True
call
SOLPRB
setup for Sun-centered motion
False
center is Sun
ncentr <= 0
True
False
nczone<=1 and nctess<=1
T
no high-order harmonics?
False
calculate central body harmonic quantities
calculate latitude quantities
calculate longitude quantities
call
LEGNOR
center is Sun
lcentr <= 0
False
center not Earth
lcentr != 3
False
call
ER7ORB
Earth satellite
True
True
center not Moon
lcentr != 10
False
call
MONORB
Moon satellite
True
natural satellite
nplnt <= 30
False
call
PLNORB
planet orbiter
True
call
PLNSAT
natural planet satellite
return to
SBFN_B


SBFN_C
partials calculations once per iteration
iteration j
> 2
set j = 1
set gravity gradient flag
mpstep = 0
True
False
j >= 2 and mpstep<0
T
False
npstep modulo mpstep
!=0
= 0
print gravity gradient matrix
set
pntflg = .true.
number of parameters wrt which partials are integrated
kount <= 0
True
return to SBFN
False
partials of motion relative to central body
SBFN_C1
target body harmonic quantities for partials
no target planets
numtar <= 0
True
False
begin loop
i = 1, numtar
ntopt(i) <= 0
T
max number of harmonic orders
False
call
LEGND2
next i
end loop
once-per-iteration setup
SBFN_C2
return to SBFN


partials of motion relative to central body
SBFN_C1
begin loop
i = 1, kount
k = i*6
begin loop
j = 1, index2
increment k
set derivatives of integrated body (r,v) equal to those from integrator
ictrl(i) > -31
True
control constants for integration of partials are the initial conditions?
False
kp(100) < 0
T
the usual equations of motion?
False
add to derivatives the derivatives calculated in ELIPT
next j
end loop
next i
end loop
store dot product of r and v of integrated bodies
return to
SBFN_C



once-per-iteration setup
SBFN_C2
lcentr <= 0
True
call SOLPRB
Sun-centered probe
False
center is Sun
ncentr <= 0
True
False
set
k1 = min(nczone,nczonp+1)
and
k2 = min(nctess,nctesp+1)
no high-order harmonics?
k1 <= 1 and k2 <= 1
T
False
central body harmonics
call LEGND2
center is Sun
lcentr <= 0
False
center not Earth
lcentr != 3
False
call ERTORB
Earth satellite
True
True
center not Moon
lcentr != 10
False
call MONORB
Moon satellite
True
natural satellite
nplnt <= 30
False
call PLNORB
planet orbiter
True
call PLNSAT
natural planet satellite
return to SBFN_C


SBFN_D
rhs of eqs of motion
equation k
position
velocity
call SBFNI
get coordinate derivatives from integration vector
turn off gravity gradient flag
return to SBFN

SBFN1

subroutine **SBFN1 (k)**

Description:	SBFN1 evaluates the equations of motion and partial derivatives during numerical integrations. This routine is the natural extension of the setup computations performed in SBFN.		
Arguments:	k	Equation number. There are six equations for the motion and six for each partial derivative. A detailed description of k is provided elsewhere.	
Calls:	DOT, ERTORB, MONORB, NEWPGT, PLNORB, PLNSAT, PRODCT, SOLPRB		
Called by:	SBFN		
Includes:	CNTHAR, DRGPRT, ELLIPS, EMMIPS, FCNTRL, FMMIPS, INCON, INODTA, NAMTIMQ, PARAM, PETUNA, RTPARS, SBEMBR, SBROT, SBSTUF, SBTHNG, TRGHAR, YVECTPLP		
Equivalence:	dadxc(3,3)	/sbstuf/	SBSTUF
	ggfg	/sbstuf/	SBSTUF
	ggfgs(2)	/sbstuf/	SBSTUF

SBFN1 Equations

Subroutine SBFN1 does most of the rhs calculations of the equations of motion, both for the position accelerations and for the partials "accelerations". The equations of motion, both in this routine and in PEP in general, are integrated in an inertial frame of reference. For output purposes, though, coordinates are expressed as differences, usually with respect to the position of the central body.

This document is in three parts. The first part describes the equations of motion integrated by PEP for integrations other than the N-body integration performed by BODFN. The effects included in the equations of motion fall into three categories. First, there is the acceleration due to the central body. Second, we have various perturbing accelerations, including pertubing planets, perturbing asteroids, harmonics of the gravitational potentials of the target and central bodies, and acceleration arising from violation of the principle of equivalence. Third, there are various accelerations whose nature depends on the particular environment through which the integrated body finds itself traveling; these are dealt with via calls to appropriate special-case routines. The order in which the following is presented is that found in the source code itself.

The second part of the document deals with the calculation of the "gravity gradient" matrices. The physical effects giving rise to the contributions to these matrices include the perturbing planets, perturbing asteroids, target and central body harmonics, and the Sun and the central body. These are explained in detail in section II.

Finally, section III presents the equations that describe the acceleration of the derivatives of the motion with respect to various parameters. PEP uses these derivatives to calculate the sensitivity of results to changes in the parameters. These parameters include the masses of the perturbing planets, masses of the perturbing asteroids, time variation of the gravitational constant, the principle of equivalence violation parameter, target and central body zonal and tesseral harmonic coefficients, target and central body masses, the masses of the target planet central bodies, the central body rotation parameters, parameters of the equations in the special-cases routines referred to above, and the initial conditions of the central and target bodies.

In PEP, planetary masses are in units of the mass of the Sun. To emphasize this, the equations described here explicitly show a division by the Sun's mass. These mass ratios are the parameters actually used in PEP, and partial derivatives with respect to planet masses are actually with respect to the mass ratios. Similarly, the masses of satellites (including the Moon) are parameterized as a fraction of the system mass (planet + satellites); this also is explicitly shown in this document.

In several of the equations which follow, the term $\left(1+\frac{M_B}{M_C}\right)$ appears. M_C is the central mass, either the mass of the Sun or the mass of the system containing M_C; and M_B is the mass of the integrated body. This term is actually a shorthand for a more general expression which reduces to this one in the usual case. The exception is the case of an integrated natural satellite, for which the general expression is $\left(\frac{M_C-\Sigma M_{sat}}{M_C}+\frac{M_B}{M_C}\right)$

Throughout this document, the term "scaled acceleration" will refer to the actual acceleration divided by the (possibly time-varying) gravitational constant.

I. Equations of Motion

A. Perturbing Accelerations

1. Perturbing Planets

The first acceleration of the integrated body (planet) that is taken into account is that due to the other 8 planets. If the integrated body is the Earth-Moon barycenter, then, its scaled acceleration towards its center (the Sun) due to the other planets is

$$\vec{a}_{CP} = \sum_{\substack{k=1 \\ k\neq 3}}^{9} \frac{M_k}{M_\odot}\left(\mu_E \frac{\vec{r}_{\oplus k}}{r_{\oplus k}^3} + \mu_M \frac{\vec{r}_{Mk}}{r_{Mk}^3} - \frac{\vec{r}_{Ck}}{r_{Ck}^3} \right)$$

where $\vec{r}_{\alpha\beta} \equiv \vec{r}_\beta - \vec{r}_\alpha$, $\vec{r}_\alpha$ is in general the position vector of body α with respect to the central body, and

$$\mu_E = \frac{M_\oplus}{M_\oplus + M_M} \qquad \mu_M = \frac{M_M}{M_\oplus + M_M}$$

For any other integrated body (planet) B, the scaled acceleration with respect to the central body due to all other planets is

$$\vec{a}_{CP} = \sum_{\substack{k=1 \\ k\neq B}}^{10} \frac{M_k}{M_C}\left(\frac{\vec{r}_{Bk}}{r_{Bk}^3} - \frac{\vec{r}_{Ck}}{r_{Ck}^3} \right)$$

where body 10 is the Moon.

2. Principle of Equivalence Violation

The scaled acceleration of the integrated body B with respect to the central body C due to violation of the principle of equivalence for the planets is

$$\vec{a}_{C\eta P} = \sum_{\substack{k=1 \\ k\neq B}}^{10} \frac{M_k}{M_\odot}\left[\left(\eta\Delta_B + \eta'\Delta_B'\right)\frac{\vec{r}_{Bk}}{r_{Bk}^3} - \left(\eta\Delta_\odot + \eta'\Delta_\odot'\right)\frac{\vec{r}_{Ck}}{r_{Ck}^3} \right]$$

where $\eta = 4\beta - \gamma - 3$ is the Nordtvedt parameter, and Δ is the ratio of internal gravitational energy to total energy. The code is written as above, but it is possible that $\Delta_\odot$ refers to the central body, which is not necessarily the Sun. The formalism incorporated by PEP includes only the PPN parameters β and γ. The parameter γ is a measure of the curvature induced per unit mass, and β measures the nonlinearity in the superposition law for gravity. In GR, both β and γ are equal to one. The other parameters in the PPN formalism involve preferred-frame and nonconservation effects, and are zero in conservative theories of gravity, such as GR, with no preferred-frame effects. The primed quantities in the above equation are included in order to allow these effects in PEP; they are to be thought of as an additional contribution to η.

3. Perturbing Asteroids

The scaled acceleration of the integrated body with respect to the central body due to perturbing asteroids is

$$\vec{a}_{Cast} = \sum_{k=1}^{N_{ast}} \frac{M_k}{M_{\odot}} \left(\frac{\vec{r}_{Bk}}{r_{Bk}^3} - \frac{\vec{r}_{Ck}}{r_{Ck}^3} \right)$$

where N_{ast} is the number of perturbing asteroids taken into account.

The accelerations corresponding to a_{CP}, $a_{C\eta P}$, and a_{Cast} are found by multiplying by

$$\gamma(t) \equiv k_G^2 \left[1 + \alpha_G (t - t_0) \right]$$

where α_G is the time variation factor for k_G, and k_G is the Gaussian gravitational constant. Thus,

$$\vec{F}_P + \vec{F}_\eta + \vec{F}_{ast} = \gamma(t) \left(\vec{a}_{CP} + \vec{a}_{C\eta P} + \vec{a}_{Cast} \right)$$

4. Target Body Gravitational Harmonics

Before illustrating the implementation of gravitational harmonics in PEP, it is worthwhile to develop the neccessary formalism. Consider the potential of a spheroidal body of mass M and equatorial radius R_e. The point-mass potential plus the expansions in zonal and tesseral harmonics can be written in the standard form

$$U = U_0 + U_{zone} + U_{tess}$$

axisymmetric

$$= -\frac{GM}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\phi)\right.$$

non axisymmetric

$$\left. + \sum_{n=2}^{\infty}\sum_{m=1}^{n} \left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right)\left(\frac{R_e}{r}\right)^n P_n^m(\sin\phi)\right]$$

where **r** is the position vector from the center of mass of M to the field point, J_n is the zonal harmonic coefficient, $P_n(z)$ is the Legendre polynomial of degree n of z, ϕ is the latitude of the field point in the body coordinates, λ is the longitude in these same coordinates, $P_n^m(z)$ is the associated Legendre function of degree n and order m of z, and G is the gravitational constant. The first sum is the expansion in zonal harmonics, representing an axisymmetric field. The double sum is the expansion in tesseral harmonics, allowing nonaxisymmetric modes. For the tesseral expansion, it is convenient (and standard practice) to split the coefficients into sine and cosine terms, as shown.

The body coordinates are such that the origin is at the center of mass and the z axis is coincident with the symmetry axis of M. Due to the location of the origin at the center of mass, the n=1 term is zero. The acceleration due to this potential field is found by taking the gradient of U,

$$\vec{F} = -\vec{\nabla} U = \vec{F}_0 + \vec{F}_{zone} + \vec{F}_{tess}$$

where

$$\vec{F}_0 = -\frac{GM}{r^2}\frac{\vec{r}}{r}$$

$$\vec{F}_{zone} = +\frac{GM}{r^2}\sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n \left[(n+1)\,P_n(\sin\phi)\,\frac{\vec{r}}{r} - r\,\frac{\partial \sin\phi}{\partial \vec{r}}\,\frac{\partial P_n(\sin\phi)}{\partial \sin\phi}\right]$$

$$\vec{F}_{tess} = -\frac{GM}{r^2}\sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \left(T_{nm}\vec{A}_{nm} + T'_{nm}\vec{B}_{nm}\right)$$

and

$$T_{nm} = C_{nm}\cos m\lambda + S_{nm}\sin m\lambda$$

$$T'_{nm} = -C_{nm}\sin m\lambda + S_{nm}\cos m\lambda$$

$$\vec{A}_{nm} = (n+1)P_n^m(\sin\phi)\frac{\vec{r}}{r} - r\frac{\partial\sin\phi}{\partial\vec{r}}\frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi}$$

$$\vec{B}_{nm} = -m\,P_n^m(\sin\phi)\,r\frac{\partial\lambda}{\partial\vec{r}}$$

Now, if a different coordinate frame is being used than the local body frame just mentioned, an appropriate transformation must be made. Let **u** be the body frame position vector. Then

$$\vec{u} = \mathbb{R}\vec{r}$$

where $\mathbb{R}$ is the transformation matrix from the external coordinates **r** to the body coordinates (In PEP, $\mathbb{R}$ includes rotations from the equator and equinox of 1950.0, planet wobble [Earth only], planet nutation [Earth only], and precession). The sine of ϕ is then

$$\sin\phi = \frac{u_3}{r} = \frac{1}{r}\sum_{i=1}^{3}\mathbb{R}_{3i}x_i$$

and the components of the gradient required for the force equations above become

$$r\frac{\partial\sin\phi}{\partial x_i} = \mathbb{R}_{3i} - \frac{x_i}{r}\sin\phi$$

The components of the longitude gradient can be found as follows. Using $\mathbb{R}$ to transform to body coordinates, we have

$$\sin\lambda = \frac{1}{r\cos\phi}\sum_{i=1}^{3}\mathbb{R}_{2i}x_i$$

$$\cos\lambda = \frac{1}{r\cos\phi}\sum_{i=1}^{3}\mathbb{R}_{1i}x_i$$

From the sine term, we find

$$r\frac{\partial\lambda}{\partial x_i} = \frac{1}{\cos\phi\,\cos\lambda}\mathbb{R}_{2i} - \frac{x_i}{r}\tan\lambda - \frac{1}{\cos\phi}\,r\frac{\partial\cos\phi}{\partial x_i}\tan\lambda$$

From the cosine term,

$$r\frac{\partial\lambda}{\partial x_i} = -\frac{1}{\cos\phi\,\sin\lambda}\mathbb{R}_{1i} + \frac{x_i}{r}\cot\lambda + \frac{1}{\cos\phi}\,r\frac{\partial\cos\phi}{\partial x_i}\cot\lambda$$

Combine these to get

$$r\frac{\partial\lambda}{\partial x_i} = \frac{1}{\cos\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)$$

which is the equation used by PEP.

Now we turn to the implementation of the force terms in PEP. The acceleration of the integrated body with respect to the central body due to gravitational harmonics of the target bodies is

$$\vec{F}_{TH} = \gamma(t)\sum_{k=1}^{N_{tar}}\frac{M_k}{M_\odot}\left(\vec{a}^{\,k}_{CTZ} + \vec{a}^{\,k}_{CTT}\right)$$

where the acceleration terms are given below.

a) Zonal Harmonics

The acceleration of the integrated body with respect to the central body due to the zonal harmonics of target body k is coded in the form

$$\vec{a}^{\,k}_{CTZ} = \frac{1}{|\vec{r}_{Bk}|^2} \sum_{n=1}^{N_z - 1} J^k_{n+1} \left(\frac{R_k}{|\vec{r}_{Bk}|} \right)^{n+1} \left[-(n+2)\, P_{n+1}(\sin\phi_k) \frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|} - |\vec{r}_{Bk}| \frac{\partial \sin\phi_k}{\partial(-\vec{r}_{Bk})} \frac{\partial P_{n+1}(\sin\phi_k)}{\partial z_k} \right]$$

where J_n^k is the zonal harmonic coefficient for the n^{th} zone of target body k, and ϕ_k is the latitude of the integrated body as seen in the body-fixed coordinate frame of body k. Let **u** be the position vector in this frame. The origin is at the center of mass, the u_3 axis is aligned with the rotation axis, and the u_1 axis points toward the intersection of the equator of target body k and the mean ecliptic of 1950.0.

b) Tesseral Harmonics

As implemented in PEP, the acceleration of the integrated body with respect to the central body due to the target body tesseral harmonics is

$$\vec{a}^{\,k}_{CTT} = \frac{1}{|\vec{r}_{Bk}|^2} \sum_{n=1}^{N_t - 1} \sum_{m=1}^{n+1} \Big[A^k_{nm} \left(C^k_{nm} \cos m\lambda_k + S^k_{nm} \sin m\lambda_k \right) + B^k_{nm} \left(-C^k_{nm} \sin m\lambda_k + S^k_{nm} \cos m\lambda_k \right) \Big]$$

where S_{nm}^k is the sine coefficient and C_{nm}^k is the cosine coefficient for the tesseral harmonic of degree n and order m for body k, and where

$$A^k_{nm} = \left(\frac{R_k}{|\vec{r}_{Bk}|} \right)^{n+1} \left[(n+2)\, P^m_{n+1}(\sin\phi_k) \frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|} + |\vec{r}_{Bk}| \frac{\partial \sin\phi_k}{\partial(-\vec{r}_{Bk})} \frac{\partial P^m_{n+1}(\sin\phi_k)}{\partial \sin\phi_k} \right]$$

$$B^k_{nm} = \left(\frac{R_k}{|\vec{r}_{Bk}|} \right)^{n+1} m\, P^m_{n+1}(\sin\phi_k)\, |\vec{r}_{Bk}| \frac{\partial \lambda_k}{\partial(-\vec{r}_{Bk})}$$

which agrees with what we expect from F_{tess} above.

5. *Sun*

The acceleration due to the Sun for non-Sun-centered bodies is

$$\vec{F}_{\odot} = \gamma(t)\,\vec{a}_{C\odot} = \gamma(t)\left(\frac{\vec{r}_{\odot C}}{r^3_{\odot C}} - \frac{\vec{r}_{\odot B}}{r^3_{\odot B}}\right)$$

6. *Central Body Gravitational Harmonics*

The acceleration of the integrated body with respect to the central body due to gravitational harmonics of the central body itself is

$$\vec{F}_{CH} = \gamma(t)\,\frac{M_C}{M_{\odot}}\left(1 + \frac{M_B}{M_{\odot}}\right)(\vec{a}_{CCZ} + \vec{a}_{CCT})$$

where the acceleration terms are as follows. The equations have the same derivation as shown above for the target body harmonics.

a. Zonal Harmonics

The acceleration term of the integrated body with respect to the central body due to the zonal harmonics of the central body is implemented in PEP as

$$\vec{a}_{CCZ} = \frac{1}{|\vec{r}_{CB}|^2}\sum_{n=1}^{N_z-1} J^C_{n+1}\left(\frac{R_C}{|\vec{r}_{CB}|}\right)^{n+1}\left[(n+2)\,P_{n+1}(\sin\phi_C)\,\frac{\vec{r}_{CB}}{|\vec{r}_{CB}|} - |\vec{r}_{CB}|\,\frac{\partial \sin\phi_C}{\partial \vec{r}_{CB}}\,\frac{\partial P_{n+1}(\sin\phi_C)}{\partial \sin\phi_C}\right]$$

where J^C_n is the central body zonal harmonic coefficient for the n^{th} zone, and R_C is the equatorial radius of the central body.

b. Tesseral Harmonics

The acceleration of the integrated body with respect to the central body due to the central body tesseral harmonics is coded as

$$\vec{a}_{CCT} = \frac{1}{|\vec{r}_{CB}|^2} \sum_{n=1}^{N_t-1} \sum_{m=1}^{n+1} \Big[A^C_{nm} \left(C^C_{nm} \cos m\lambda_C + S^C_{nm} \sin m\lambda_C \right)$$

$$+ B^C_{nm} \left(- C^C_{nm} \sin m\lambda_C + S^C_{nm} \cos m\lambda_C \right) \Big]$$

where S^C_{nm} is the sine coefficient and C^C_{nm} is the cosine coefficient for the tesseral harmonic of degree n and order m, and where

$$A^C_{nm} = \left(\frac{R_C}{|\vec{r}_{CB}|} \right)^{n+1} \left[-(n+2)\, P^m_{n+1}(\sin\phi_C) \frac{\vec{r}_{CB}}{|\vec{r}_{CB}|} + |\vec{r}_{CB}| \frac{\partial \sin\phi_C}{\partial \vec{r}_{CB}} \frac{\partial P^m_{n+1}(\sin\phi_C)}{\partial \sin\phi_C} \right]$$

$$B^C_{nm} = \left(\frac{R_C}{|\vec{r}_{CB}|} \right)^{n+1} m\, P^m_{n+1}(\sin\phi_C) |\vec{r}_{CB}| \frac{\partial \lambda_C}{\partial \vec{r}_{CB}}$$

B. Special-Case Accelerations

Accelerations specific to a

Sun-centered probe
Earth satellite
Lunar orbiter
planetary orbiter
natural planetary satellite

are evaluated via calls to subroutines SOLPRB, ERTORB, MONORB, PLNORB, and PLNSAT.

C. Central Force

1. Earth-Moon Barycenter

The acceleration, due to the Sun, of the Earth-Moon barycenter with respect to the Sun is

$$\vec{F}_C = \gamma(t)\,\vec{a}_{CC} = -\gamma(t)\left(1 + \frac{M_{\oplus} + M_M}{M_{\odot}}\right)\left(\mu_E \frac{\vec{r}_{C\oplus}}{r_{C\oplus}^3} + \mu_M \frac{\vec{r}_{CM}}{r_{CM}^3}\right)$$

2. All Other Bodies

For any body but the Earth-Moon barycenter as integrated body, the force with respect to the central body due to the central body is

$$\vec{F}_C = \gamma(t)\,\frac{M_C}{M_{\odot}}\left(1 + \frac{M_B}{M_C}\right)\vec{a}_{CC} = -\gamma(t)\,\frac{M_C}{M_{\odot}}\left(1 + \frac{M_B}{M_C}\right)\frac{\vec{r}_{CB}}{r_{CB}^3}$$

3. Subtraction of Mean Orbit Acceleration

Currently not implemented.

4. Subtraction of Elliptic Orbit Acceleration

The scaled acceleration to be added is that due to the central body of an elliptical orbit,

$$\vec{a}_{\varepsilon} = -\frac{\vec{r}_{\varepsilon}}{r_{\varepsilon}^3}$$

where r_{ε} is the Encke orbit position with respect to the central body. The acceleration is then

$$\vec{F}_e = \gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\vec{a}_{Ce}$$

The Encke integration was added to PEP in order to provide more accuracy.

5. Violation of Principle of Equivalence

The acceleration with respect to the central body due to violation of the equivalence principle is

$$\vec{F}_{\eta C} = \gamma(t)\,\vec{a}_{C\eta} = -\gamma(t)\left[(\eta\Delta_B + \eta'\Delta'_B) + \left(\eta\Delta_\odot + \frac{M_B}{M_\odot}\eta'\Delta'_\odot\right)\right]\cdot\vec{a}_{CC}$$

II. Partial Derivatives--Gravity Gradient Matrices

The derivatives of the motion (position) with respect to parameters α can be found by integrating the derivatives of the acceleration along with the equations of motion. Thus, for a position vector **x**,

$$\frac{dx}{d\alpha} = \int\int \frac{d\ddot{x}}{d\alpha}\, dt\, dt$$

The integrand is

$$\frac{d\ddot{x}}{d\alpha} = \frac{\partial \ddot{x}}{\partial x} \cdot \frac{\partial x}{\partial \alpha} + \frac{\partial \ddot{x}}{\partial \dot{x}} \cdot \frac{\partial \dot{x}}{\partial \alpha} + \frac{\partial \ddot{x}}{\partial \alpha}$$

Or, in component form,

$$\frac{d\ddot{x}_j}{d\alpha} = \frac{\partial \ddot{x}_j}{\partial x_k} \frac{\partial x_k}{\partial \alpha} + \frac{\partial \ddot{x}_j}{\partial \dot{x}_k} \frac{\partial \dot{x}_k}{\partial \alpha} + \frac{\partial \ddot{x}_j}{\partial \alpha}$$

with implied summation over repeated index k. The third term is zero if α is an initial condition, since the acceleration functions are independent of the initial conditions. If there is no velocity dependence in the acceleration functions, then the second term is also zero. We are then left with just the first term, the first part of which we will refer to as the gravity gradient matrix:

$$G_{jk} \equiv \frac{\partial \ddot{x}_j}{\partial x_k}$$

Since **G** is the second derivative of the gravitational potential, which satisfies Laplace's equation, it is therefore a symmetric matrix.

In this section we list all the gravity gradient matrix contributions calculated by PEP. The gravity gradient matrix is composed of the sum of the individual contributions $\mathbf{G}^{\alpha}$:

$$G = \sum_{\alpha} G^{\alpha}$$

$$\alpha \in \{P, ast, TZ, TT, \odot, CZ, CT, C\}$$

where

P = perturbing planets

ast = perturbing asteroids

TZ = target body zonal harmonics

TT = target body tesseral harmonics

$\odot$ = Sun

CZ = central body zonal harmonics

CT = central body tesseral harmonics

C = central body

A. Perturbing Bodies

1. Earth-Moon Barycenter

The contribution to the gravitational gradient matrix for the Earth-Moon barycenter due to the other planets is

$$G_{jk}^{P} = -\gamma(t) \sum_{\substack{\lambda=1 \\ \lambda \neq 3}}^{9} \frac{M_{\lambda}}{M_{\odot}} \left\{ \left[\mu_E \frac{3x_{\oplus\lambda}^{j} x_{\oplus\lambda}^{k}}{|\vec{r}_{\oplus\lambda}|^5} + \mu_M \frac{3x_{M\lambda}^{j} x_{M\lambda}^{k}}{|\vec{r}_{\oplus\lambda}|^5} \right] + \delta_{jk} \left[\mu_E \frac{1}{|\vec{r}_{\oplus\lambda}|^3} + \mu_M \frac{1}{|\vec{r}_{M\lambda}|^3} \right] \right\}$$

where $x_{\alpha\beta}^{k}$ is the kth component of the position vector $\vec{r}_{\alpha\beta}$, and $\gamma(t) = k_G^2[1 + \alpha_G(t - t_0)]$

where α_G is the time variation factor for k_G, and k_G is the Gaussian gravitational constant. In PEP, we have

$$k_G = 0.01720209895\ (AU)^{\frac{3}{2}}\ (ephemeris\ day)^{-1}\ (M_\odot)^{-\frac{1}{2}}$$

which is the standard value as specified by the IAU (1976) System of Astronomical Constants.

2. All Other Planets

For all planets other than the Earth-Moon system, we have, for integrated planet B,

$$G_{jk}^{P} = \gamma(t) \sum_{\substack{\lambda=1 \\ \lambda \neq B}}^{10} \frac{M_\lambda}{M_\odot} \left\{ \frac{3 x_{B\lambda}^{j} x_{B\lambda}^{k}}{|\vec{r}_{B\lambda}|^5} - \delta_{jk} \frac{1}{|\vec{r}_{B\lambda}|^3} \right\}$$

3. Perturbing Asteroids

The contribution due to perturbing asteroids is, for planet B,

$$G_{jk}^{ast} = \gamma(t) \sum_{\lambda=1}^{N_{ast}} \frac{M_\lambda}{M_\odot} \left\{ \frac{3 x_{B\lambda}^{j} x_{B\lambda}^{k}}{|\vec{r}_{B\lambda}|^5} - \frac{\delta_{jk}}{|\vec{r}_{B\lambda}|^3} \right\}$$

B. Target Bodies

Before showing the PEP implementation of the gravity gradient matrices for the target and central body harmonics, we briefly review the formalism. Remember the acceleration vectors due to a spheroid,

$$\vec{F}_0 = -\frac{GM}{r^2}\frac{\vec{r}}{r}$$

$$\vec{F}_{zone} = +\frac{GM}{r^2}\sum_{n=2}^{\infty} J_n\left(\frac{R_e}{r}\right)^n\left[(n+1)\,P_n(\sin\phi)\,\frac{\vec{r}}{r} - r\,\frac{\partial\sin\phi}{\partial\vec{r}}\,\frac{\partial P_n(\sin\phi)}{\partial\sin\phi}\right]$$

$$\vec{F}_{tess} = -\frac{GM}{r^2}\sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n\left(T_{nm}\vec{A}_{nm} + T'_{nm}\vec{B}_{nm}\right)$$

where

$$T_{nm} = C_{nm}\cos m\lambda + S_{nm}\sin m\lambda$$

$$T'_{nm} = -C_{nm}\sin m\lambda + S_{nm}\cos m\lambda$$

$$\vec{A}_{nm} = (n+1)\,P_n^m(\sin\phi)\,\frac{\vec{r}}{r} - r\,\frac{\partial\sin\phi}{\partial\vec{r}}\,\frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi}$$

$$\vec{B}_{nm} = -m\,P_n^m(\sin\phi)\,r\,\frac{\partial\lambda}{\partial\vec{r}}$$

and **r** is the position vector referenced from the center of mass of the spheroid. The gradient of the spherical contribution produces the matrix

$$G^0_{ik} = \frac{GM}{r^3}\left(\frac{3\,x_i x_k}{r^2} - \delta_{ik}\right)$$

The gradient of the zonal harmonic term results in

$$G_{ik}^{zone} = \frac{GM}{r^3}\sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n \Bigg\{ \left[\delta_{ik} - (n+3)\,\frac{x_i x_k}{r^2} \right](n+1)\,P_n(\sin\phi)$$

$$+ \left(\frac{\partial \sin\phi}{\partial x_i}\,x_k + \frac{\partial \sin\phi}{\partial x_k}\,x_i \right)(n+1)\,\frac{\partial P_n(\sin\phi)}{\partial \sin\phi}$$

$$- r^2 \left(\frac{\partial \sin\phi}{\partial x_i}\,\frac{\partial \sin\phi}{\partial x_k}\,\frac{\partial^2 P_n(\sin\phi)}{(\partial \sin\phi)^2} + \frac{\partial^2 \sin\phi}{\partial x_i\,\partial x_k}\,\frac{\partial P_n(\sin\phi)}{\partial \sin\phi} \right) \Bigg\}$$

The gradient of the tesseral harmonic term is

$$G_{ik}^{tess} = -\frac{GM}{r^3}\sum_{n=2}^{\infty}\sum_{m=1}^{n} \left(\frac{R_e}{r}\right)^n \Bigg\{ T_{nm}\left[\delta_{ik} - (n+3)\,\frac{x_i x_k}{r^2} \right](n+1)\,P_n^m(\sin\phi)$$

$$+ T_{nm}\left(\frac{\partial \sin\phi}{\partial x_i}\,x_k + \frac{\partial \sin\phi}{\partial x_k}\,x_i \right)(n+1)\,\frac{\partial P_n^m(\sin\phi)}{\partial \sin\phi}$$

$$+ m\,T'_{nm}\left(\frac{\partial \lambda}{\partial x_i}\,x_k + \frac{\partial \lambda}{\partial x_k}\,x_i \right)(n+1)\,P_n^m(\sin\phi)$$

$$- T_{nm}\,r^2 \left(\frac{\partial \sin\phi}{\partial x_i}\,\frac{\partial \sin\phi}{\partial x_k}\,\frac{\partial^2 P_n^m(\sin\phi)}{(\partial \sin\phi)^2} + \frac{\partial^2 \sin\phi}{\partial x_i\,\partial x_k}\,\frac{\partial P_n^m(\sin\phi)}{\partial \sin\phi} - m^2\,\frac{\partial \lambda}{\partial x_i}\,\frac{\partial \lambda}{\partial x_k}\,P_n^m(\sin\right.$$

$$- m\,T'_{nm}\,r^2 \left[\left(\frac{\partial \sin\phi}{\partial x_k}\,\frac{\partial \lambda}{\partial x_i} + \frac{\partial \sin\phi}{\partial x_i}\,\frac{\partial \lambda}{\partial x_k} \right) \frac{\partial P_n^m(\sin\phi)}{\partial \sin\phi} + \frac{\partial^2 \lambda}{\partial x_i\,\partial x_k}\,P_n^m(\sin\phi) \right] \Bigg\}$$

Using the coordinate transformation matrix, we find that the second derivatives of the latitude and longitude can be written

$$r^2 \frac{\partial^2 \sin\phi}{\partial x_i \partial x_k} = \left(\frac{x_i x_k}{r^2} - \delta_{ik} \right) \sin\phi - \left(\mathbb{R}_{3k} \frac{x_i}{r} + \mathbb{R}_{3i} \frac{x_k}{r} \right)$$

$$r^2 \frac{\partial^2 \lambda}{\partial x_i \partial x_k} = - \frac{1}{\cos^2\phi} \left[\left(\mathbb{R}_{2i} \cos\lambda - \mathbb{R}_{1i} \sin\lambda \right) \left(\mathbb{R}_{2k} \sin\lambda + \mathbb{R}_{1k} \cos\lambda \right) \right.$$
$$\left. + \left(\mathbb{R}_{2k} \cos\lambda - \mathbb{R}_{1k} \sin\lambda \right) \left(\mathbb{R}_{2i} \sin\lambda + \mathbb{R}_{1i} \cos\lambda \right) \right]$$

Insertion into the expressions for the zonal and tesseral components of the gravity gradient matrices results in the expressions

$$G_{ik}^{zone} = \frac{GM}{r^3} \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r} \right)^n \left\{ \delta_{ik} \left[(n+1)\, P_n(\sin\phi) + \frac{\partial P_n(\sin\phi)}{\partial \sin\phi} \sin\phi \right] \right.$$
$$- \frac{x_i x_k}{r^2} \left[(n+1)\,(n+2)\, P_n(\sin\phi) + (2n+5) \frac{\partial P_n(\sin\phi)}{\partial \sin\phi} \sin\phi + \frac{\partial^2 P_n(\sin\phi)}{(\partial \sin\phi)^2} \sin^2\phi \right]$$
$$+ \left(\mathbb{R}_{3i} \frac{x_k}{r} + \mathbb{R}_{3k} \frac{x_i}{r} \right) \left[(n+2) \frac{\partial P_n(\sin\phi)}{\partial \sin\phi} + \frac{\partial^2 P_n(\sin\phi)}{(\partial \sin\phi)^2} \sin\phi \right]$$
$$\left. - \mathbb{R}_{3i} \mathbb{R}_{3k} \frac{\partial^2 P_n(\sin\phi)}{(\partial \sin\phi)^2} \right\}$$

and

$$G_{ik}^{tess} = -\frac{GM}{r^3}\sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \left\{ T_{nm}\,\delta_{ik}\left[(n+1)\,P_n^m(\sin\phi) + \frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi}\sin\phi\right]\right.$$

$$- T_{nm}\frac{x_i x_k}{r^2}\left[(n+1)\,(n+3)\,P_n^m(\sin\phi) + (2n+5)\frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi}\sin\phi + \frac{\partial^2 P_n^m(\sin\phi)}{(\partial\sin\phi)^2}\sin^2\phi\right]$$

$$+ T_{nm}\left(\mathbb{R}_{3i}\frac{x_k}{r} + \mathbb{R}_{3k}\frac{x_i}{r}\right)\left[(n+2)\frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi} + \frac{\partial^2 P_n^m(\sin\phi)}{(\partial\sin\phi)^2}\sin\phi\right]$$

$$- T_{nm}\,\mathbb{R}_{3i}\,\mathbb{R}_{3k}\frac{\partial^2 P_n^m(\sin\phi)}{(\partial\sin\phi)^2}$$

$$+ T_{nm}\frac{m^2}{\cos^2\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)P_n^m(\sin\phi)$$

$$+ T'_{nm}\frac{m}{\cos\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left[\frac{x_k}{r}(n+1)\,P_n^m(\sin\phi) - \left(\mathbb{R}_{3k} - \frac{x_k}{r}\sin\phi\right)\frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi}\right]$$

$$+ T'_{nm}\frac{m}{\cos\phi}\left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)\left[\frac{x_i}{r}(n+1)\,P_n^m(\sin\phi) - \left(\mathbb{R}_{3i} - \frac{x_i}{r}\sin\phi\right)\frac{\partial P_n^m(\sin\phi)}{\partial\sin\phi}\right]$$

$$+ T'_{nm}\frac{m}{\cos^2\phi}\Big[\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left(\mathbb{R}_{2k}\sin\lambda + \mathbb{R}_{1k}\cos\lambda\right)$$

$$\left. + \left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)\left(\mathbb{R}_{2i}\sin\lambda + \mathbb{R}_{1i}\cos\lambda\right)\Big]P_n^m(\sin\phi) \right\}$$

1. Zonal Harmonics

In PEP, the matrix components from the zonal gravitational harmonics of target body α are coded as

$$
\begin{aligned}
G_{ik}^{TZ(\alpha)} = \gamma(t)\,\frac{M_\alpha}{M_\odot}\,\frac{1}{r^3}\sum_{n=1}^{N_z-1} J_n\left(\frac{R_\epsilon}{r}\right)^{n+1} \Bigg\{ & \delta_{ik}\left[(n+2)\,P_{n+1}(\sin\phi) + \frac{\partial P_{n+1}(\sin\phi)}{\partial \sin\phi}\sin\phi\right] \\
& + \frac{x_i x_k}{r^2}\left[(n+2)\,(n+4)\,P_{n+1}(\sin\phi) + (2n+7)\,\frac{\partial P_{n+1}(\sin\phi)}{\partial \sin\phi}\sin\phi + \frac{\partial^2 P_{n+1}(\sin\phi)}{(\partial \sin\phi)^2}\sin^2\phi\right] \\
& + \left(\mathbf{R}_{3i}\frac{x_k}{r} + \mathbf{R}_{3k}\frac{x_i}{r}\right)\left[(n+3)\,\frac{\partial P_{n+1}(\sin\phi)}{\partial \sin\phi} + \frac{\partial^2 P_{n+1}(\sin\phi)}{(\partial \sin\phi)^2}\sin\phi\right] \\
& + \mathbf{R}_{3i}\mathbf{R}_{3k}\,\frac{\partial^2 P_{n+1}(\sin\phi)}{(\partial \sin\phi)^2} \Bigg\}
\end{aligned}
$$

where, in terms of previous notation,

$$
r \equiv \left|\vec{r}_{B\alpha}\right| \qquad\qquad x_i \equiv x^i_{B\alpha}
$$

2. Tesseral Harmonics

The tesseral harmonics for target body α lead to the gravity gradient matrix components

$$
\begin{aligned}
G_{ik}^{tess} = & -\gamma(t)\frac{M_\alpha}{M_\odot}\frac{1}{r^3}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\left(\frac{R_\alpha}{r}\right)^n \Bigg\{ T_{nm}\,\delta_{ik}\left[(n+2)\,P_{n+1}^m(\sin\phi) + \frac{\partial P_{n+1}^m(\sin\phi)}{\partial\sin\phi}\sin\phi\right] \\
& - T_{nm}\frac{x_i x_k}{r^2}\left[(n+2)\,(n+4)\,P_{n+1}^m(\sin\phi) + (2n+7)\frac{\partial P_{n+1}^m(\sin\phi)}{\partial\sin\phi}\sin\phi + \frac{\partial^2 P_{n+1}^m(\sin\phi)}{(\partial\sin\phi)^2}\sin^2\phi\right] \\
& - T_{nm}\left(\mathbb{R}_{3i}\frac{x_k}{r} + \mathbb{R}_{3k}\frac{x_i}{r}\right)\left[(n+3)\frac{\partial P_{n+1}^m(\sin\phi)}{\partial\sin\phi} + \frac{\partial^2 P_{n+1}^m(\sin\phi)}{(\partial\sin\phi)^2}\sin\phi\right] \\
& - T_{nm}\,\mathbb{R}_{3i}\,\mathbb{R}_{3k}\frac{\partial^2 P_{n+1}^m(\sin\phi)}{(\partial\sin\phi)^2} \\
& + T_{nm}\frac{m^2}{\cos^2\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)P_{n+1}^m(\sin\phi) \\
& - T_{nm}'\frac{m}{\cos\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left[\frac{x_k}{r}(n+2)\,P_{n+1}^m(\sin\phi) + \left(\mathbb{R}_{3k} + \frac{x_k}{r}\sin\phi\right)\frac{\partial P_{n+1}^m(\sin\phi)}{\partial\sin\phi}\right] \\
& - T_{nm}'\frac{m}{\cos\phi}\left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)\left[\frac{x_i}{r}(n+2)\,P_{n+1}^m(\sin\phi) + \left(\mathbb{R}_{3i} + \frac{x_i}{r}\sin\phi\right)\frac{\partial P_{n+1}^m(\sin\phi)}{\partial\sin\phi}\right] \\
& + T_{nm}'\frac{m}{\cos^2\phi}\Big[\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left(\mathbb{R}_{2k}\sin\lambda + \mathbb{R}_{1k}\cos\lambda\right) \\
& \qquad + \left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)\left(\mathbb{R}_{2i}\sin\lambda + \mathbb{R}_{1i}\cos\lambda\right)\Big]P_{n+1}^m(\sin\phi) \Bigg\}
\end{aligned}
$$

C. The Sun

The effect of the Sun on the partial derivatives for non-Sun-centered bodies is

$$G_{jk}^{\odot} = \gamma(t)\left\{ \frac{3x_{\odot B}^{j}x_{\odot B}^{k}}{|\vec{r}_{\odot B}|^{5}} - \delta_{jk}\frac{1}{|\vec{r}_{\odot B}|^{3}} \right\}$$

D. Central Body

1. Zonal Harmonics

The zonal harmonics for the central body result in the matrix components

$$\begin{aligned} G_{ik}^{CZ} = -\gamma(t)\frac{M_C}{M_C}\left(1+\frac{M_B}{M_\odot}\right)\frac{1}{r^3}\sum_{n=1}^{N_z-1} J_{n+1}\left(\frac{R_C}{r}\right)^{n+1} \Bigg\{ & \delta_{ik}\left[(n+2)\,P_{n+1}(\sin\phi) + \frac{\partial P_{n+1}(\sin\phi)}{\partial\sin\phi}\sin\phi\right] \\ & - \frac{x_i x_k}{r^2}\left[(n+2)\,(n+3)\,P_{n+1}(\sin\phi) + (2n+7)\frac{\partial P_{n+1}(\sin\phi)}{\partial\sin\phi}\sin\phi + \frac{\partial^2 P_{n+1}(\sin\phi)}{(\partial\sin\phi)^2}\sin^2\phi\right] \\ & + \left(\mathbb{R}_{3i}\frac{x_k}{r} + \mathbb{R}_{3k}\frac{x_i}{r}\right)\left[(n+3)\frac{\partial P_{n+1}(\sin\phi)}{\partial\sin\phi} + \frac{\partial^2 P_{n+1}(\sin\phi)}{(\partial\sin\phi)^2}\sin\phi\right] \\ & - \mathbb{R}_{3i}\mathbb{R}_{3k}\frac{\partial^2 P_{n+1}(\sin\phi)}{(\partial\sin\phi)^2} \Bigg\} \end{aligned}$$

where

$$r \equiv |\vec{r}_{CB}| \qquad\qquad x_i \equiv x_{CB}^{i}$$

2. Tesseral Harmonics

For the central body tesseral harmonics, the gravity gradient matrix components are

$$
\begin{aligned}
G_{ik}^{tess} = -\gamma(t)\frac{M_C}{M_C}\left(1+\frac{M_B}{M_\odot}\right)\frac{1}{r^3}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\left(\frac{R_C}{r}\right)^n \Bigg\{ & T_{nm}\,\delta_{ik}\left[(n+2)\,P_{n+1}^{m}(\sin\phi) + \frac{\partial P_{n+1}^{m}(\sin\phi)}{\partial\sin\phi}\sin\phi\right] \\
& - T_{nm}\frac{x_i x_k}{r^2}\left[(n+2)(n+4)\,P_{n+1}^{m}(\sin\phi) + (2n+7)\frac{\partial P_{n+1}^{m}(\sin\phi)}{\partial\sin\phi}\sin\phi + \frac{\partial^2 P_{n+1}^{m}(\sin\phi)}{(\partial\sin\phi)^2}\sin^2\phi\right] \\
& + T_{nm}\left(\mathbb{R}_{3i}\frac{x_k}{r} + \mathbb{R}_{3k}\frac{x_i}{r}\right)\left[(n+3)\frac{\partial P_{n+1}^{m}(\sin\phi)}{\partial\sin\phi} + \frac{\partial^2 P_{n+1}^{m}(\sin\phi)}{(\partial\sin\phi)^2}\sin\phi\right] \\
& - T_{nm}\,\mathbb{R}_{3i}\mathbb{R}_{3k}\frac{\partial^2 P_{n+1}^{m}(\sin\phi)}{(\partial\sin\phi)^2} \\
& + T_{nm}\frac{m^2}{\cos^2\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)P_{n+1}^{m}(\sin\phi) \\
& + T_{nm}'\frac{m}{\cos\phi}\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left[\frac{x_k}{r}(n+2)\,P_{n+1}^{m}(\sin\phi) - \left(\mathbb{R}_{3k} - \frac{x_k}{r}\sin\phi\right)\frac{\partial P_{n+1}^{m}(\sin\phi)}{\partial\sin\phi}\right] \\
& + T_{nm}'\frac{m}{\cos\phi}\left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)\left[\frac{x_i}{r}(n+2)\,P_{n+1}^{m}(\sin\phi) - \left(\mathbb{R}_{3i} - \frac{x_i}{r}\sin\phi\right)\frac{\partial P_{n+1}^{m}(\sin\phi)}{\partial\sin\phi}\right] \\
& + T_{nm}'\frac{m}{\cos^2\phi}\left[\left(\mathbb{R}_{2i}\cos\lambda - \mathbb{R}_{1i}\sin\lambda\right)\left(\mathbb{R}_{2k}\sin\lambda + \mathbb{R}_{1k}\cos\lambda\right)\right. \\
& \qquad\left. + \left(\mathbb{R}_{2k}\cos\lambda - \mathbb{R}_{1k}\sin\lambda\right)\left(\mathbb{R}_{2i}\sin\lambda + \mathbb{R}_{1i}\cos\lambda\right)\right]P_{n+1}^{m}(\sin\phi) \quad \Bigg\}
\end{aligned}
$$

E. Effect of the Central Force

1. Earth-Moon System

The effect of the force due to the central body on the partial derivatives for the Earth-Moon barycenter is

$$G_{jk}^{C} = \gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\left\{\left[\mu_E\,\frac{3x_{C\oplus}^{j}x_{C\oplus}^{k}}{|\vec{r}_{C\oplus}|^{5}} + \mu_M\,\frac{3x_{CM}^{j}x_{CM}^{k}}{|\vec{r}_{C\oplus}|^{5}}\right] - \delta_{jk}\left[\mu_E\,\frac{1}{|\vec{r}_{C\oplus}|^{3}} + \mu_M\,\frac{1}{|\vec{r}_{CM}|^{3}}\right]\right\}$$

2. All Other Planets

The effect of the force due to the central body on the partial derivatives for the other bodies is

$$G_{jk}^{C} = \gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\left\{\frac{3x_{CB}^{j}x_{CB}^{k}}{|\vec{r}_{CB}|^{5}} - \frac{\delta_{jk}}{|\vec{r}_{CB}|^{3}}\right\}$$

III. Partial Derivatives Accelerations

This section documents the accelerations arising from the time derivatives of the derivatives of the motion with respect to the parameters α,

$$\vec{f}_\alpha = \frac{d\ddot{x}}{d\alpha} = \frac{d^2}{dt^2}\left(\frac{dx}{d\alpha}\right)$$

A. Perturbing Bodies

1. α is the Mass of a Perturbing Planet

The acceleration of the derivative with respect to the mass of perturbing body P is as follows. If body P is the Earth and the central body C is not the Moon, the acceleration is

$$\vec{f}_{PP} = \gamma(t)\left[\mu_E\left(\frac{\vec{r}_{B\oplus}}{|\vec{r}_{B\oplus}|^3} - \frac{\vec{r}_{C\oplus}}{|\vec{r}_{C\oplus}|^3}\right) + \mu_M\left(\frac{\vec{r}_{BM}}{|\vec{r}_{BM}|^3} - \frac{\vec{r}_{CM}}{|\vec{r}_{CM}|^3}\right)\right]$$

while if the central body *is* the Moon,

$$\vec{f}_{PP} = \gamma(t)\left[\mu_E\left(\frac{\vec{r}_{B\oplus}}{|\vec{r}_{B\oplus}|^3} - \frac{\vec{r}_{C\oplus}}{|\vec{r}_{C\oplus}|^3}\right) - \mu_M\frac{\vec{r}_{BM}}{|\vec{r}_{BM}|^3}\right]$$

If the perturbing planet is the Moon, then if the central body is the Earth the acceleration is

$$\vec{f}_{PP} = \gamma(t)\,\frac{M_\oplus + M_M}{M_\odot}\left(\frac{\vec{r}_{BM}}{|\vec{r}_{BM}|^3} - \frac{\vec{r}_{EM}}{|\vec{r}_{EM}|^3} - \frac{\vec{r}_{B\oplus}}{|\vec{r}_{B\oplus}|^3}\right)$$

while if the central body is not the Earth it is

$$\vec{f}_{PP} = \gamma(t)\,\frac{M_\oplus + M_M}{M_\odot}\left(\frac{\vec{r}_{BM}}{|\vec{r}_{BM}|^3} - \frac{\vec{r}_{CM}}{|\vec{r}_{CM}|^3} - \frac{\vec{r}_{B\oplus}}{|\vec{r}_{B\oplus}|^3} + \frac{\vec{r}_{C\oplus}}{|\vec{r}_{C\oplus}|^3}\right)$$

For all other perturbing bodies,

$$\vec{f}_{PP} = \gamma(t)\left(\frac{\vec{r}_{BP}}{|\vec{r}_{BP}|^3} - \frac{\vec{r}_{CP}}{|\vec{r}_{CP}|^3}\right)$$

2. α is a System Mass

When the derivative is with respect to the mass of a system S which includes perturbing satellites, we have

$$\vec{f}_{PS} = \gamma(t)\sum_{k=1}^{Nast}\frac{M_k}{M_S}\left(\frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|^3} - \frac{\vec{r}_{Ck}}{|\vec{r}_{Ck}|^3}\right)$$

3. α is an Asteroid Mass

For the derivative with respect to the mass of asteroid k,

$$\vec{f}^{\,k}_{Pa} = \gamma(t)\left(\frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|^3} - \frac{\vec{r}_{Ck}}{|\vec{r}_{Ck}|^3}\right)$$

4. α is the Time Variation of G

When the derivative is with respect to time variation of the gravitational constant, the force is

$$\vec{f}_{PG} = k_G^2 (t - t_0) \cdot \left(\vec{a}_{CP} + \vec{a}_{C\eta P} + \vec{a}_{Cast} \right)$$

5. α is the Principle of Equivalence Violation Parameter

If the derivative is with respect to the Solar ratio of internal gravitational to total energy, the force is

$$\vec{f}_{P\eta_\odot} = -\gamma(t) \sum_{k=1}^{10} \frac{M_k}{M_\odot} \frac{\vec{r}_{Ck}}{|\vec{r}_{Ck}|^3}$$

If the derivative is with respect to the integrated body ratio of internal gravitational to total energy, the force is

$$\vec{f}_{P\eta_B} = +\gamma(t) \sum_{k=1}^{10} \frac{M_k}{M_\odot} \frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|^3}$$

B. Target Bodies

1. α is a Zonal Harmonic Coefficient

When the derivative is with respect to the zonal harmonic coefficient of degree n of target body k, we have the corresponding acceleration

$$\vec{f}_{TZ}^{\,k,n} = \gamma(t) \frac{M_k}{M_\odot} \frac{1}{|\vec{r}_{Bk}|^2} \left(\frac{R_k}{|\vec{r}_{Bk}|} \right)^n$$

$$\cdot \left[-(n+1) P_n(\sin\phi_k) \frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|} - |\vec{r}_{Bk}| \frac{\partial \sin\phi_k}{\partial(-\vec{r}_{Bk})} \frac{\partial P_n(\sin\phi_k)}{\partial \sin\phi_k} \right]$$

where ϕ_k is the latitude of the integrated body with respect to target body k.

2. α is a Tesseral Harmonic Cosine Coefficient

When the derivative is with respect to the tesseral harmonic cosine coefficient of degree n and order m of target body k, we have

$$\vec{f}_{TTC}^{\,k,n,m} = \gamma(t) \frac{M_k}{M_\odot} \frac{1}{|\vec{r}_{Bk}|^2} \left(A_{nm}^k \cos m\lambda_k - B_{nm}^k \sin m\lambda_k \right)$$

where

$$A_{nm}^k = \left(\frac{R_k}{|\vec{r}_{Bk}|}\right)^n \left[(n+1)\, P_n^m(\sin\phi_k) \frac{\vec{r}_{Bk}}{|\vec{r}_{Bk}|} + |\vec{r}_{Bk}| \frac{\partial \sin\phi_k}{\partial(-\vec{r}_{Bk})} \frac{\partial P_n^m(\sin\phi_k)}{\partial \sin\phi_k} \right]$$

$$B_{nm}^k = \left(\frac{R_k}{|\vec{r}_{Bk}|}\right)^n m\, P_n^m(\sin\phi_k)\, |\vec{r}_{Bk}| \frac{\partial \lambda_k}{\partial(-\vec{r}_{Bk})}$$

3. α is a Tesseral Harmonic Sine Coefficient

When the derivative is with respect to the tesseral harmonic sine coefficient of degree n and order m of target body k,

$$\vec{f}_{TTS}^{\,k,n,m} = \gamma(t) \frac{M_k}{M_\odot} \frac{1}{|\vec{r}_{Bk}|^2} \left[A_{nm}^k \sin m\lambda_k + B_{nm}^k \cos m\lambda_k \right]$$

4. α is a Target Planet Mass

When the parameter is the mass of target planet k,

$$\vec{f}_{TM}^{\,k} = \gamma(t) \frac{M_k^{TC}}{M_\odot} \left(\vec{a}_{CTZ}^{\,k} + \vec{a}_{CTT}^{\,k} \right)$$

where the acceleration towards the central body due to target body k harmonics has been given previously in section I. Here, M_k^{TC} is the mass of the central body of target body k.

5. α is the Mass of a Target Planet Central Body

When the parameter is the mass of the central body of a target body,

$$\vec{f}_{tcm} = \gamma(t) \sum_{k=1}^{N_{targ}} \frac{M_k^{ast}}{M_\odot} \vec{a}_{cth}^{\,k}$$

where M_k^{ast} is the mass of target satellite k.

6. α is the Time Variation of the Gravitational Constant

When the parameter is the time variation of G,

$$\vec{f}_{tG} = k_G^2 (t - t_0) \sum_{k=1}^{N_{targ}} \frac{M_k^{targ}}{M_\odot} \vec{a}_{cth}^{\,k}$$

C. The Sun

1. α is the Time Variation of the Gravitational Constant

When the parameter is the time variation of G,

$$\vec{f}_{\odot G} = k_G^2 (t - t_0) \vec{a}_{C\odot}$$

D. The Central Body

1. α is a Zonal Harmonic Coefficient

When α is the zonal harmonic coefficient of degree n for the central body, we have

$$\vec{f}_{CZ} = \gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\frac{1}{|\vec{r}_{CB}|^2}\left(\frac{R_C}{|\vec{r}_{CB}|}\right)^n\left[(n+1)\,P_n(\sin\phi_C)\,\frac{\vec{r}_{CB}}{|\vec{r}_{CB}|} - |\vec{r}_{CB}|\,\frac{\partial \sin\phi_C}{\partial \vec{r}_{CB}}\,\frac{\partial P_n(\sin\phi_C)}{\partial \sin\phi_C}\right]$$

where R_C is the equatorial radius of the central body.

2. α is a Tesseral Harmonic Cosine Coefficient

When α is the tesseral harmonic cosine coefficient of degree n and order m for the central body, we have

$$\vec{f}_{CTC} = \gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\frac{1}{|\vec{r}_{CB}|^2}\left(A^C_{nm}\cos m\lambda_C - B^C_{nm}\sin m\lambda_C\right)$$

where

$$A^C_{nm} = \left(\frac{R_C}{|\vec{r}_{CB}|}\right)^n\left[-(n+1)\,P^m_n(\sin\phi_C)\,\frac{\vec{r}_{CB}}{|\vec{r}_{CB}|} + |\vec{r}_{CB}|\,\frac{\partial \sin\phi_C}{\partial \vec{r}_{CB}}\,\frac{\partial P^m_n(\sin\phi_C)}{\partial \sin\phi_C}\right]$$

$$B^C_{nm} = \left(\frac{R_C}{|\vec{r}_{CB}|}\right)^n m\,P^m_n(\sin\phi_C)\,|\vec{r}_{CB}|\,\frac{\partial \lambda_C}{\partial \vec{r}_{CB}}$$

3. α is a Tesseral Harmonic Sine Coefficient

When α is the tesseral harmonic sine coefficient of degree n and order m for the central body, we have

$$\vec{f}_{CTS} = \gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\frac{1}{|\vec{r}_{CB}|^2}\left(A^C_{nm}\sin m\lambda_C + B^C_{nm}\cos m\lambda_C\right)$$

4. α is one of the Central Body Rotation Parameters

When α is one of the central body rotation parameters,

$$\vec{f}^{\,\alpha}_{c\,rot} = -\gamma(t)\,\frac{M_C}{M_\odot}\left(1 + \frac{M_B}{M_C}\right)\frac{\partial \mathbb{R}^C}{\partial \alpha}\cdot \vec{a}_{CCH} + \left(G^{CH}\right)^T \cdot \frac{\partial \mathbb{R}^C}{\partial \alpha}\cdot \vec{r}_{CB}$$

for rotation parameter α. Here, $\mathbb{R}^C$ is the rotation matrix that transforms from central body coordinates to the fixed frame with celestial equator of 1950.0; $\mathbf{a}_{CCH}$ is the acceleration with respect to the central body due to central body gravitational harmonics; and

$$\left(G^{Ch}\right)^T \equiv \left(G^{CT} + G^{CZ}\right)^T$$

where superscript T stands for matrix transpose.

E. Additional Accelerations

The effects on the partial derivatives due to accelerations specific to

Sun-centered probe
Earth satellite
Lunar orbiter
planetary orbiter
natural planetary satellite

are determined via calls to subroutines SOLPRB, ERTORB, MONORB, PLNORB, and PLNSAT, respectively.

F. Initial Conditions

1. α is an Initial Condition of a Target Planet

When the parameter is an initial condition of target planet k, the partials acceleration is

$$\vec{f}_{TI}^{\,k} = \gamma(t)\left\{ \frac{M_k}{M_\odot}\left[\frac{1}{|\vec{r}_{Bk}|^3}\left(\frac{\partial \vec{r}_k}{\partial \alpha_k} - 3\vec{r}_{Bk}\frac{A_{Bk}}{|\vec{r}_{Bk}|^2} \right) - \frac{1}{|\vec{r}_{Ck}|^3}\left(\frac{\partial \vec{r}_k}{\partial \alpha_k} - 3\vec{r}_{Ck}\frac{A_{Ck}}{|\vec{r}_{Ck}|^2} \right)\right] - \frac{\partial \vec{r}_k}{\partial \alpha_k}\cdot\left(G_k^{TZ} + G_k^{TT}\right) \right\}$$

where

$$A_{Bk} \equiv \vec{r}_{Bk}\cdot\frac{\partial \vec{r}_k}{\partial \alpha_k} \qquad\qquad A_{Ck} \equiv \vec{r}_{Ck}\cdot\frac{\partial \vec{r}_k}{\partial \alpha_k}$$

and α_k is an initial condition (orbital element) of the target planet k.

2. α is an Initial Condition of the Central Body

When α is an initial condition of the central body, the following effects contribute to the acceleration of the partial derivative of the central body position with respect to α.

a. The Sun's Effect on Integrated Initial Condition Partial Derivative

$$\vec{f}_{CI}^{\,\odot} = \frac{\partial \vec{r}_C}{\partial \alpha_C}\cdot G^\odot + \gamma(t)\frac{1}{|\vec{r}_C|^3}\left(\frac{\partial \vec{r}_C}{\partial \alpha_C} - 3\frac{\vec{r}_C}{|\vec{r}_C|^2}\left(\vec{r}_C\cdot\frac{\partial \vec{r}_C}{\partial \alpha_C} \right)\right)$$

b. Effects of Perturbing Planets

$$\vec{f}_{CI}^{\,P} = \frac{\partial \vec{r}_C}{\partial \alpha_C}\cdot G^P + \gamma(t)\sum_{k=1}^{10}\frac{M_k}{M_\odot}\frac{1}{|\vec{r}_{Ck}|^3}\left[\frac{\partial \vec{r}_{Ck}}{\partial \alpha_C} - 3\frac{\vec{r}_{Ck}}{|\vec{r}_{Ck}|^2}\left(\vec{r}_{Ck}\cdot\frac{\partial \vec{r}_{Ck}}{\partial \alpha_C} \right)\right]$$

c. Effects of Perturbing Asteroids

Currently not implemented.

d. Effects of the Target Body Harmonics

$$\vec{f}_{CI}^{\,TH} = \left[\sum_{k=1}^{N_{targ}} \left(G^{TZ} + G^{TT} \right) \right] \cdot \frac{\partial \vec{r}_C}{\partial \alpha_C}$$

G. Effect of Central Forces on Partial Derivatives

1. Earth-Moon Barycenter

The gravity gradient matrix for the Earth-Moon system orbiting a central body is

$$G_{jk}^{C} = \gamma(t) \left(1 + \frac{M_B}{M_{\odot}} \right) \left\{ \left[\mu_E \frac{3 x_{C\oplus}^{j} x_{C\oplus}^{k}}{\left| \vec{r}_{C\oplus} \right|^5} + \mu_M \frac{3 x_{CM}^{j} x_{CM}^{k}}{\left| \vec{r}_{CM} \right|^5} \right] - \delta_{jk} \left[\mu_E \frac{1}{\left| \vec{r}_{C\oplus} \right|^3} + \mu_M \frac{1}{\left| \vec{r}_{CM} \right|^3} \right] \right\}$$

2. All Other Bodies

The gravity gradient matrix for any other body is

$$G_{jk}^{C} = \gamma(t) \frac{M_C}{M_{\odot}} \left(1 + \frac{M_B}{M_C} \right) \left[\frac{3 x_{CB}^{j} x_{CB}^{k}}{\left| \vec{r}_{CB} \right|^5} - \frac{\delta_{jk}}{\left| \vec{r}_{CB} \right|^3} \right]$$

In the special case of the Jovian system, the expression for the mass in the above equation is more complicated.

H. Gravity Gradient Matrix

The total gravity gradient matrix consists of the sum of all the preceding contributions:

$$G = \sum G^{\alpha}$$

where

$$\alpha \in \{P, ast, TZ, TT, \odot, CZ, CT, C\}$$

as listed at the beginning of section II.

I. Partial Derivative is with respect to an Initial Condition

1. Subtract Mean Orbit Acceleration Partial

Currently not implemented.

2. Subtract Elliptic Orbit Acceleration Partial

The Encke orbit force to be subtracted:

$$\vec{f}_{IE}^{\beta} = -\gamma(t)\frac{M_C}{M_\odot}\left(1+\frac{M_B}{M_C}\right)\frac{1}{|\vec{r}_E|^3}\left[3\frac{\vec{r}_E}{|\vec{r}_E|}\left(\vec{r}_E \cdot \frac{\partial \vec{r}_E}{\partial \alpha}\right)\right]$$

3. α is Time Variation of G

If the parameter is the variation of the gravitational constant, then

$$\vec{f}_{IG} = -k_G^2(t-t_0)\frac{M_C}{M_\odot}\left(1+\frac{M_B}{M_C}\right)\vec{a}_{CC}$$

where a_{CC} is the acceleration due to the central body, given in section I.C.

4. α is Mass of Another Satellite

If the parameter is the mass of another satellite,

$$\vec{f}_{Isat} = \gamma(t)\,\frac{M_C}{M_\odot}\,\vec{a}_{CC}$$

5. α is Asteroid Mass

If the parameter is the mass of an asteroid,

$$\vec{f}_{Iast} = -\gamma(t)\,\frac{M_C}{M_\odot}\,\vec{a}_{CC}$$

6. α is Mass of Central Body

If the parameter is the mass of the central body, then

$$\vec{f}_{IC} = -\gamma(t)\left(1+\frac{M_B}{M_C}\right)\vec{a}_{CC}$$

7. α is the Principle of Equivalence Violation Parameter

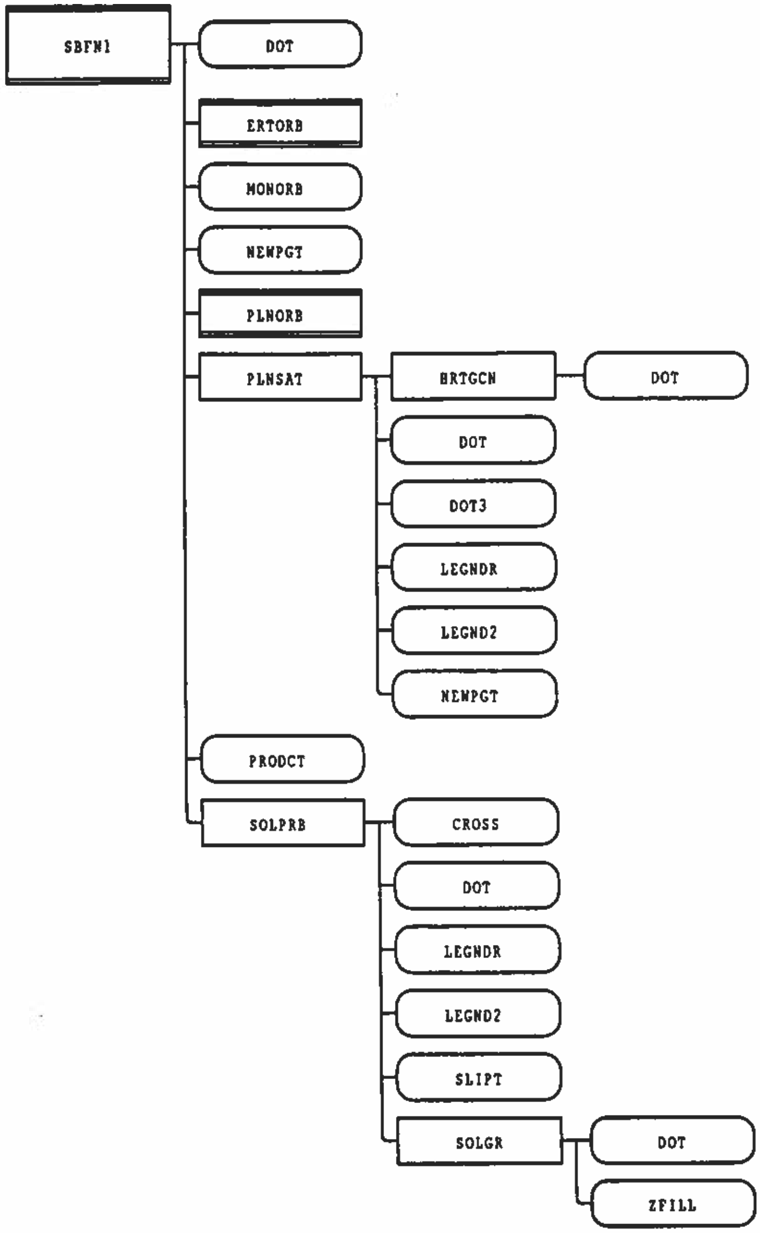
Finally, if α is the principle of equivalence violation parameter,

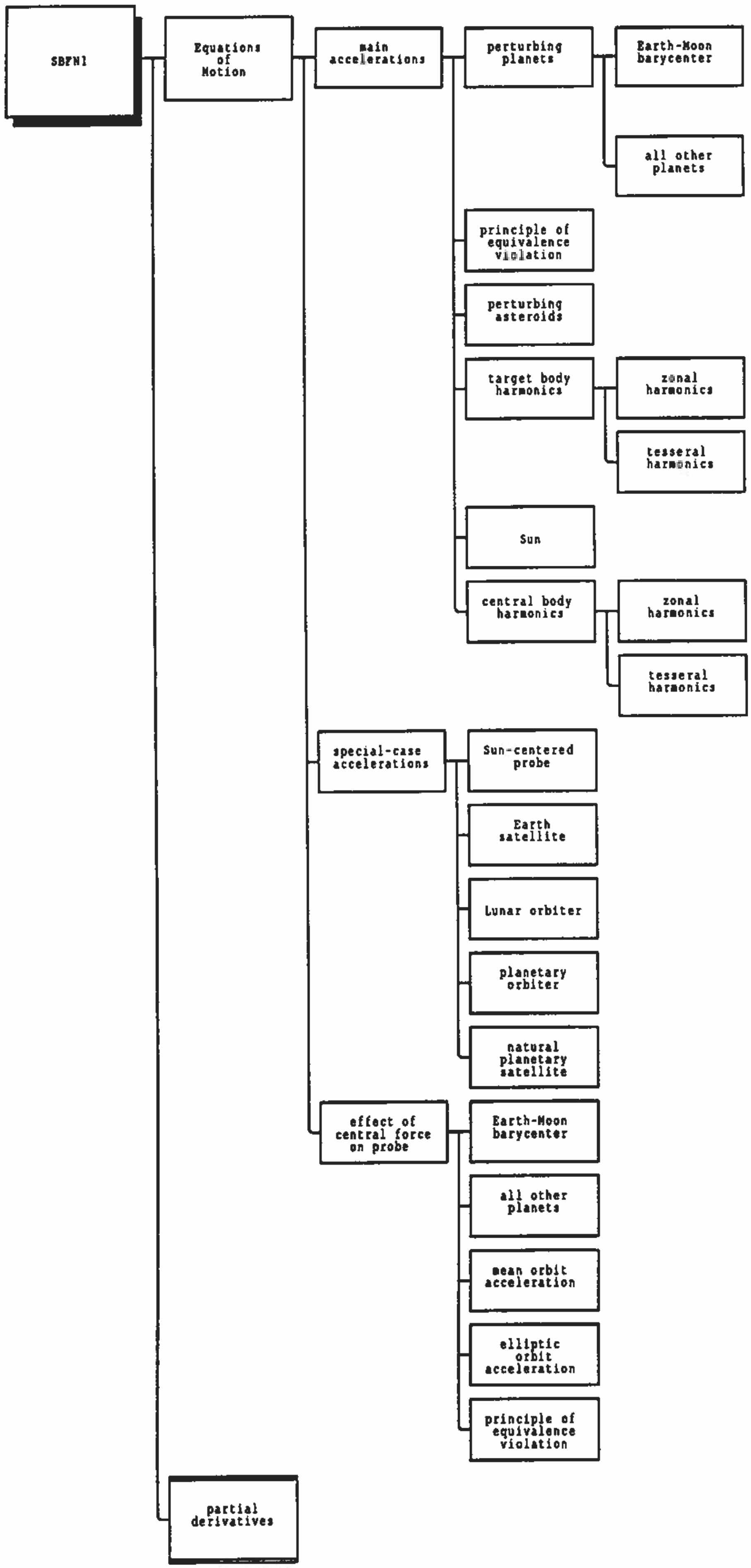
$$\vec{f}_{I\eta} = -\gamma(t)\,\frac{M_B}{M_C}\,\vec{a}_{CC}$$

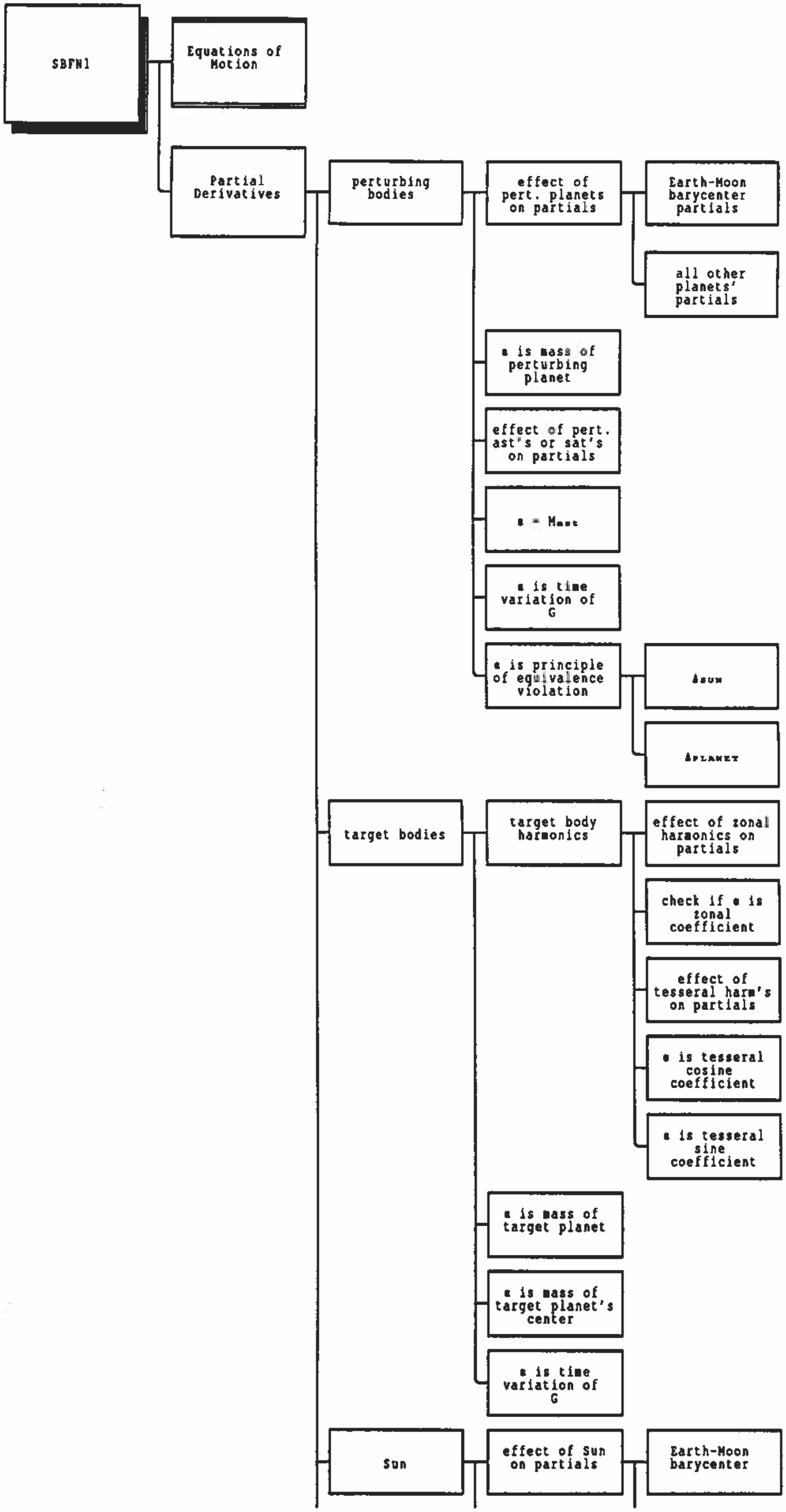
for the Sun, and

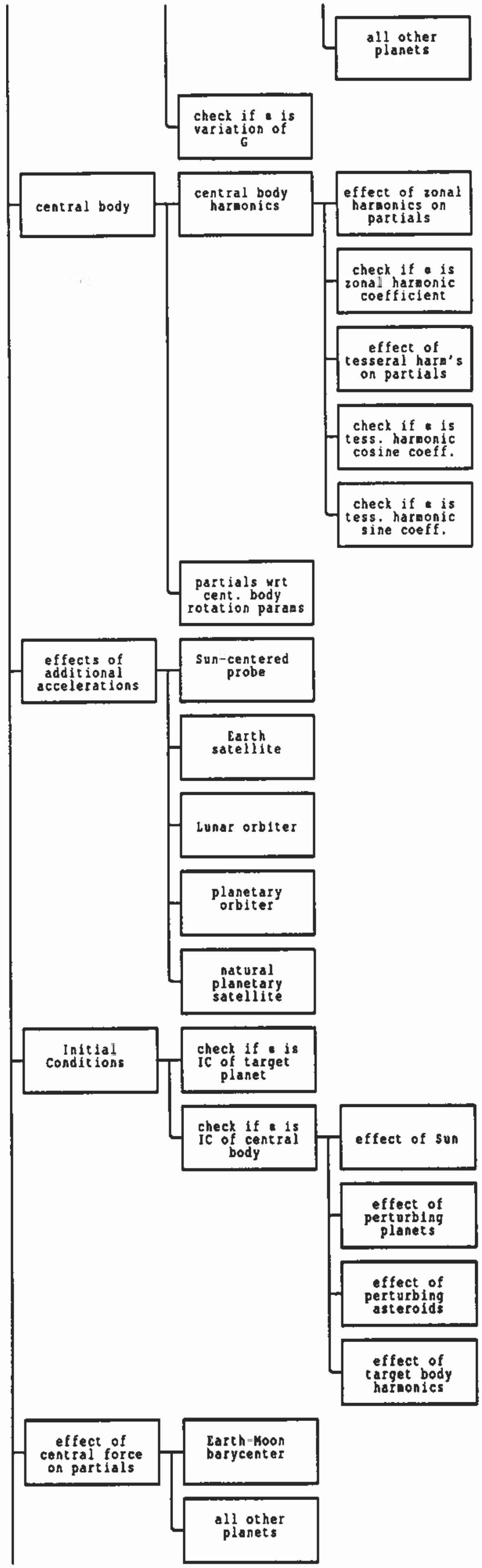
$$\vec{f}_{I\eta} = -\gamma(t)\,\vec{a}_{CC}$$

for the integrated body.

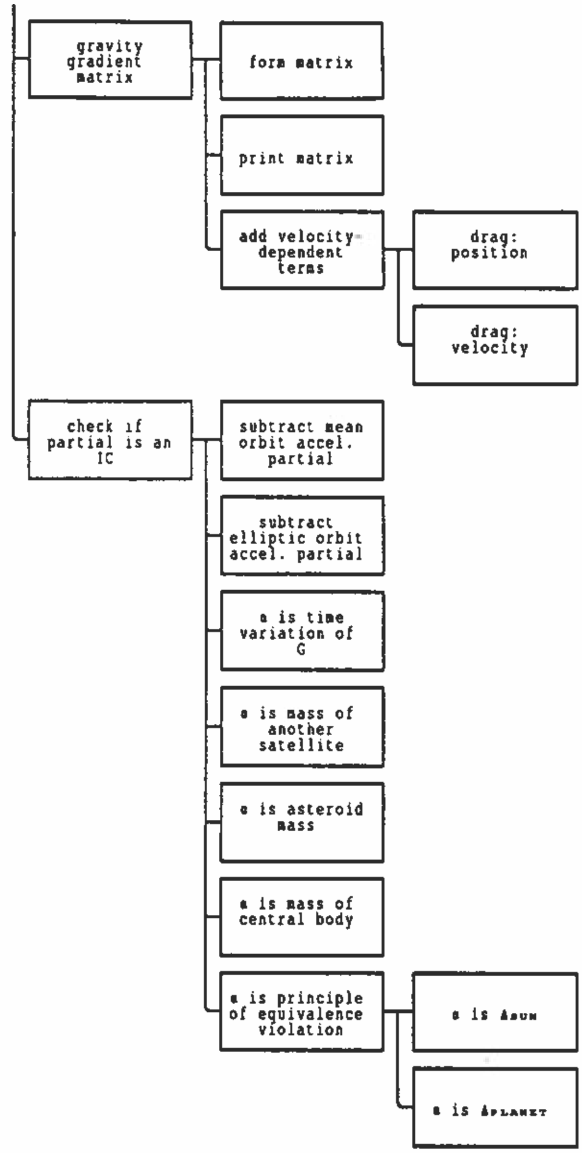

SBFN1
DOT
ERTORB
MONORB
NEWPGT
PLNORB
PLNSAT
BRTGCN
DOT
DOT
DOT3
LEGNDR
LEGND2
NEWPGT
PRODCT
SOLPRB
CROSS
DOT
LEGNDR
LEGND2
SLIPT
SOLGR
DOT
ZFILL


SBFN1
Equations of Motion
main accelerations
perturbing planets
Earth-Moon barycenter
all other planets
principle of equivalence violation
perturbing asteroids
target body harmonics
zonal harmonics
tesseral harmonics
Sun
central body harmonics
zonal harmonics
tesseral harmonics
special-case accelerations
Sun-centered probe
Earth satellite
Lunar orbiter
planetary orbiter
natural planetary satellite
effect of central force on probe
Earth-Moon barycenter
all other planets
mean orbit acceleration
elliptic orbit acceleration
principle of equivalence violation
partial derivatives

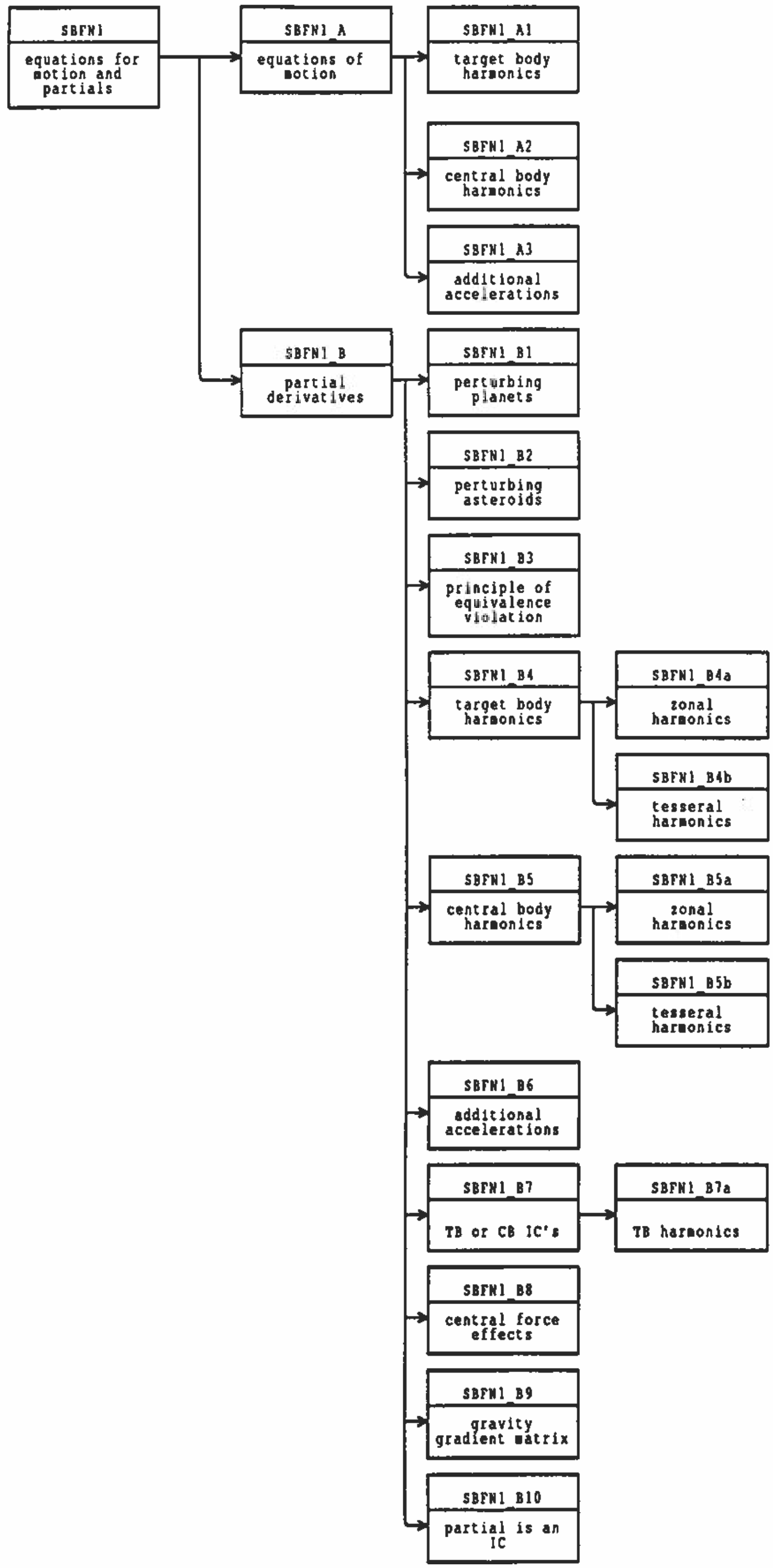

SBFN1
Equations of Motion
Partial Derivatives
perturbing bodies
effect of pert. planets on partials
Earth-Moon barycenter partials
all other planets' partials
is mass of perturbing planet
effect of pert. ast's or sat's on partials
= M
is time variation of G
is principle of equivalence violation
ΔSUN
ΔPLANET
target bodies
target body harmonics
effect of zonal harmonics on partials
check if is zonal coefficient
effect of tesseral harm's on partials
is tesseral cosine coefficient
is tesseral sine coefficient
is mass of target planet
is mass of target planet's center
is time variation of G
Sun
effect of Sun on partials
Earth-Moon barycenter



all other planets
check if ● is variation of G
central body
central body harmonics
effect of zonal harmonics on partials
check if ● is zonal harmonic coefficient
effect of tesseral harm's on partials
check if ● is tess. harmonic cosine coeff.
check if ● is tess. harmonic sine coeff.
partials wrt cent. body rotation params
effects of additional accelerations
Sun-centered probe
Earth satellite
Lunar orbiter
planetary orbiter
natural planetary satellite
Initial Conditions
check if ● is IC of target planet
check if ● is IC of central body
effect of Sun
effect of perturbing planets
effect of perturbing asteroids
effect of target body harmonics
effect of central force on partials
Earth-Moon barycenter
all other planets


gravity gradient matrix
form matrix
print matrix
add velocity-dependent terms
drag: position
drag: velocity
check if partial is an IC
subtract mean orbit accel. partial
subtract elliptic orbit accel. partial
α is time variation of G
α is mass of another satellite
α is asteroid mass
α is mass of central body
α is principle of equivalence violation
α is ΔSUN
α is ΔPLANET

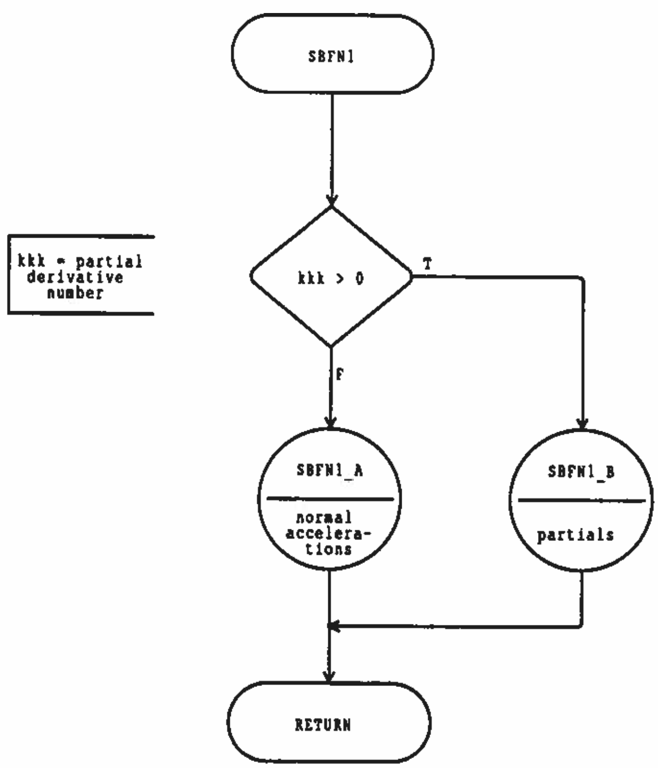
This chart shows the tree structure of the SBFN1 flow charts.

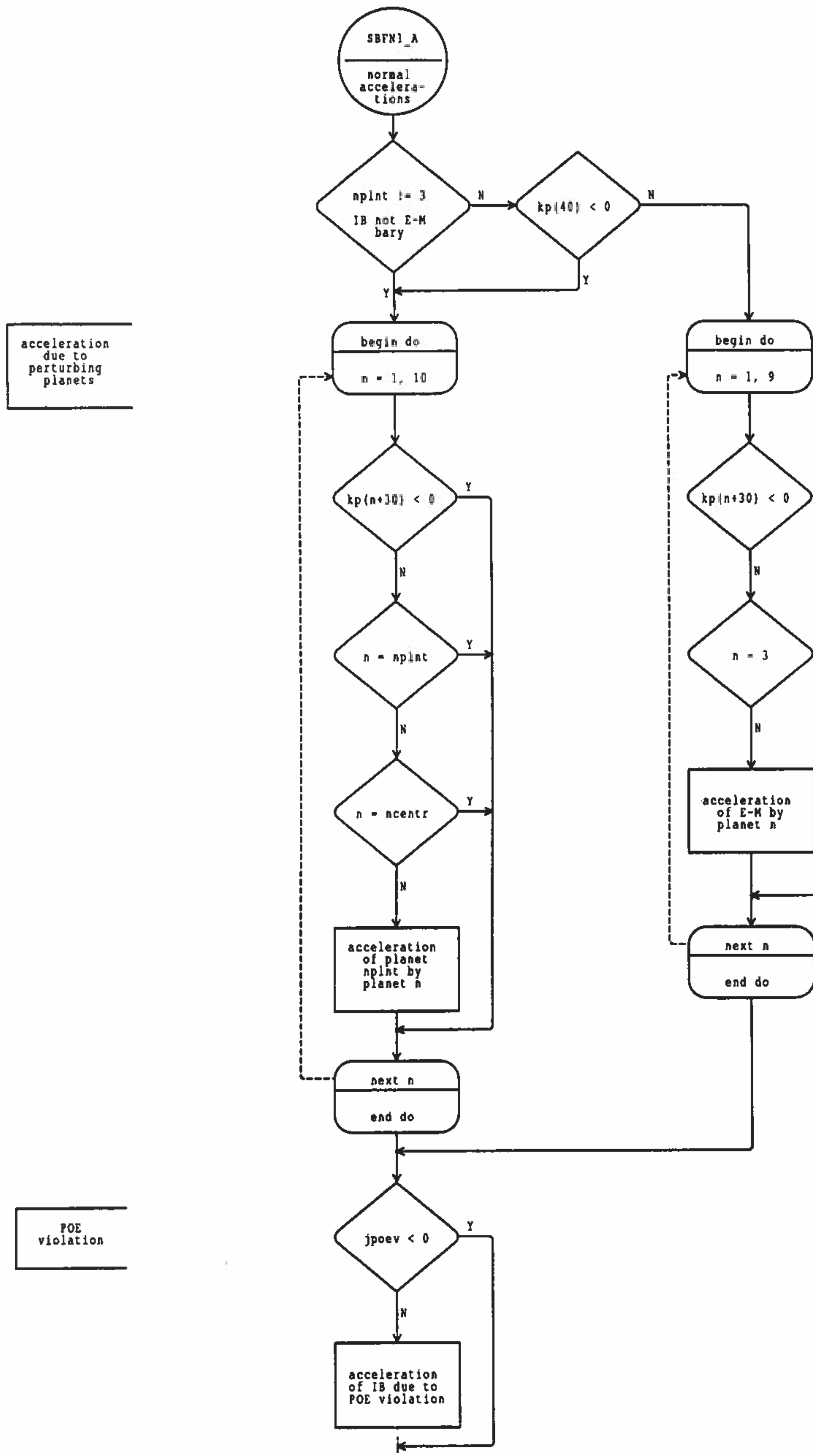


ABBREVIATIONS:

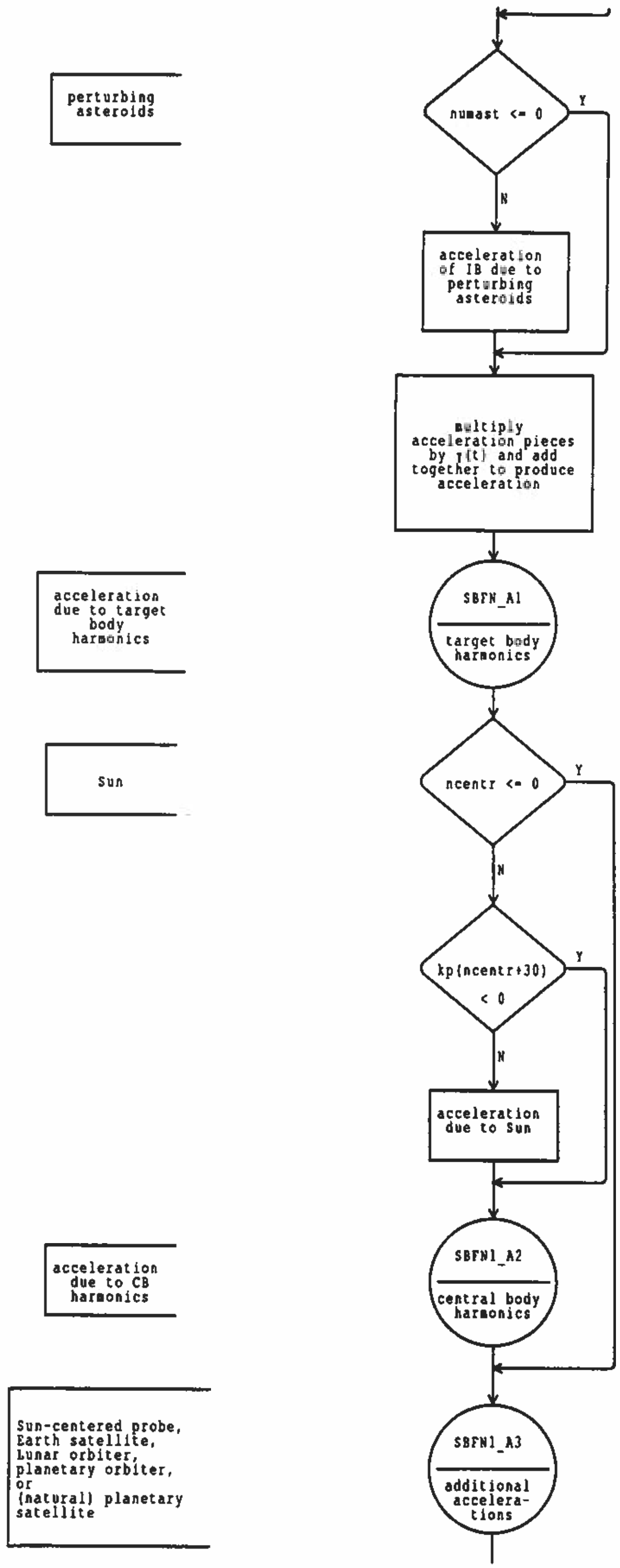
IB	Integrated Body
CB	Central Body
TB	Target Body
E-M	Earth-Moon barycenter
POE	Principle Of Equivalence
N_z	number of zonal harmonics
J_n	zonal harmonic coefficient of degree n
S_{nm}	tesseral harmonic sine coefficient of degree n and order m
C_{nm}	tesseral harmonic cosine coefficient of degree n and order m
N_T	number of tesseral harmonics
N_{TARG}	number of target bodies
GG	Gravity Gradient matrix
k	partial derivative parameter

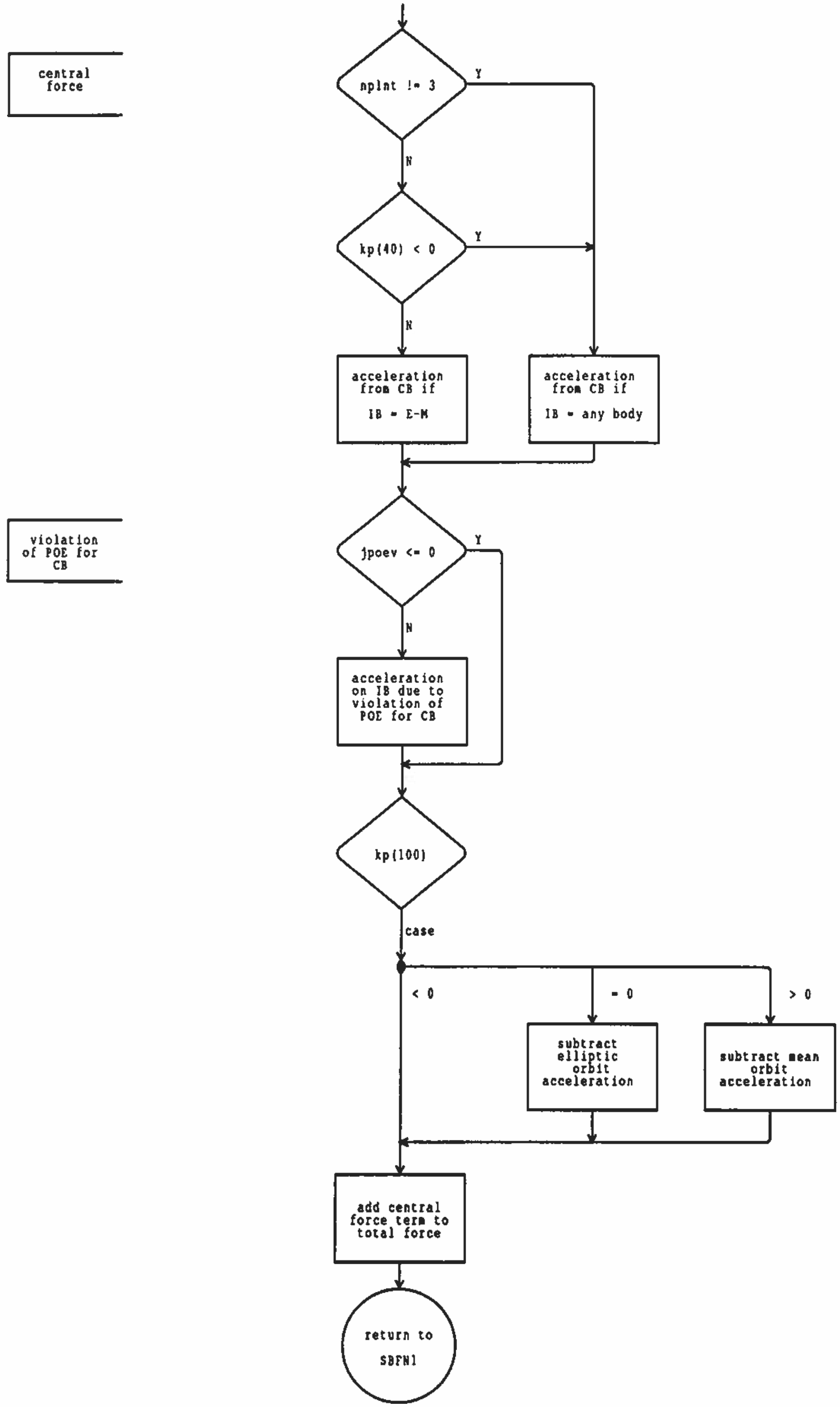
Also, for all conditionals, "Y" and "N" are equivalent to "T" and "F", respectively.



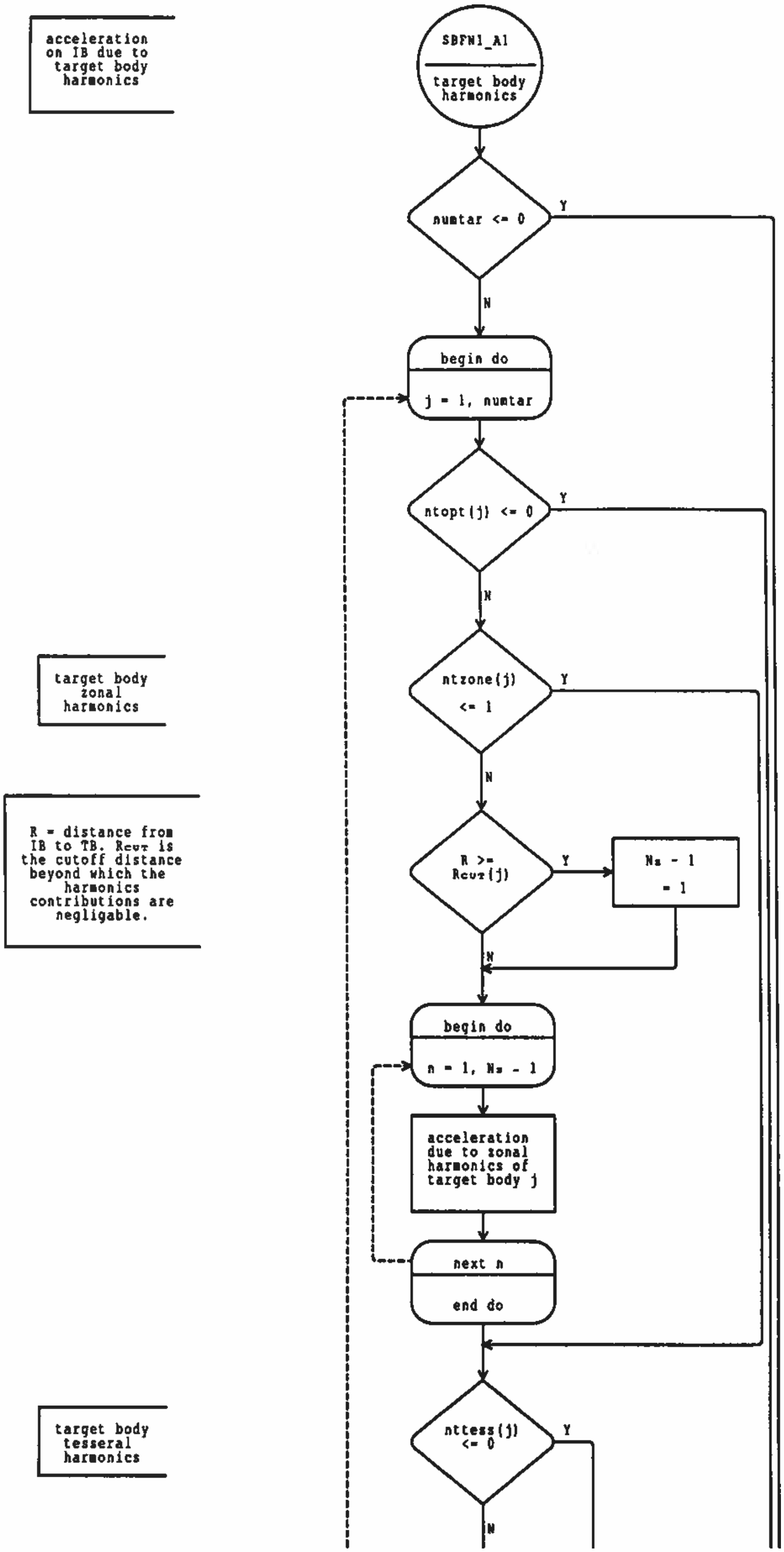


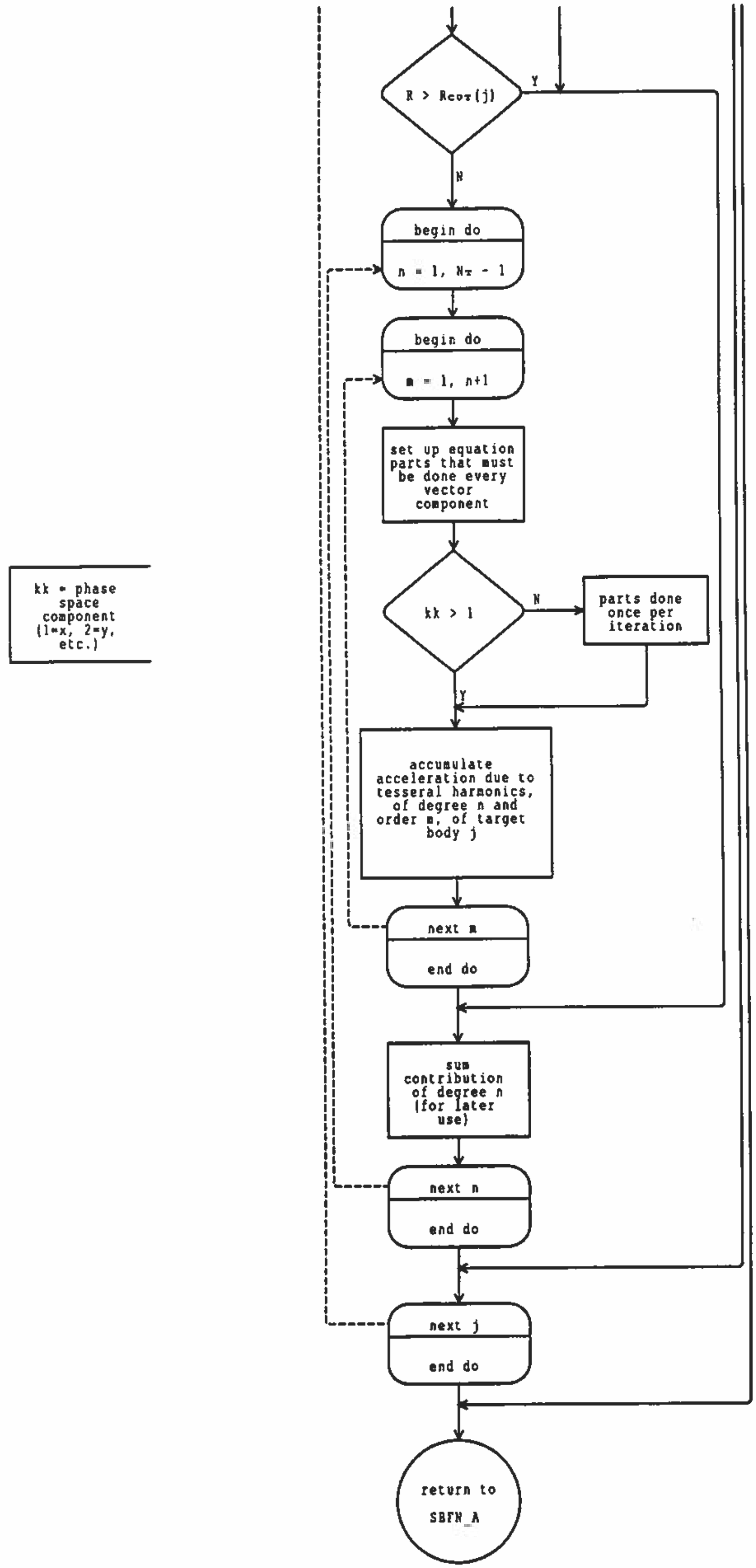
SBFN1_A
normal accelera-tions
nplnt != 3
IB not E-M bary
N
kp(40) < 0
N
Y
Y
acceleration due to perturbing planets
begin do
n = 1, 10
begin do
n = 1, 9
kp(n+30) < 0
Y
N
n = nplnt
Y
N
n = ncentr
Y
N
acceleration of planet nplnt by planet n
next n
end do
kp(n+30) < 0
Y
N
n = 3
Y
N
acceleration of E-M by planet n
next n
end do
POE violation
jpoev < 0
Y
N
acceleration of IB due to POE violation

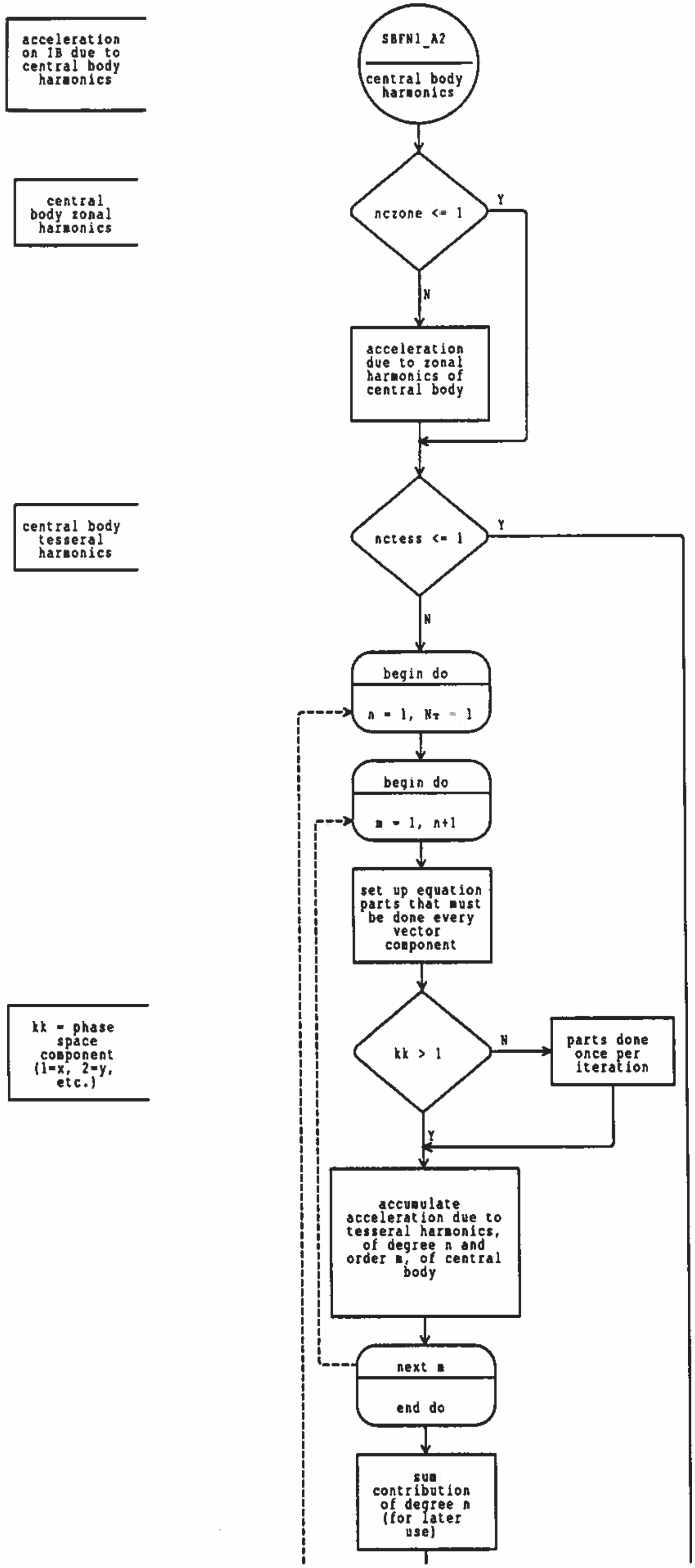

perturbing asteroids
numast <= 0
Y
N
acceleration of IB due to perturbing asteroids
multiply acceleration pieces by γ(t) and add together to produce acceleration
acceleration due to target body harmonics
SBFN_A1
target body harmonics
Sun
ncentr <= 0
Y
N
kp(ncentr+30) < 0
Y
N
acceleration due to Sun
acceleration due to CB harmonics
SBFN1_A2
central body harmonics
Sun-centered probe, Earth satellite, Lunar orbiter, planetary orbiter, or (natural) planetary satellite
SBFN1_A3
additional accelera-tions

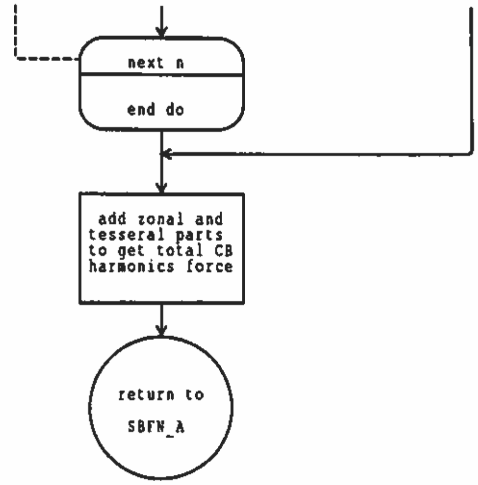


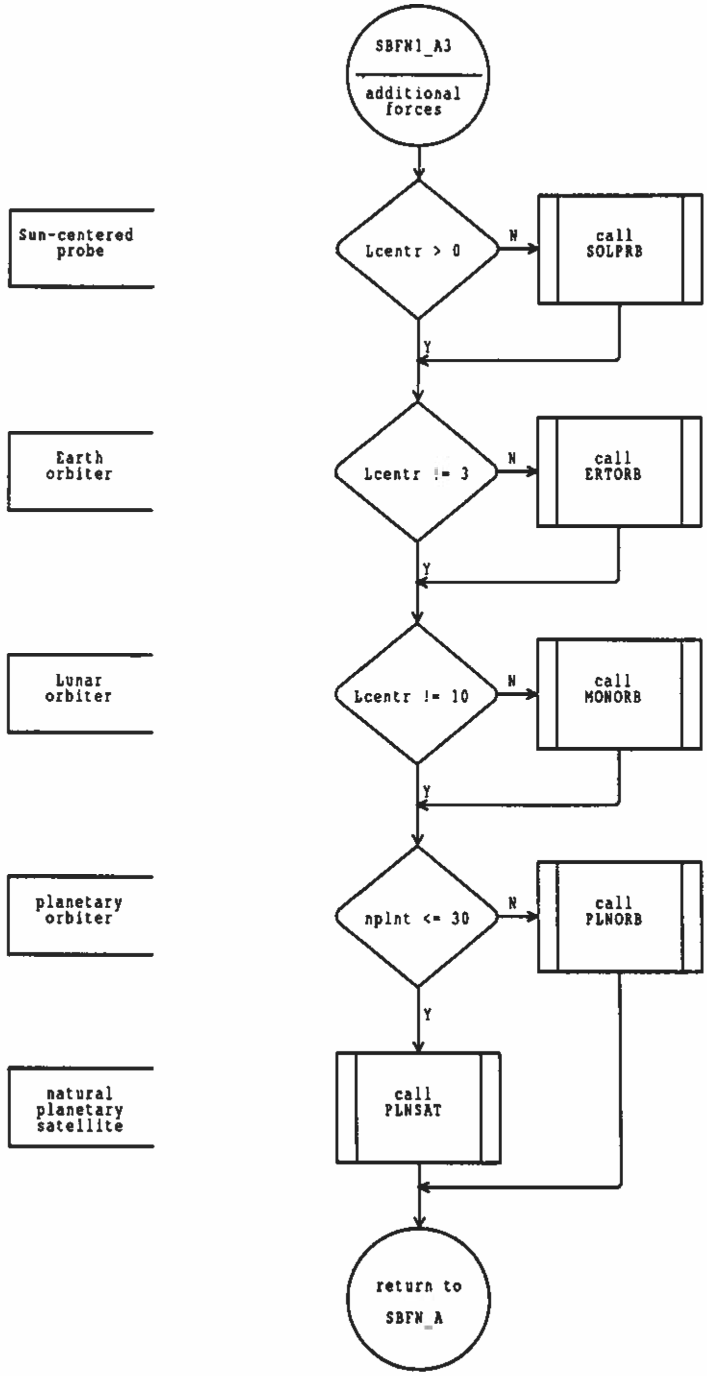
central force
nplnt != 3
Y
N
kp(40) < 0
Y
N
acceleration from CB if IB = E-M
acceleration from CB if IB = any body
violation of POE for CB
jpoev <= 0
Y
N
acceleration on IB due to violation of POE for CB
kp(100)
case
< 0
= 0
> 0
subtract elliptic orbit acceleration
subtract mean orbit acceleration
add central force term to total force
return to SBFN1

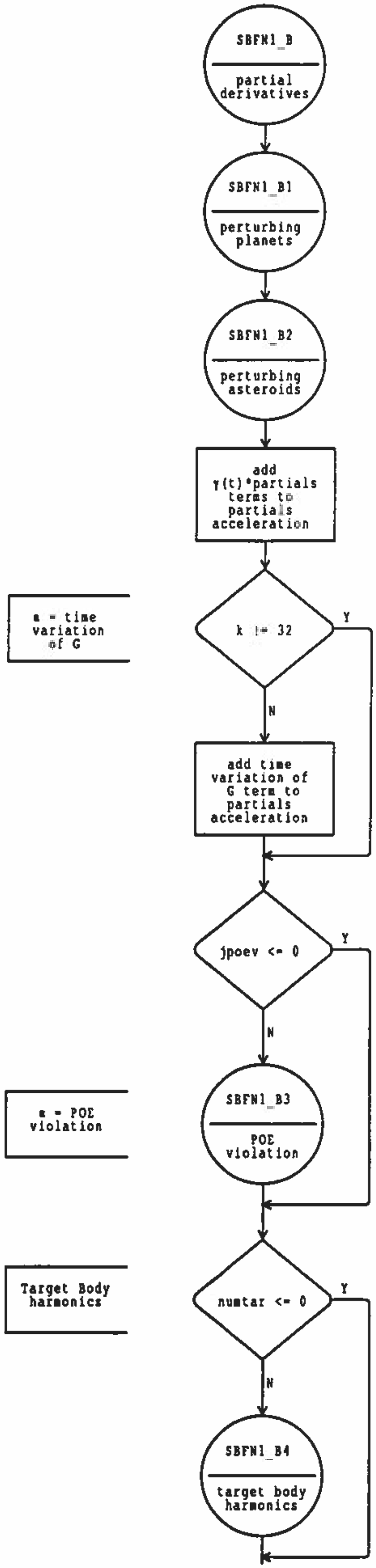

acceleration on IB due to target body harmonics
SBFN1_A1
target body harmonics
numtar <= 0
Y
N
begin do
j = 1, numtar
ntopt(j) <= 0
Y
N
target body zonal harmonics
ntzone(j) <= 1
Y
N
R = distance from IB to TB. Rcut is the cutoff distance beyond which the harmonics contributions are negligable.
R >= Rcut(j)
Y
Nz - 1 = 1
N
begin do
n = 1, Nz - 1
acceleration due to zonal harmonics of target body j
next n
end do
target body tesseral harmonics
nttess(j) <= 0
Y
N

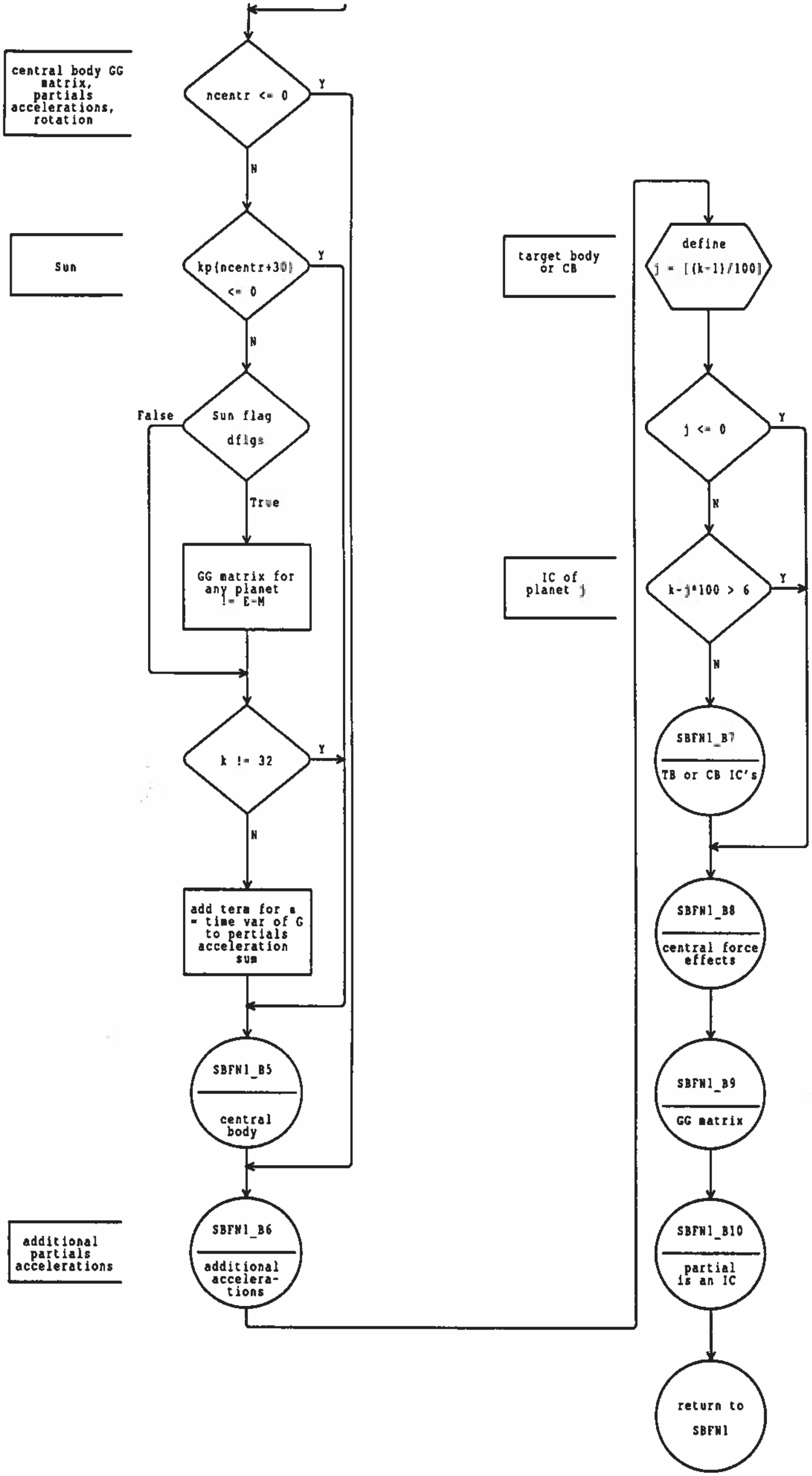

R > Rcor(j)
Y
N
begin do
n = 1, NT - 1
begin do
m = 1, n+1
set up equation parts that must be done every vector component
kk = phase space component (1=x, 2=y, etc.)
kk > 1
N
parts done once per iteration
Y
accumulate acceleration due to tesseral harmonics, of degree n and order m, of target body j
next m
end do
sum contribution of degree n (for later use)
next n
end do
next j
end do
return to SBFN_A

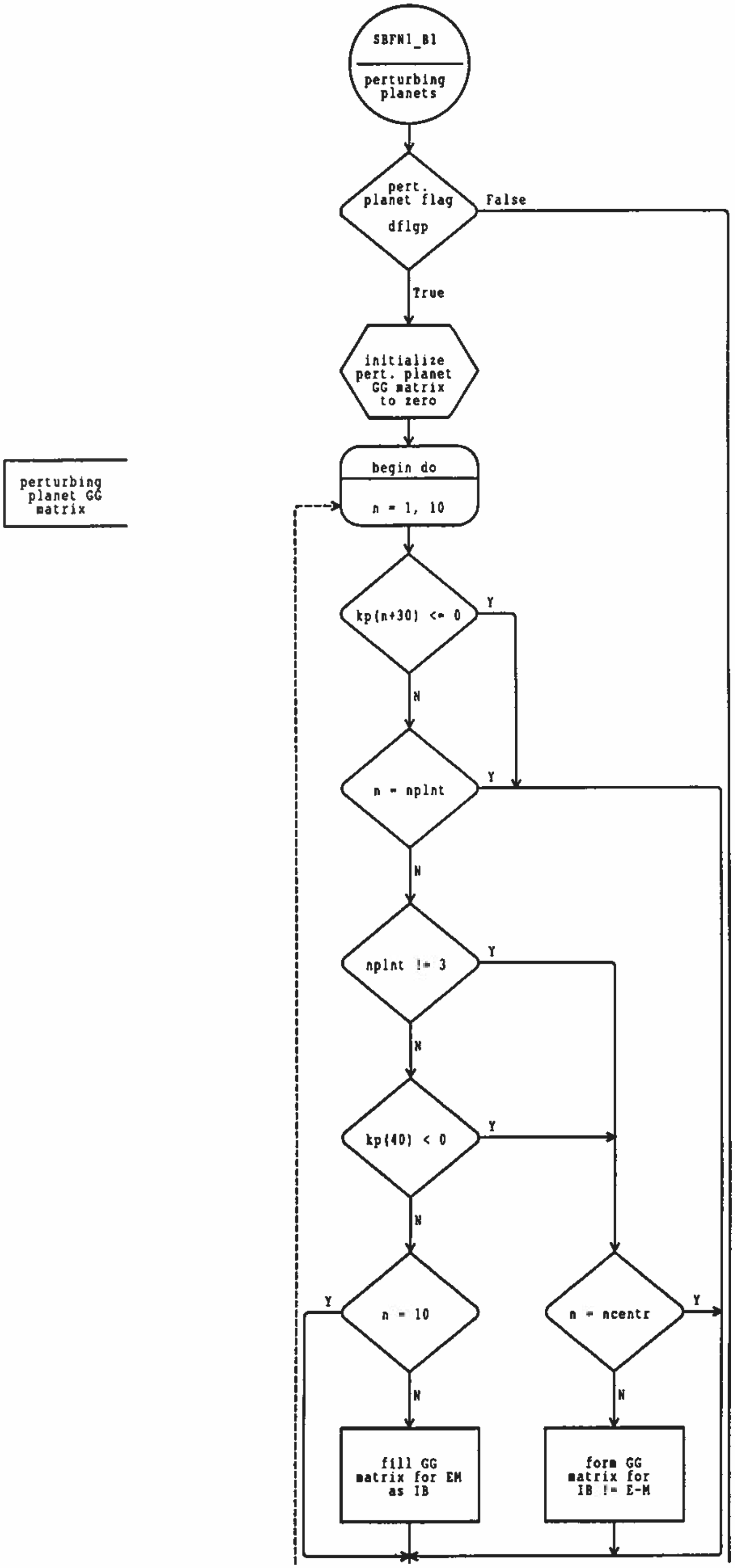

acceleration on IB due to central body harmonics
SBFN1_A2
central body harmonics
central body zonal harmonics
nczone <= 1
Y
N
acceleration due to zonal harmonics of central body
central body tesseral harmonics
nctess <= 1
Y
N
begin do
n = 1, N_T - 1
begin do
m = 1, n+1
set up equation parts that must be done every vector component
kk = phase space component (1=x, 2=y, etc.)
kk > 1
N
parts done once per iteration
Y
accumulate acceleration due to tesseral harmonics, of degree n and order m, of central body
next m
end do
sum contribution of degree n (for later use)


next n
end do
add zonal and tesseral parts to get total CB harmonics force
return to SBFN_A

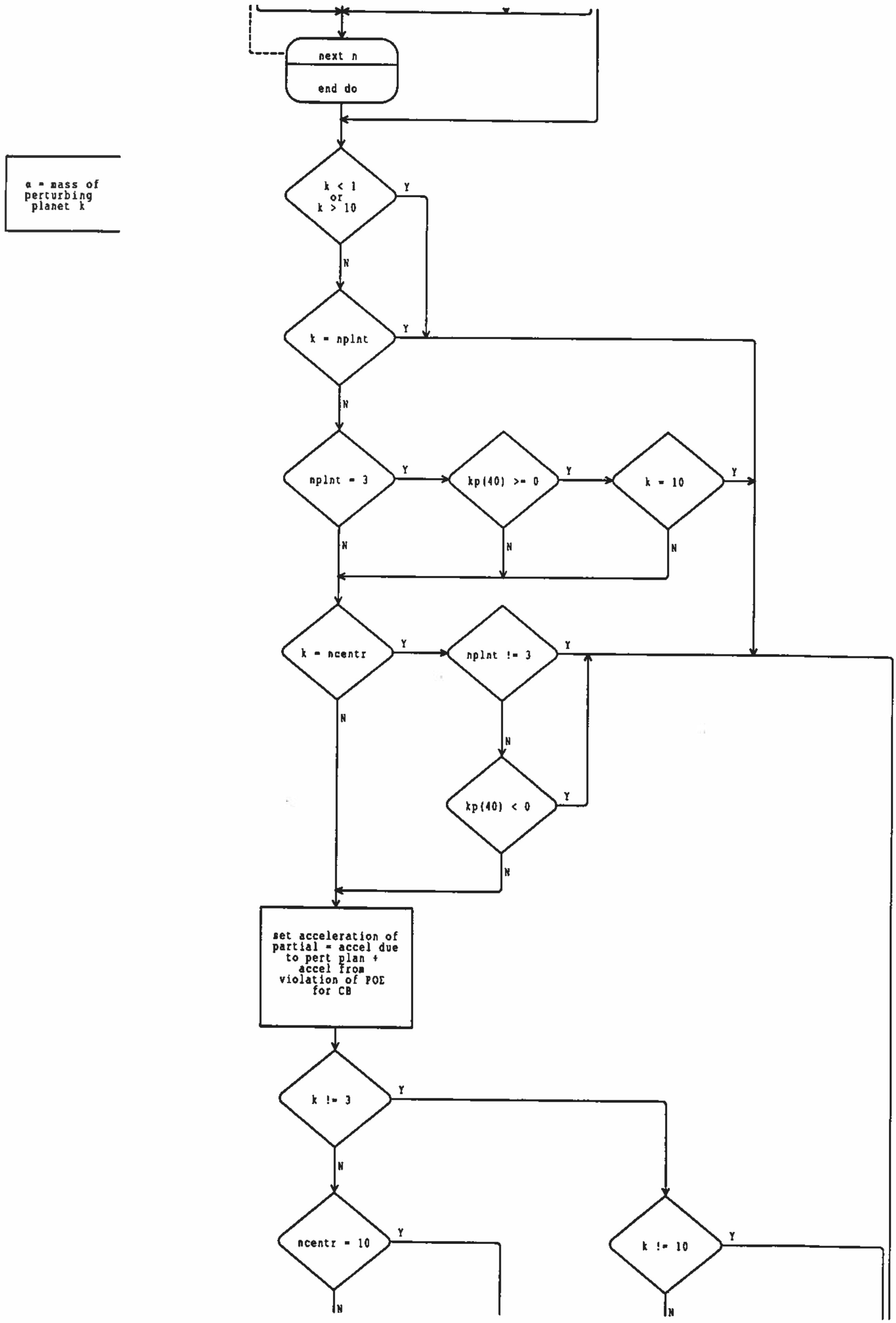

SBFN1_A3
additional forces
Sun-centered probe
Lcentr > 0
N
Y
call SOLPRB
Earth orbiter
Lcentr != 3
call ERTORB
Lunar orbiter
Lcentr != 10
call MONORB
planetary orbiter
nplnt <= 30
call PLNORB
natural planetary satellite
call PLNSAT
return to SBFN_A


SBFN1_B
partial derivatives
SBFN1_B1
perturbing planets
SBFN1_B2
perturbing asteroids
add γ(t)*partials terms to partials acceleration
a = time variation of G
k != 32
Y
N
add time variation of G term to partials acceleration
jpoev <= 0
Y
N
a = POE violation
SBFN1_B3
POE violation
Target Body harmonics
numtar <= 0
Y
N
SBFN1_B4
target body harmonics

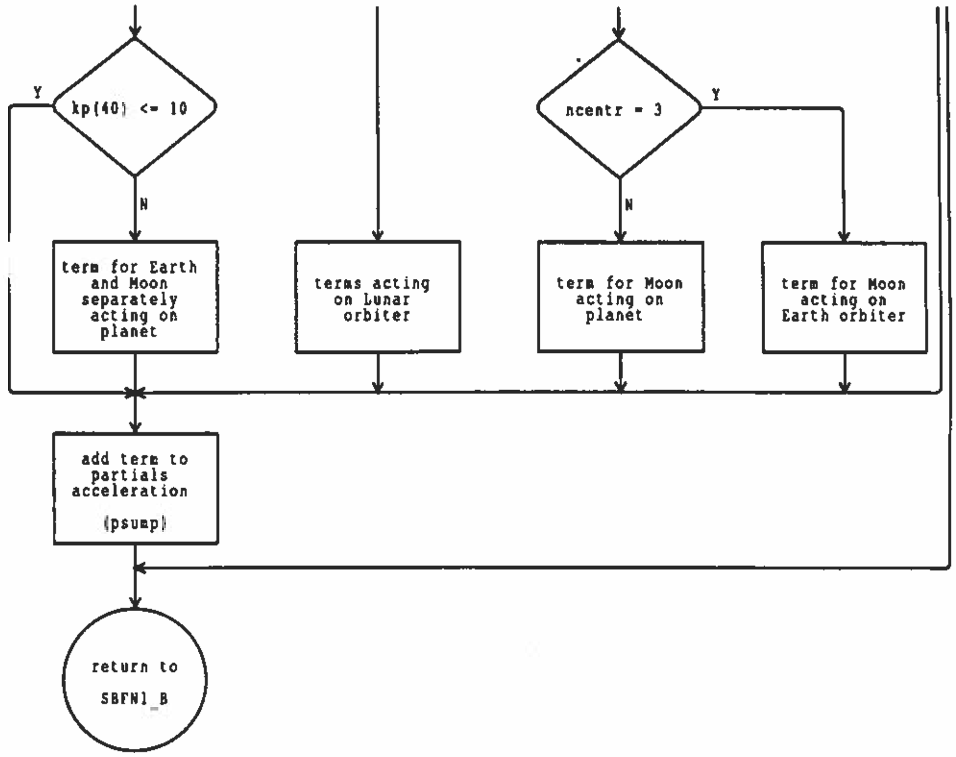

central body GG matrix, partials accelerations, rotation
ncentr <= 0
Y
N
Sun
kp(ncentr+30) <= 0
Y
N
Sun flag dflgs
False
True
GG matrix for any planet != E-M
k != 32
Y
N
add term for a = time var of G to pertials acceleration sum
SBFN1_B5
central body
additional partials accelerations
SBFN1_B6
additional accelera-tions
target body or CB
define j = [(k-1)/100]
j <= 0
Y
N
IC of planet j
k-j*100 > 6
Y
N
SBFN1_B7
TB or CB IC's
SBFN1_B8
central force effects
SBFN1_B9
GG matrix
SBFN1_B10
partial is an IC
return to SBFN1

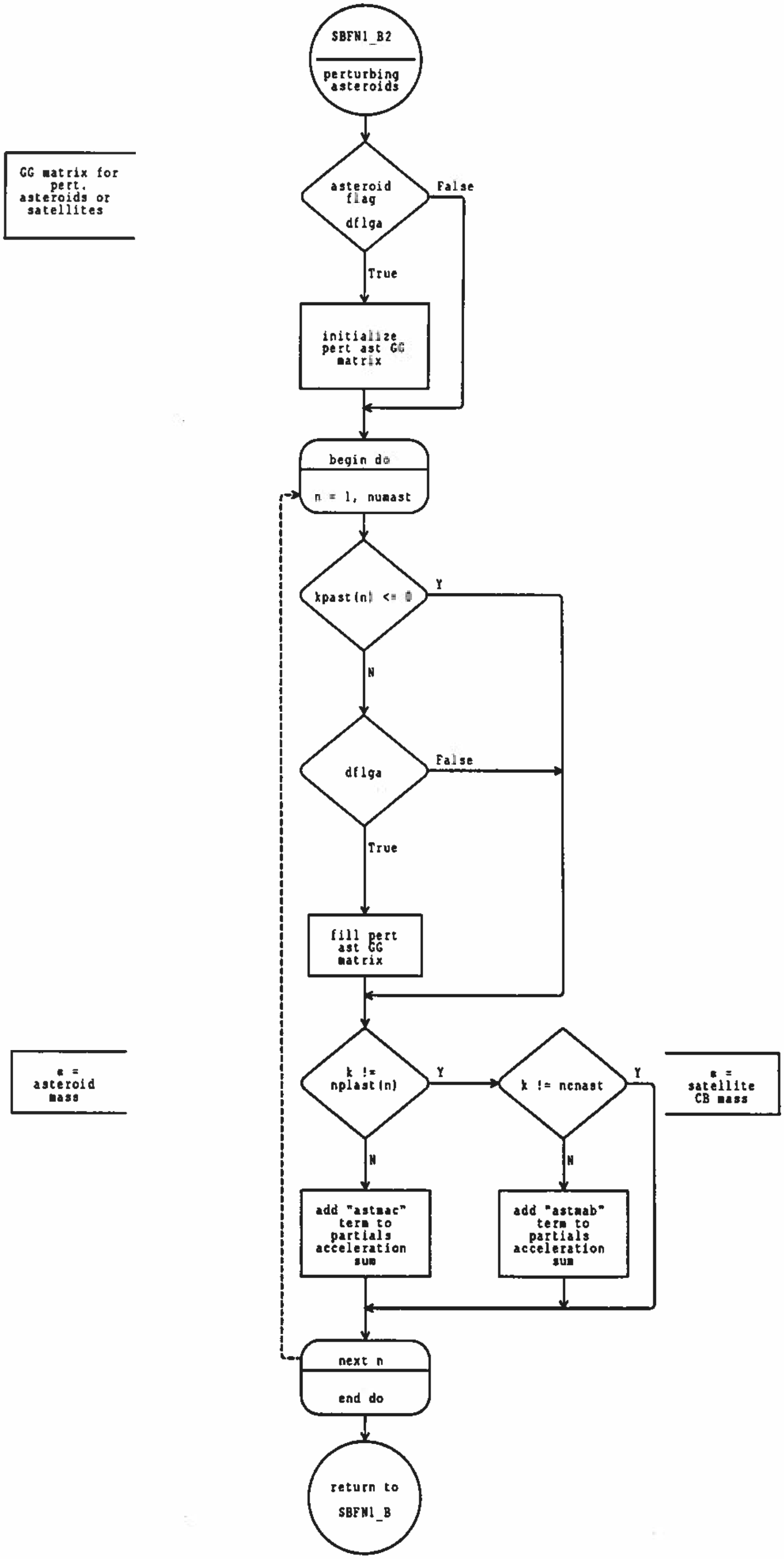


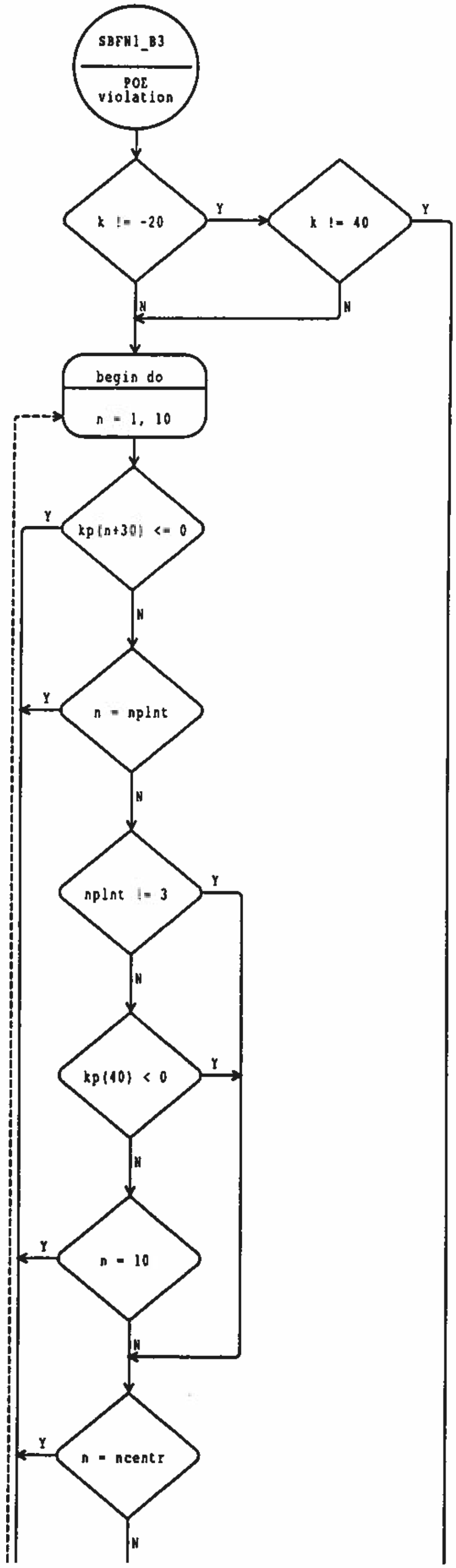
SBFN1_B1
perturbing planets
pert. planet flag dflgp
False
True
initialize pert. planet GG matrix to zero
perturbing planet GG matrix
begin do
n = 1, 10
kp(n+30) <= 0
Y
N
n = nplnt
Y
N
nplnt != 3
Y
N
kp(40) < 0
Y
N
n = 10
Y
N
n = ncentr
Y
N
fill GG matrix for EM as IB
form GG matrix for IB != E-M

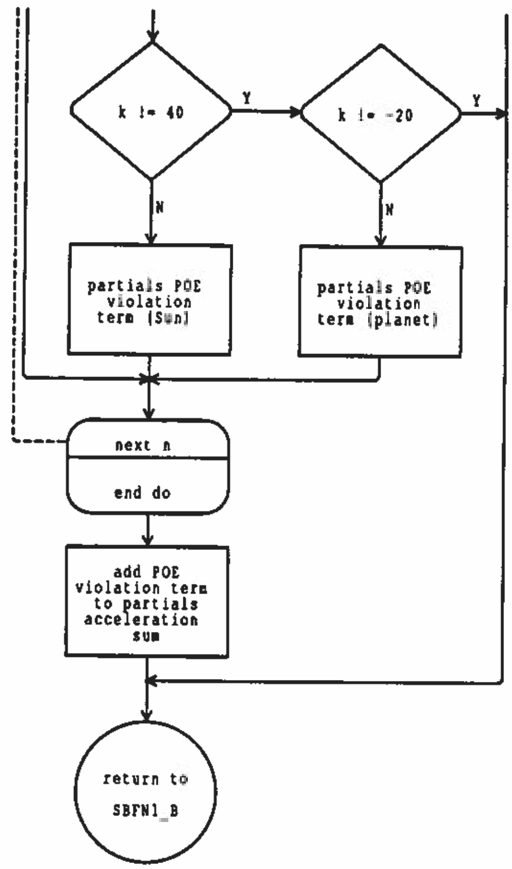


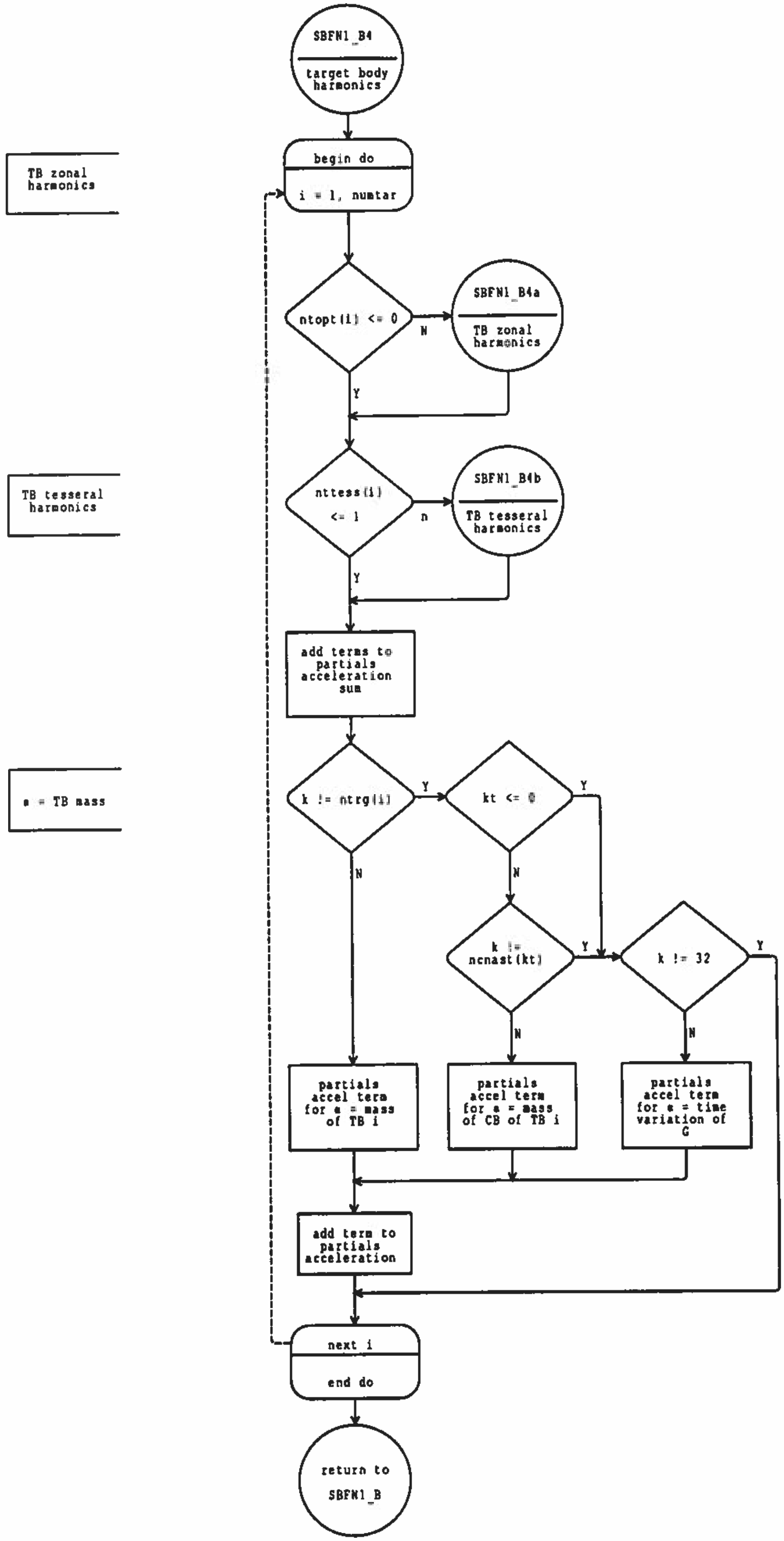
a = mass of perturbing planet k
next n
end do
k < 1 or k > 10
Y
N
k = nplnt
Y
N
nplnt = 3
Y
N
kp(40) >= 0
Y
N
k = 10
Y
N
k = ncentr
Y
N
nplnt != 3
Y
N
kp(40) < 0
Y
N
set acceleration of partial = accel due to pert plan + accel from violation of POE for CB
k != 3
Y
N
ncentr = 10
Y
N
k != 10
Y
N

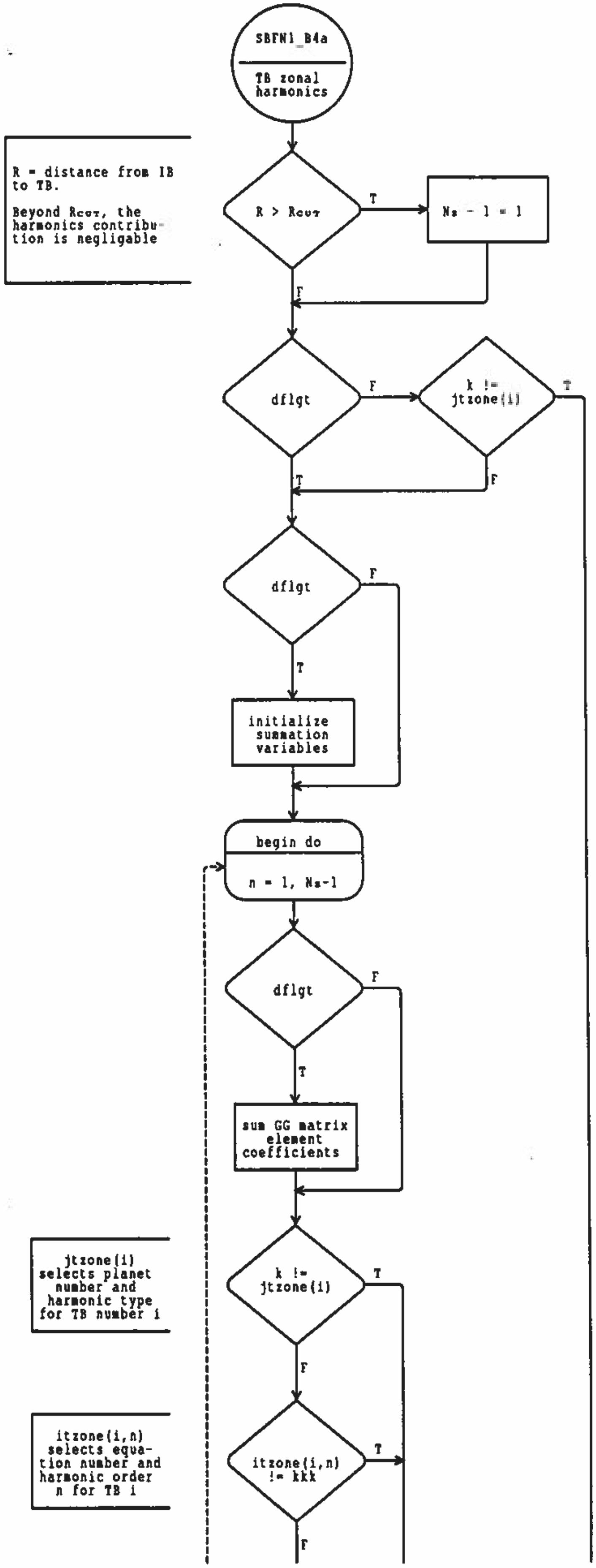

kp(40) <= 10
Y
N
term for Earth and Moon separately acting on planet
terms acting on Lunar orbiter
ncentr = 3
Y
N
term for Moon acting on planet
term for Moon acting on Earth orbiter
add term to partials acceleration
(psump)
return to
SBFN1_B

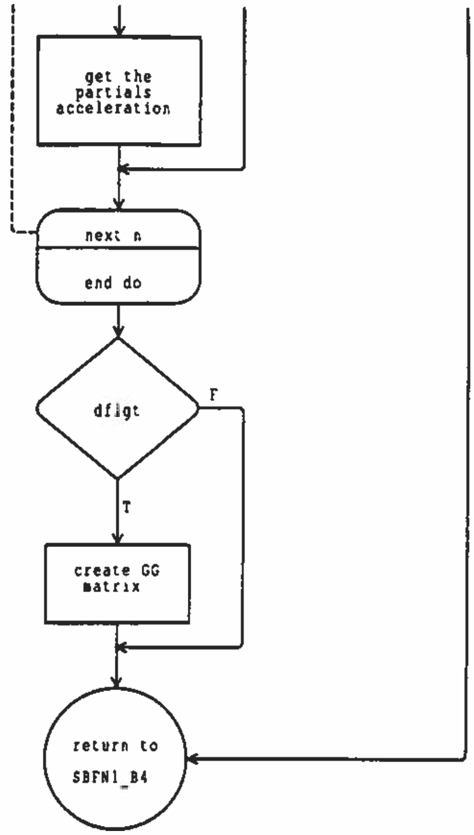

SBFN1_B2
perturbing asteroids
GG matrix for pert. asteroids or satellites
asteroid flag dflga
False
True
initialize pert ast GG matrix
begin do
n = 1, numast
kpast(n) <= 0
Y
N
dflga
False
True
fill pert ast GG matrix
a = asteroid mass
k != nplast(n)
Y
k != ncnast
Y
a = satellite CB mass
N
N
add "astmac" term to partials acceleration sum
add "astmab" term to partials acceleration sum
next n
end do
return to SBFN1_B

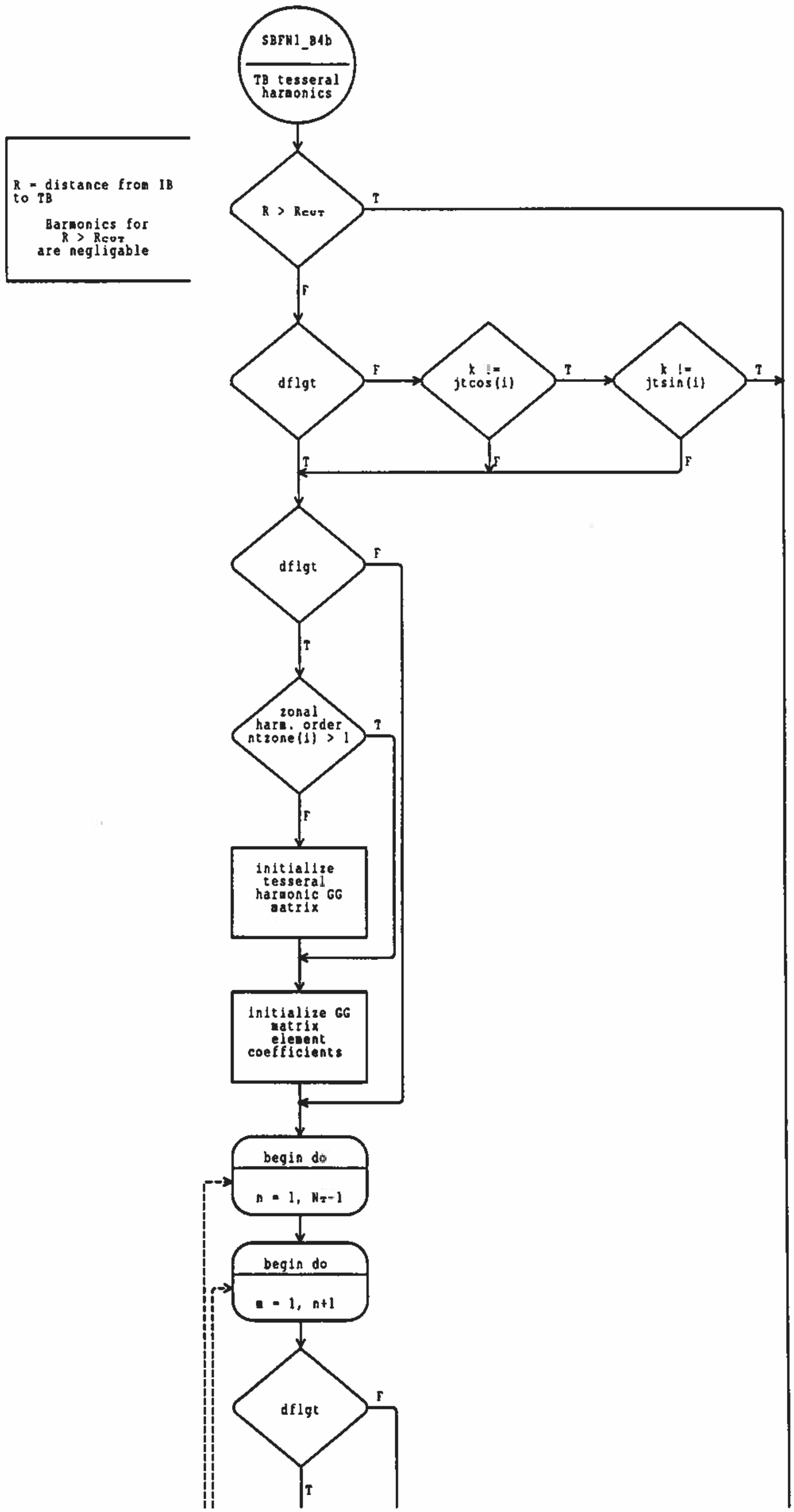

SBFN1_B3
POE violation
k != -20
k != 40
begin do
n = 1, 10
kp(n+30) <= 0
n = nplnt
nplnt != 3
kp(40) < 0
n = 10
n = ncentr
Y
N


k != 40
Y
N
k != -20
Y
N
partials POE violation term (Sun)
partials POE violation term (planet)
next n
end do
add POE violation term to partials acceleration sum
return to SBFN1_B

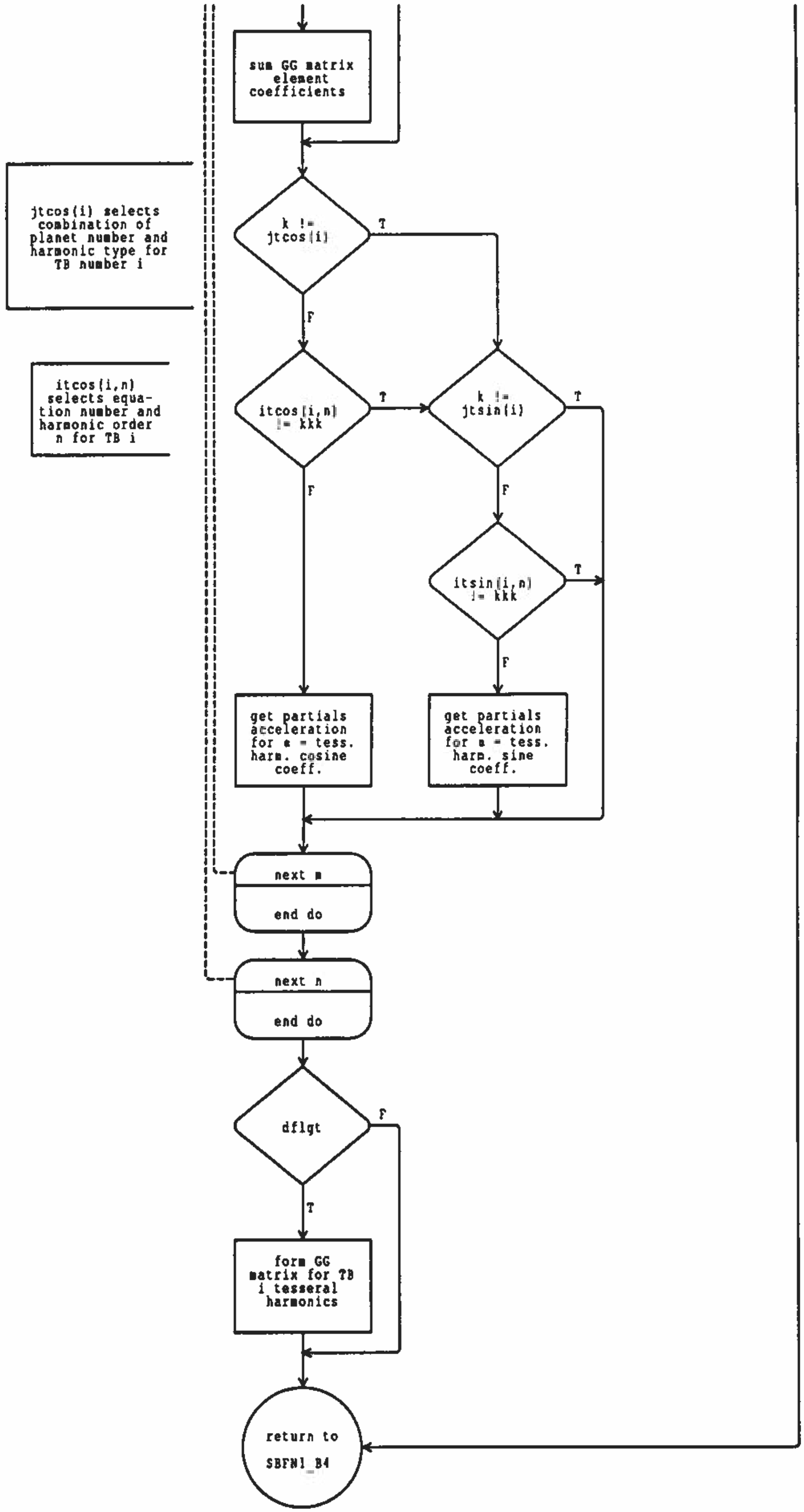

SBFN1_B4
target body harmonics
TB zonal harmonics
begin do
i = 1, numtar
ntopt(i) <= 0
N
SBFN1_B4a
TB zonal harmonics
Y
TB tesseral harmonics
nttess(i) <= 1
n
SBFN1_B4b
TB tesseral harmonics
Y
add terms to partials acceleration sum
a = TB mass
k != ntrg(i)
Y
kt <= 0
Y
N
N
k != ncnast(kt)
Y
k != 32
Y
N
N
partials accel term for a = mass of TB i
partials accel term for a = mass of CB of TB i
partials accel term for a = time variation of G
add term to partials acceleration
next i
end do
return to SBFN1_B

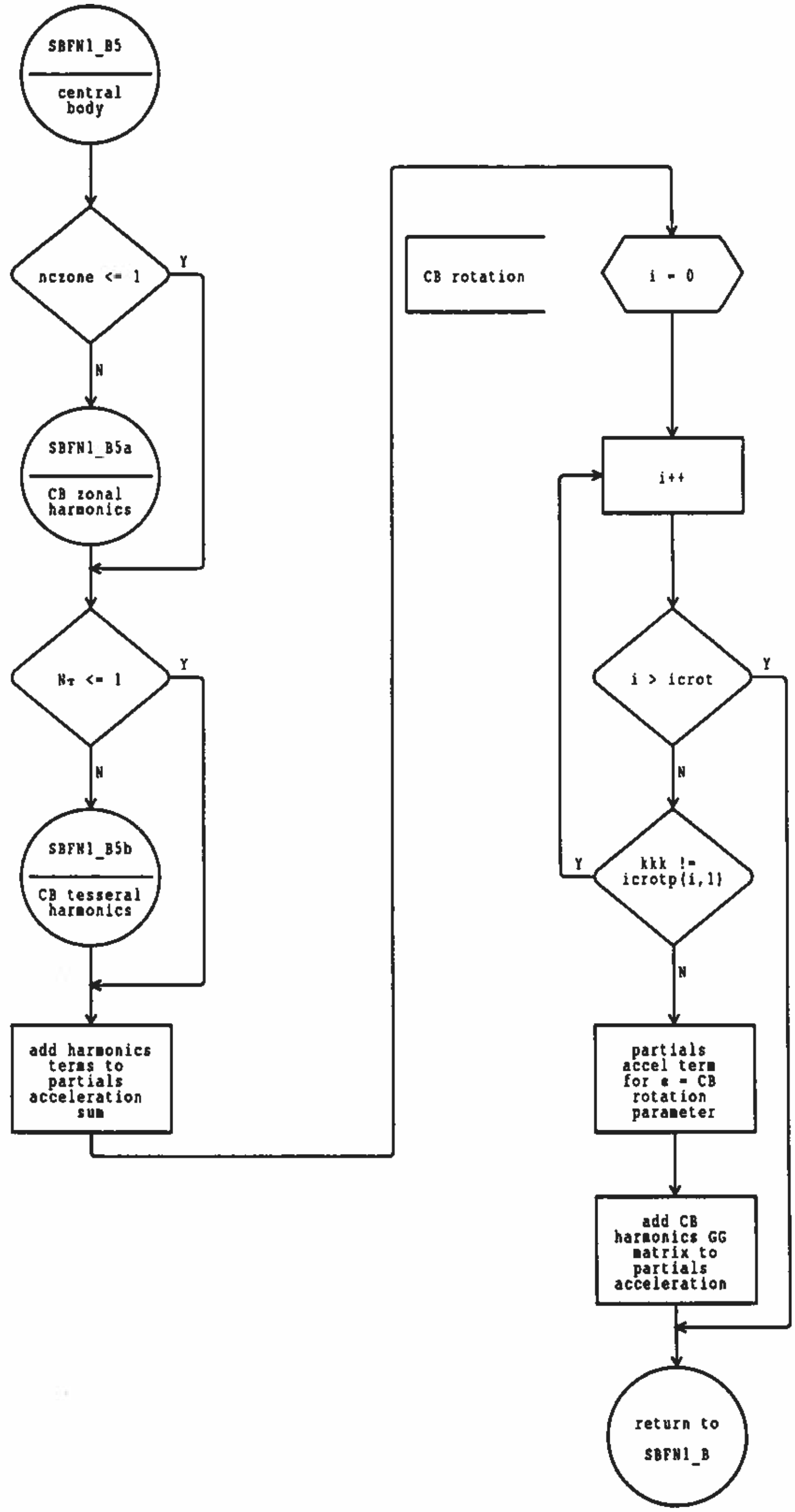

SBFN1_B4a
TB zonal harmonics
R = distance from IB to TB.
Beyond RCUT, the harmonics contribution is negligable
R > RCUT
T
Ns - 1 = 1
F
dflgt
F
k != jtzone(i)
T
F
T
dflgt
F
T
initialize summation variables
begin do
n = 1, Ns-1
dflgt
F
T
sum GG matrix element coefficients
jtzone(i) selects planet number and harmonic type for TB number i
k != jtzone(i)
T
F
itzone(i,n) selects equation number and harmonic order n for TB i
itzone(i,n) != kkk
T
F

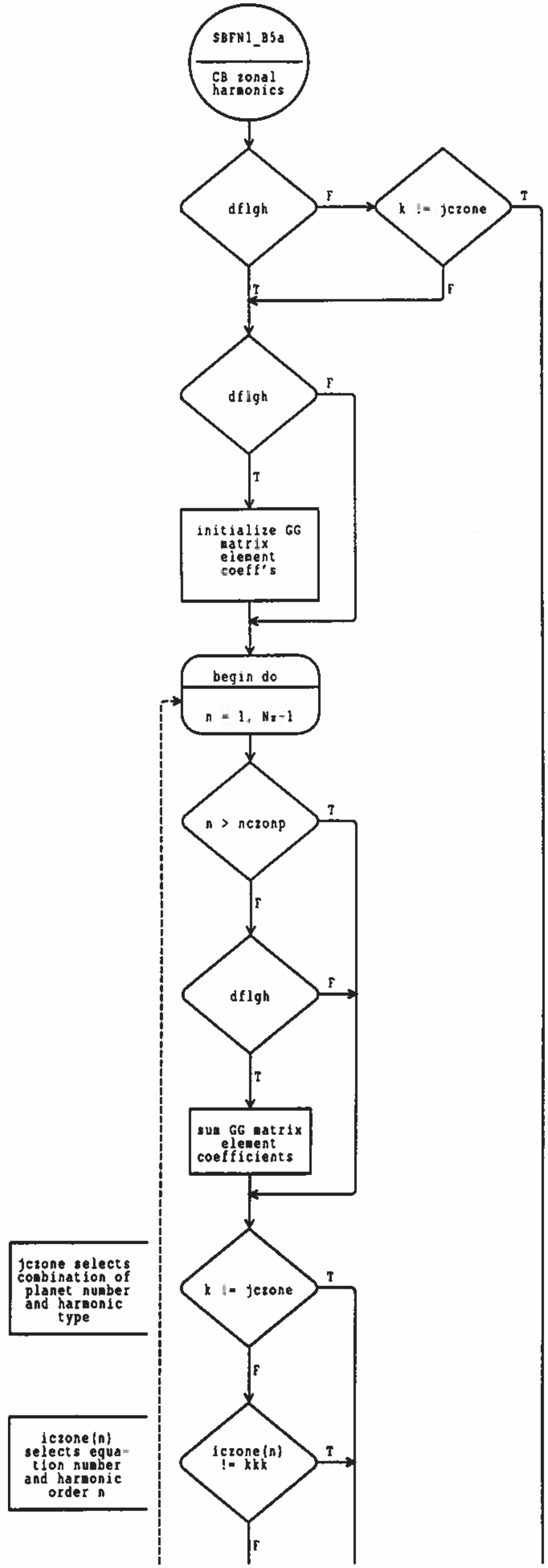

get the partials acceleration
next n
end do
dflgt
F
T
create GG matrix
return to SBFN1_B4



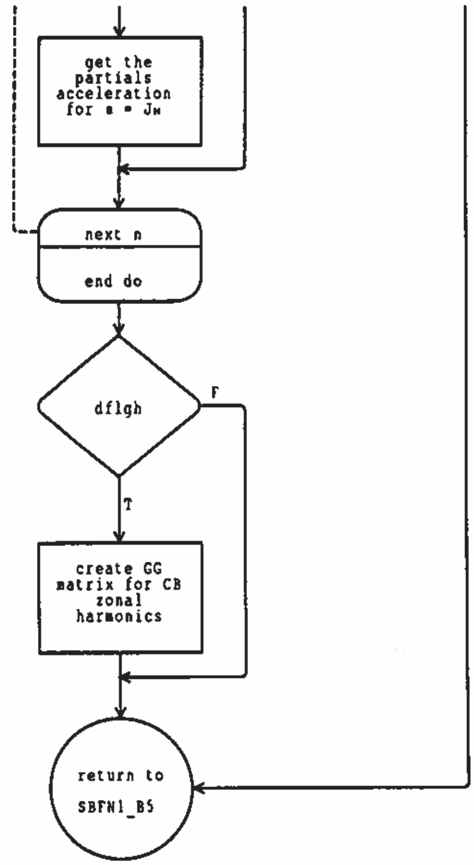
SBFN1_B4b
TB tesseral harmonics
R = distance from IB to TB
Barmonics for R > Rcut are negligable
R > Rcut
T
F
dflgt
F
k != jtcos(i)
T
k != jtsin(i)
T
F
F
T
dflgt
F
T
zonal harm. order ntzone(i) > 1
T
F
initialize tesseral harmonic GG matrix
initialize GG matrix element coefficients
begin do
n = 1, NT-1
begin do
m = 1, n+1
dflgt
F
T

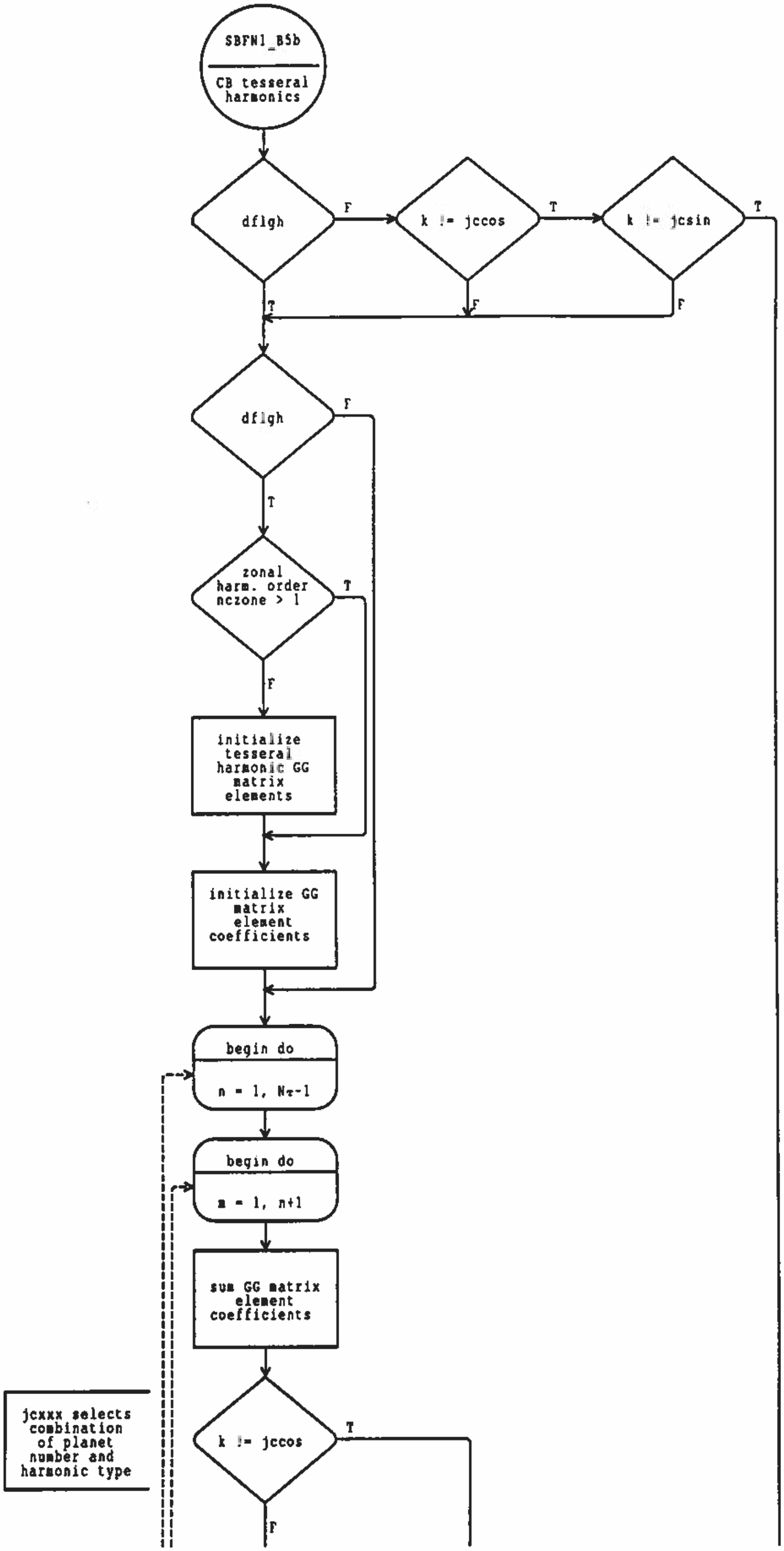

sum GG matrix element coefficients
jtcos(i) selects combination of planet number and harmonic type for TB number i
k != jtcos(i)
T
F
itcos(i,n) selects equation number and harmonic order n for TB i
itcos(i,n) != kkk
k != jtsin(i)
itsin(i,n) != kkk
get partials acceleration for m = tess. harm. cosine coeff.
get partials acceleration for m = tess. harm. sine coeff.
next m
end do
next n
end do
dflgt
form GG matrix for TB i tesseral harmonics
return to SBFN1_B4

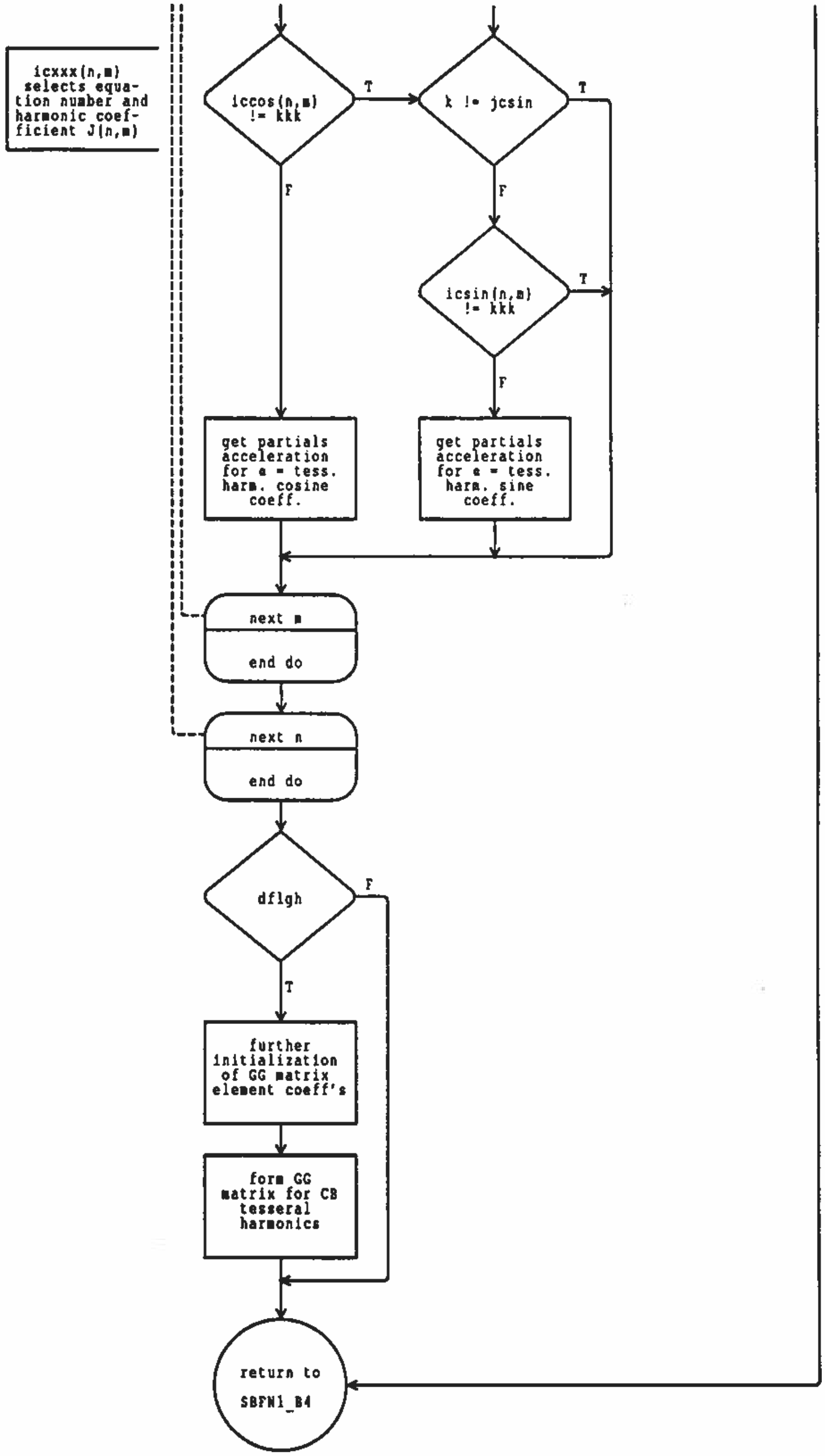

SBFN1_B5
central body
nczone <= 1
Y
N
SBFN1_B5a
CB zonal harmonics
N_T <= 1
Y
N
SBFN1_B5b
CB tesseral harmonics
add harmonics terms to partials acceleration sum
CB rotation
i = 0
i++
i > icrot
Y
N
kkk != icrotp(i,1)
Y
N
partials accel term for ε = CB rotation parameter
add CB harmonics GG matrix to partials acceleration
return to SBFN1_B

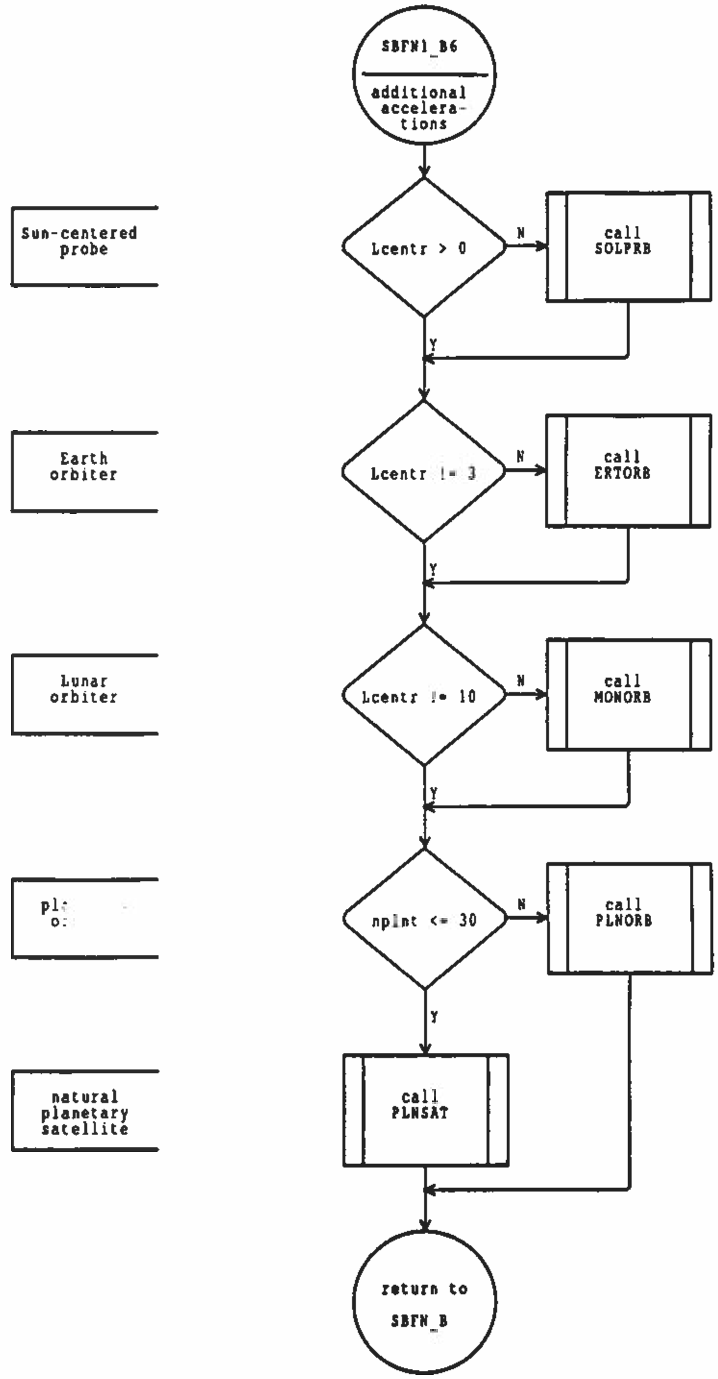


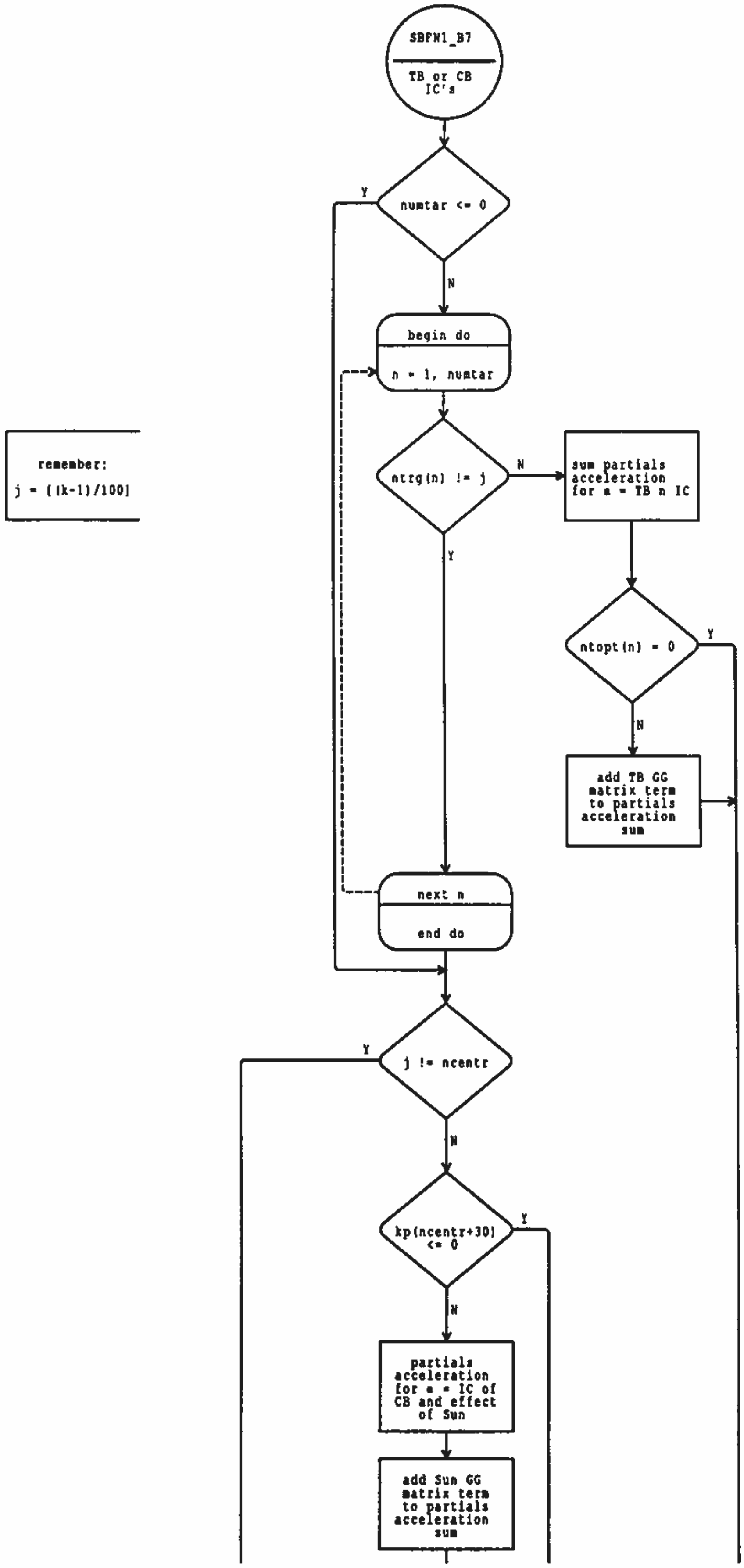
SBFN1_B5a
CB zonal harmonics
dflgh
F
k != jczone
T
T
F
dflgh
F
T
initialize GG matrix element coeff's
begin do
n = 1, Nz-1
n > nczonp
T
F
dflgh
F
T
sum GG matrix element coefficients
jczone selects combination of planet number and harmonic type
k != jczone
T
F
iczone(n) selects equation number and harmonic order n
iczone(n) != kkk
T
F

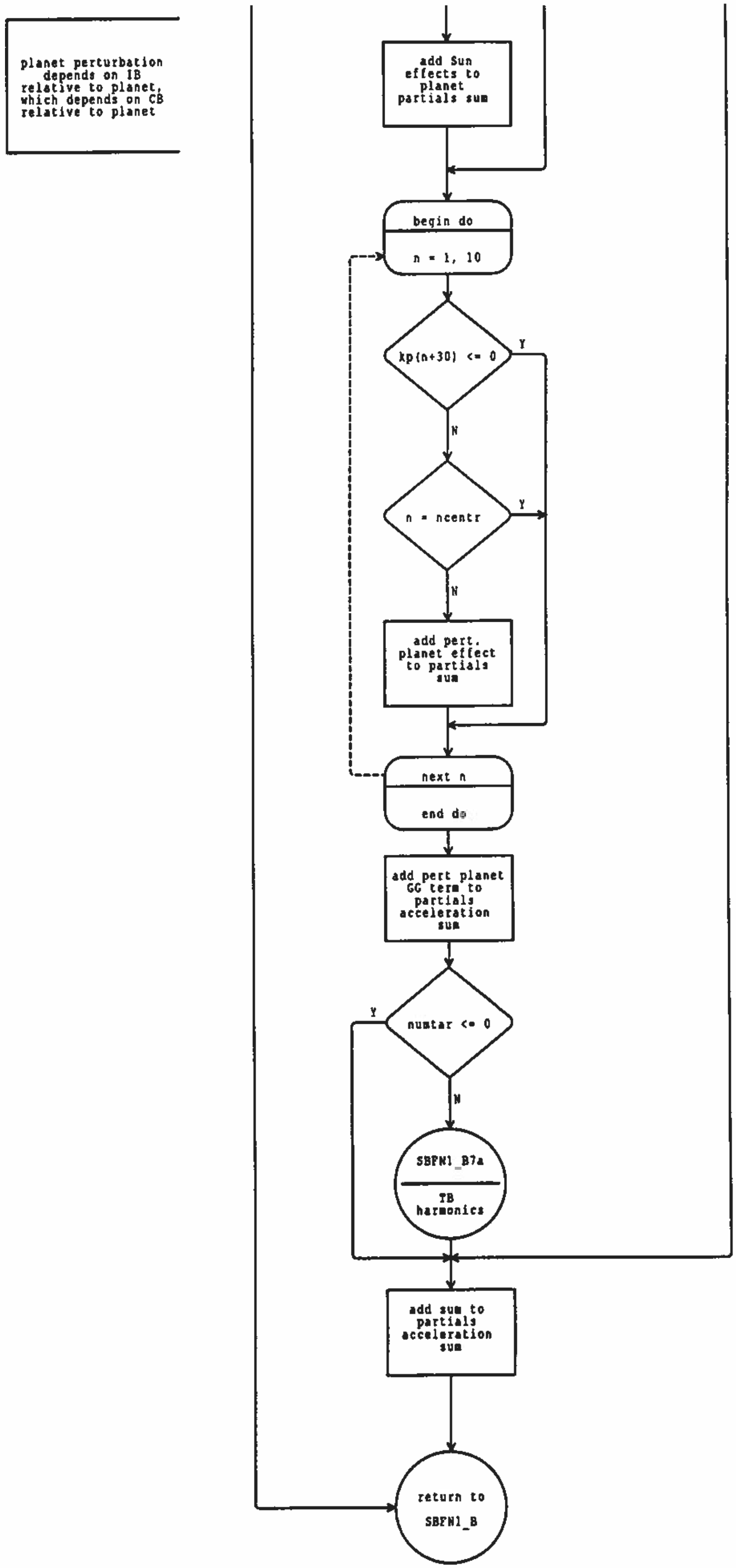

get the partials acceleration for m = J_M
next n
end do
dflgh
F
T
create GG matrix for CB zonal harmonics
return to
SBFN1_B5

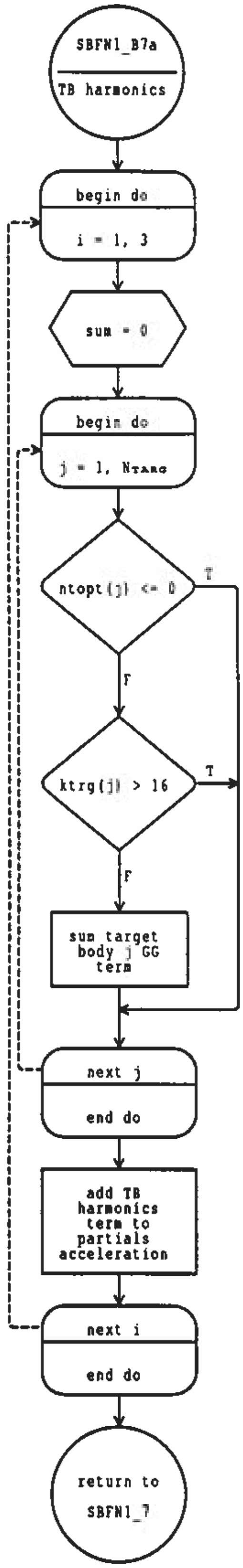

SBFN1_B5b
CB tesseral harmonics
dflgh
k != jccos
k != jcsin
dflgh
zonal harm. order nczone > 1
initialize tesseral harmonic GG matrix elements
initialize GG matrix element coefficients
begin do
n = 1, Nτ-1
begin do
m = 1, n+1
sum GG matrix element coefficients
jcxxx selects combination of planet number and harmonic type
k != jccos
T
F


icxxx(n,m) selects equation number and harmonic coefficient J(n,m)
iccos(n,m) != kkk
T
k != jcsin
T
F
F
icsin(n,m) != kkk
T
F
get partials acceleration for a = tess. harm. cosine coeff.
get partials acceleration for a = tess. harm. sine coeff.
next m
end do
next n
end do
dflgh
F
T
further initialization of GG matrix element coeff's
form GG matrix for CB tesseral harmonics
return to SBFN1_B4

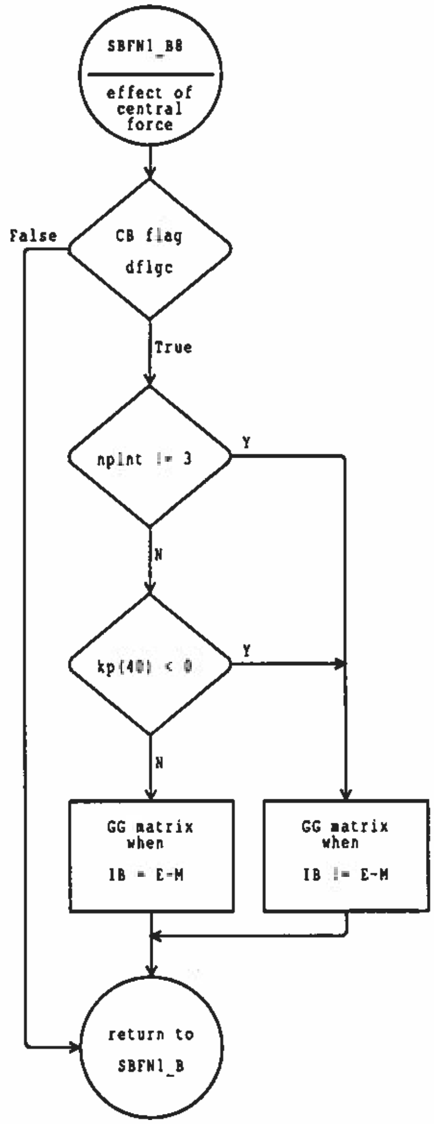

SBFN1_B6
additional accelera-tions
Sun-centered probe
Lcentr > 0
N
call SOLPRB
Y
Earth orbiter
Lcentr != 3
N
call ERTORB
Y
Lunar orbiter
Lcentr != 10
N
call MONORB
Y
nplnt <= 30
N
call PLNORB
Y
natural planetary satellite
call PLNSAT
return to SBFN_B

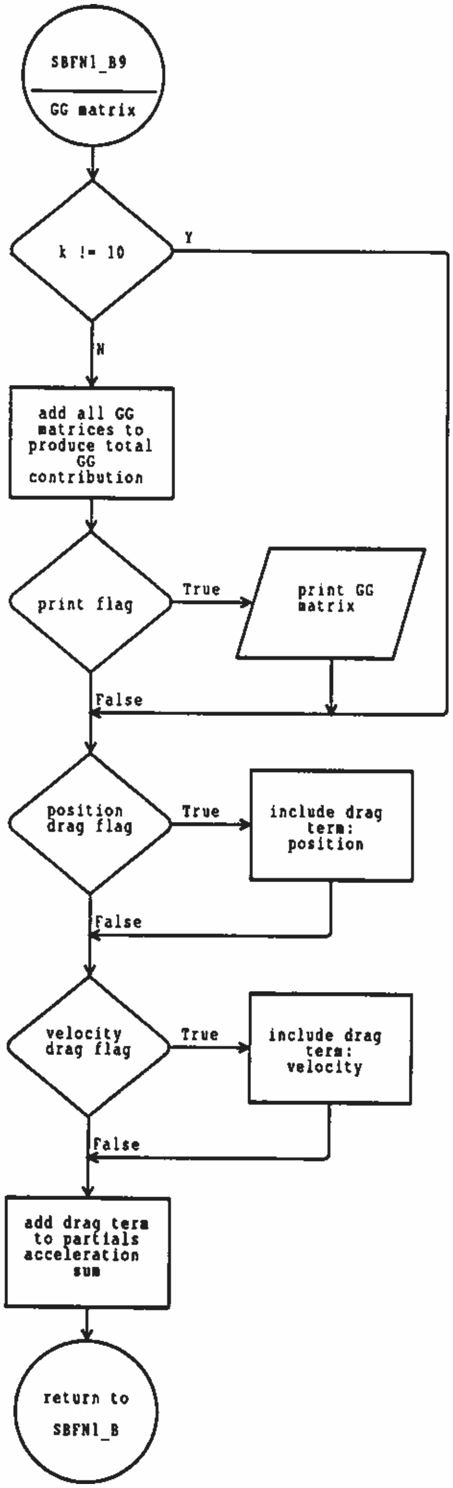

SBFN1_B7
TB or CB IC's
numtar <= 0
Y
N
begin do
n = 1, numtar
remember:
j = ((k-1)/100)
ntrg(n) != j
N
Y
sum partials acceleration for a = TB n IC
ntopt(n) = 0
Y
N
add TB GG matrix term to partials acceleration sum
next n
end do
j != ncentr
Y
N
kp(ncentr+30) <= 0
Y
N
partials acceleration for a = IC of CB and effect of Sun
add Sun GG matrix term to partials acceleration sum

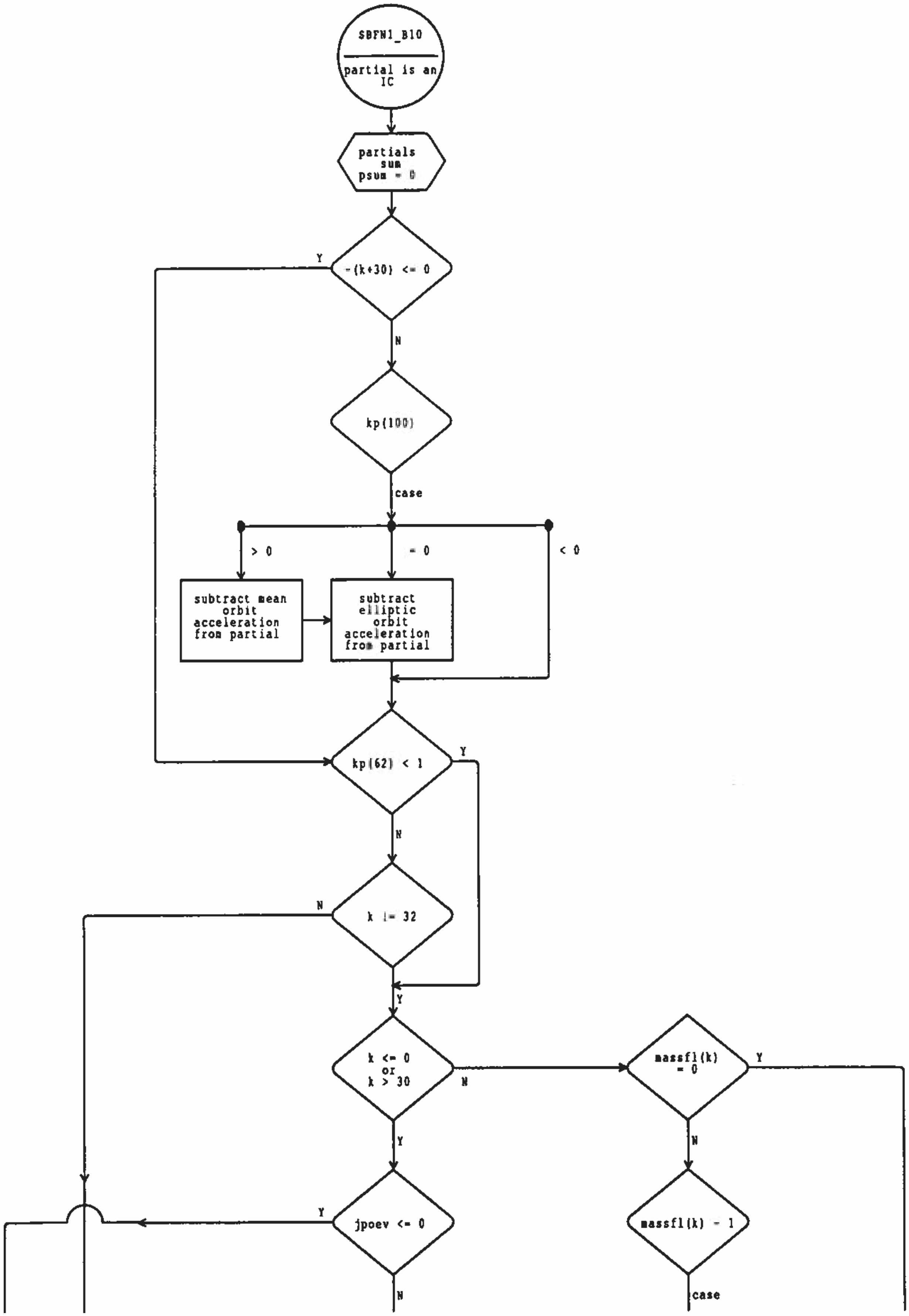

planet perturbation depends on IB relative to planet, which depends on CB relative to planet
add Sun effects to planet partials sum
begin do
n = 1, 10
kp(n+30) <= 0
Y
N
n = ncentr
Y
N
add pert. planet effect to partials sum
next n
end do
add pert planet GG term to partials acceleration sum
numtar <= 0
Y
N
SBFN1_B7a
TB harmonics
add sum to partials acceleration sum
return to SBFN1_B

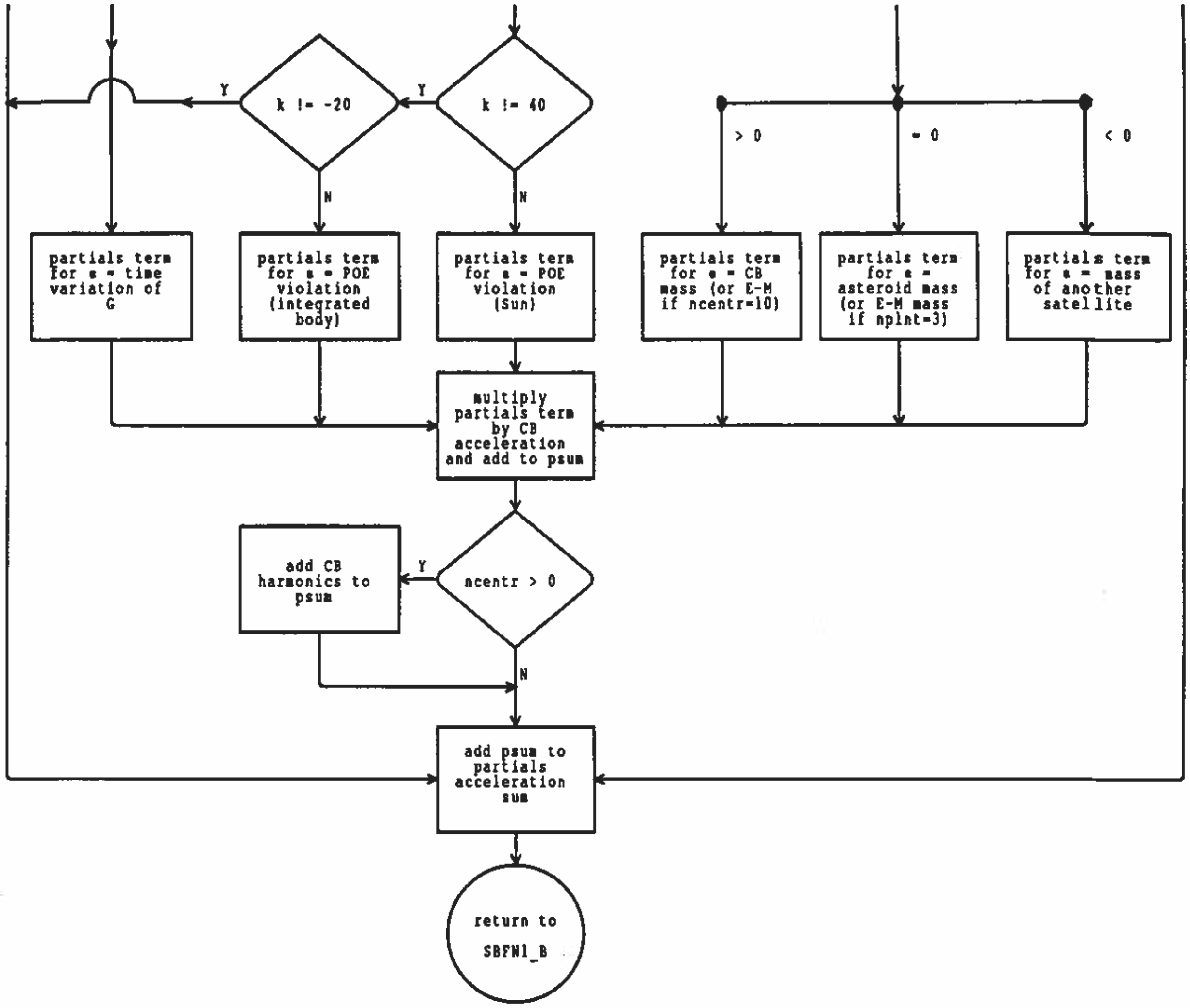


SBFN1_B7a
TB harmonics
begin do
i = 1, 3
sum = 0
begin do
j = 1, NTARG
ntopt(j) <= 0
T
F
ktrg(j) > 16
T
F
sum target body j GG term
next j
end do
add TB harmonics term to partials acceleration
next i
end do
return to SBFN1_7

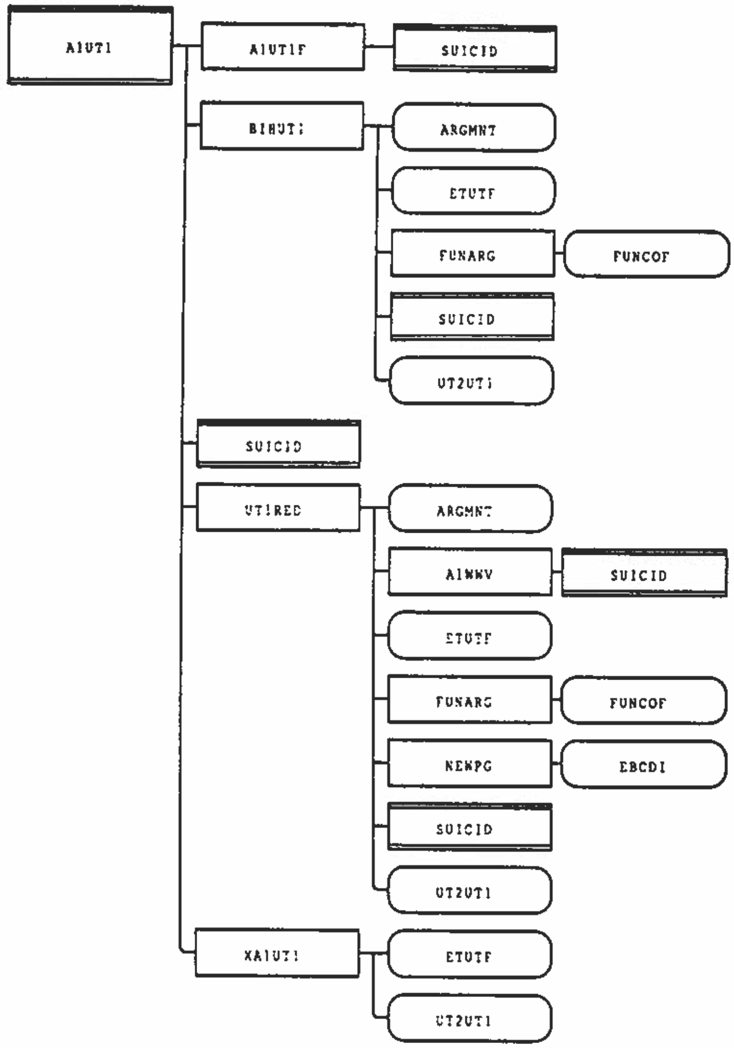

SBFN1_B8
effect of central force
CB flag dflgc
False
True
npint != 3
Y
N
kp(40) < 0
Y
N
GG matrix when IB = E-M
GG matrix when IB != E-M
return to SBFN1_B


SBFN1_B9
GG matrix
k != 10
Y
N
add all GG matrices to produce total GG contribution
print flag
True
print GG matrix
False
position drag flag
True
include drag term: position
False
velocity drag flag
True
include drag term: velocity
False
add drag term to partials acceleration sum
return to SBFN1_B

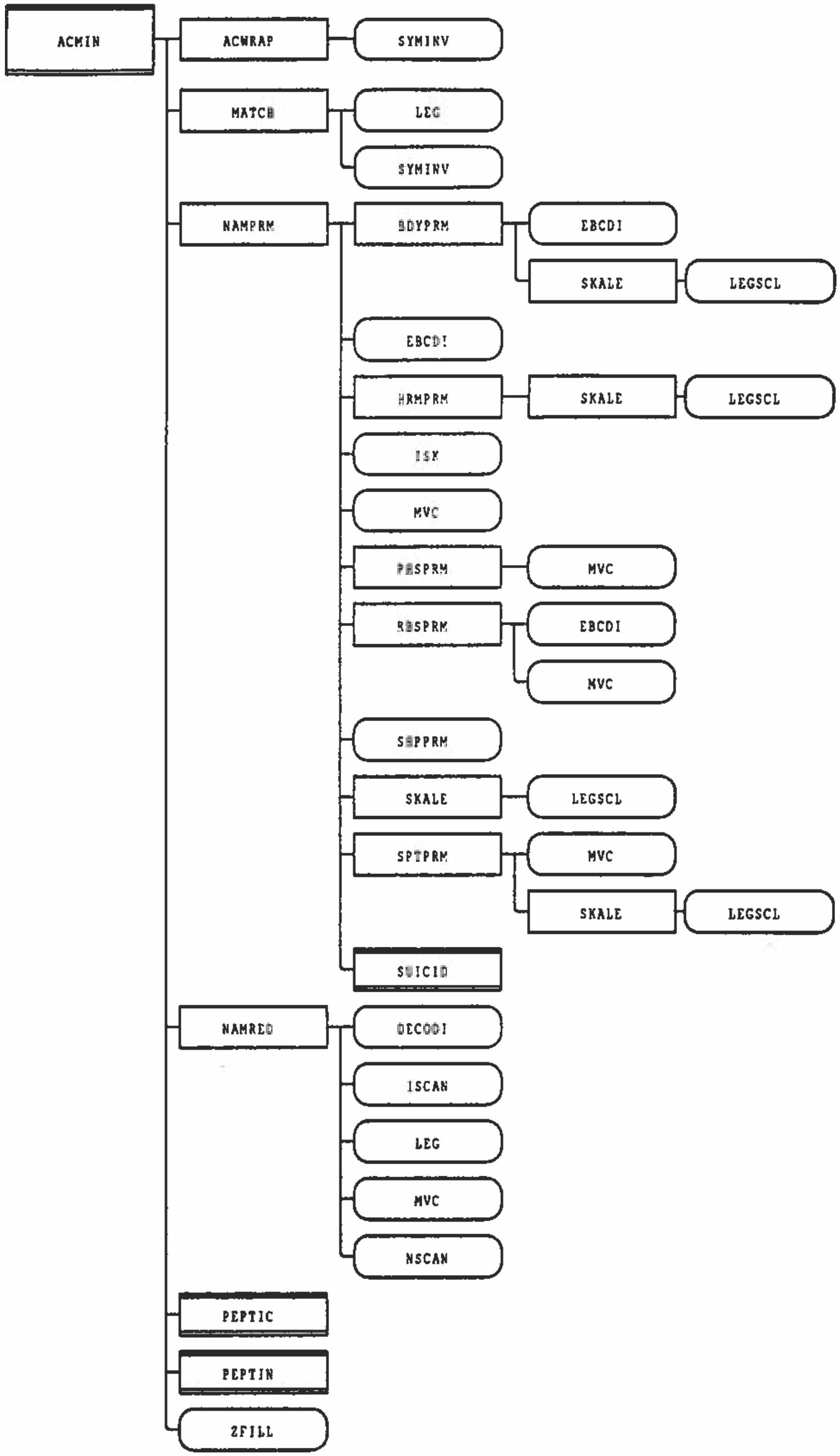

SBFN1_B10
partial is an IC
partials sum
psum = 0
-(k+30) <= 0
Y
N
kp(100)
case
> 0
= 0
< 0
subtract mean orbit acceleration from partial
subtract elliptic orbit acceleration from partial
kp(62) < 1
Y
N
k != 32
N
Y
k <= 0 or k > 30
N
massfl(k) = 0
Y
N
Y
jpoev <= 0
Y
N
massfl(k) - 1
case

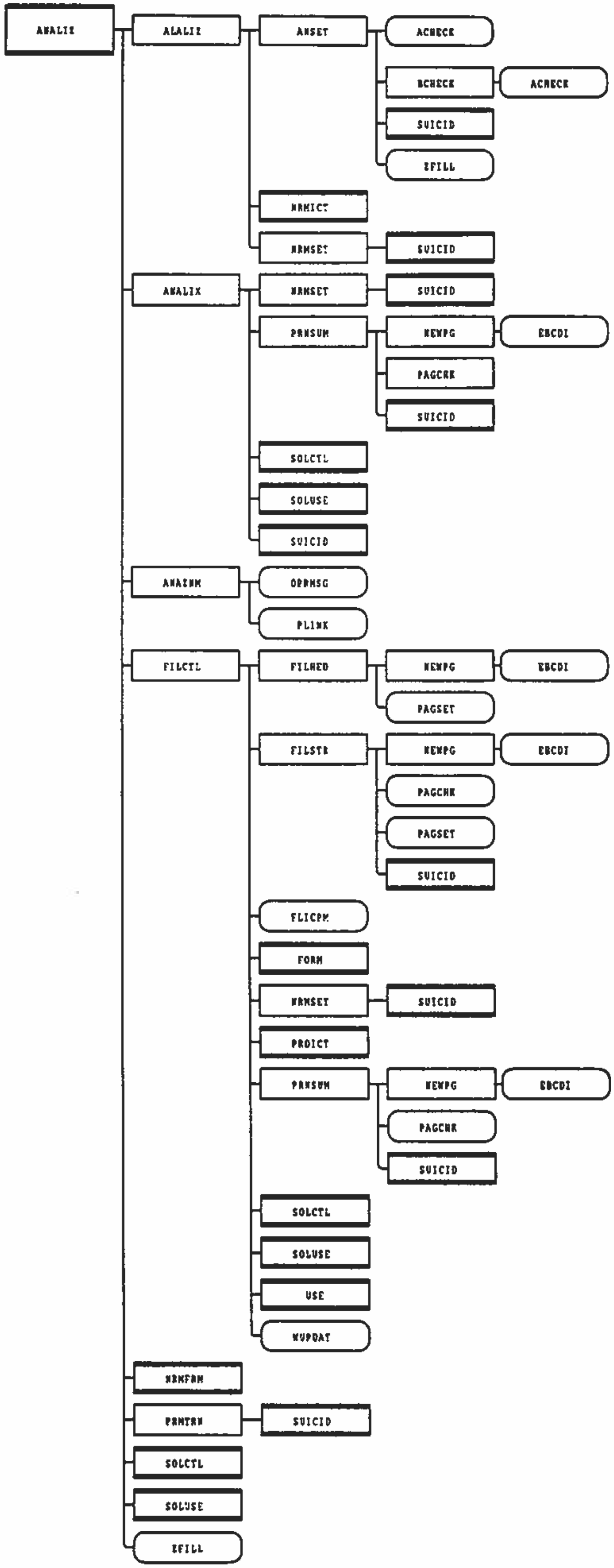

k != -20
k != 40
Y
Y
N
N
> 0
= 0
< 0
partials term for s = time variation of G
partials term for s = POE violation (integrated body)
partials term for s = POE violation (Sun)
partials term for s = CB mass (or E-M if ncentr=10)
partials term for s = asteroid mass (or E-M mass if nplnt=3)
partials term for s = mass of another satellite
multiply partials term by CB acceleration and add to psum
ncentr > 0
add CB harmonics to psum
add psum to partials acceleration sum
return to SBFN1_B

Subroutine Charts

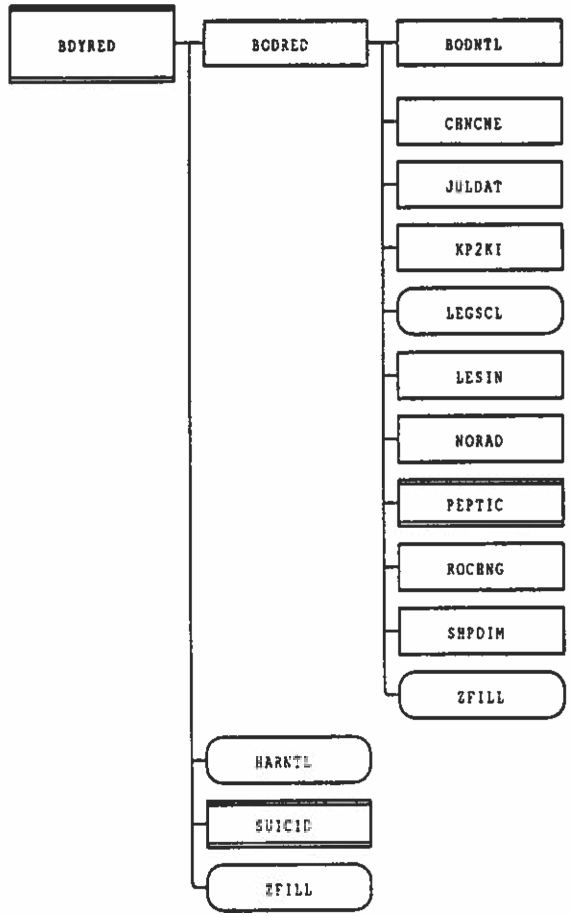


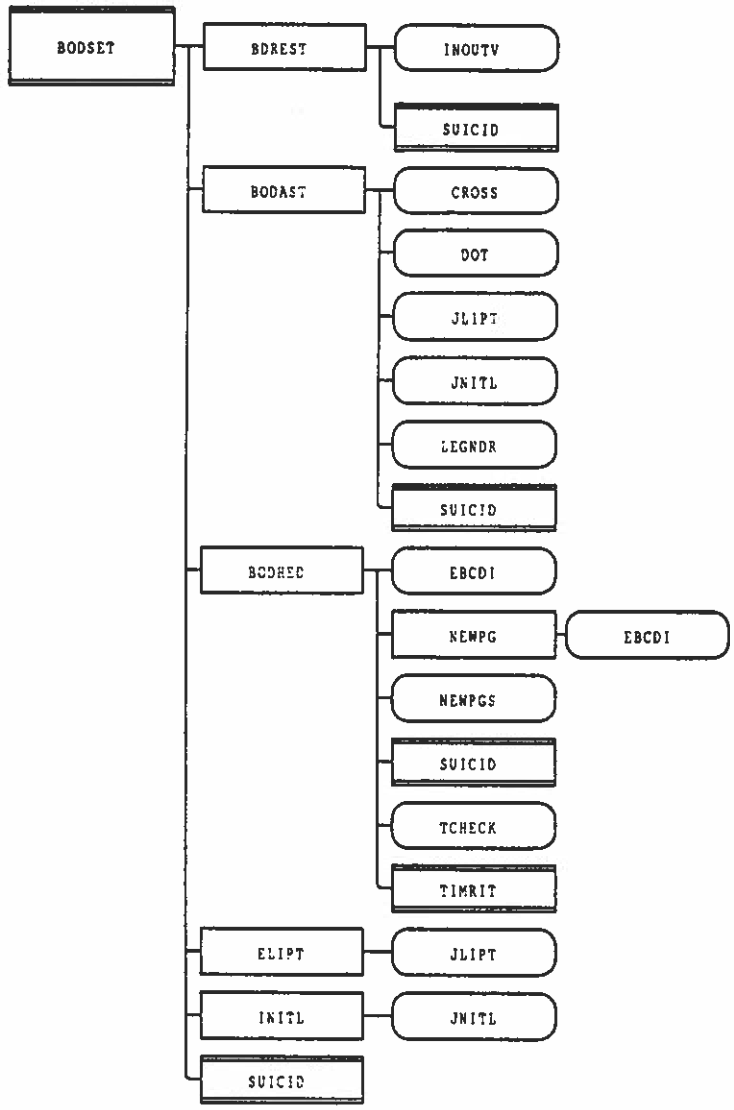
AIUTI
AIUTIF
SUICID
BIHUTI
ARGMNT
ETUTF
FUNARG
FUNCOF
SUICID
UT2UTI
SUICID
UTIRED
ARGMNT
AINWV
SUICID
ETUTF
FUNARG
FUNCOF
NEWPG
EBCDI
SUICID
UT2UTI
XAIUTI
ETUTF
UT2UTI

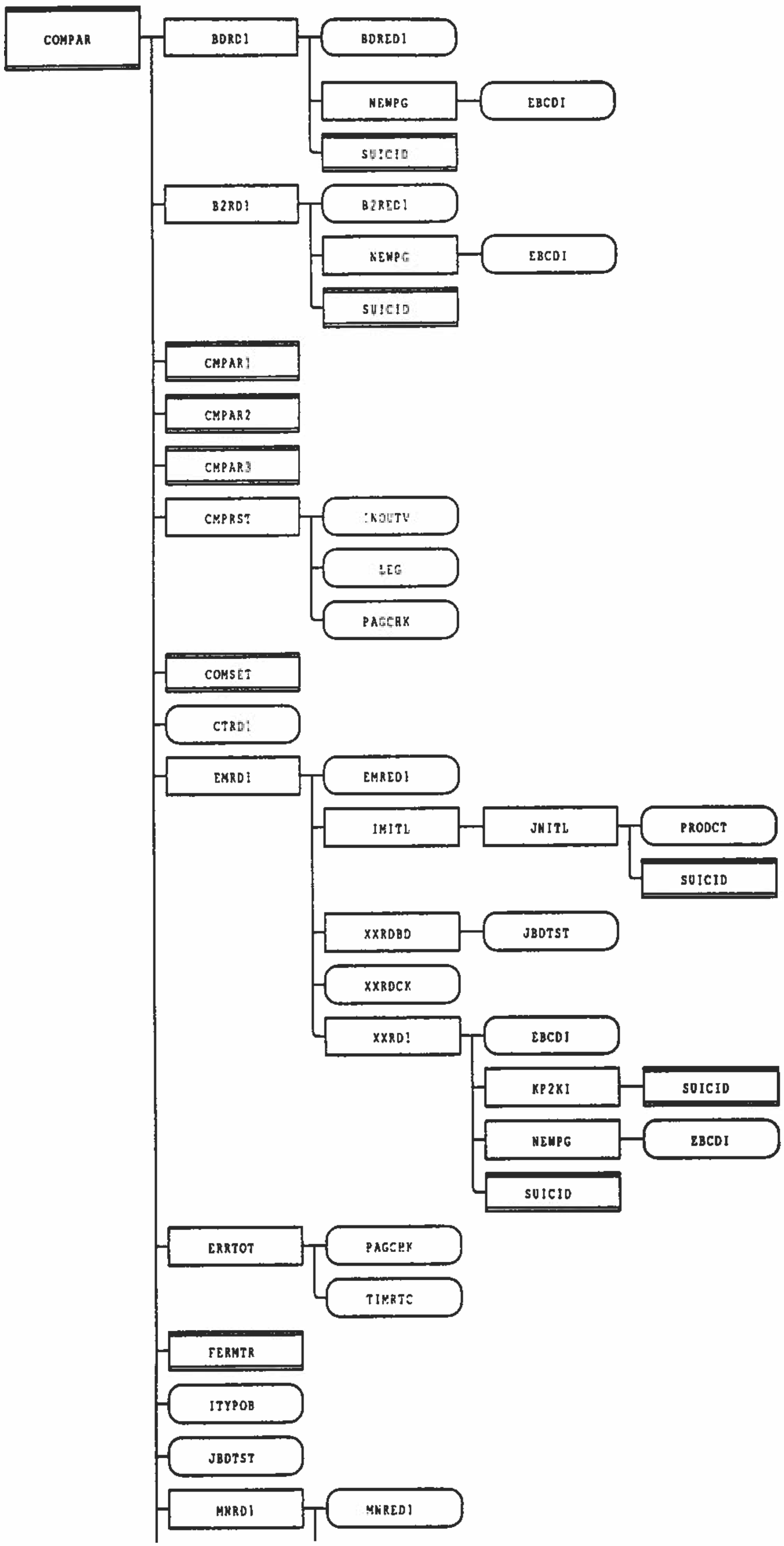

ACMIN
ACWRAP
SYMINV
MATCH
LEG
SYMINV
NAMPRM
BDYPRM
EBCDI
SKALE
LEGSCL
EBCDI
HRMPRM
SKALE
LEGSCL
ISK
MVC
PRSPRM
MVC
RBSPRM
EBCDI
MVC
SAPPRM
SKALE
LEGSCL
SPTPRM
MVC
SKALE
LEGSCL
SUICID
NAMRED
DECODI
ISCAN
LEG
MVC
NSCAN
PEPTIC
PEPTIN
ZFILL

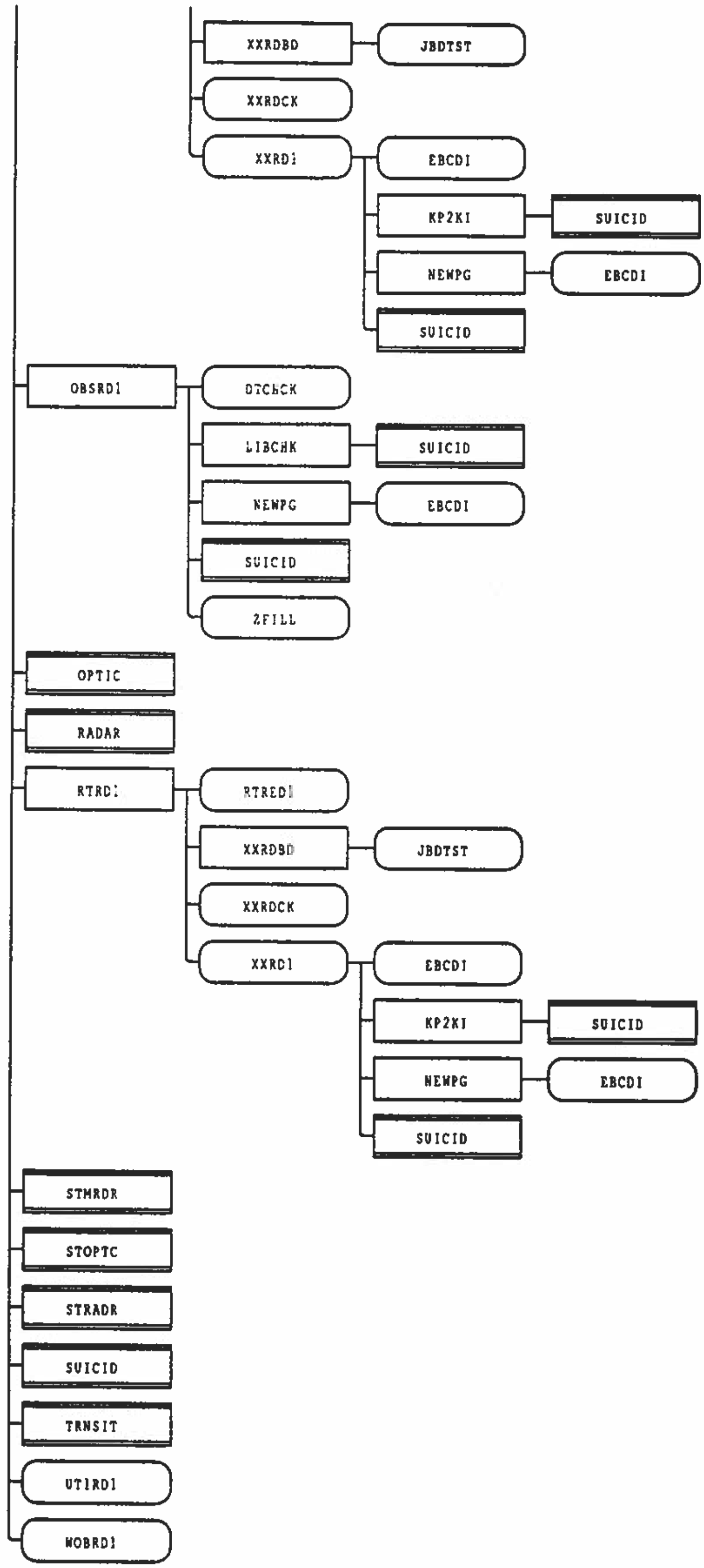


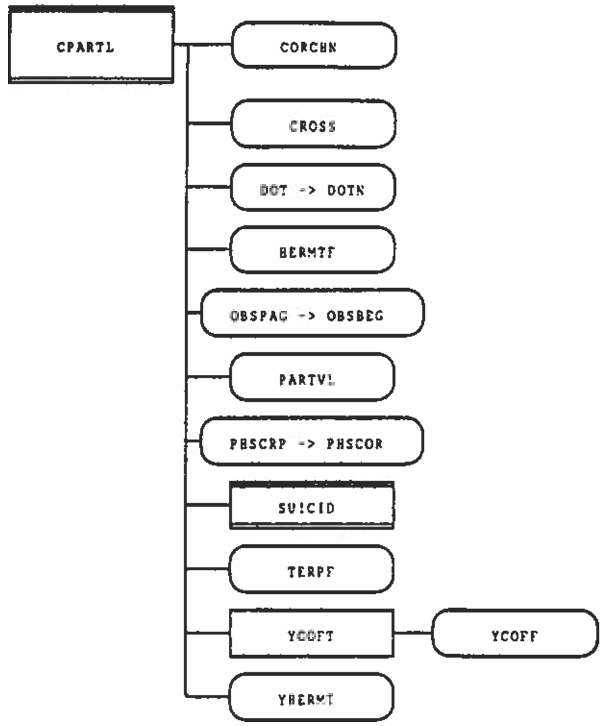
ANALIZ
ALALIZ
ANSET
ACHECK
BCHECK
ACHECK
SUICID
ZFILL
NRMICT
NRMSET
SUICID
ANALIX
NRMSET
SUICID
PRNSUM
NEWPG
EBCDI
PAGCRK
SUICID
SOLCTL
SOLUSE
SUICID
ANAZNM
OPRMSG
PLINK
FILCTL
FILHED
NEWPG
EBCDI
PAGSET
FILSTR
NEWPG
EBCDI
PAGCHR
PAGSET
SUICID
FLICPM
FORM
NRMSET
SUICID
PROICT
PRNSUM
NEWPG
EBCDI
PAGCHR
SUICID
SOLCTL
SOLUSE
USE
WUPDAT
NBMFRM
PRNTRN
SUICID
SOLCTL
SOLUSE
ZFILL

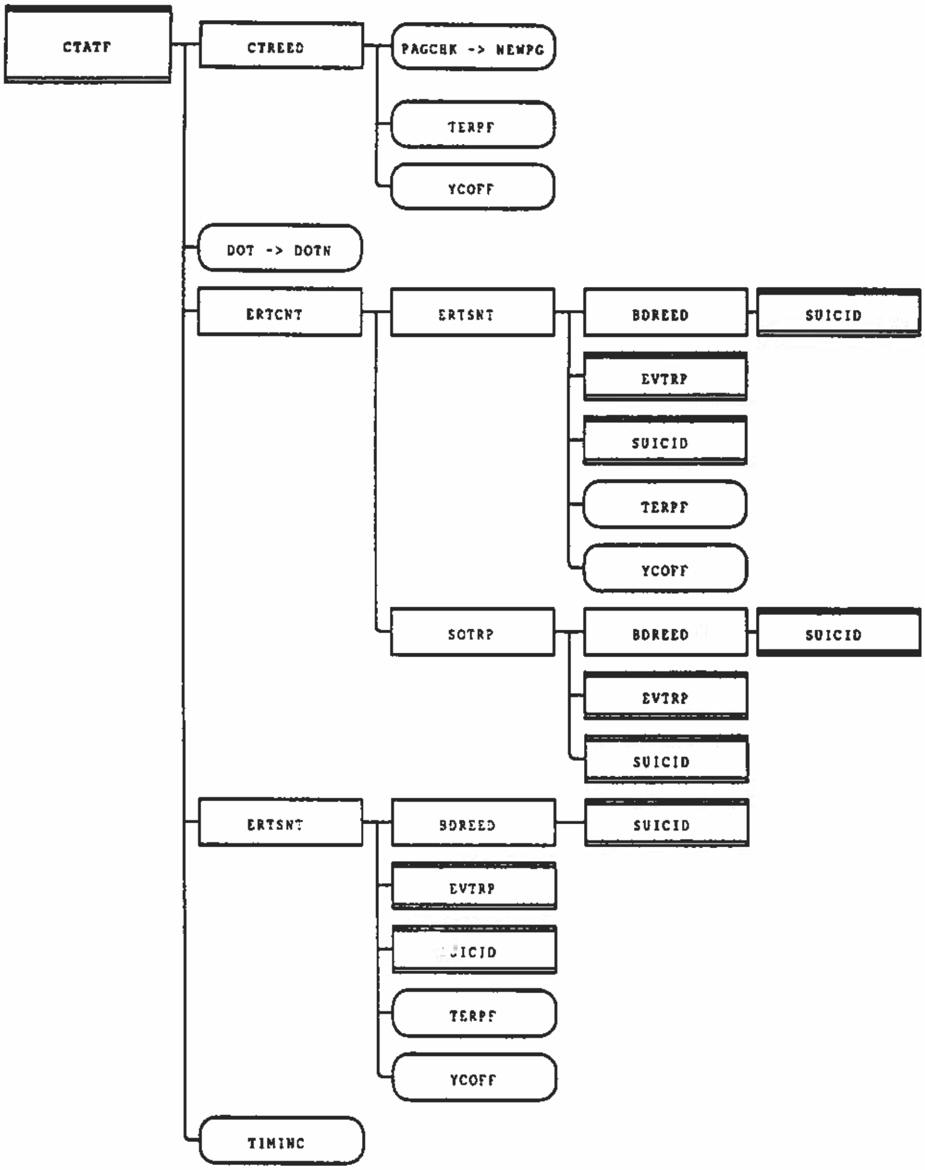

BDYRED
BODREC
BODNTL
CBNCNE
JULDAT
KP2KI
LEGSCL
LESIN
NORAD
PEPTIC
ROCBNG
SHPDIM
ZFILL
HARKTL
SUICID
ZFILL

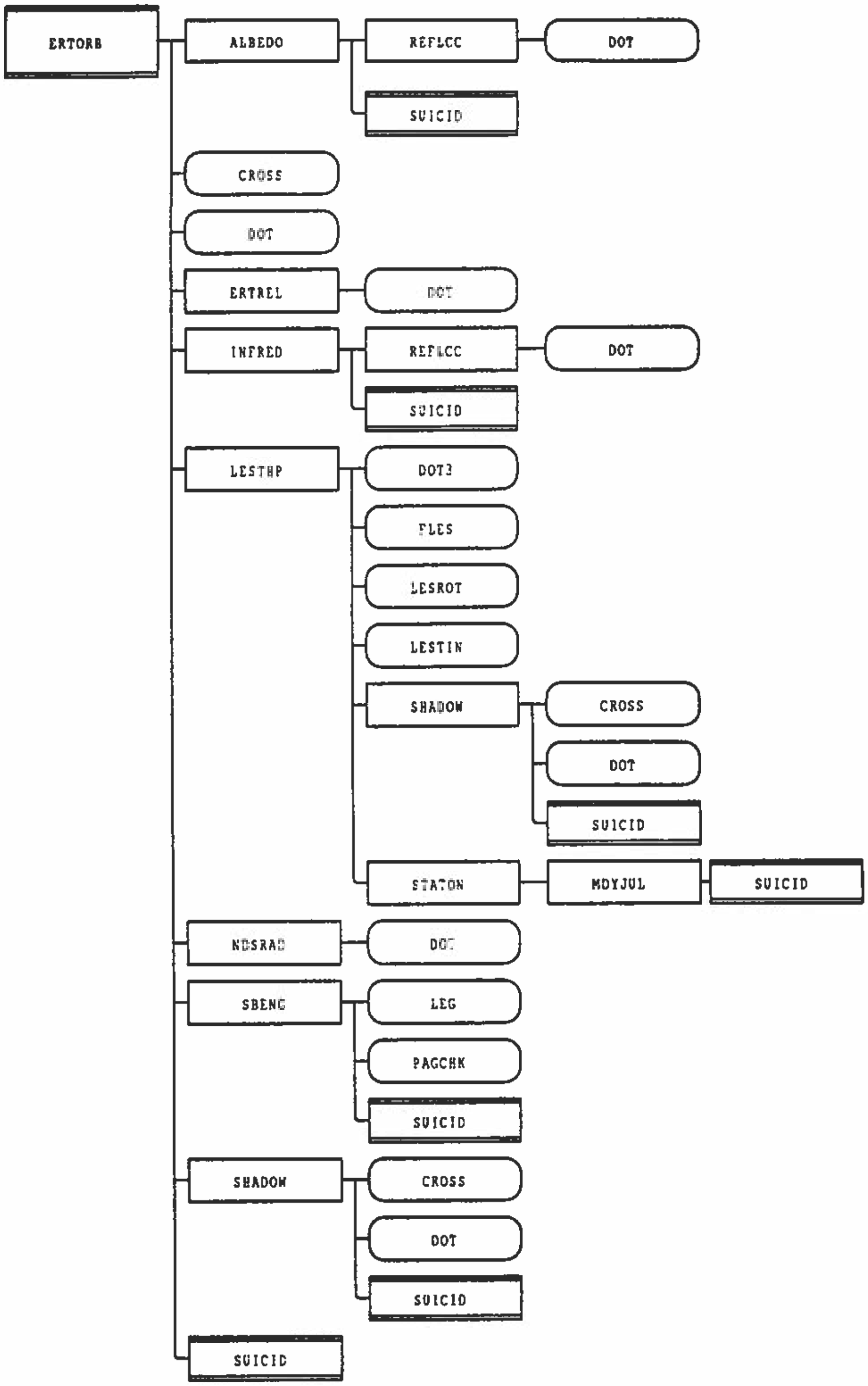

BODSET
BDREST
INOUTV
SUICID
BODAST
CROSS
DOT
JLIPT
JNITL
LEGNDR
SUICID
BODHED
EBCDI
NEWPG
EBCDI
NEWPGS
SUICID
TCHECK
TIMRIT
ELIPT
JLIPT
INITL
JNITL
SUICID

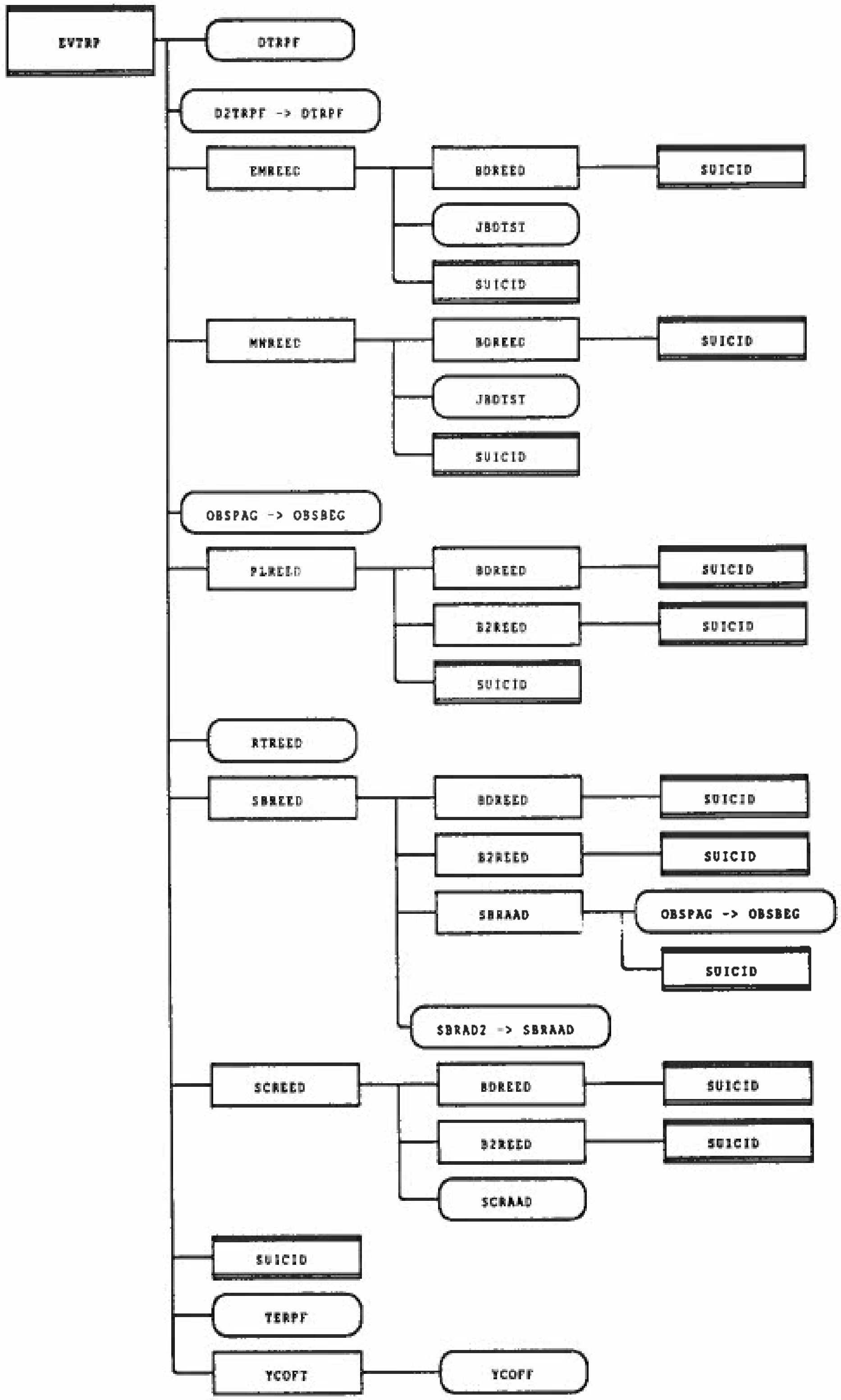

COMPAR
BDRD1
BDRED1
NEWPG
EBCDI
SUICID
B2RD1
B2RED1
NEWPG
EBCDI
SUICID
CMPAR1
CMPAR2
CMPAR3
CMPRST
INOUTV
LEG
PAGCHK
COMSET
CTRD1
EMRD1
EMRED1
INITL
JNITL
PRODCT
SUICID
XXRDBD
JBDTST
XXRDCK
XXRD1
EBCDI
KP2KI
SUICID
NEWPG
EBCDI
SUICID
ERRTOT
PAGCHK
TIMRTC
FERMTR
ITYPOB
JBDTST
MNRD1
MNRED1


XXRDBD
JBDTST
XXRDCK
XXRDI
EBCDI
KP2KI
SUICID
NEWPG
EBCDI
SUICID
OBSRDI
DTCHCK
LIBCHK
SUICID
NEWPG
EBCDI
SUICID
ZFILL
OPTIC
RADAR
RTRDI
RTREDI
XXRDBD
JBDTST
XXRDCK
XXRDI
EBCDI
KP2KI
SUICID
NEWPG
EBCDI
SUICID
STMRDR
STOPTC
STRADR
SUICID
TRNSIT
UTIRDI
WOBRDI

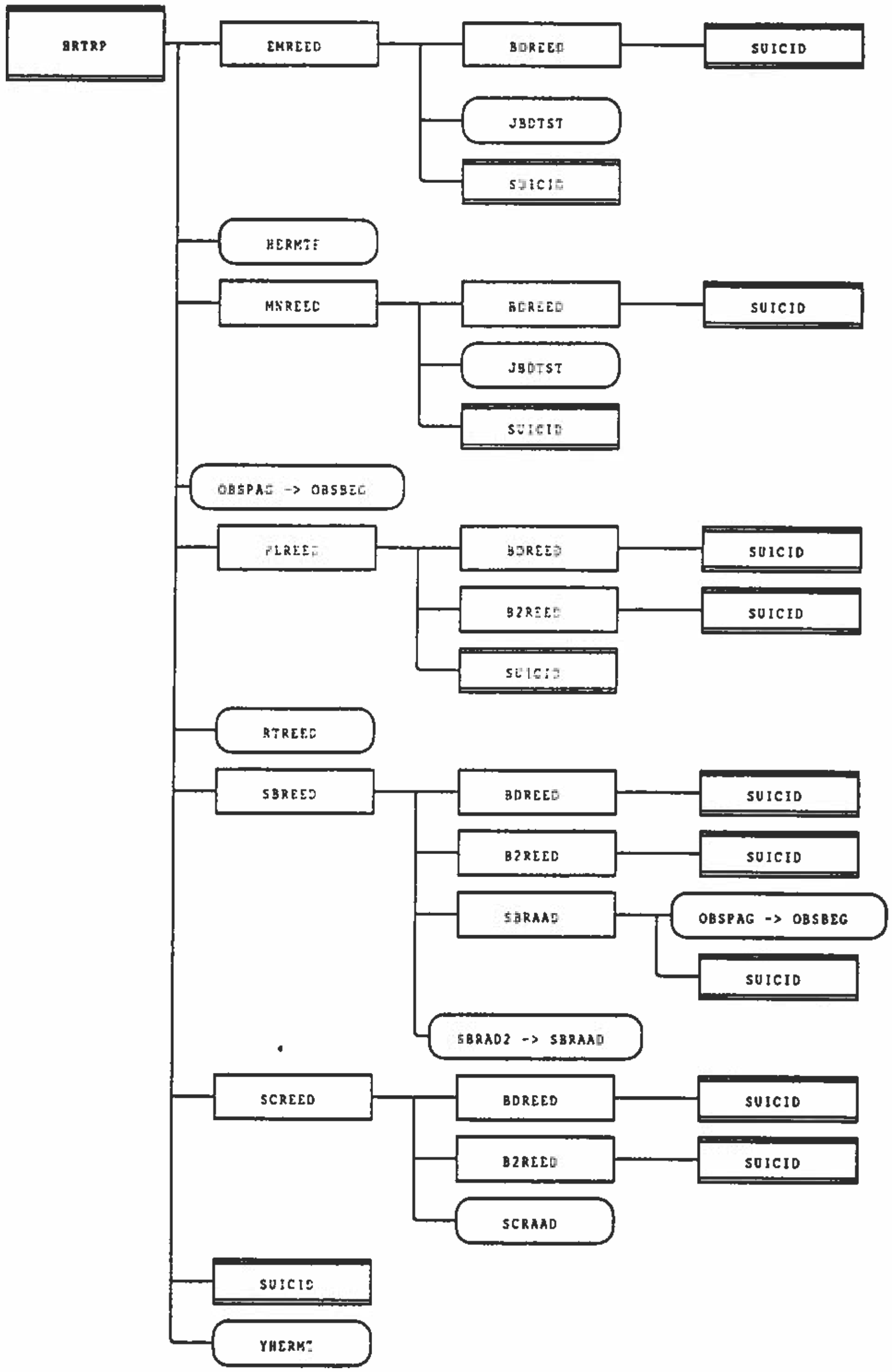

CPARTL
CORCHN
CROSS
DOT -> DOTN
HERMTF
OBSPAG -> OBSBEG
PARTVL
PHSCRP -> PHSCOR
SUICID
TERPF
YCOFT
YCOFF
YHERMT

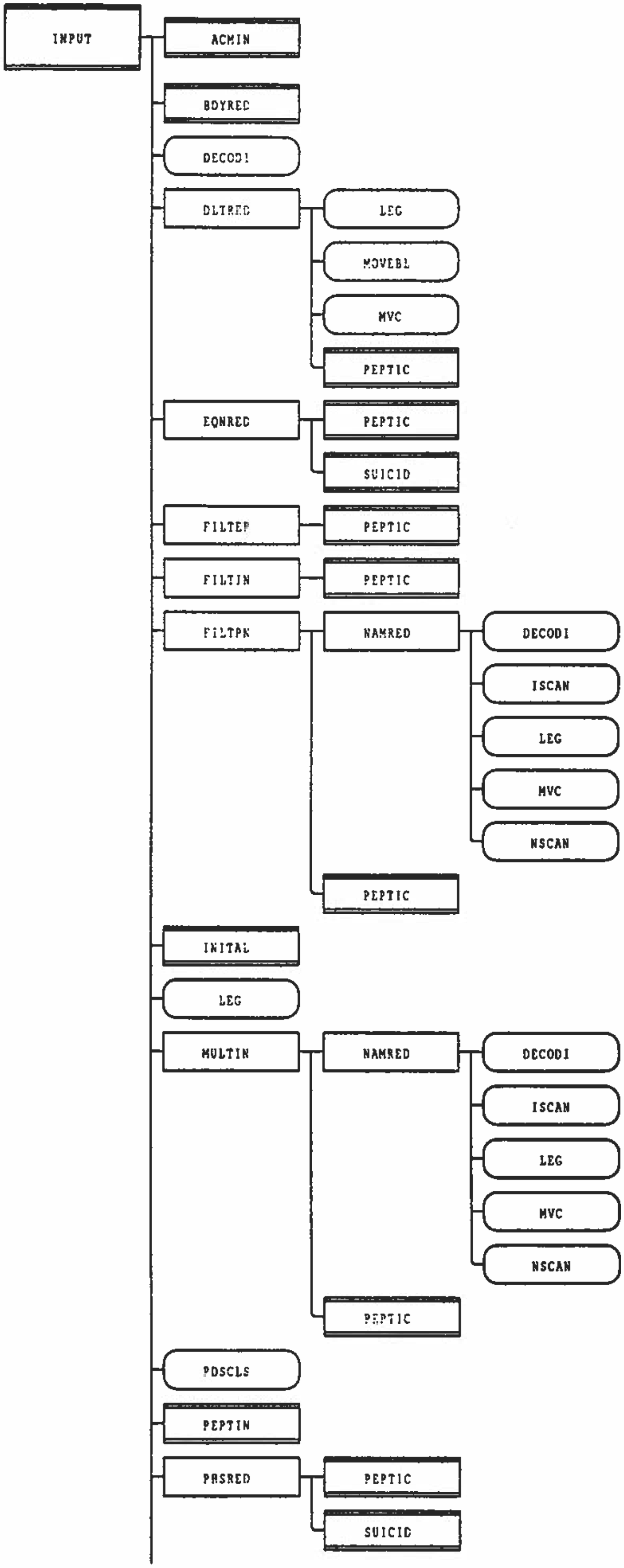

CTATF
CTREED
PAGCHK -> NEWPG
TERPF
YCOFF
DOT -> DOTN
ERTCNT
ERTSNT
BDREED
SUICID
EVTRP
SUICID
TERPF
YCOFF
SOTRP
BDREED
SUICID
EVTRP
SUICID
ERTSNT
BDREED
SUICID
EVTRP
SUICID
TERPF
YCOFF
TIMINC

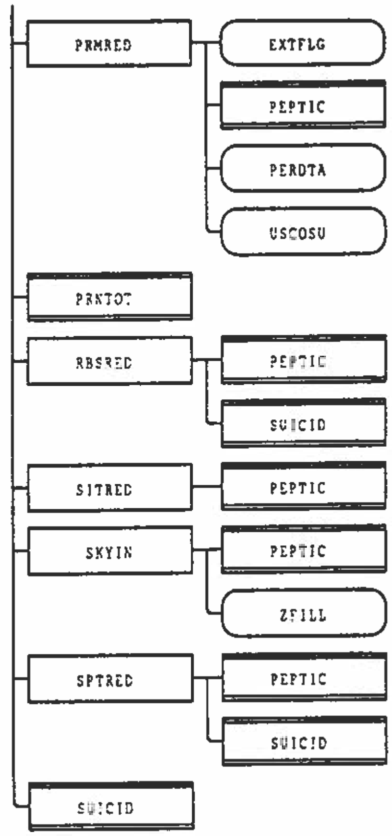

ERTORB
ALBEDO
REFLCC
DOT
SUICID
CROSS
DOT
ERTREL
DOT
INFRED
REFLCC
DOT
SUICID
LESTHP
DOT3
FLES
LESROT
LESTIN
SHADOW
CROSS
DOT
SUICID
STATON
MDYJUL
SUICID
NDSRAD
DOT
SBENG
LEG
PAGCHK
SUICID
SHADOW
CROSS
DOT
SUICID
SUICID

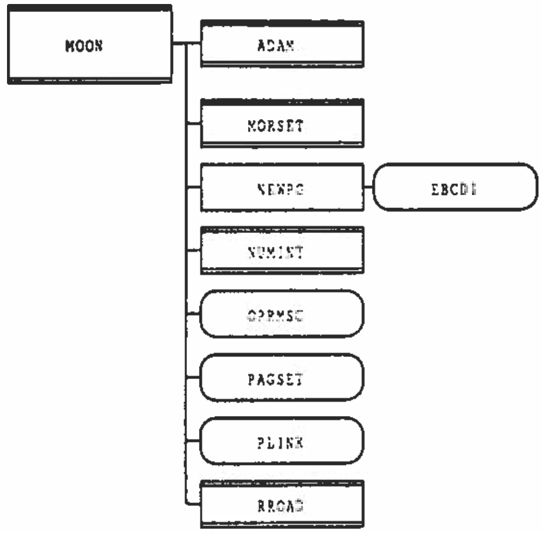


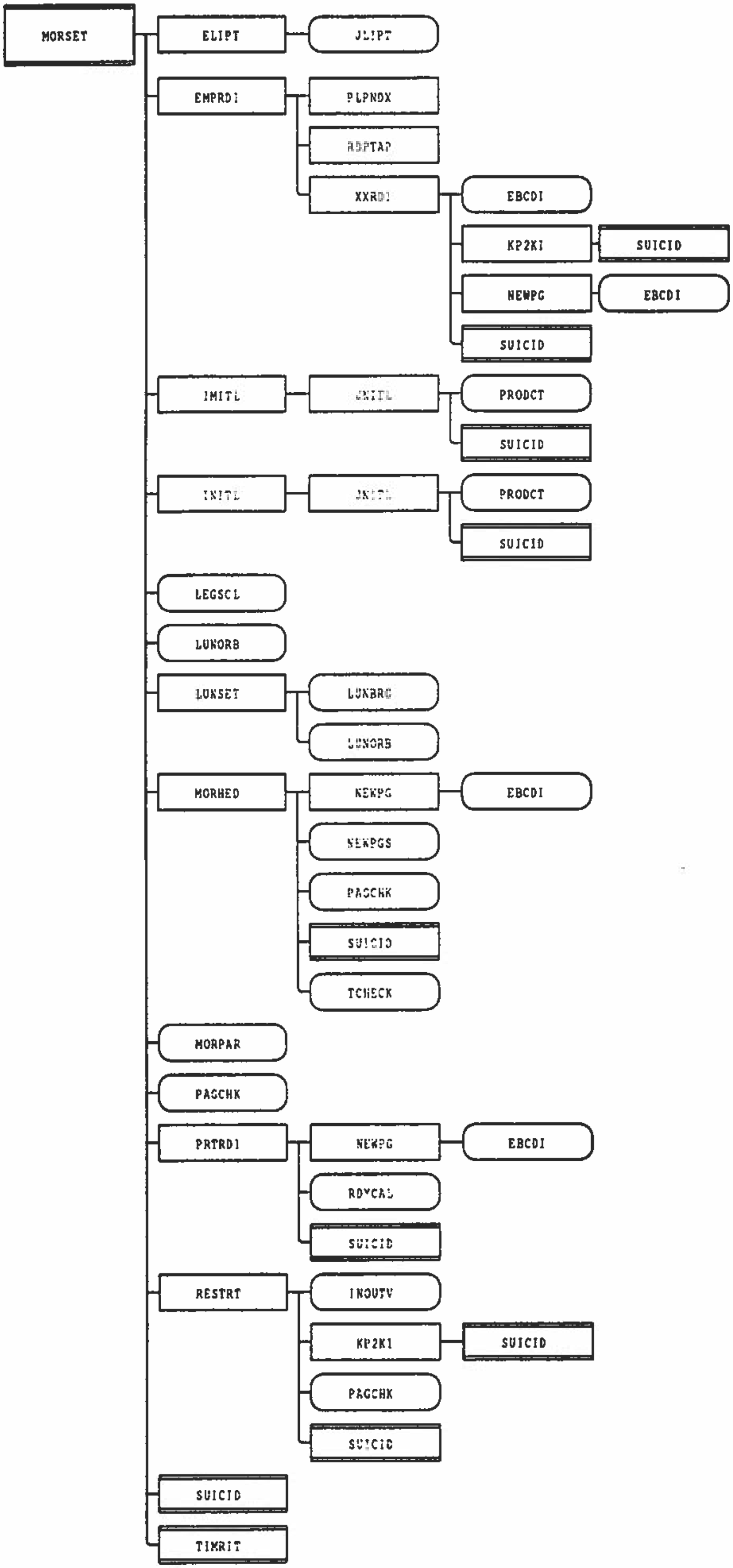
EVTRP
DTRPF
D2TRPF -> DTRPF
EMREEC
BDREED
SUICID
JBDTST
SUICID
MNREED
BDREED
SUICID
JBDTST
SUICID
OBSPAG -> OBSBEG
PLREED
BDREED
SUICID
B2REED
SUICID
SUICID
RTREED
SBREED
BDREED
SUICID
B2REED
SUICID
SBRAAD
OBSPAG -> OBSBEG
SUICID
SBRAD2 -> SBRAAD
SCREED
BDREED
SUICID
B2REED
SUICID
SCRAAD
SUICID
TERPF
YCOFT
YCOFF

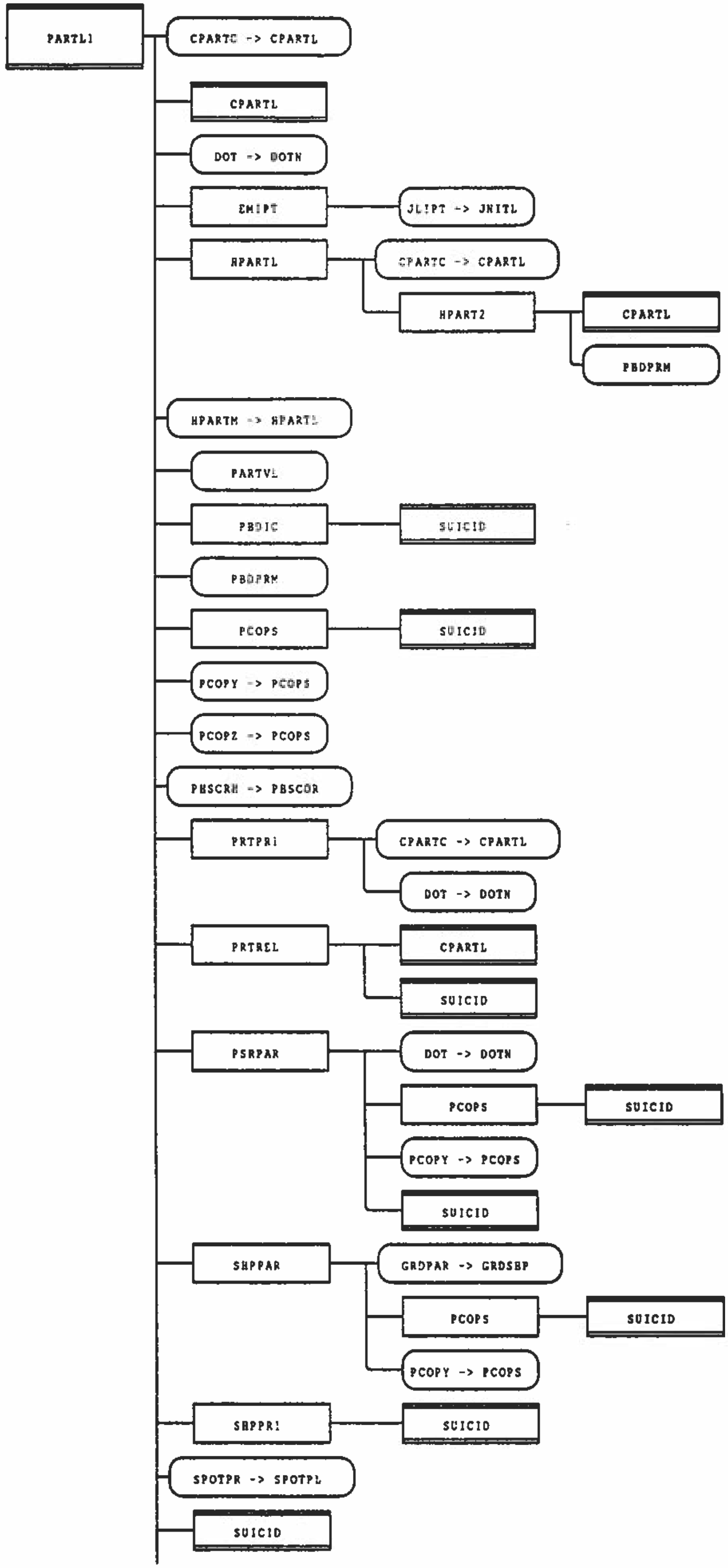

BRTRP
EMREED
BDREED
SUICID
JBDTST
SUICID
HERMTF
MNREED
BDREED
SUICID
JBDTST
SUICID
OBSPAG -> OBSBEG
FLREED
BDREED
SUICID
B2REED
SUICID
SUICID
RTREED
SBREED
BDREED
SUICID
B2REED
SUICID
SBRAAD
OBSPAG -> OBSBEG
SUICID
SBRAD2 -> SBRAAD
SCREED
BDREED
SUICID
B2REED
SUICID
SCRAAD
SUICID
YHERMT

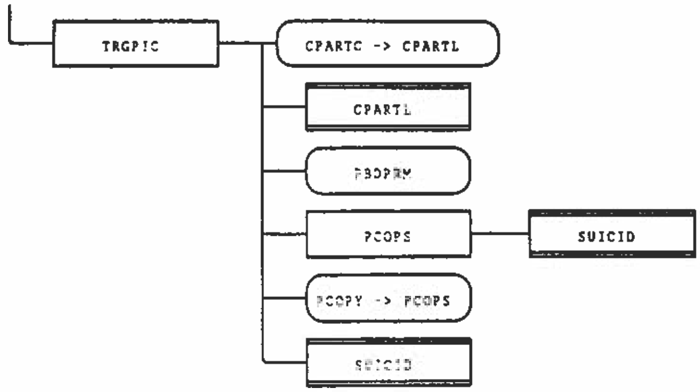

INPUT
ACMIN
BDYRED
DECODI
DLTRED
LEG
MOVEBL
MVC
PEPTIC
EQNRED
PEPTIC
SUICID
FILTEP
PEPTIC
FILTIN
PEPTIC
FILTPK
NAMRED
DECODI
ISCAN
LEG
MVC
NSCAN
PEPTIC
INITAL
LEG
MULTIN
NAMRED
DECODI
ISCAN
LEG
MVC
NSCAN
PEPTIC
PDSCLS
PEPTIN
PRSRED
PEPTIC
SUICID

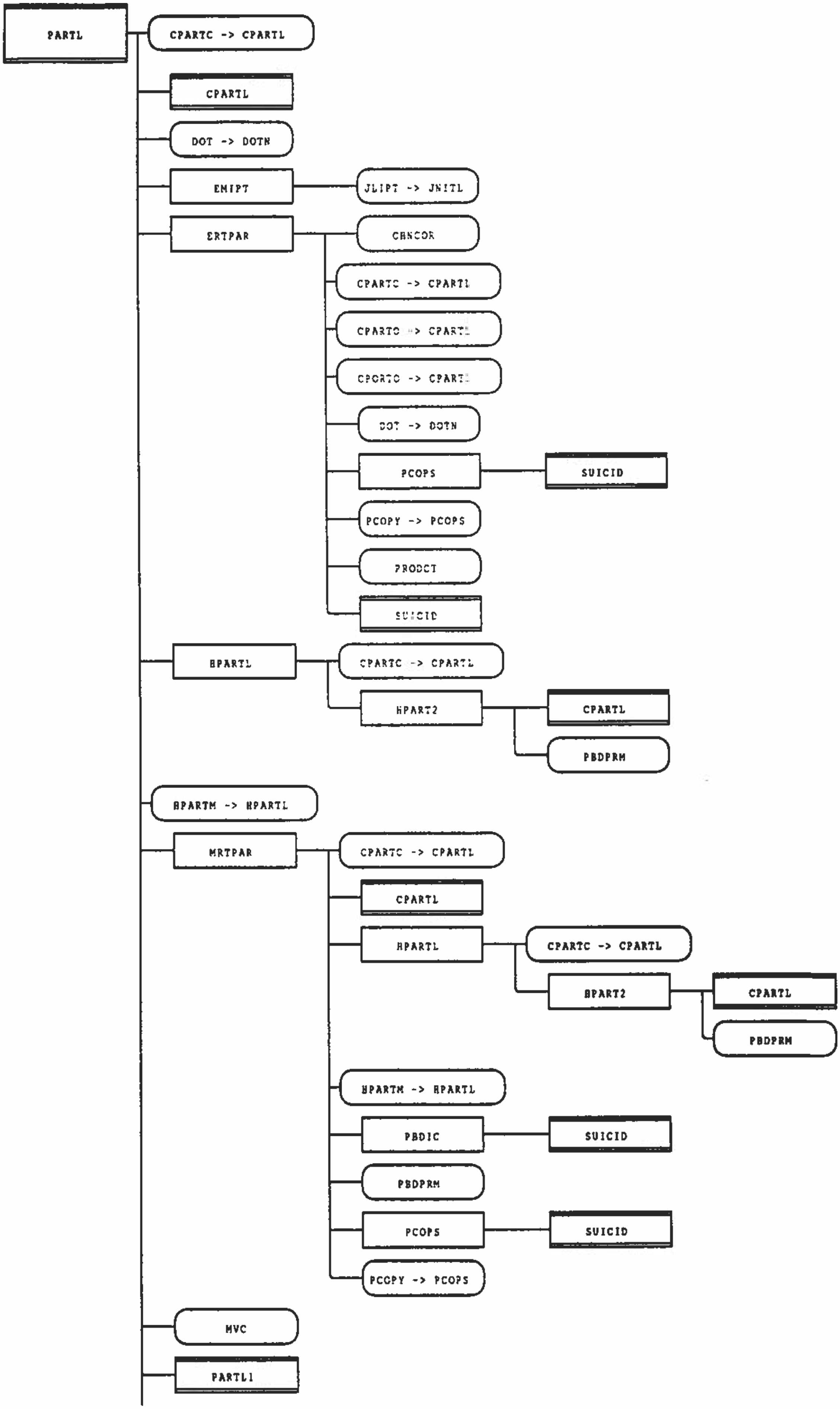

PRMRED
EXTFLG
PEPTIC
PERDTA
USCOSU
PRNTOT
RBSRED
PEPTIC
SUICID
SITRED
PEPTIC
SKYIN
PEPTIC
ZFILL
SPTRED
PEPTIC
SUICID
SUICID


MOON
ADAM
NORSET
NEWPG
EBCDI
NUMINT
OPRMSG
PAGSET
PLINK
RROAD

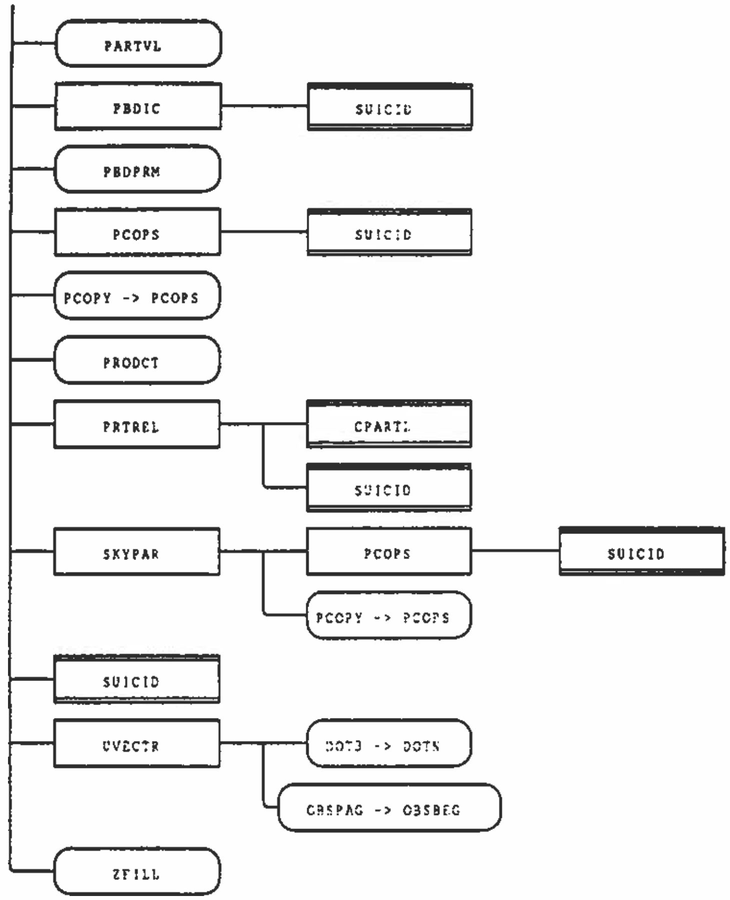

MORSET
ELIPT
JLIPT
EMPRDI
PLPNDX
RDPTAP
XXRDI
EBCDI
KP2KI
SUICID
NEWPG
EBCDI
SUICID
IMITL
PRODCT
SUICID
INITL
JNITL
PRODCT
SUICID
LEGSCL
LUNORB
LUNSET
LUNBRG
LUNORB
MORHED
NEWPG
EBCDI
NEWPGS
PAGCHK
SUICID
TCHECK
MORPAR
PAGCHK
PRTRDI
NEWPG
EBCDI
RDYCAL
SUICID
RESTRT
INOUTV
KP2KI
SUICID
PAGCHK
SUICID
SUICID
TIMRIT

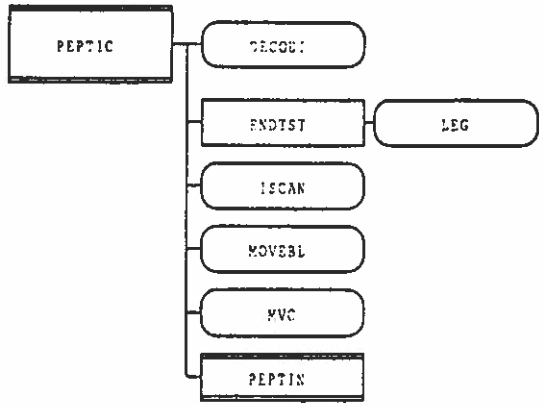

PARTL1
CPARTC -> CPARTL
CPARTL
DOT -> DOTN
EMIPT
JLIPT -> JNITL
HPARTL
CPARTC -> CPARTL
HPART2
CPARTL
PBDPRM
HPARTM -> HPARTL
PARTVL
PBDIC
SUICID
PBDPRM
PCOPS
SUICID
PCOPY -> PCOPS
PCOPZ -> PCOPS
PBSCRN -> PBSCOR
PRTPR1
CPARTC -> CPARTL
DOT -> DOTN
PRTREL
CPARTL
SUICID
PSRPAR
DOT -> DOTN
PCOPS
SUICID
PCOPY -> PCOPS
SUICID
SHPPAR
GRDPAR -> GRDSHP
PCOPS
SUICID
PCOPY -> PCOPS
SHPPR1
SUICID
SPOTPR -> SPOTPL
SUICID

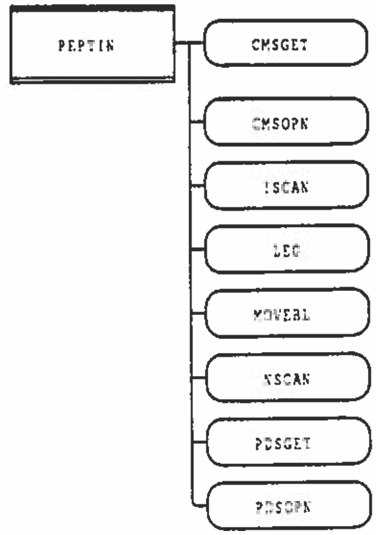

TRGPIC
CPARTC -> CPARTL
CPARTL
PBOPRM
PCOPS
SUICID
PCOPY -> PCOPS
SUICID

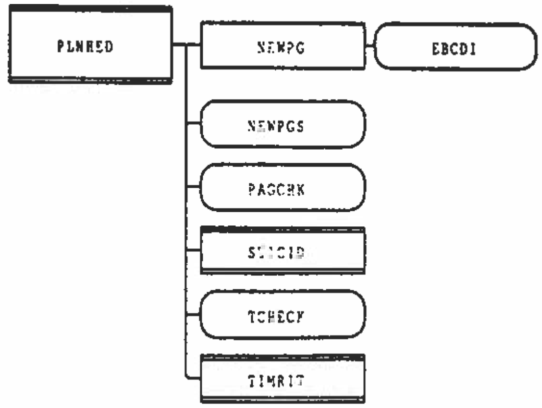


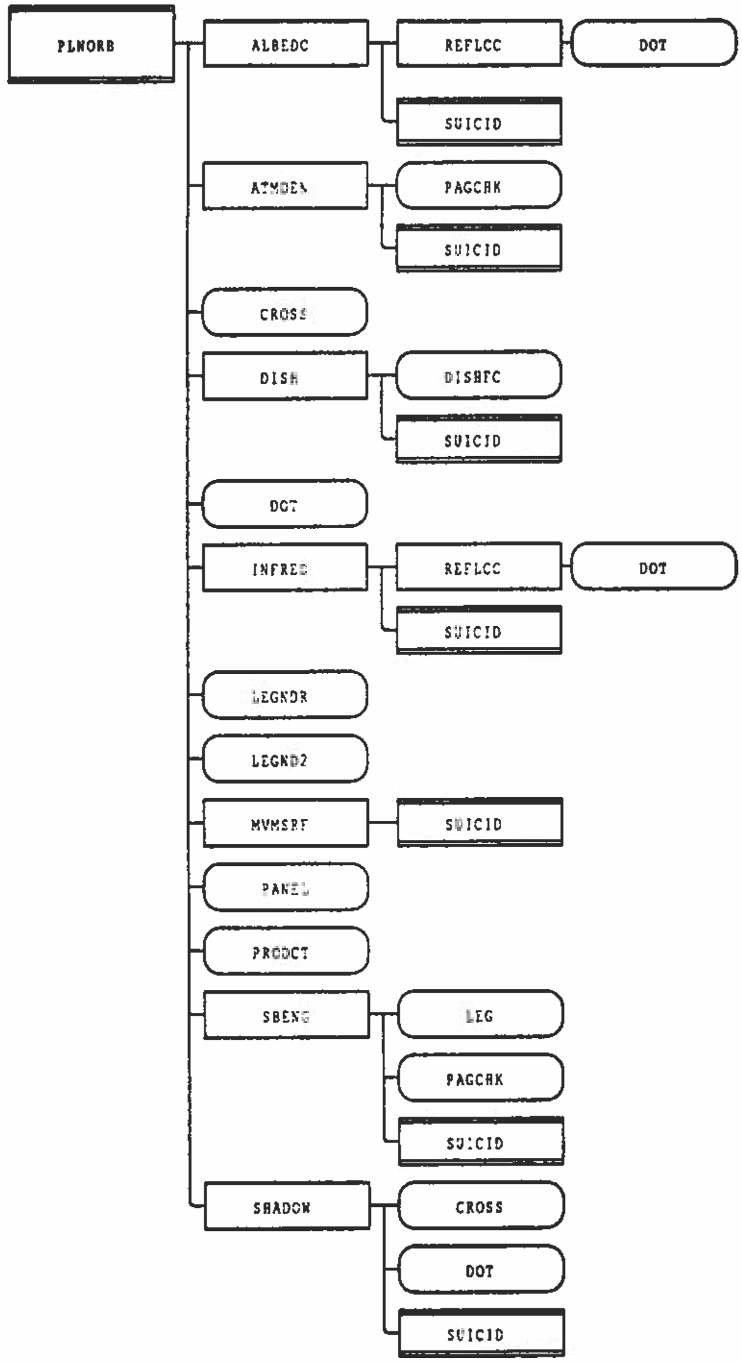
PARTL
CPARTC -> CPARTL
CPARTL
DOT -> DOTN
EMIPT
JLIPT -> JNITL
ERTPAR
CHNCOR
CPARTC -> CPARTL
CPARTC -> CPARTL
CPARTC -> CPARTL
DOT -> DOTN
PCOPS
SUICID
PCOPY -> PCOPS
PRODCT
SUICID
HPARTL
CPARTC -> CPARTL
HPART2
CPARTL
PBDPRM
HPARTM -> HPARTL
MRTPAR
CPARTC -> CPARTL
CPARTL
HPARTL
CPARTC -> CPARTL
HPART2
CPARTL
PBDPRM
HPARTM -> HPARTL
PBDIC
SUICID
PBDPRM
PCOPS
SUICID
PCOPY -> PCOPS
MVC
PARTL1

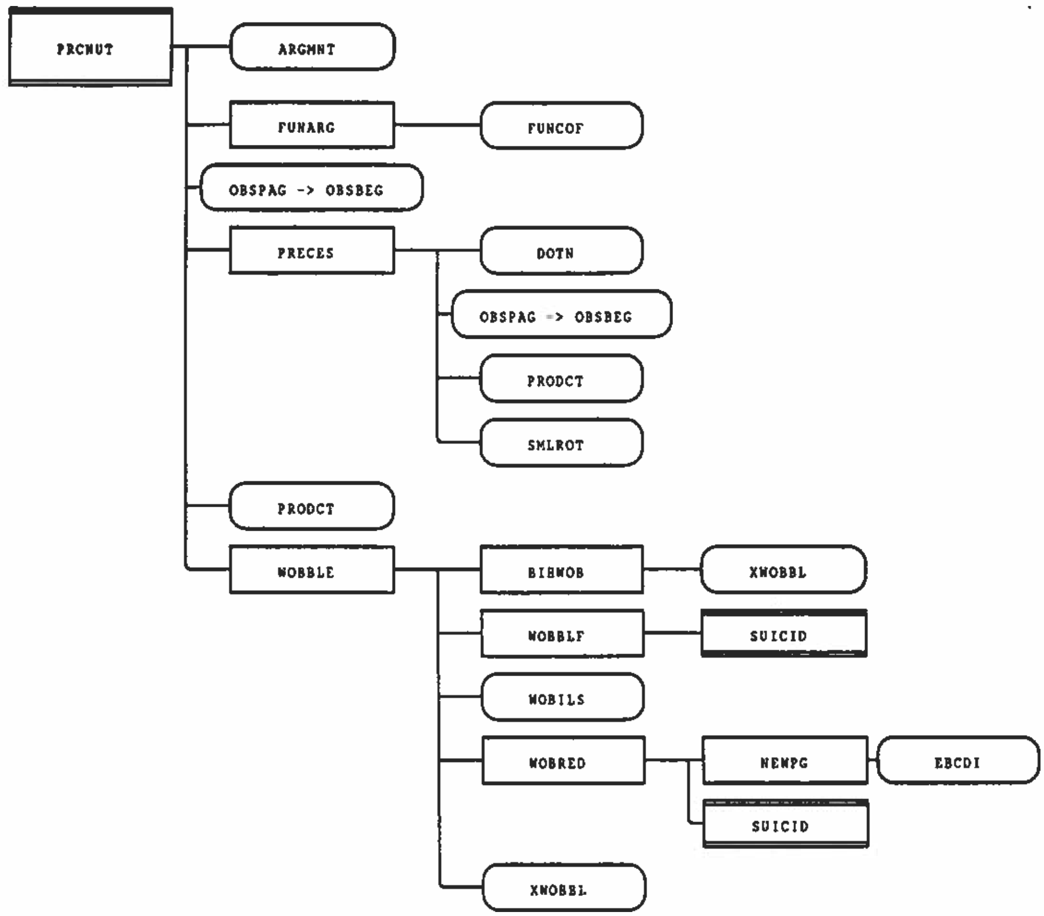

PARTVL
PBDIC
SUICID
PBDPRM
PCOPS
SUICID
PCOPY -> PCOPS
PRODCT
PRTREL
CPARTL
SUICID
SKYPAR
PCOPS
SUICID
PCOPY -> PCOPS
SUICID
UVECTR
DOTB -> DOTN
OBSPAG -> OBSBEG
ZFILL

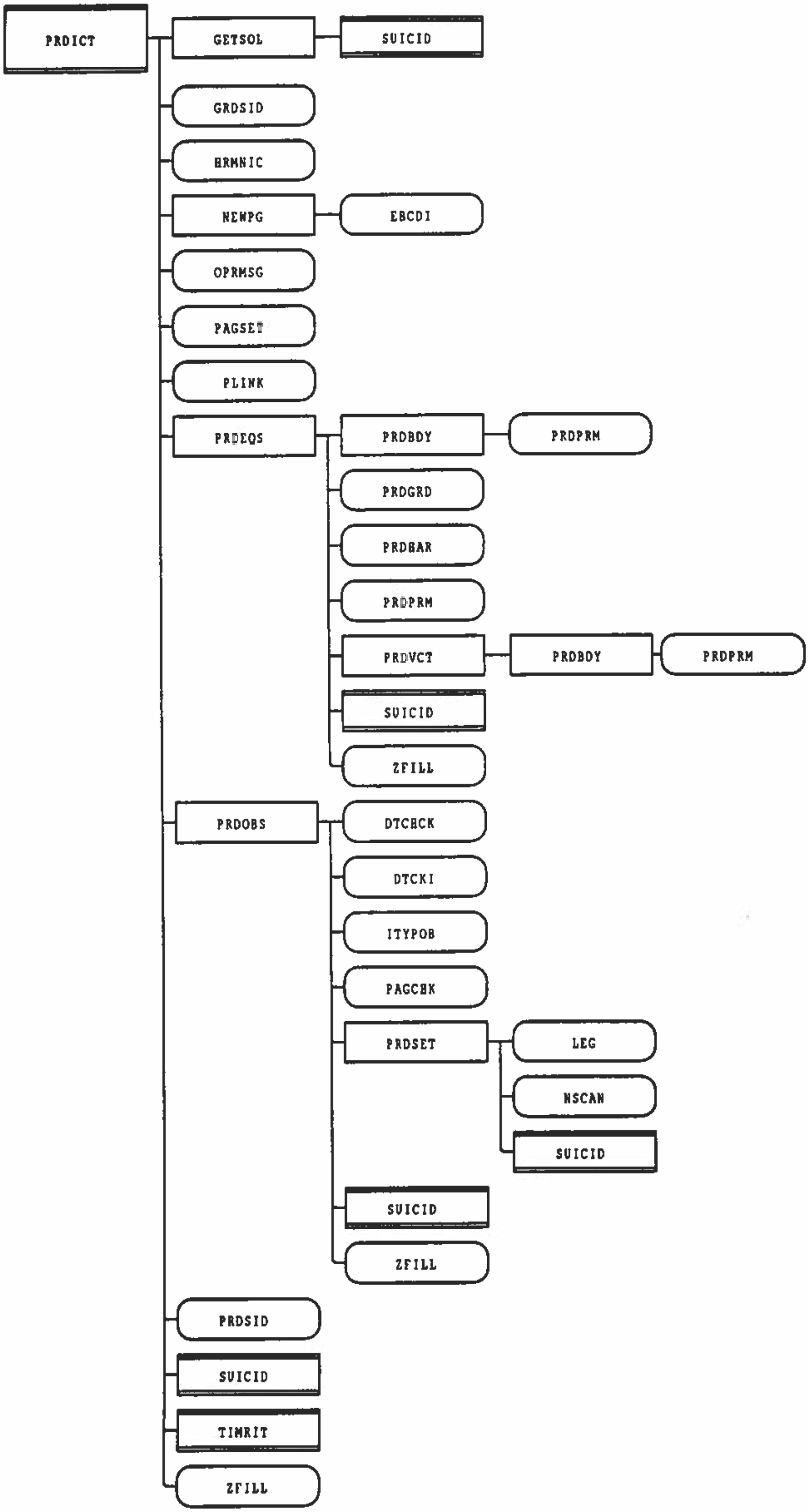

PEPTIC
ENDTST
LEG
ISCAN
MOVEBL
MVC
PEPTIN

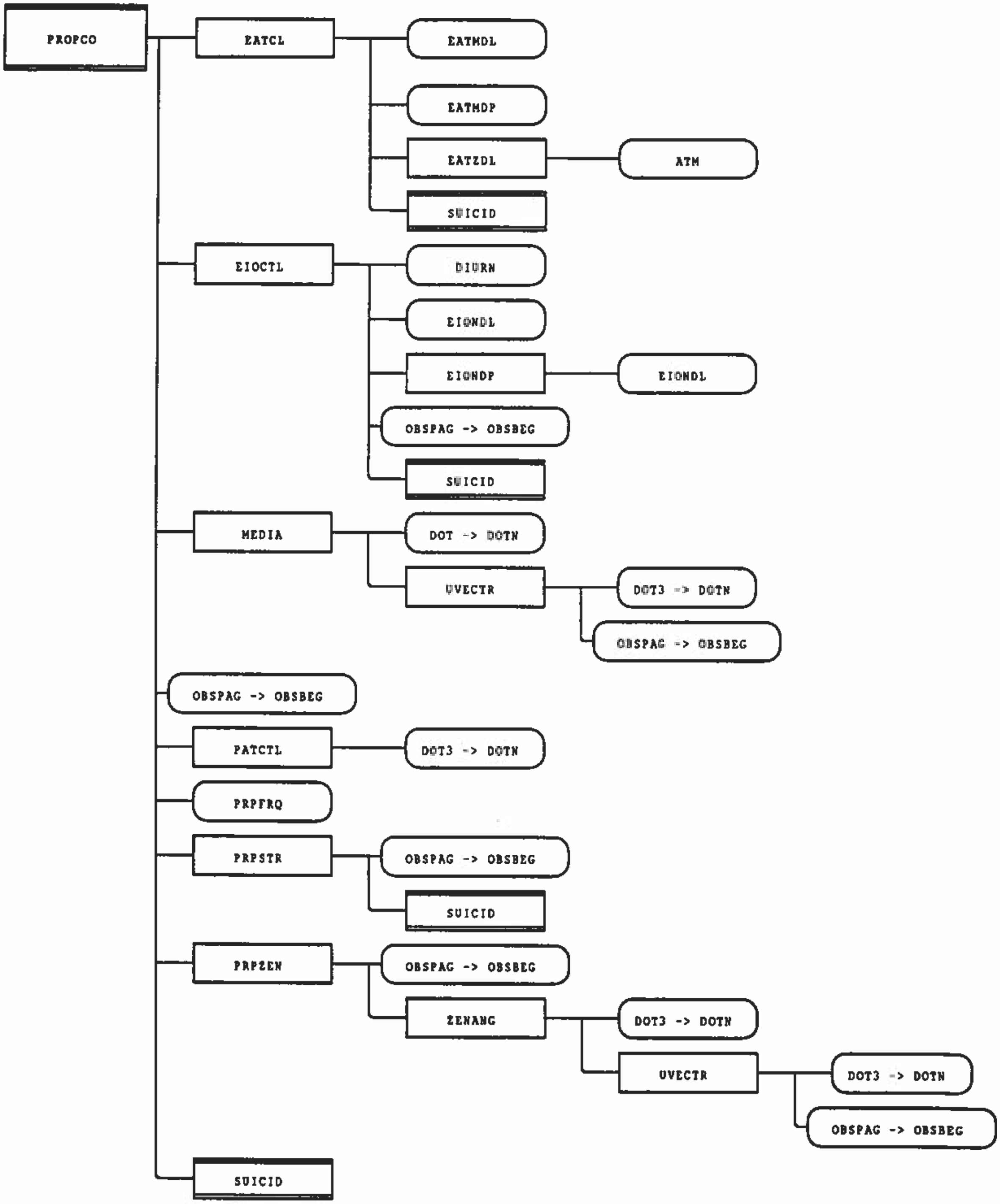

PEPTIN
CMSGET
CMSOPN
ISCAN
LEG
MOVEBL
NSCAN
PDSGET
PDSOPN


PLNRED
NEWPG
EBCDI
NEWPGS
PAGCHK
TCHECK
TIMRIT

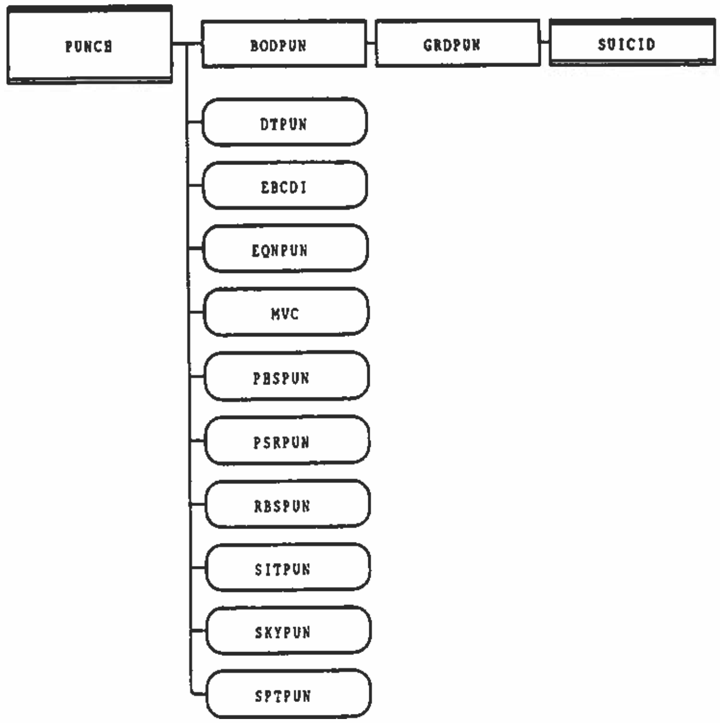

PLNORB
ALBEDC
REFLCC
DOT
SUICID
ATMDEN
PAGCHK
SUICID
CROSS
DISH
DISHFC
SUICID
DOT
INFRED
REFLCC
DOT
SUICID
LEGNDR
LEGND2
MVMSRF
SUICID
PANEL
PRODCT
SBENG
LEG
PAGCHK
SUICID
SHADOW
CROSS
DOT
SUICID

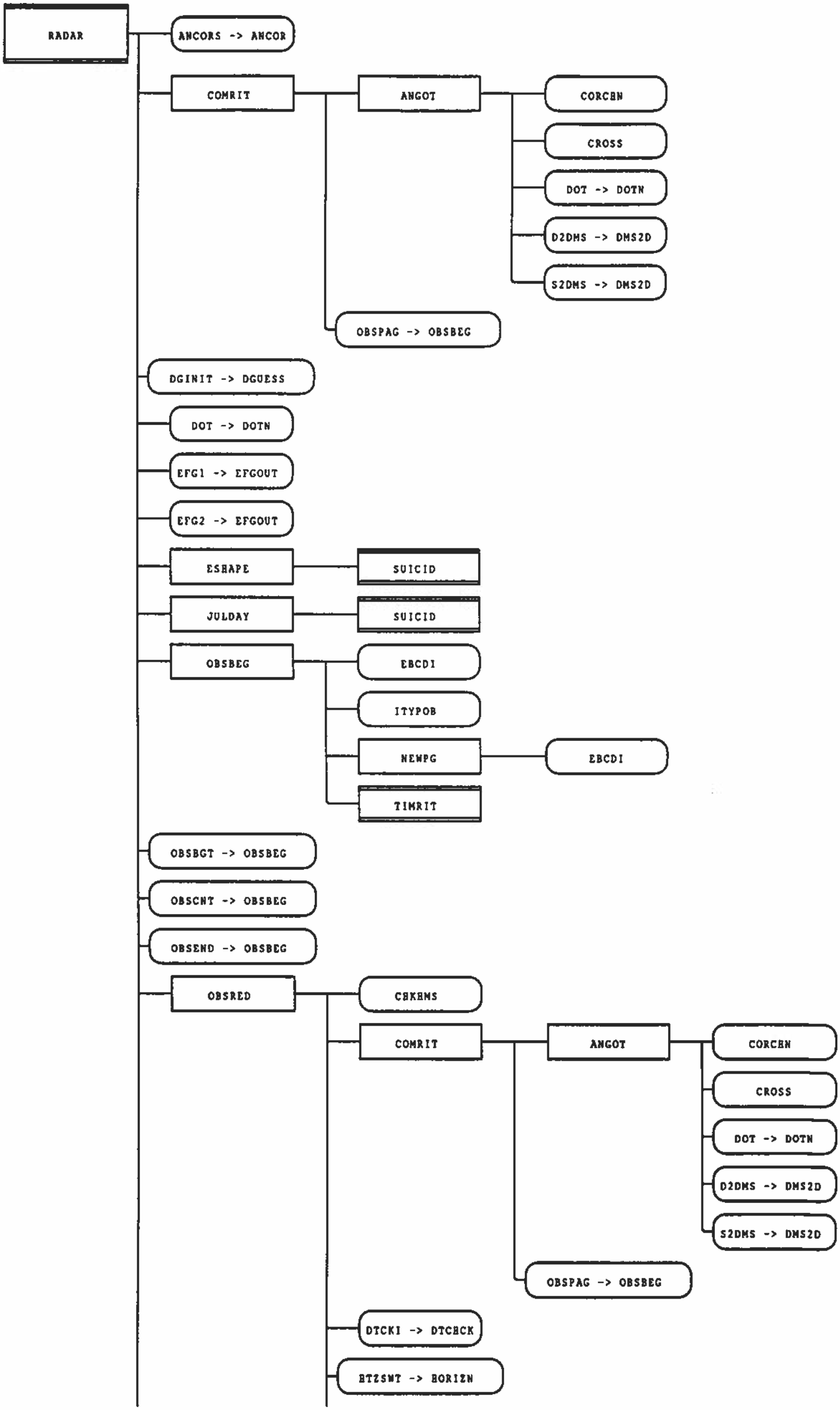

PRCNUT
ARGMNT
FUNARG
FUNCOF
OBSPAG -> OBSBEG
PRECES
DOTN
OBSPAG -> OBSBEG
PRODCT
SMLROT
PRODCT
WOBBLE
BIHWOB
XWOBBL
WOBBLF
SUICID
WOBILS
WOBRED
NEWPG
EBCDI
SUICID
XWOBBL

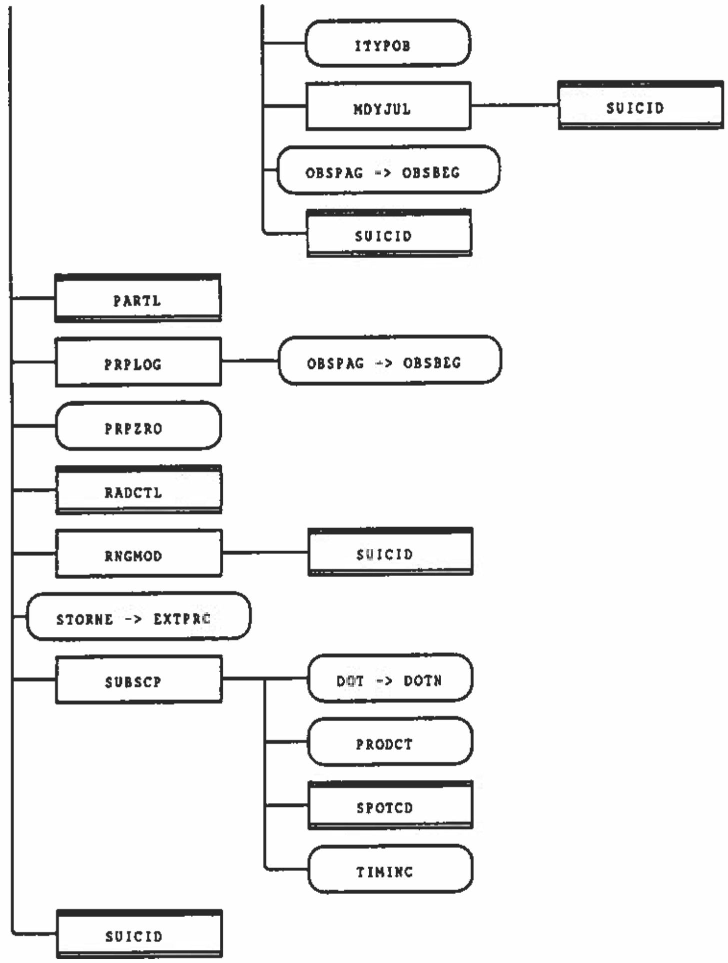

PRDICT
GETSOL
SUICID
GRDSID
HRMNIC
NEWPG
EBCDI
OPRMSG
PAGSET
PLINK
PRDEQS
PRDBDY
PRDPRM
PRDGRD
PRDHAR
PRDPRM
PRDVCT
PRDBDY
PRDPRM
SUICID
ZFILL
PRDOBS
DTCHCK
DTCKI
ITYPOB
PAGCHK
PRDSET
LEG
NSCAN
SUICID
SUICID
ZFILL
PRDSID
SUICID
TIMRIT
ZFILL

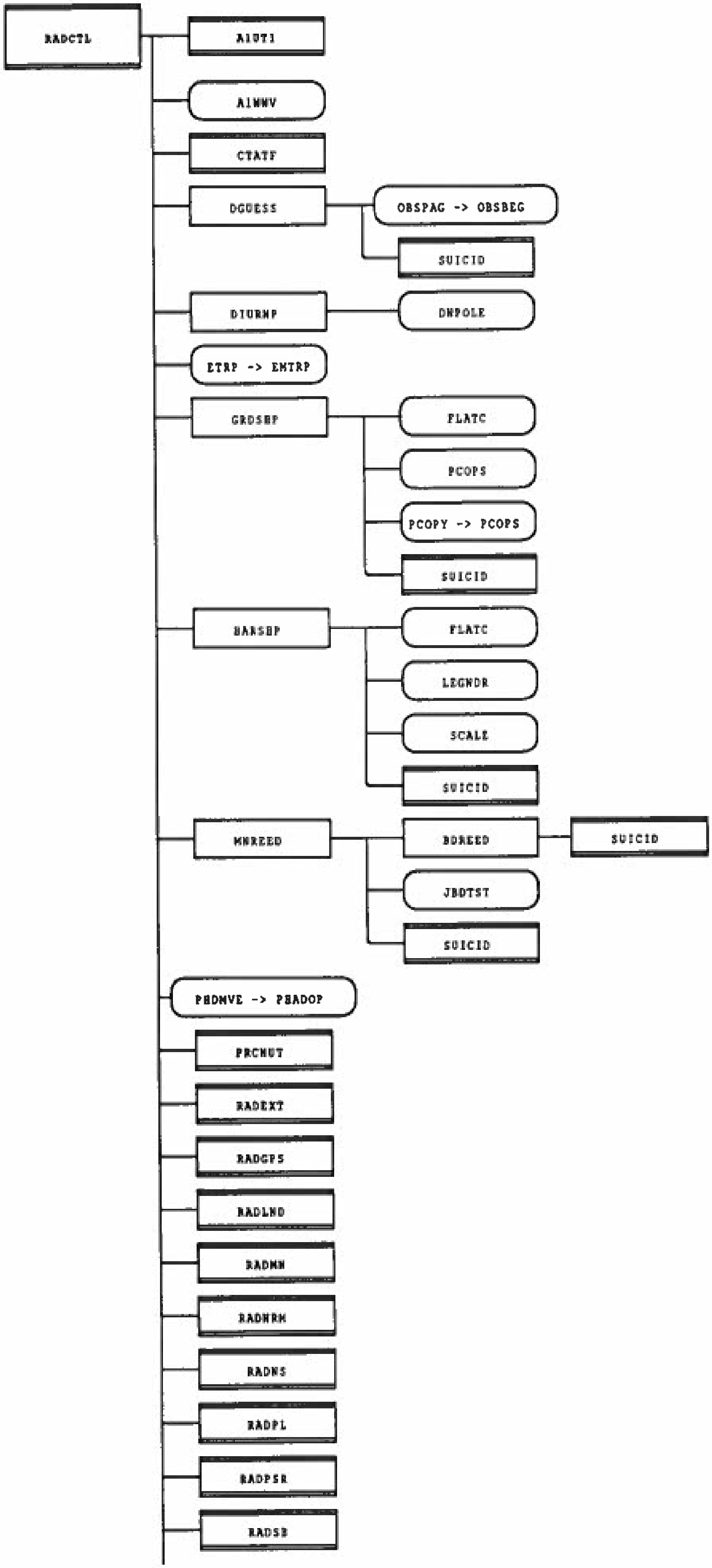


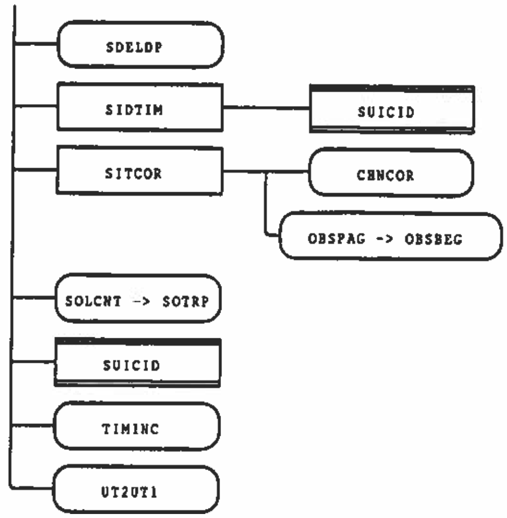
PROPCO
EATCL
EATMDL
EATMDP
EATZDL
ATM
SUICID
EIOCTL
DIURN
EIONDL
EIONDP
EIONDL
OBSPAG -> OBSBEG
SUICID
MEDIA
DOT -> DOTN
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
OBSPAG -> OBSBEG
PATCTL
DOT3 -> DOTN
PRPFRQ
PRPSTR
OBSPAG -> OBSBEG
SUICID
PRPZEN
OBSPAG -> OBSBEG
ZENANG
DOT3 -> DOTN
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
SUICID

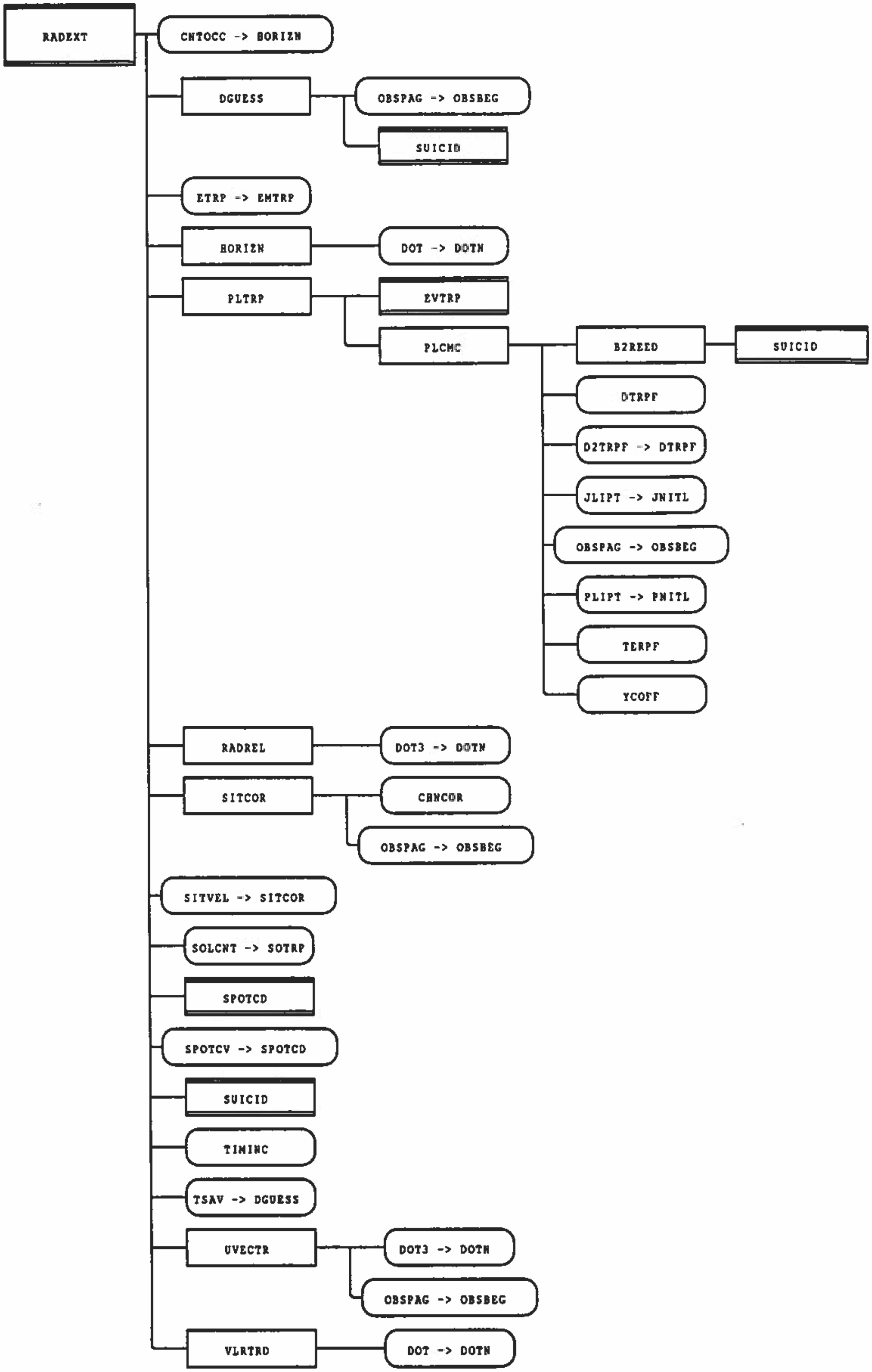

PUNCH
BODPUN
GRDPUN
SUICID
DTPUN
EBCDI
EQNPUN
MVC
PBSPUN
PSRPUN
RBSPUN
SITPUN
SKYPUN
SPTPUN

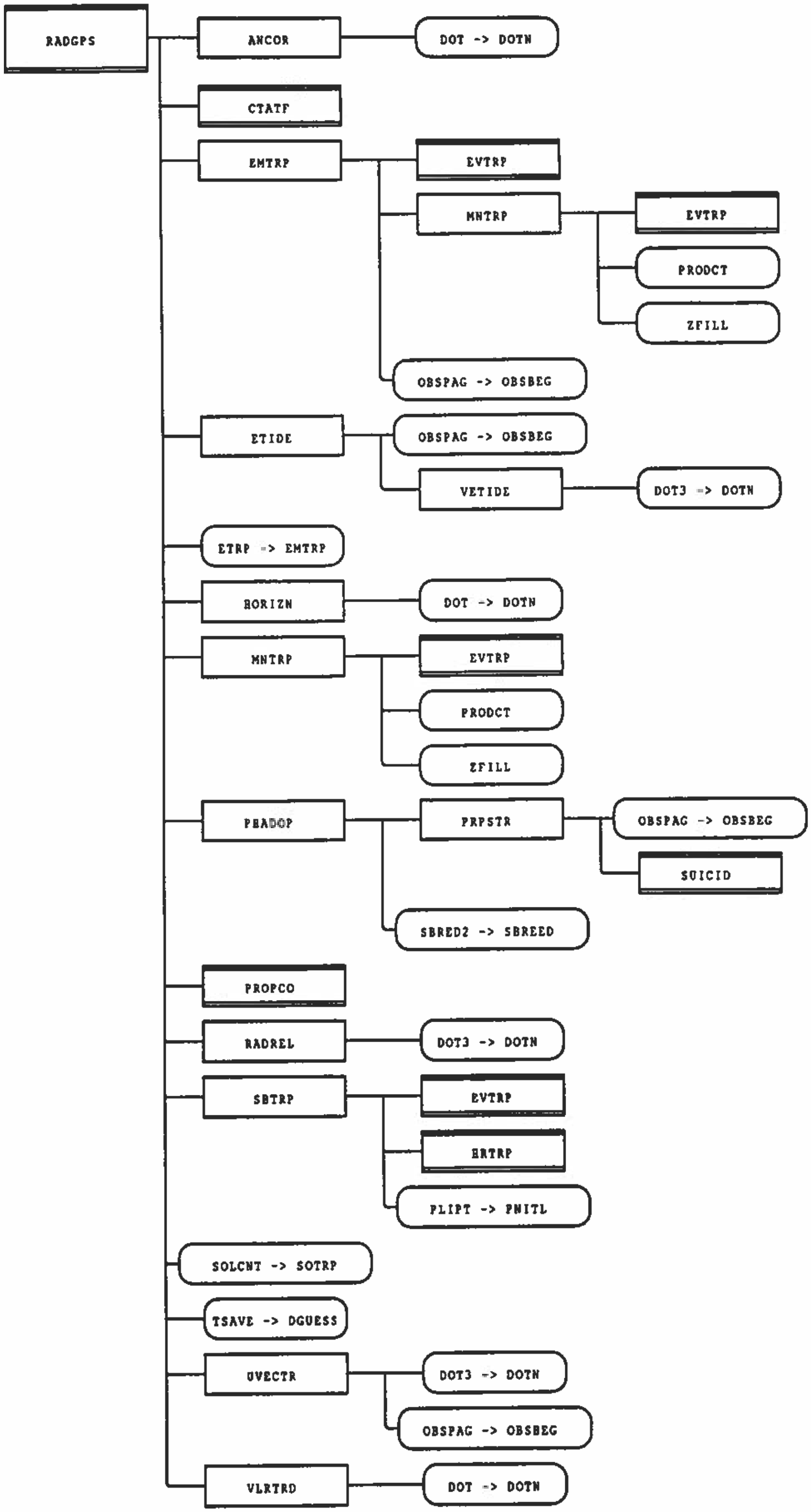

RADAR
ANCORS -> ANCOR
COMRIT
ANGOT
CORCEN
CROSS
DOT -> DOTN
D2DMS -> DMS2D
S2DMS -> DMS2D
OBSPAG -> OBSBEG
DGINIT -> DGUESS
DOT -> DOTN
EFG1 -> EFGOUT
EFG2 -> EFGOUT
ESHAPE
SUICID
JULDAY
SUICID
OBSBEG
EBCDI
ITYPOB
NEWPG
EBCDI
TIMRIT
OBSBGT -> OBSBEG
OBSCNT -> OBSBEG
OBSEND -> OBSBEG
OBSRED
CHKEMS
COMRIT
ANGOT
CORCEN
CROSS
DOT -> DOTN
D2DMS -> DMS2D
S2DMS -> DMS2D
OBSPAG -> OBSBEG
DTCKI -> DTCHCK
HTZSWT -> HORIZN

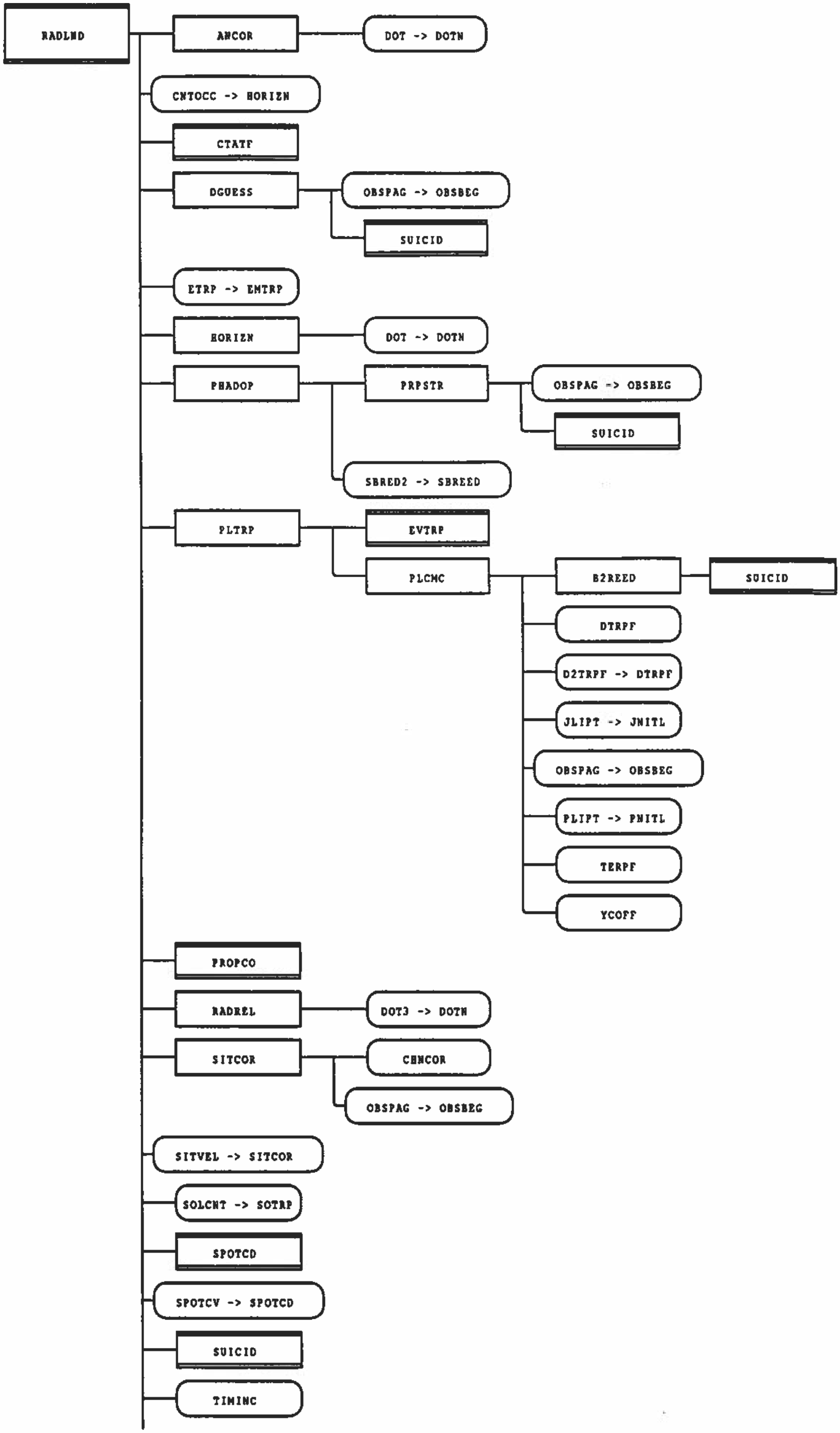

ITYPOB
MDYJUL
SUICID
OBSPAG -> OBSBEG
SUICID
PARTL
PRPLOG
OBSPAG -> OBSBEG
PRPZRO
RADCTL
RNGMOD
SUICID
STORNE -> EXTPRC
SUBSCP
DOT -> DOTN
PRODCT
SPOTCD
TIMINC
SUICID

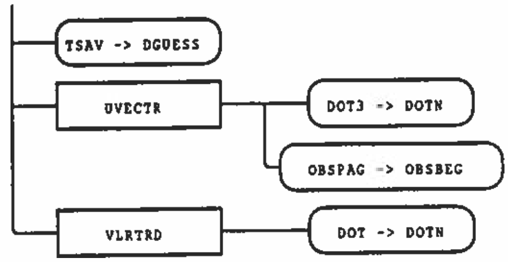

RADCTL
A1UT1
A1MNV
CTATF
DGUESS
OBSPAG -> OBSBEG
SUICID
DIURNP
DNPOLE
ETRP -> ENTRP
GRDSHP
FLATC
PCOPS
PCOPY -> PCOPS
SUICID
HARSHP
FLATC
LEGNDR
SCALE
SUICID
MNREED
BDREED
SUICID
JBDTST
SUICID
PHDMVE -> PSADOP
PRCHUT
RADEXT
RADGPS
RADLND
RADMN
RADNRM
RADNS
RADPL
RADPSR
RADSB

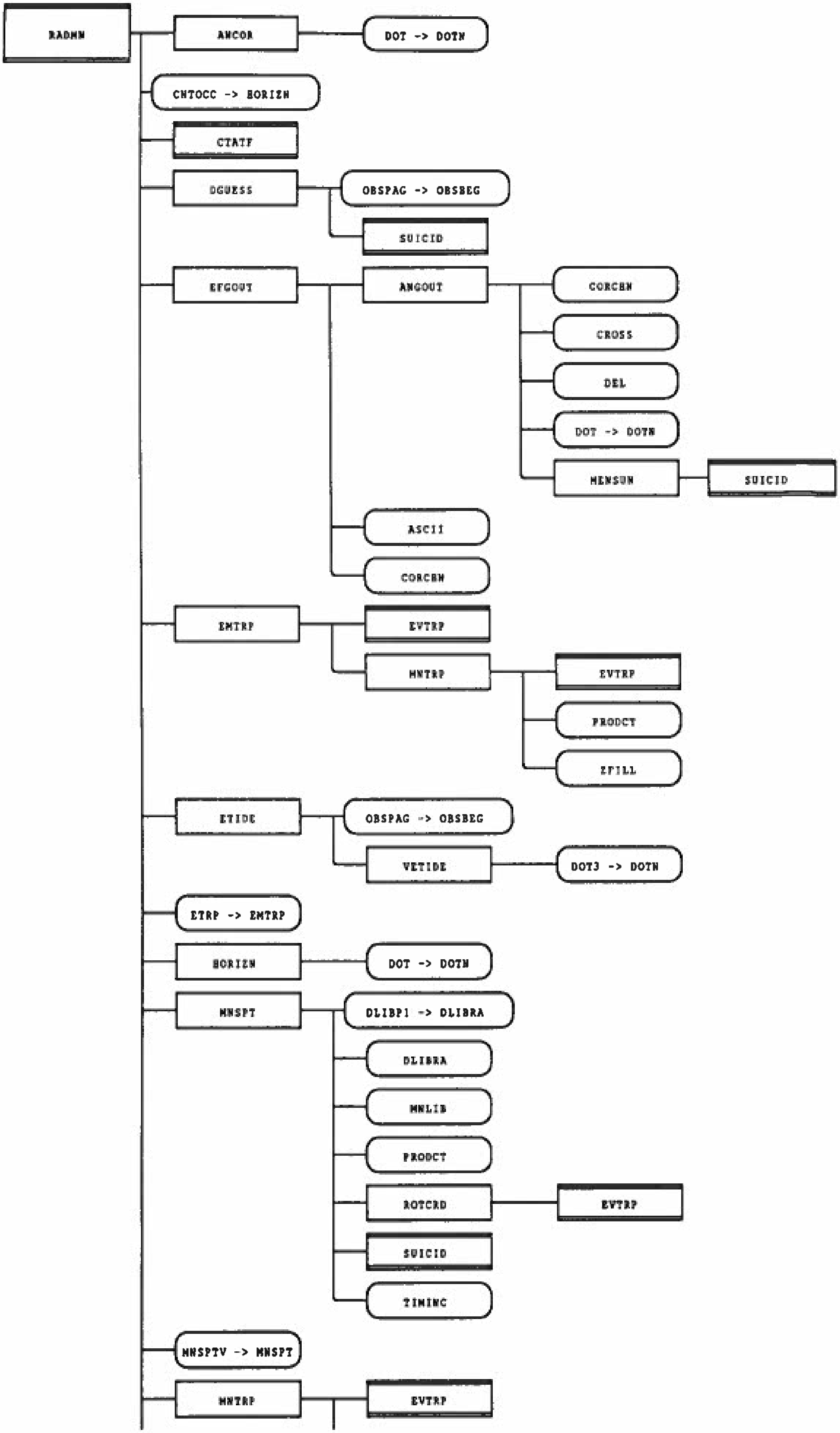

SDELDP
SIDTIM
SUICID
SITCOR
CHNCOR
OBSPAG -> OBSBEG
SOLCNT -> SOTRP
SUICID
TIMINC
UT2UT1


RADEXT
CNTOCC -> HORIZN
DGUESS
OBSPAG -> OBSBEG
SUICID
ETRP -> EMTRP
HORIZN
DOT -> DOTN
PLTRP
EVTRP
PLCMC
B2REED
SUICID
DTRPF
D2TRPF -> DTRPF
JLIPT -> JNITL
OBSPAG -> OBSBEG
PLIPT -> PNITL
TERPF
YCOFF
RADREL
DOT3 -> DOTN
SITCOR
CHNCOR
OBSPAG -> OBSBEG
SITVEL -> SITCOR
SOLCNT -> SOTRP
SPOTCD
SPOTCV -> SPOTCD
SUICID
TIMINC
TSAV -> DGUESS
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN

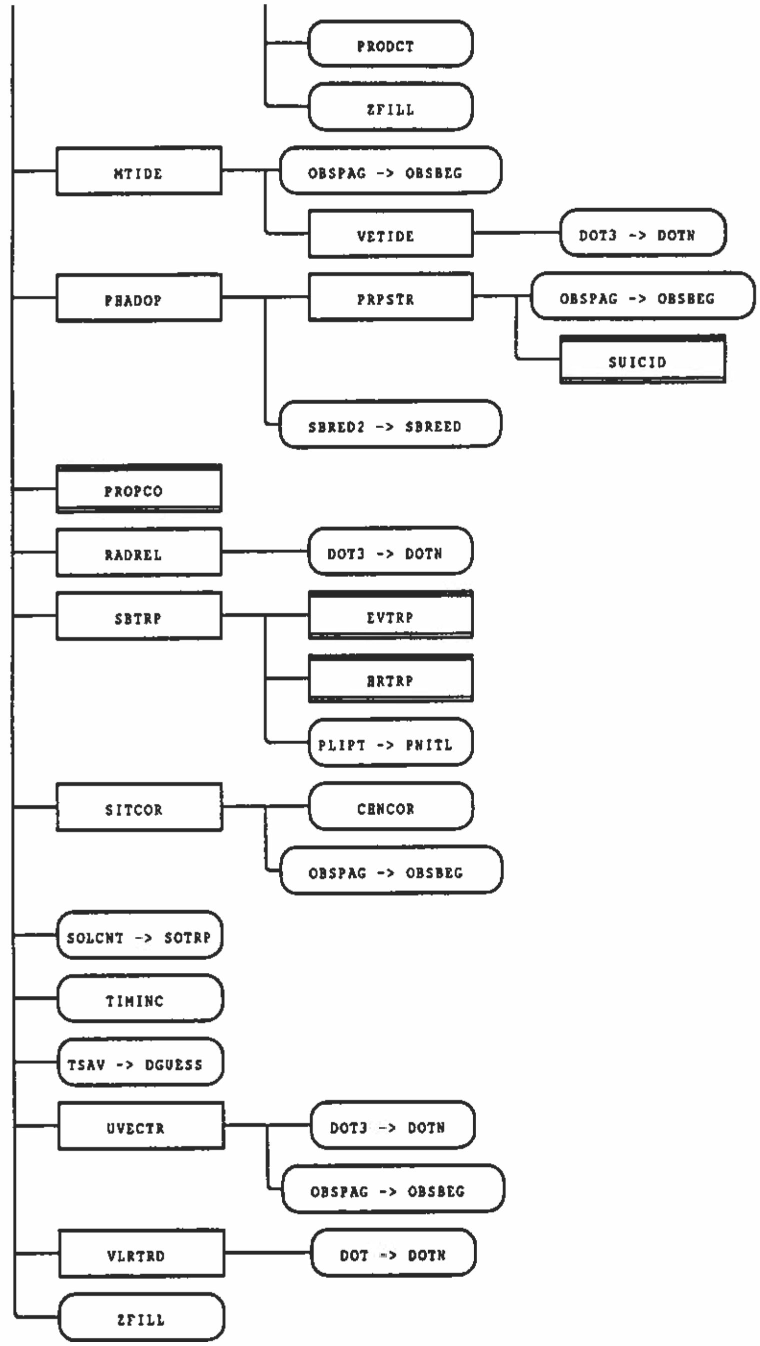

RADGPS
ANCOR
DOT -> DOTN
CTATF
EMTRP
EVTRP
MNTRP
EVTRP
PRODCT
ZFILL
OBSPAG -> OBSBEG
ETIDE
OBSPAG -> OBSBEG
VETIDE
DOT3 -> DOTN
ETRP -> EMTRP
HORIZN
DOT -> DOTN
MNTRP
EVTRP
PRODCT
ZFILL
PHADOP
PRPSTR
OBSPAG -> OBSBEG
SUICID
SBRED2 -> SBREED
PROPCO
RADREL
DOT3 -> DOTN
SBTRP
EVTRP
HRTRP
PLIPT -> PNITL
SOLCNT -> SOTRP
TSAVE -> DGUESS
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN

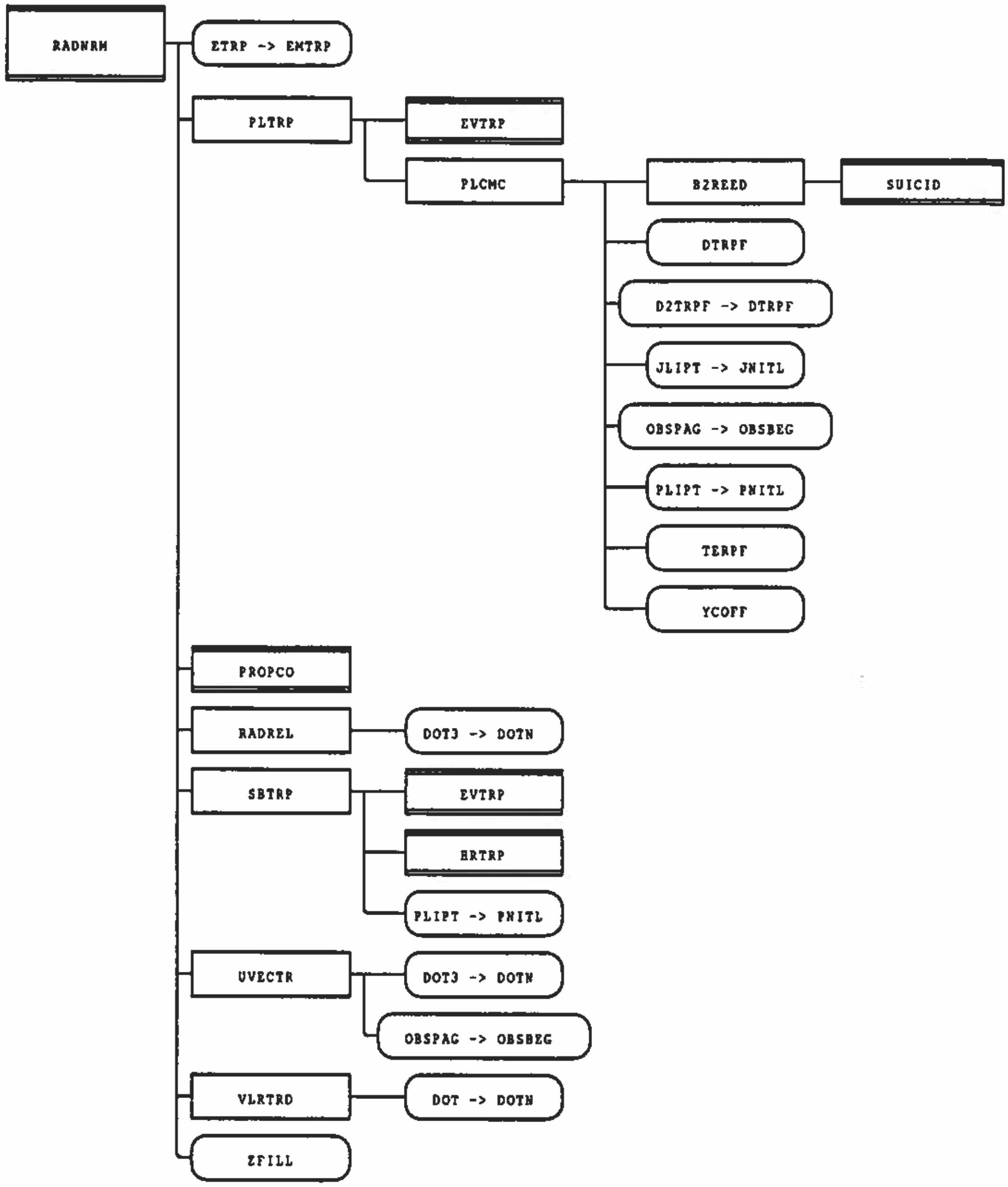

RADLND
ANCOR
DOT -> DOTN
CNTOCC -> HORIZN
CTATF
DGUESS
OBSPAG -> OBSBEG
SUICID
ETRP -> EMTRP
HORIZN
DOT -> DOTN
PHADOP
PRPSTR
OBSPAG -> OBSBEG
SUICID
SBRED2 -> SBREED
PLTRP
EVTRP
PLCMC
B2REED
SUICID
DTRPF
D2TRPF -> DTRPF
JLIPT -> JNITL
OBSPAG -> OBSBEG
PLIPT -> PNITL
TERPF
YCOFF
PROPCO
RADREL
DOT3 -> DOTN
SITCOR
CHNCOR
OBSPAG -> OBSBEG
SITVEL -> SITCOR
SOLCNT -> SOTRP
SPOTCD
SPOTCV -> SPOTCD
SUICID
TIMINC

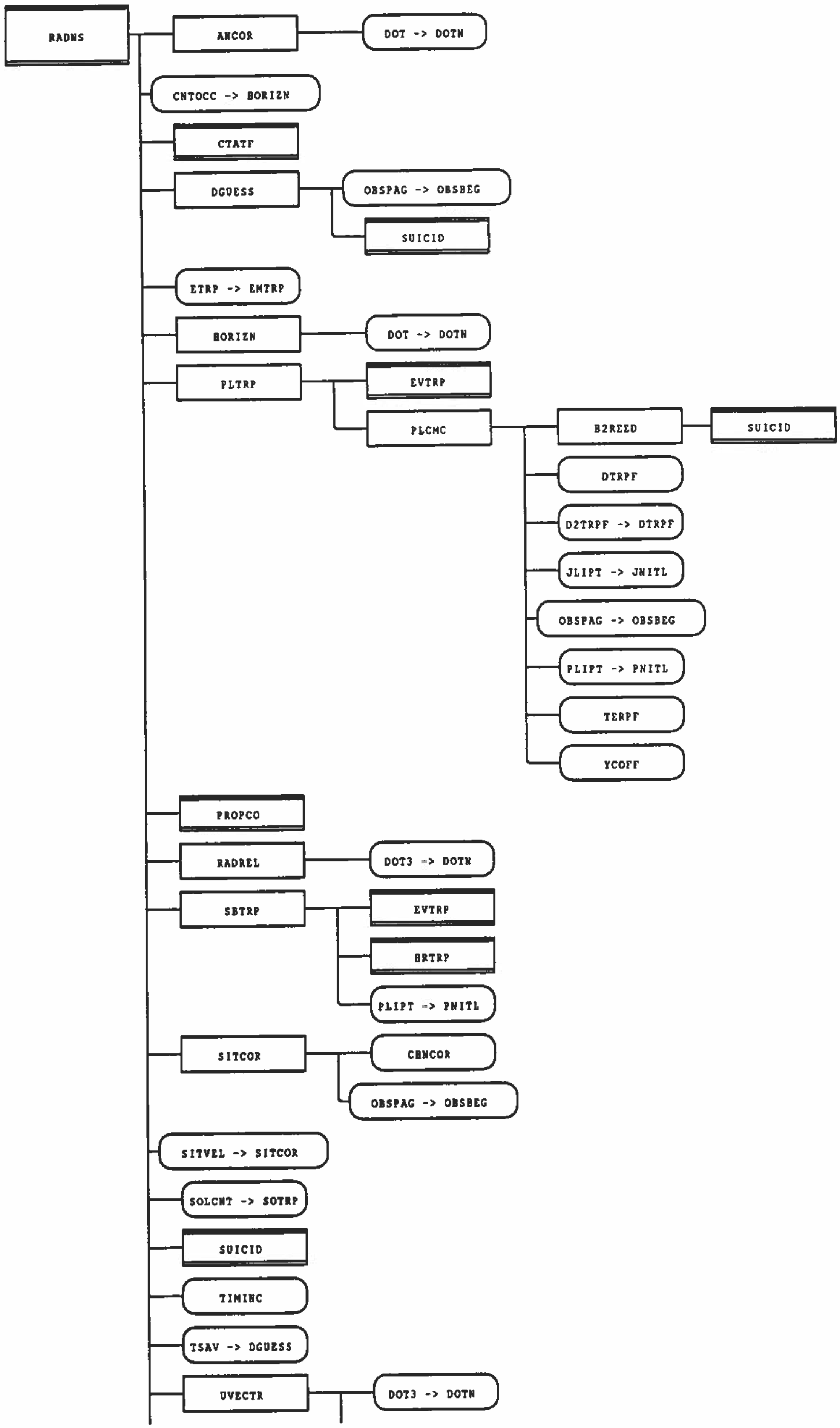

TSAV -> DGUESS
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN

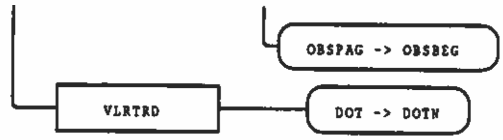


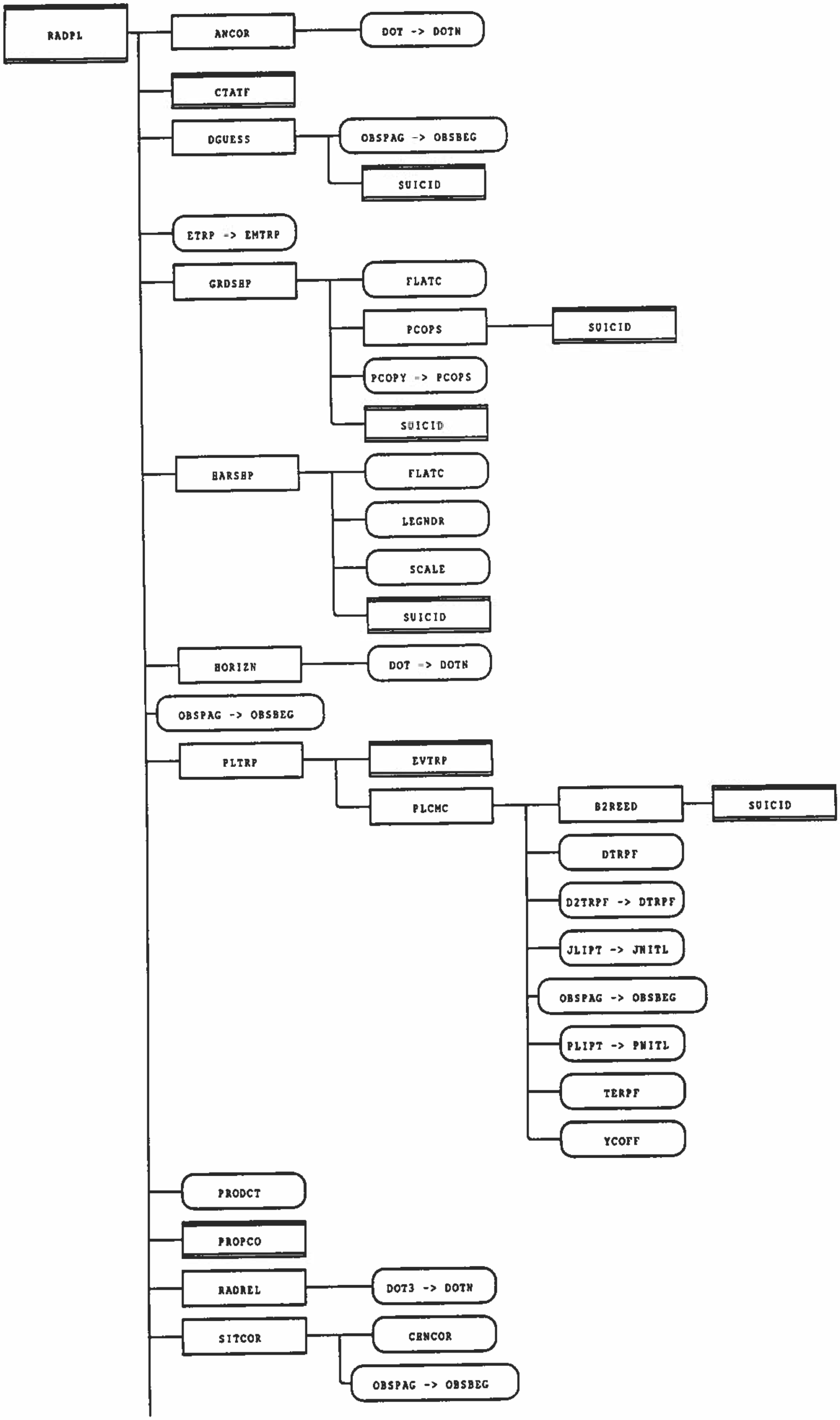
RADMN
ANCOR
DOT -> DOTN
CNTOCC -> HORIZN
CTATF
DGUESS
OBSPAG -> OBSBEG
SUICID
EFGOUT
ANGOUT
CORCHN
CROSS
DEL
DOT -> DOTN
MENSUN
SUICID
ASCII
CORCHN
EMTRP
EVTRP
MNTRP
EVTRP
PRODCT
ZFILL
ETIDE
OBSPAG -> OBSBEG
VETIDE
DOT3 -> DOTN
ETRP -> EMTRP
HORIZN
DOT -> DOTN
MNSPT
DLIBP1 -> DLIBRA
DLIBRA
MNLIB
PRODCT
ROTCRD
EVTRP
SUICID
TIMINC
MNSPTV -> MNSPT
MNTRP
EVTRP

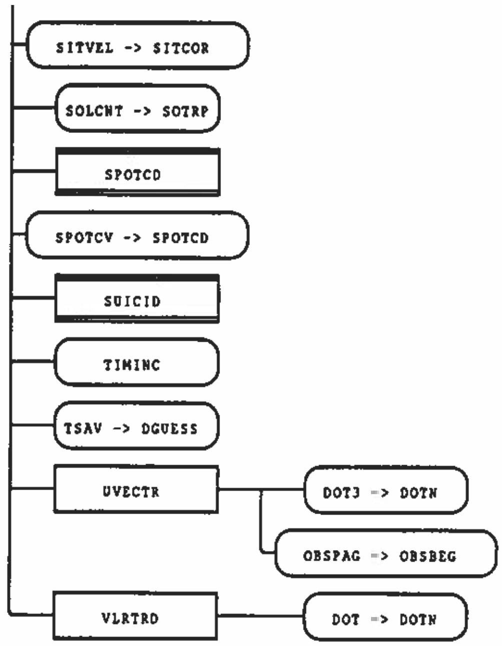

PRODCT
ZFILL
MTIDE
OBSPAG -> OBSBEG
VETIDE
DOT3 -> DOTN
PHADOP
PRPSTR
OBSPAG -> OBSBEG
SUICID
SBRED2 -> SBREED
PROPCO
RADREL
DOT3 -> DOTN
SBTRP
EVTRP
ERTRP
PLIPT -> PNITL
SITCOR
CENCOR
OBSPAG -> OBSBEG
SOLCNT -> SOTRP
TIMINC
TSAV -> DGUESS
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN
ZFILL

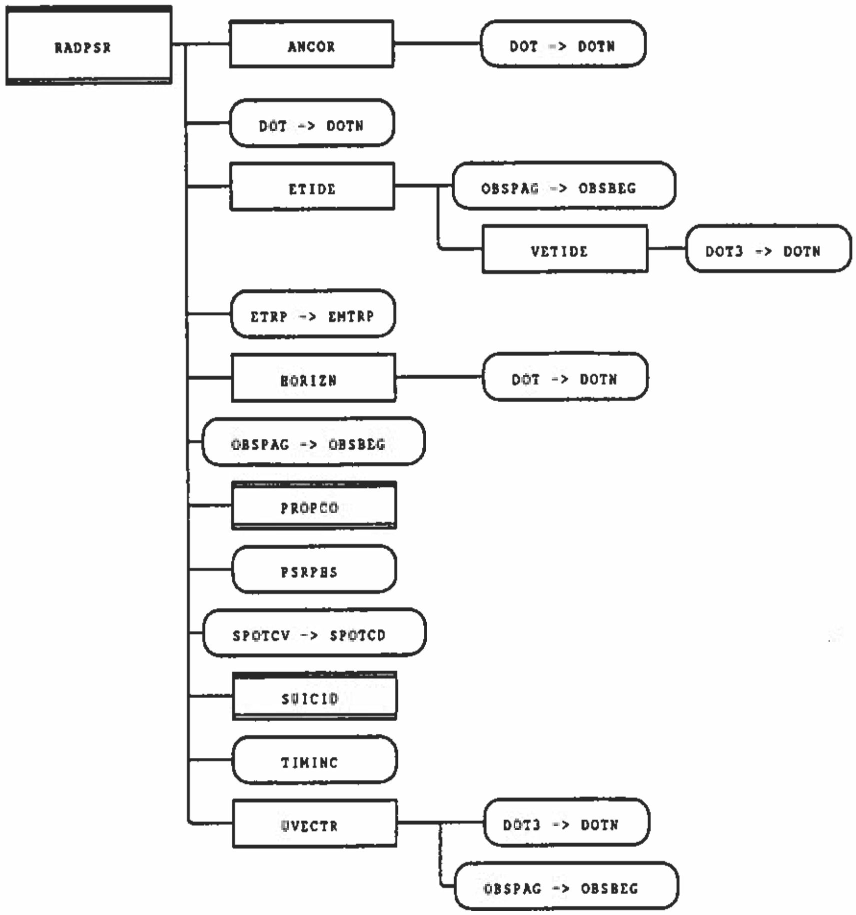

RADNRM
ETRP -> EMTRP
PLTRP
EVTRP
PLCMC
B2REED
SUICID
DTRPF
D2TRPF -> DTRPF
JLIPT -> JNITL
OBSPAG -> OBSBEG
PLIPT -> PNITL
TERPF
YCOFF
PROPCO
RADREL
DOT3 -> DOTN
SBTRP
EVTRP
HRTRP
PLIPT -> PNITL
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN
ZFILL

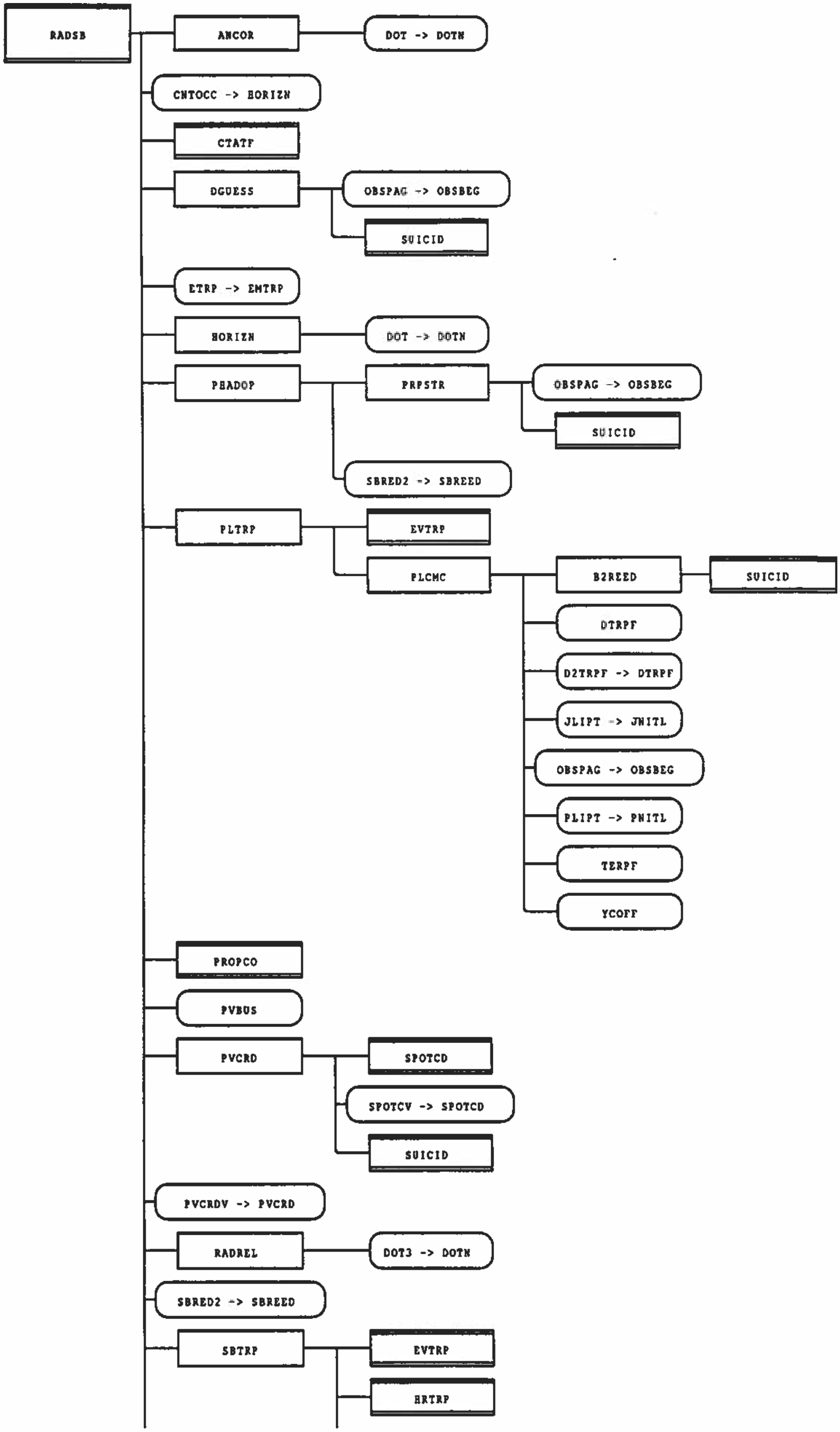

RADNS
ANCOR
DOT -> DOTN
CNTOCC -> HORIZN
CTATF
DGUESS
OBSPAG -> OBSBEG
SUICID
ETRP -> EMTRP
HORIZN
DOT -> DOTN
PLTRP
EVTRP
PLCMC
B2REED
SUICID
DTRPF
D2TRPF -> DTRPF
JLIPT -> JNITL
OBSPAG -> OBSBEG
PLIPT -> PNITL
TERPF
YCOFF
PROPCO
RADREL
DOT3 -> DOTN
SBTRP
EVTRP
HRTRP
PLIPT -> PNITL
SITCOR
CHNCOR
OBSPAG -> OBSBEG
SITVEL -> SITCOR
SOLCNT -> SOTRP
SUICID
TIMINC
TSAV -> DGUESS
UVECTR
DOT3 -> DOTN

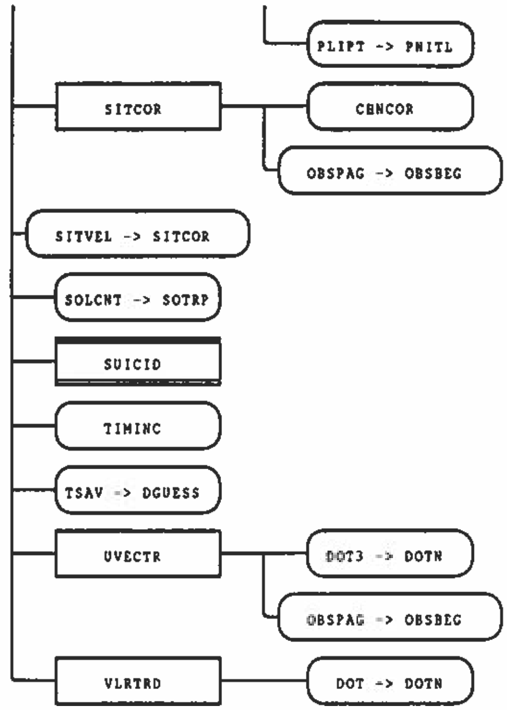

OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN

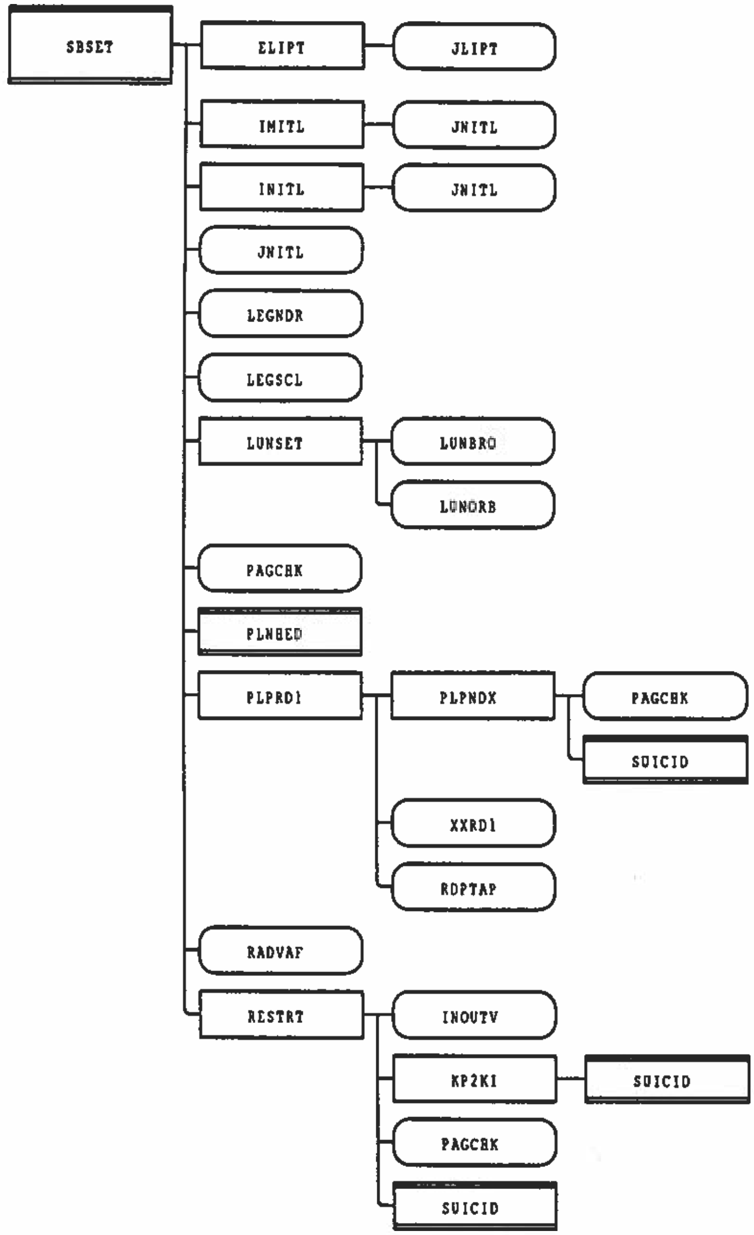

RADPL
ANCOR
DOT -> DOTN
CTATF
DGUESS
OBSPAG -> OBSBEG
SUICID
ETRP -> EMTRP
GRDSHP
FLATC
PCOPS
SUICID
PCOPY -> PCOPS
SUICID
HARSHP
FLATC
LEGNDR
SCALE
SUICID
HORIZN
DOT -> DOTN
OBSPAG -> OBSBEG
PLTRP
EVTRP
PLCMC
B2REED
SUICID
DTRPF
D2TRPF -> DTRPF
JLIPT -> JNITL
OBSPAG -> OBSBEG
PLIPT -> PNITL
TERPF
YCOFF
PRODCT
PROPCO
RADREL
DOT3 -> DOTN
SITCOR
CENCOR
OBSPAG -> OBSBEG


SITVEL -> SITCOR
SOLCNT -> SOTRP
SPOTCD
SPOTCV -> SPOTCD
SUICID
TIMINC
TSAV -> DGUESS
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN

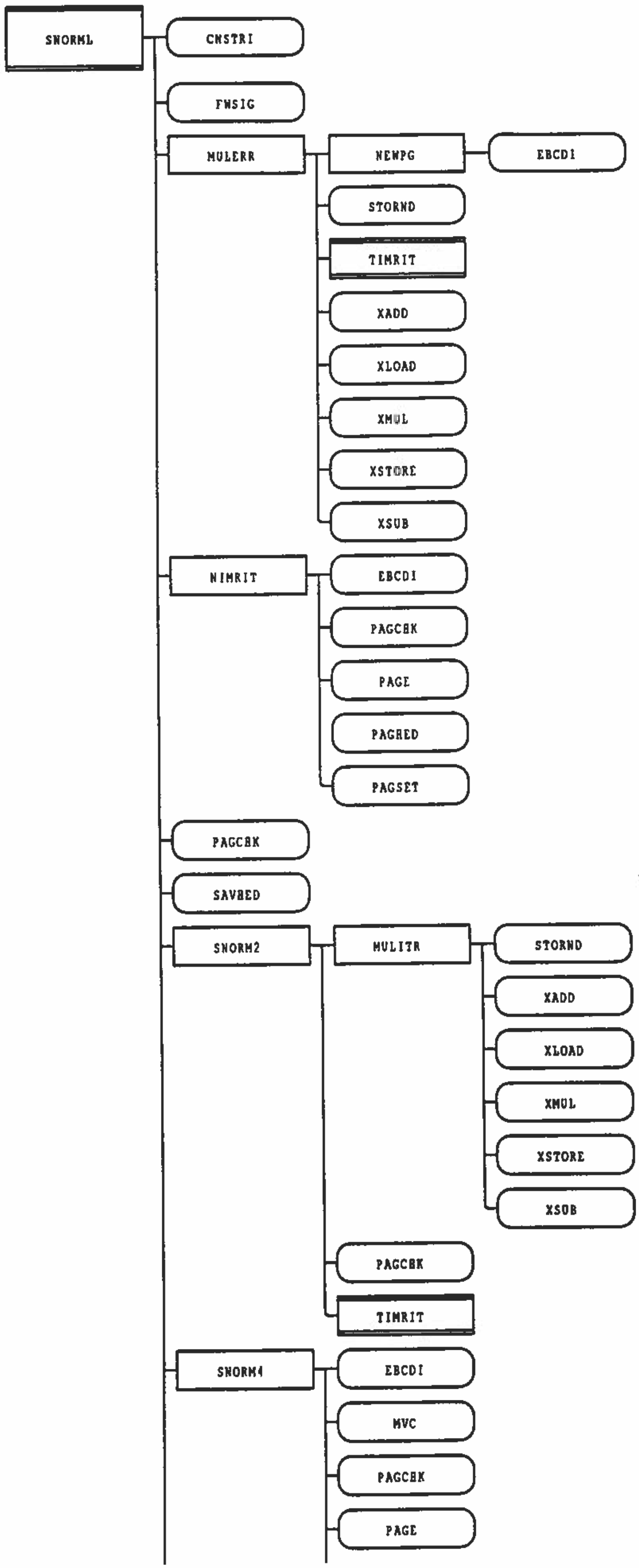

RADPSR
ANCOR
DOT -> DOTN
DOT -> DOTN
ETIDE
OBSPAG -> OBSBEG
VETIDE
DOT3 -> DOTN
ETRP -> EMTRP
HORIZN
DOT -> DOTN
OBSPAG -> OBSBEG
PROPCO
PSRPHS
SPOTCV -> SPOTCD
SUICID
TIMINC
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG

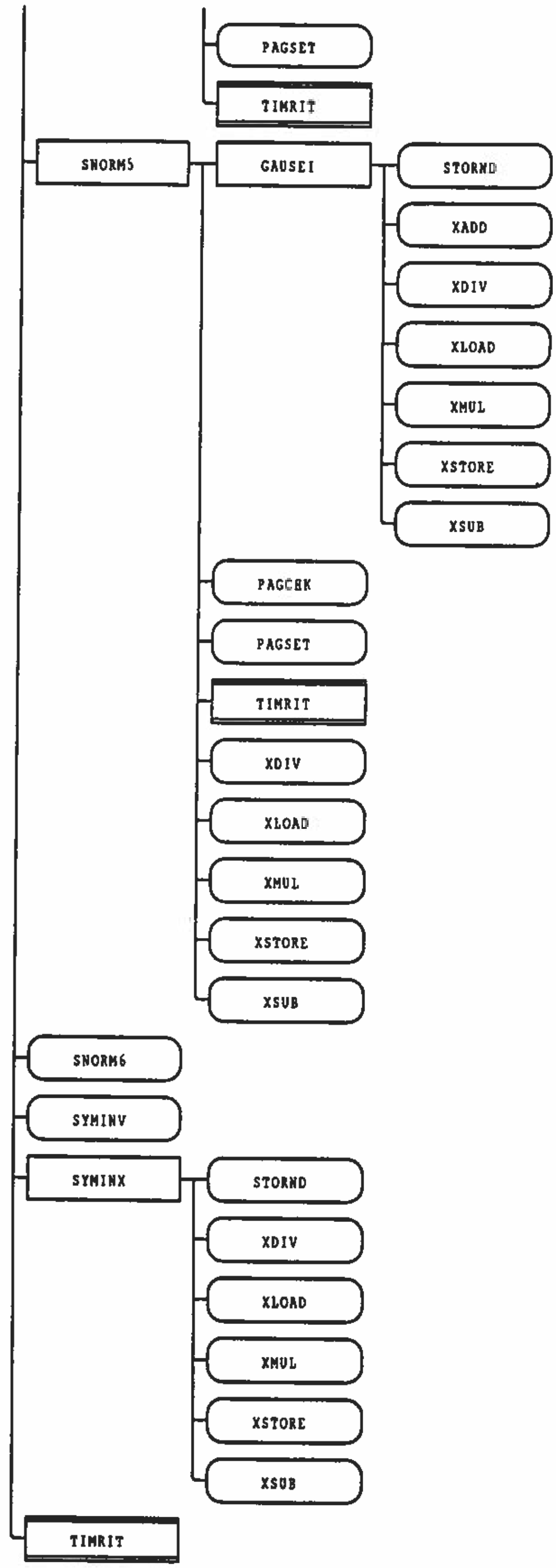

RADSB
ANCOR
DOT -> DOTN
CNTOCC -> HORIZN
CTATF
DGUESS
OBSPAG -> OBSBEG
SUICID
ETRP -> EMTRP
HORIZN
DOT -> DOTN
PHADOP
PRPSTR
OBSPAG -> OBSBEG
SUICID
SBRED2 -> SBREED
PLTRP
EVTRP
PLCMC
B2REED
SUICID
DTRPF
D2TRPF -> DTRPF
JLIPT -> JNITL
OBSPAG -> OBSBEG
PLIPT -> PNITL
TERPF
YCOFF
PROPCO
PVBUS
PVCRD
SPOTCD
SPOTCV -> SPOTCD
SUICID
PVCRDV -> PVCRD
RADREL
DOT3 -> DOTN
SBRED2 -> SBREED
SBTRP
EVTRP
HRTRP

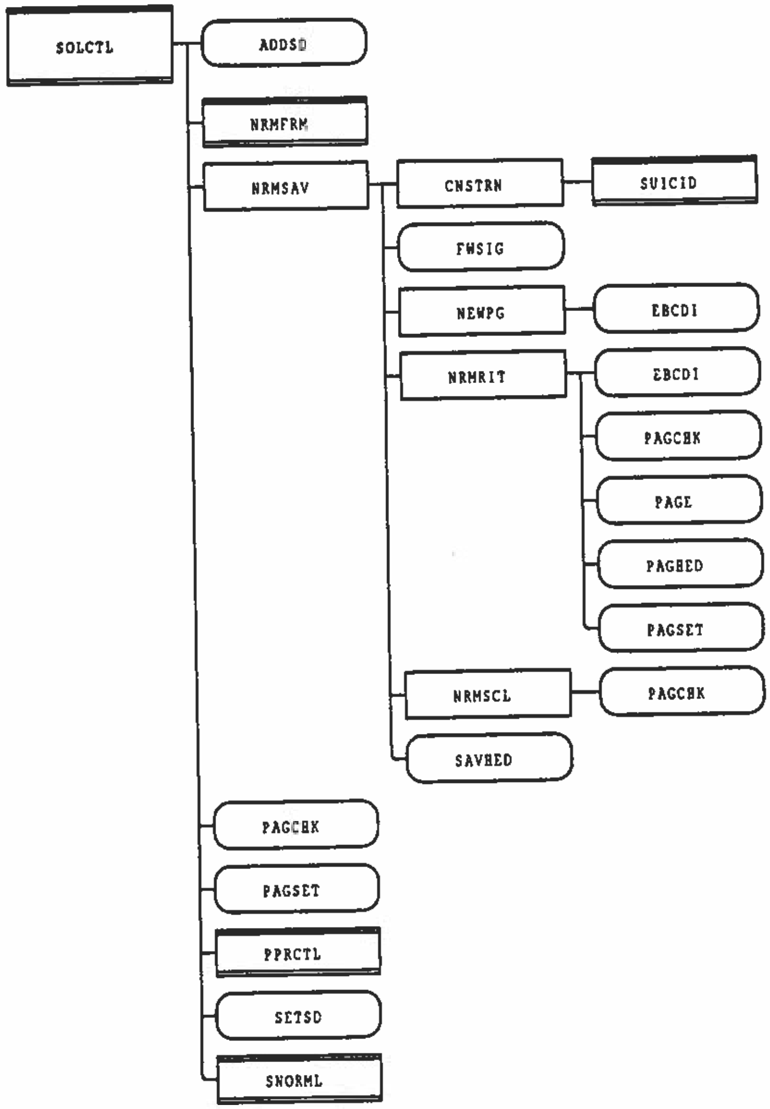

PLIPT -> PNITL
SITCOR
CBNCOR
OBSPAG -> OBSBEG
SITVEL -> SITCOR
SOLCNT -> SOTRP
SUICID
TIMINC
TSAV -> DGUESS
UVECTR
DOT3 -> DOTN
OBSPAG -> OBSBEG
VLRTRD
DOT -> DOTN

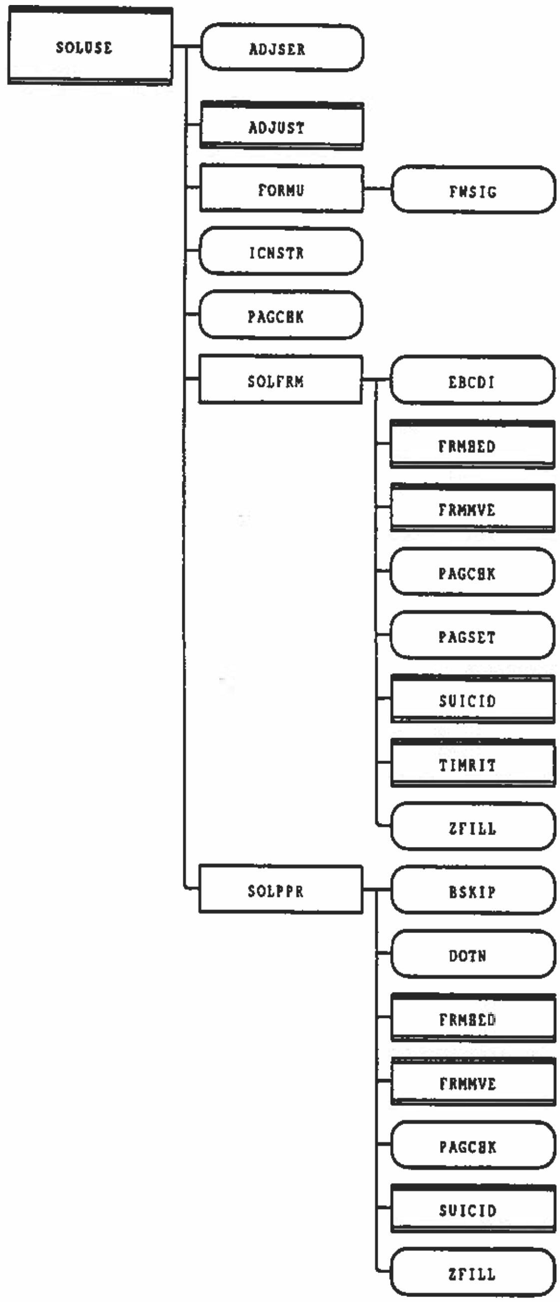


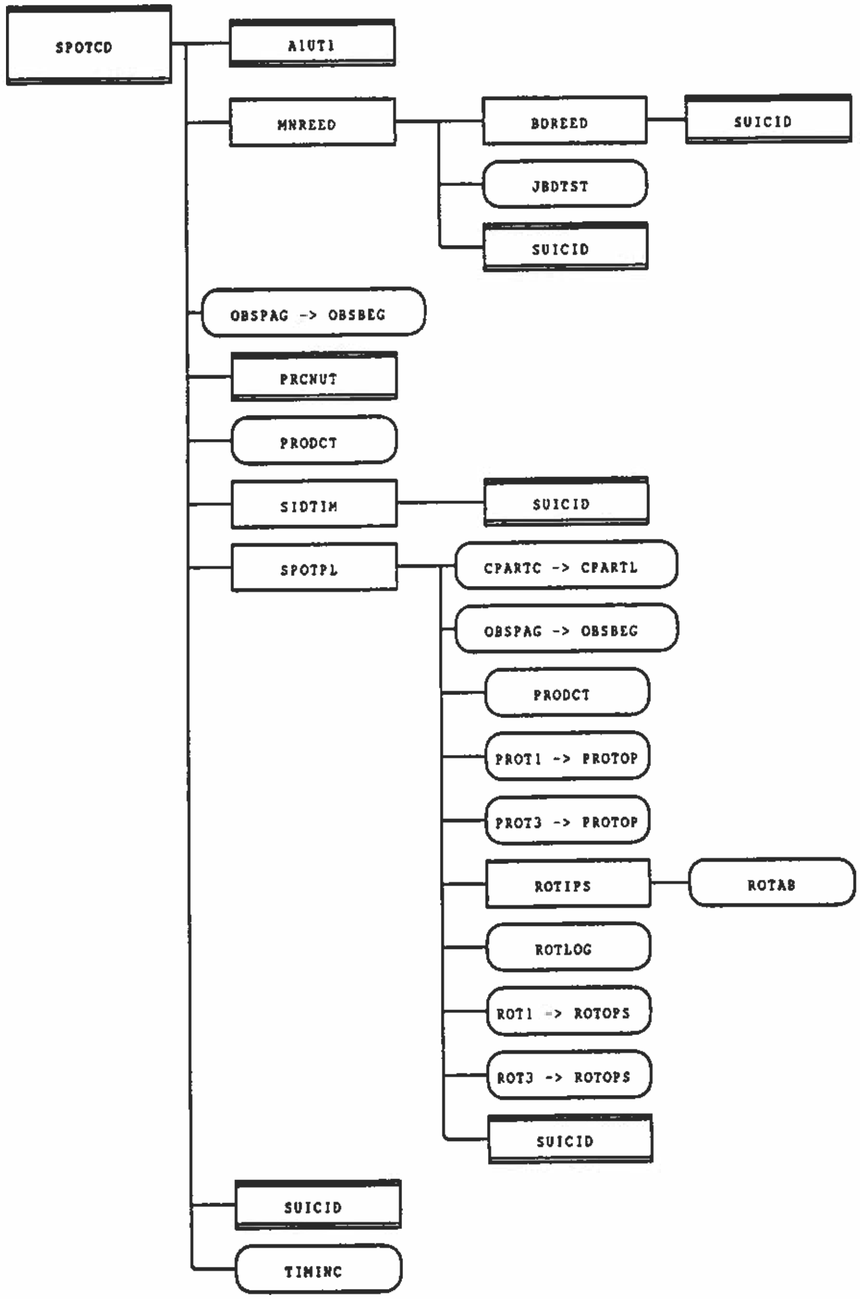
SBSET
ELIPT
JLIPT
IMITL
JNITL
INITL
JNITL
JNITL
LEGNDR
LEGSCL
LUNSET
LUNBRO
LUNORB
PAGCHK
PLNHED
PLPRD1
PLPNDX
PAGCHK
SUICID
XXRD1
RDPTAP
RADVAF
RESTRT
INOUTV
KP2K1
SUICID
PAGCHK
SUICID

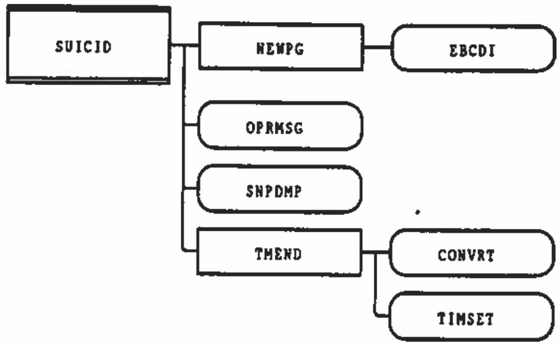

SNORML
CNSTRI
FNSIG
MULERR
NEWPG
EBCDI
STORND
TIMRIT
XADD
XLOAD
XMUL
XSTORE
XSUB
NIMRIT
EBCDI
PAGCHK
PAGE
PAGRED
PAGSET
PAGCHK
SAVRED
SNORM2
MULITR
STORND
XADD
XLOAD
XMUL
XSTORE
XSUB
PAGCHK
TIMRIT
SNORM4
EBCDI
MVC
PAGCHK
PAGE

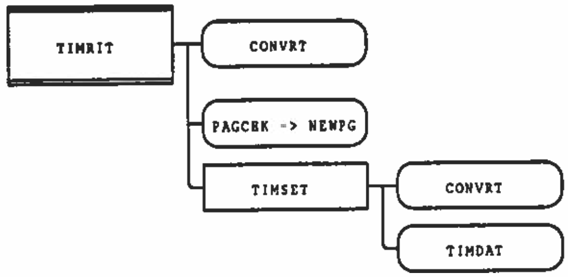

PAGSET
TIMRIT
SNORM5
GAUSEI
STORND
XADD
XDIV
XLOAD
XMUL
XSTORE
XSUB
PAGCHK
PAGSET
TIMRIT
XDIV
XLOAD
XMUL
XSTORE
XSUB
SNORM6
SYMINV
SYMINX
STORND
XDIV
XLOAD
XMUL
XSTORE
XSUB
TIMRIT


SOLCTL
ADDSD
NRMFRM
NRMSAV
CNSTRN
SUICID
FWSIG
NEWPG
EBCDI
NRMRIT
EBCDI
PAGCHK
PAGE
PAGHED
PAGSET
NRMSCL
PAGCHK
SAVHED
PAGCHK
PAGSET
PPRCTL
SETSD
SNORML


SOLUSE
ADJSER
ADJUST
FORMU
FNSIG
ICNSTR
PAGCBK
SOLFRM
EBCDI
FRMBED
FRMMVE
PAGCBK
PAGSET
SUICID
TIMRIT
ZFILL
SOLPPR
BSKIP
DOTN
FRMBED
FRMMVE
PAGCBK
SUICID
ZFILL


SPOTCD
A1UT1
MNREED
BDREED
SUICID
JBDTST
SUICID
OBSPAG -> OBSBEG
PRCNUT
PRODCT
SIDTIM
SUICID
SPOTPL
CPARTC -> CPARTL
OBSPAG -> OBSBEG
PRODCT
PROT1 -> PROTOP
PROT3 -> PROTOP
ROTIPS
ROTAB
ROTLOG
ROT1 -> ROTOPS
ROT3 -> ROTOPS
SUICID
SUICID
TIMINC


SUICID
NEWPG
EBCDI
OPRMSG
SNPDMP
TMEND
CONVRT
TIMSET


TIMRIT
CONVRT
PAGCEK => NEWPG
TIMSET
CONVRT
TIMDAT