DEPARTMENT OF EARTH AND PLANETARY SCIENCES

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77-4

Date: November 18, 1977
To: Distribution
From: R. D. Reasenberg
Subject: Implementation in PEP of a Disparate Parameter Saved Normal Equations Combiner
Reference: Memorandum of 10 Feb 1975: Distribution/RDR

Introduction

The referenced memorandum contains some examples of circumstances in which it would be useful to be able to combine two or more saved - normal-equations sets not calculated using a common set of nominal parameter values. The linear estimator equations applicable to a Disparate Parameter Saved Normal Equations Combiner (DIPSNEC) are given there and are reproduced in the appendix. The purpose of this memorandum is to present a convenient means of including a DIPSNEC in PEP. In all of the discussion that follows, the linear approximation to the estimation equations is assumed.

The values of a set of estimated parameters are independent of the nominal (i.e., initial) values of the parameters. (See Equation Ref-12.) By using Equation Ref-10, one can change the nominal values of the parameters of a set of normal equations: the coefficient matrix, B, remains unchanged and the right-hand-side vector, U, is modified by the addition of a vector proportional to Δx, the difference between the new and old nominal parameter vectors. Of course, if the new nominal parameter values are chosen to be very different from the eventual (post-fit) parameter estimates, then significant numerical errors could be introduced into the normal-equations solution process. Similarly, if the new

nominal parameter values are chosen to be much closer to the eventual parameter estimates than were the original nominal parameter values, then one may reduce the numerical errors in the solution of the normal equations. A numerical improvement of this kind can be obtained for any PEP solution of normal equations by using the PEP iterated-solution facility, i.e., by setting ICT(48) > 0.

Selection of New Nominals

There are two obvious possible sources for the new set of nominal parameter values: (1) the values contained in the input stream and (2) the nominals associated with a particular set of restored normal equations. As indicated in the previous section, for a reasonable choice of new nominals, the estimated value of the parameters will be independent of the choice of new nominals. The implementation of the DIPSNEC will be simplified if the input stream values are used as the new nominals. I recommend that this approach be taken, at least initially.

Selectable Options

For some purposes the DIPSNEC facility will be neither useful nor even acceptable. Therefore the following options should be made available to the user, and selectable using ICT(76).

ICT(76)=0	Take no action.
ICT(76)=2	Calculate the nominal parameter value vector initially and after each restore of a set of normal equations. Flag changes. (DEFAULT)
ICT(76)=4	Like ICT(76)=2, but also stop program if any changes are detected.

ICT(76)=6 Like ICT(76)=4, except that a defined subset of the parameters is to be ignored, i.e., the corresponding elements of the difference vector are to be zeroed before checking for changes.

ICT(76)=14 or 16 Like ICT(76)=4 or 6, but as they are restored, the normal equations are modified according to the procedure to be described in the next section.

Modifications to PEP

PEP now has a facility for generating vectors of nominal values, scale factors, and names for all of the parameters that are to be estimated in a given run. The code is contained in the subroutine NAMPRM and the subroutines it calls. This facility is normally invoked after the input stream is read. The first program change would be to invoke the NAMPRM facility in connection with the restore-normal-equations operation. The call would be made after all of the indicative records and nominal-parameter-value records had been read from the saved-normal-equations data set. Thus PEP would generate a vector of nominal parameter values based on each set of saved normal equations as well as the input stream quantities.

For each set of normal equations the new code would calculate $\Delta x = x'-x$ (see Equation Ref-10). For ICT(76)=6 or 16, a specified set of elements of Δx would be zeroed. For ICT(76)=14 or 16, $\delta U = -B\Delta x$ would be calculated; one element would be calculated each time a useful row of the normal equations is read in.

At the end of each normal-equations restoration it will be necessary to reset the PEP parameter arrays to the input-stream nominal values. This can be done in 2 ways: (1) Write these arrays onto a temporary data set (e.g., IPLCON) at the end of the input-stream read operation. Read back this data set to reset the parameter values. (2) Write a set of subroutines like NAMPRM,

etc. that move parameter values, but in the opposite direction. Use these routines to move the nominal values from the vector generated by NAMPRM to the usual PEP arrays.

Order of Implementation, Extras

The PEP modifications described above can be made in several stages such that the program is left at the end of each stage with a new and useful facility. These stages are related to the following sequence of values for ICT(76): 0, 2, 4, 14, 16. If it should eventually appear useful, the program could be further modified to permit the nominal parameter values to be determined by a particular set of restored normal equations. However, at this time such an added complication does not seem justified.

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Appendix: First two of three pages of referenced memorandum.

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TO: Distribution

FROM: R. D. Reasenberg

SUBJECT: Combination of Saved Normal Equations which Were Generated Using Different Nominal Parameter Values

It is often both practical and economically sound to analyze data in small batches, at least initially. This having been successful, an analysis using larger batches or all of the data is a reasonable next step. If the problem has proved to be nonlinear (e.g., if it has been necessary to iterate on a linearized estimator), then the "best" solutions for the separate batches represent linear adjustments from a disparate set of nominals. (Nonlinearity is not the only possible reason for this condition to obtain.) The Saved Normal Equations (SNE) from these "best" solutions can not be combined directly, however, they can be combined (linearly) by a simple method given below.

Many examples of the need for a Disparate Parameter Saved Normal Equation Combiner (DIPSNEC) can be found. Among them are:

1. The Mariner 71 gravity analysis yielded 3 separate (sixth degree spherical harmonic) models each with a set of spacecraft state vectors for the associated spacecraft trajectory arcs. In the absence of a DIPSNEC, the individual gravity model coefficients were averaged over the three models, ignoring the differences in the covariances of the 3 solutions. The error associated with this procedure will become more severe for the Viking data analysis because of the availability of data taken with a substantially different planet/spacecraft geometry.

2. In preparing to generate a new ephemeris and solar system model, I find that: a) the classical optical data SNE and observation library (obslib) are referred to PEP Ephemeris 311; b) the planetary radar data SNE and

obslib are referred to PEP Ephemerides 452, 311, etc.; c) there is evidence for nonlinearity in the ephemeris adjustment from these ephemerides to an improved ephemeris at the precision of the available radar data; and d) it is prohibitively expensive to reform the optical data obslib and SNE using the PEP Ephemeris 452. The question of non-linearity at the precision of the optical data remains open. However, this is of secondary importance since the optical data are swamped by the radar data for the inner planet ephemerides. (These statements are deliberate over-simplifications.)

Normal Equation Manipulations

The linearized least squares estimator operates according to

$$z = H(x,t) + \nu \qquad 1$$

$$r = z - H(\tilde{x},t) \qquad 2$$

$$R = \langle \nu\nu^{\dagger} \rangle \qquad 3$$

$$A = \partial H/\partial x \qquad 4$$

$$B = A^{\dagger}R^{-1}A \qquad 5$$

$$U = A^{\dagger}R^{-1}r \qquad 6$$

$$X = B^{-1}U \qquad 7$$

$$\hat{x} = \tilde{x} + X \qquad 8$$

where z is the observation vector which is a function (H) of a parameter vector, x, and time, t, and is corrupted by a noise, ν. In the above $\tilde{x}$ and $\hat{x}$ are the pre- and post-fit estimates of x; X is the adjustment vector; r is the pre-fit residual; R is the noise covariance; and B^{-1} is the parameter-estimate covariance.

If the pre-fit estimate had been x', then the equations would have taken the form

$$r' = z - H(x',t) \approx z - [H(\tilde{x},t) + A(x'-\tilde{x})] \qquad 9$$

$$U' = A^{\dagger}R^{-1}r' \approx U - B(x'-\tilde{x}) \qquad 10$$

$$X' = B^{-1}U' \approx B^{-1}U - (x'-\tilde{x}) \qquad 11$$

$$\hat{x}' = x' + X' \approx \tilde{x} + X = \hat{x} \qquad 12$$

Here H is taken to be approximately linear in x and A is assumed to be correct, independent of the nominal x. It is not surprising that $\hat{x}' = \hat{x}$ with the linear approximation.