Center for Astrophysics

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MEMORANDUM

To: Distribution

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Subject: Deflection of the Beam Emerging from an Imperfect Corner Cube

I present a calculation of the deflection of the laser beam emerging from a corner cube whose faces are not exactly orthogonal.

A beam whose propagation direction is $\hat{k}$ reflecting from a mirror whose normal vector is $\hat{n}$ undergoes the operation

$$\hat{k} \to \hat{k} - 2\hat{n}\,(\hat{n}\cdot\hat{k}) \tag{1}$$

A corner cube reflector can be described as three such reflections from surfaces whose normal vectors are $\hat{x}$, $\hat{y}$, and $\hat{z}$. A perfect corner cube thus performs the operation

$$\begin{aligned}\hat{k} \to \hat{k} &- 2\hat{x}\,(\hat{k}\cdot\hat{x}) - 2\hat{y}\,(\hat{k}\cdot\hat{y}) - 2\hat{z}\,(\hat{k}\cdot\hat{z}) \\ &= -\hat{k}\end{aligned} \tag{2}$$

Now suppose the corner cube's faces are not orthogonal. Let us use

$$\begin{aligned}\hat{n}_1 &\equiv \hat{x} \\ \hat{n}_2 &\equiv \hat{y} + \varepsilon_3\hat{x} \\ \hat{n}_3 &\equiv \hat{z} - \varepsilon_2\hat{x} + \varepsilon_1\hat{y}\end{aligned} \tag{3}$$

for the three normal vectors: this allows the second face to have some angle with the first, and the third to have some angles with each of the first two. Then the $\hat{k}_n$ vectors of the beam after reflecting from n faces are given by

$$
\begin{aligned}
\hat{k}_1 &= \hat{k}_0 - 2\hat{n}_1\,(\hat{k}_0\cdot\hat{n}_1)\\
\hat{k}_2 &= \hat{k}_1 - 2\hat{n}_2\,(\hat{k}_1\cdot\hat{n}_2)\\
\hat{k}_3 &= \hat{k}_2 - 2\hat{n}_3\,(\hat{k}_2\cdot\hat{n}_3)
\end{aligned}
\tag{4}
$$

Turning the crank, we get

$$
\begin{aligned}
\hat{k}_2 = \hat{k}_0 &- 2\hat{x}\,(\hat{k}_0\cdot\hat{x}) - 2\hat{y}\,(\hat{k}_0\cdot\hat{y})\\
&+ 2\varepsilon_3\hat{y}\,(\hat{k}_0\cdot\hat{x}) - 2\varepsilon_3\hat{x}\,(\hat{k}_0\cdot\hat{y})\\
\hat{k}_3 = \hat{k}_0 &- 2\hat{x}\,(\hat{k}_0\cdot\hat{x}) - 2\hat{y}\,(\hat{k}_0\cdot\hat{y}) - 2\hat{z}\,(\hat{k}_0\cdot\hat{z})\\
&+ 2\hat{x}\left[\varepsilon_2\,(\hat{k}_0\cdot\hat{z}) - \varepsilon_3\,(\hat{k}_0\cdot\hat{y})\right] + 2\hat{y}\left[\varepsilon_3\,(\hat{k}_0\cdot\hat{x}) - \varepsilon_1\,(\hat{k}_0\cdot\hat{z})\right]\\
&+ 2\hat{z}\left[\varepsilon_1\,(\hat{k}_0\cdot\hat{y}) - \varepsilon_2\,(\hat{k}_0\cdot\hat{x})\right]
\end{aligned}
\tag{5}
$$

We can then write the emerging beam propagation direction as

$$
\hat{k}_3 = -\hat{k}_0 + 2\,\vec{\varepsilon}\times\hat{k}_0 \tag{6}
$$

where $\vec{\varepsilon} \equiv \varepsilon_1\hat{x} + \varepsilon_2\hat{y} + \varepsilon_3\hat{z}$. When the beam strikes the three mirrors in a different order, however, the vector $\vec{\varepsilon}$ is different — by a sign reversal of one or more of its components along $\hat{x}$, $\hat{y}$, and $\hat{z}$. The figure shows the correct vector $\vec{\varepsilon}$ to use for an incident laser beam in each of the six open sectors of the corner cube.

We can make the following observations:

(1) In general, each time the incident beam crosses a glue line (an $\hat{x}$, $\hat{y}$, or $\hat{z}$ axis) or its image, the corresponding element of $\vec{\varepsilon}$ changes sign.

(2) Under rotations of the corner cube around the laser beam (pivoting around the beam rather than the apex), the $2\,\vec{\varepsilon}\times\hat{k}_0$ deflection rotates with the corner cube.

(3) Mathematically there exists a laser input direction $\hat{k}_0$ for which the beam emerges exactly parallel to the input ($\hat{k}_3 = -\hat{k}_0$). However, only for $\vec{\varepsilon}$ within two (of eight) octants of the 4π solid angle does this $\hat{k}_o$ direction fall within the aperture of the corner cube. By moving

the laser to another of the six sectors we can get another of the three distinct $\vec{\varepsilon}$ directions; this allows alignment of $\hat{K}_0$ and $\hat{K}_3$ in another two octants. By moving to a third sector, we can get the third $\vec{\varepsilon}$ direction; this allows alignment of $\hat{K}_0$ and $\hat{K}_3$ in another two octants. But there is 25% of the 4π solid angle for $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ for which there is no solution. Stated another way, 25% of all corner cubes produced are expected to have no such solution.

(4) For the all-flat corner-cube interferometer endpoints with integral Fabry-Perot mirrors, requiring alignment of the Fabry-Perot mirrors eliminates all possibility of tilting the corner cubes to adjust this deflection to zero.

In addition to the all-flat corner cube interferometer (a marginally unstable interferometer design), we have considered using a curved mirror in one of the corner cubes (a stable interferometer design). One great advantage of the stable resonator can be demonstrated from the above discussion. If the incident $\hat{K}_0$ direction is fixed, then there are only two remaining degrees of freedom in the deflection, and hence in the vector $\vec{\varepsilon}$ (specifically, the components of $\vec{\varepsilon}$ perpendicular to $\hat{K}_0$). Thus a tilt of a single mirror of the corner cube is sufficient to repair any direction error. This means that the use of a spherical mirror allows one to find a spot on the corner cube for which $\hat{K}_3$ is exactly parallel to $\hat{K}_0$ (a necessary condition for building a resonator).

This then requires that the laser strike the curved face at a specific place in order that the emerging beam is parallel to the incident beam; thus we have a strict requirement on the "impact parameter" of the beams from the apex of the corner cube. If we were to place a curved mirror in the other corner cube of our interferometer, we would have an equally rigid constraint on that impact parameter. Since the lateral separation of the incident and emerging beams at each corner cube is just twice this impact parameter, we would have to compensate for the mismatch of impact parameter by forcing the incident and emerging beams to be far from parallel. This in turn produces a large cosine error in our distance determination. Our measurements will hopefully determine the absolute optical path length, but this should be measured parallel to the line between apices.

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