Data Structures and Algorithms : Maximum Satisfaction Homework 2

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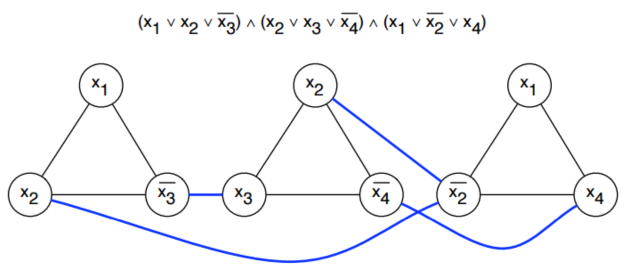
Introduction

The goal of this project is to develop a Las Vegas type algorithm that maximizes the number of satisfied clauses in a Boolean formula of conjunctive nominal form (CNF).



Requierements

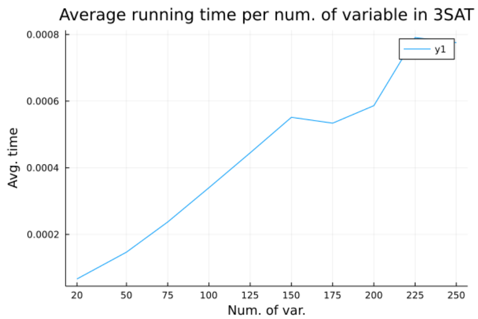
This project was written in **Julia 1.7.0**, it uses the standard packages as well as the ***Plots*** package which you will need to install as follows :

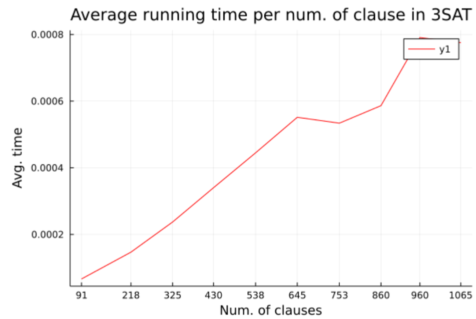
```
julia
using Pkg
Pkg.add("Plots")
```

The entire source code of this project is available via this GitHub repository. This algorithm has been tested on the instances of the 3SAT problem available via this link.

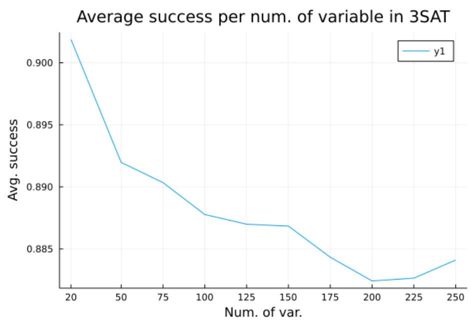
Experimental results

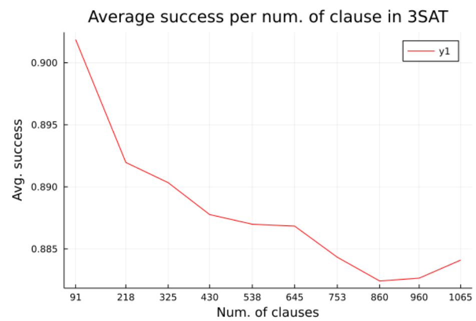
At each instance the number of variables and clauses varies as well as the number of files in each instance. The graphs below show for each number of variables and each number of clauses the averages obtained by the algorithm on all the files of this instance.



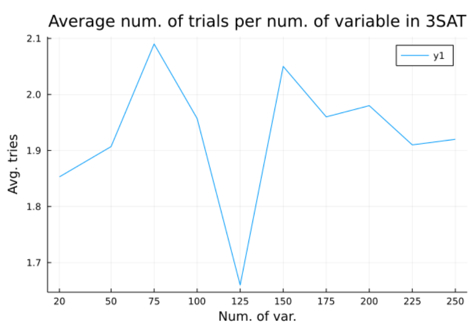


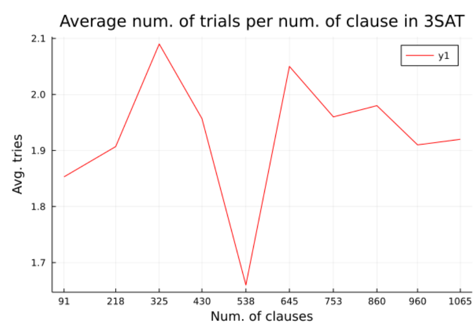
We can easily notice that the more the number of variables (and thus of clauses) increases in the formula, the longer the time taken by the algorithm to find a solution that satisfies 7/8 of the clauses.





In the same way, the more the number of variables (and clauses) increases, the more the success will decrease. By success, we mean the ratio between the number of satisfied clauses and the total number of clauses.





Finally, by number of trials we mean the number of times the algorithm will have to reassign random truth values to obtain a solution where 7/8 clauses are satisfied. We notice here that this number varies on average between 1.66 and 2.1.

Moreover we notice that all the instances of the 3-SAT problem have obtained a solution which satisfies the lemma that says that there exists an assignment of truth which satisfies at least $\frac{7k}{8}$ of the clauses.

Analysis

Given a 3-SAT formula with k clauses, the expected number of clauses satisfied by a random assignement of the truth values to their variables is $\frac{7k}{8}$. Indeed, if we put X_j the random variable equal to 1 if the clause C_j is satisfied, 0 otherwise. Then we can put $Pr[X_j = 1] = 1 - (\frac{1}{2})^3$, the probability that the clause C_j is satisfied.