Optimization problems in graphs with locational uncertainty

Michaël Poss

Topic : Optimization

Julien Baudru - 000460130

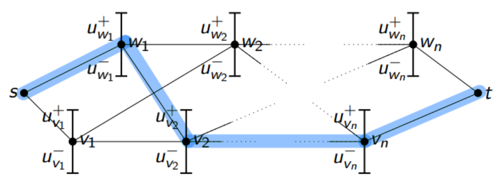
November 5, 2021

Contents

1 Introduction

This document the synthesis of the seminar given by Michael Poss during which he presented the research done with his team in their paper : **Optimization problems in graphs with locational uncertainty** [8].

Among the many topics often treated in the field of optimization, one of the best known is the one consisting in the selection of a path in a subgraph in order to minimize the cost of this path. These pages will also focus on this subject but in the particular context where the positions of the vertices are uncertain. In other words, we are in a context where each node of the graph is a set of vertices for which the positions and distances from each other are unknown. The objective of the authors of the paper treated in this seminar is to minimize the sum of the distances in the chosen subgraph for the worst positions of the vertices in their uncertainty sets.



During this talk, the presenter began by presenting a formal definition of the stated problem as well as explaining the motivation for their research. Then, the contributions brought by the three authors have been presented. These breakthroughs are the following : They proved that the problem defined here belongs to the class of NP-hard problems, they provided an algorithm to obtain an exact solution, they experimented around the guarantee approximation of the maximum distance in the context of more general graphs and distances and finally they provide an algorithm which gives an optimal solution in polynomial time. Each of these four contributions was then presented in detail during the rest of the seminar.

(1574 characters)

2 Biography

Michael Poss, the presenter of the seminar, started his academic career at the Université Libre de Bruxelles with a master in mathematics. He then obtained a degree in operational research at the University of Edinburgh. In February 2011, under the supervision of Bernard Fortz, Martine Labbé, and François Louveaux, he defended his PhD thesis entitled : **Models and Algorithms for Network Design Problems** [9]. During these years spent in Brussels, he was a member of the GOM (Graphs and Mathematical Optimisation) research group, which covers many aspects of mathematical programming with a particular focus on combinatorial and network optimisation. In 2012, after a couple of months at the University of Aveiro, he became a research assistant in the Mathematics Center of the University of Coimbra in Portugal. At the same time, he joined the Heudiasyc unit of the CNRS in France, this unit operates in the field of information and digital sciences, including computer science, automation, robotics and artificial intelligence. In 2015, he left the CNRS and became director of research at the LIRMM (Laboratory of Computer Science, Robotics and Microelectronics of Montpellier), he still holds this function at the time of the production of this document. In 2018, Michael Poss received the first **Robert Faure Prize*** ex aequo with Thibaut Vidal, this award is named in honor of Robert Faure, a pioneer of operations research in France, this distinction is conferred by ROADEF (French Operations Research Decision Support Society). In 2019, he won the award of the **Best paper of INFORMS telecommunications section***, this distinction rewards outstanding paper using operational research techniques in the context of telecommunications. During his career, he has written and co-authored over fifty journal papers, including several publications in the best operational research journals such as **Operations Research**, **Computational Management Science** or **Networks**.

3 The major points discussed

The main points discussed during this seminar are listed and detailed below. They mainly concern the breakthroughs made by the speaker and his team.

- NP**-hardness :** As a reminder, NP-problems are a set of problems whose solutions are hard to find but easy to verify and which are solved by a non-deterministic machine in polynomial time, NP-hard problems are at least as hard as the hardest NP problems. Formally, we say that a problem H is NP-hard when every problem L in NP can be reduced in polynomial time to H.

  In their paper, the authors have proved that the problem treated here is NP-hard. To achieve this, the authors use a more generic definition of the initial problem in which each $\Pi \in S$ represents an instance of a problem. This instance is defined by its input $I = (G, \alpha, u, D)$ where G is the graph, a contains the data specific to the problem, u the position of the nodes and D the distance matrix in each pair (u, u').

  Next, the authors showed that given F a set of vertices, M the set of positions and F the family of feasible sets, G a graph, s a source vertex and t a traget vertex, this problem remains NP-hard even when F consists of all $s - t$ paths and (M, d) is the one-dimensional Euclidean metric space or when F consists of all spanning trees of G.

- **Experimentation on maximum distance :** The three authors experimented on the bounds of the value d^{max}, to do so they set the value c^{max} such that $c^{max}(F) < \rho 2c(F)$ with F being the maximum pairwise distance and $\rho 2c(F)$ being a polynomial approximation for Robust-Π. In a theorem, they will show that the value of $\rho 2$ is 4 in the general case, 2 when F is a $CLIQUE$ and 3 when F is a $STAR$ graph. This work of approximation of the value of d^{max} serves as a basis for the algorithm treated in the next point.

- **Exact solution algorithm :** The presenter and his team designed an algorithm to obtain an exact solution to the stated problem. This algorithm is based on the principle of cutting plane, which consists in adding constraints to the problem to refine it, and bring it closer to integer solutions. This technique was introduced by Gomory and Johnson [4]. The main idea behind this algorithm is to replace large uncertainty sets by an approximation of small cardinality, allowing to simplify, to relax, the initial problem.

  Then, this algorithm, called ***exact***, was compared with other known algorithms by varying different parameters of the graph used as input. These algorithms are : ***center*** a heuristic algorithm based on barycenters [5], ***dmax*** an algorithm based on maximum pairwise distances and ***adr*** based on affine decision rules [1]. The authors concluded from the comparison results that the **exact** algorithm was able to solve the same problems as the other algorithms for the same time limit.

- **Optimal solution algorithm :** The second algorithm they have developed allows to approximate the exact solution of the problem. This one is based on the notion of d^{max}. E being the set of edges, u_i the position of vertex i, for each $\{i, j\} \in E$, d^{max} is given by : $max_{u_i \in U_i, u_j \in U_j} d(u_i, u_j)$. This algorithm also uses the value c^{max} which is the sum of all d^{max}. Note that the value d^{max} represents the worst-case distance.

  Then, given the comparisons detailed in the previous point, the authors concluded that ***dmax*** returns almost always slightly better quality solutions than ***center***. Moreover, the latter two algorithms were largely preferable to ***exact*** for larger instances.

4 Other papers

4.1 Related papers

Among the various papers related to the subject treated during the seminars we can retain the paper by Correia, da Gama entitled **Facility location under uncertainty** [6] and the paper by Citovsky, Mayer and Mitchell entitled **TSP With Locational Uncertainty : The Adversarial Model** [3]. The first article presents a summary of the knowledge related to optimization problems with positional uncertainties. It presents and differentiates problems in the areas of robust optimization, stochastic programming and chance-constrained programming. The second paper focuses on the special case of the Traveling Salesman Problem (TSP) and more particularly on a model where an adversary can choose a point in each of the regions (set of uncertain positions) in order to maximize the distance to travel for the saleman.

These two papers are directly related to the presentation because they both deal with cases where linear programs have uncertainties, such as the positions of vertices in the case of the problem presented in these pages.

4.2 Closely related papers

Among the many articles closely related to the subject treated here, we can mention the one by Bezanson, Karpinski, Shah and Edelman entitled **Julia: A fast dynamic language for technical computing** [7]. This article is an overview of the innovations brought by the Julia language, it explains how this language has been designed and proposes a comparison with other high level programming languages. This article is relevant in the sense that all the algorithms produced by the three authors were written in Julia. An other closely related paper is the one by Fischetti and Monaci entitled **Cutting plane versus compact formulations for uncertain (integer) linear programs** [2]. In this second paper, the authors compare the two options (cutting plane and compact formulation) on some problems with different characteristics. I think this paper is even more relevant because the algorithm created by the authors, allowing to obtain an exact solution, is based on the cutting-plane method.

5 Other groups

Among other research groups focusing on robust optimization with uncertainty, in France we find the LAAS (Laboratory for Analysis and Architecture of Systems - CNRS) and Belgium we find the GOM team from the ULB mentioned earlier. Abroad, we can mention the Technion Institute (Israel Institute of Technology) to which Aharon Ben-Tal belongs, an often referenced author in the field of robust optimization. Among the other influential authors in the field of optimization with uncertainty we can mention Dimitris Bertsimas and the many co-authors with which he collaborated to bring new advances in the field.

6 Open science

It seems that the author of the seminar supports open science. Indeed, the last slide of this presentation presents the Open Journal of Mathematical Optimization, a journal providing free and open access to scientific publications in the field of optimization. However, according to the journal's website, only one of the three authors of the paper presented at this conference, Jérémy Omer, seems to have submitted a paper to this journal.

In addition the presenter of this seminar has a series of articles available on HAL, a rather famous French and foreign open archive site. Furthermore, the paper on which this seminar is based is freely downloadable via this link. Finally, and this is rare enough to be noticed, all the code used during their research is available on GitHub.

7 Two scientific questions

During the seminar the presenter indicated that the different algorithms had been tested and compared on different instances of Steiner Tree, which are undirected graphs, hence the following question: **How do the exact algorithm and the algorithm for approximating the solution behave in directed graphs ?** Will the ***exact*** algorithm still be able to solve the same problems as the other algorithms for the same time limit ? How much should the algorithms be modified to be efficient in this case ? I find this question and the ones that follow particularly interesting because they were not treated during the seminar nor in the article on which it is based. Indeed, there could very well be cases of application where the graphs with uncertitude are of the directed type.

It was explained that the algorithm ***dmax*** sought to minimize the worst-case distance, hence the following question : **What would be the effect on the algorithm *dmax* if the metric used could take negative values ?** I find this question relevant because, although a distance cannot in theory be negative, there might be cases where it would be practical or even necessary. Also, there is a variety of definitions of the d^{max} value, so I wonder what the effect on the algorithm would be if we changed the way the distance is calculated.

8 Criticisms about the scientific

One of the only remarks I can make about the scientific content of this seminar is the lack of concrete examples, despite the fact that the authors detail three examples in their article. I believe that the connection to real life examples allows a better understanding of the importance of the discoveries made by the authors. To conclude, I found the material particularly dense given the time allotted to the presenter.

References

[1] E. Delage and D.A. Iancu. "Robust Multistage Decision Making". In: *Operations Research* (2015). URL: https://web.stanford.edu/~daniancu/Papers/Other/tutorial-multistage-RO.pdf.

[2] M. Fischetti and M. Monaci. "Cutting plane versus compact formulations for uncertain (integer) linear programs." In: *Mathematical Programming Computer* 4.3 (2012), pp. 239–273. URL: https://link.springer.com/article/10.1007/s12532-012-0039-y.

[3] T. Mayer G. Citovsky and J.S.B. Mitchell. "TSP With Locational Uncertainty: The Adversarial Model." In: *International Symposium on Computational Geometry* 77 (2017). URL: https://drops.dagstuhl.de/opus/volltexte/2017/7233/.

[4] R. Gomory and E.L. Johnson. "T-space and cutting planes". In: *Mathematical Programming* 96.2 (2003), pp. 341–375. URL: https://www.researchgate.net/publication/225155939_T-space_and_cutting_planes.

[5] M. Ferrer I. Bardaji and A. Sanfeliu. "Computing the Barycenter Graph by means of the Graph Edit Distance". In: *International Conference on Pattern Recognition* (2010). URL: http://www.iri.upc.edu/files/scidoc/1215-Computing-the-Barycenter-Graph-by-Means-of-the-Graph-Edit-Distance.pdf.

[6] F.S. da Gama I. Correia. "Facility location under uncertainty." In: *Location science* (2015), pp. 177–203. URL: https://link.springer.com/chapter/10.1007/978-3-030-32177-2_8.

[7] V.B. Shah J. Bezanson S. Karpinski and A. Edelman. *Julia: A fast dynamic language for technical computing.* 2012. URL: arXiv%20preprint%20arXiv:1209.5145.

[8] J. Omer M. Bougeret and M. Poss. *Optimization problems in graphs with locational uncertainty.* 2021. URL: https://arxiv.org/abs/2109.00389.

[9] M. Poss. "Models and algorithms for network design problems". PhD thesis. Université Libre de Bruxelles, 2011. URL: https://www.semanticscholar.org/paper/Models-and-algorithms-for-network-design-problems-Poss/67e0fef6f1731d5c7421697a3bc6b183576f8b8e.