Rectilinear convex hull of points in 3D

Carlos Seara

Topic : Algorithms

Julien Baudru - 000460130

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1 Introduction

This document the synthesis of the seminar given by Carlos Seara during which he presented the research done with his team in their paper : **Rectilinear convex hull of points in 3D** [7].

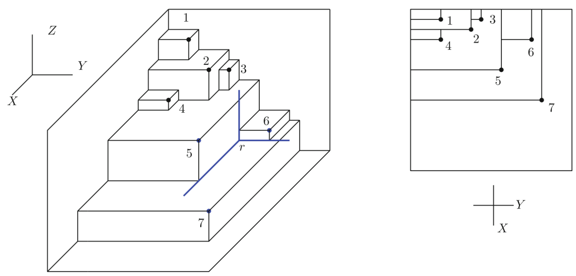

Figure 1: Illustration taken from the original paper

In computational geometry, one of the most common problems is to calculate the Convex Hull (CH) of a set of points (P). This concept can be defined simplistically by saying that the CH is the chord surrounding all points of the plane. In their paper, the authors have focused on several algorithms allowing to compute the CH but in the precise context of a set of points in space and not in 2D as it is classically the case. Moreover, here the authors have concentrated their research on calculating the Rectilinear Convex Hull (RCH), also called orthogonal convex hull, which is a variant of the traditional Convex Hull problem in which each line of the CH is perpendicular to each other in the classical case or intersects with an angle θ in the case of $RCH_\theta(P)$. Since it is a three dimensional space, it is no longer a question of calculating the CH of a polygon but of a three dimensional shape called an octant. In addition to this, the seminar presenter and his colleagues also produced an algorithm for calculating the Rectilinear Convex Hull when the octant is rotated on the z-axis. Moreover, in order to support his speech, the presenter put forward the different theorems that he and his team had elaborated. Finally, during the seminar several interactive demonstration widgets were shown to the audience, both of them were for the two dimensional version of the problem, the first for $RCH(P)$ and the second for $RCH_\theta(P)$.

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2 Biography

Carlos Seara Ojea, the presenter of the seminar, currently teaches in Barcelona in different faculties such as the one of Mathematics of the Politechnical University of Catalonia and the one of Computer Science of the same university. The speaker teaches students the following subjects: Mathematics, Algebra, Discrete Mathematics and Computational Geometry. In 2002 he published his doctoral thesis *On Geometric Separability* [8] supervised by Ferran Hurtado Diaz, although the publication of his first articles dates back to 1992. Throughout his career, he has published a total of almost 73 scientific papers in different well-known journals such as Computer Science, Computational Geometry or Discrete Mathematics. According to ResearchGate his articles have been cited more than 1460 times, and he has an h-index of 17 (a measure of productivity and impact of publications) and according to Google Scholaar his articles would have been cited around 2000 times and his h-index would be 21. Regarding the collaborations, it seems that the scientists with whom he has co-authored the most articles are Jorge Urrutia (≈ 20), Pablo Pérez-Lantero (≈ 18), Mercè Mora (≈ 16), David Orden (≈ 16) and Ferran Hurtado (≈ 15). In addition, he has participated as a presenter in almost 90 international conferences and about 30 Spanish conferences, among the most notable, we can mention the American Mathematical Society Meeting or the European Workshop on Computational Geometry. Carlos Seara has also published two books, both in Catalan, entitled Algebra 1 and Algebra 2, the latter two are used as reference books for his students. Finally, regarding his private life, it seems that the presenter is invested in music, in fact Carlos Seara practices the piano and has played in a concert for the British Combinatorial Conference in 2003, in addition he has a family of musicians who form a band.

3 The major points discussed

The main points discussed during this seminar are listed and detailed below. They mainly concern the breakthroughs made by the speaker and his team.

- **Computing $RCH(P)$:** In this section, the presenter explained how the RCH of a set of points P is calculated. The main idea of this method is to use a horizontal plane that we will name $\mathbb{Q}_c$ and to vary the position of this plane, here we vary the value c of the z-coordinate, $z = c$, of the plane, i.e. it will move progressively from top to bottom. Thus this plane will meet the various points of P, at each meeting, the $RCH(P)$ will have to be updated in the following way:

  $$RCH(P) = \bigcap_{i=1...8} \mathbb{P}_i$$

  where $\mathbb{P}_i$ is the set of faces and vertices of the orthogonal polyhedron, i.e., the faces meet at right angles and edges are parallel to the axes. (Remark : Here we take an example with 8 points as in the original paper.) Note that the elements composing $\mathbb{P}_i$ could be computed easily because these calculations are based on the well known *maxima problem* which consists in finding the maximal point in a set of points. It has been proved by the author in a paper entitled *Maximum Rectilinear Convex Subsets* [3] that an optimal solution to this problem can be found with time complexity in $O(n\ log\ n)$ and space complexity in $O(n)$. Finally, using the type of algorithm described above, called a sweep plane algorithm, the presenter and his colleagues were able to create an efficient algorithm with time complexity in $O(n\ log\ n)$ and space complexity in $O(n)$ for a set P with n points in space.

- **Maintaining $RCH_\theta(P)$:** In this section, based on the previous results, the presenter explained the solution they found to calculate the $RCH(P)$ in the particular case where the angle θ of the octants is varied, i.e. a rotation between 0 and 2π around the z axis is performed. Here we use the terms θ-octant and $RCH_\theta(P)$. Note that the resolution of this problem in the particular case of $\mathbb{R}^3$ builds on the results obtained for $\mathbb{R}^2$ by the author as well as his colleagues, Alegría-Galicia, Orden and Urrutia in 2019 in their paper *On the $\mathbb{O}\beta$-hull of a planar point set* [1]. To do so, the author has defined the following concepts. We separate the points $p \in P$ in eight classes according to their disposition in space, these will be called the $p^\theta - octants$ if they have p as their apex. We can then subdivide these $p^\theta - octants$ into two classes, the high and the low, according to their position with respect to the horizontal plane λ_p. We will say that an up $p^\theta - octant$ is $non - P - free$ if it contains elements of P above λ_p and vice versa for down $p^\theta - octant$. Moreover, we will say that a point $p \in P$ is a vertex of $RCH_\theta(P)$ if there is a $P - free$ octant, in this case we will say that p is a $\theta - active$ point, otherwise p is $\theta - inactive$. The author also showed that the angle intervals at which p_i is up-active is equivalent to finding whether there exist $P_i - free$ wedges in λ_{p_i} whose apex is p_i of angular size at least $\frac{\Pi}{2}$. Then, since the number of points grows as we go through the space from top to bottom, in order to keep the algorithm efficient, the active and non-active points are stored in a balanced binary tree. Finally the presenter showed the pseudo code of the algorithm based on the notions described above, which has a time complexity of $O(n\ log^2\ n)$ and a space complexity of $O(n\ log\ n)$.

- **The Combinatorics of Rectilinear Convex Hulls in $\mathbb{R}^3$:** In this section the presenter showed that there are examples of set of points P in $\mathbb{R}^3$ such that the rectilinear convex hull number can be at least $\Omega(n^2)$ while the vertice number of $RCH_\theta(P)$ remains constant. To be more accurate, the presenter and these colleagues deduced the following theorem from their results:

  Theorem 1. *There are configurations of points in $\mathbb{R}^3$ such that for an angular interval $[\alpha, \beta]$ while $\theta \in [\alpha, \beta]$, $RCH_\theta(P)$ will maintain the same vertices while it changes a quadratic number of times.*

  He also showed that $RCH_\theta(P)$ is not necessarily connected, that the rectilinear convex hull of a set of points that is not simply connected and that the problem of maintaining $RCH_\theta(P)$ does not follow directly from their previous results.

4 Other papers

4.1 Related papers

Among the different papers related to the subject treated during this seminar, the one that seems to be at the base of everything is the very well known and quoted paper published by Graham in 1972 entitled *An Efficient Algorithm for Determining the Convex Hull of a Finite Planar Set* [4]. In this groundbreaking paper, the author describes one of the first techniques, if not the first technique, for computing the convex hull of a 2-dimensional point set. This paper is the basis of many other works in the field of computational geometry, especially those dealing with CH in a 3-dimensional space.

Another paper related to the topic presented during this seminar is the one published in 1996 by Chan entitled *Optimal output-sensitive convex hull algorithms in two and three dimensions* [2]. In this paper the author presents an algorithm to compute the CH of a set of points in a 2 and 3 dimensional space. To do so, he bases himself partly on the algorithm introduced by Graham, the innovation of this algorithm at the time it was published lies in the fact that it is simpler than the analogous algorithms for the 3-dimensional cases and in complexity $O(n\, log\, h)$.

It does not seem necessary here to explain in detail the obvious link between these two articles and the subject presented at the seminar.

4.2 Closely related papers

Among the many articles closely related to the subject treated here, we can mention the one by Ben Kenwright entitled *Convex Hulls Surface Mapping onto a Sphere* [5]. In this paper, the author focuses on presenting a new iterative method for computing the CH of 2 and 3 dimensional objects. The particularity of this method is not that it is optimal but rather that it is simple and especially applicable in practice on complex examples and not only point clouds. This article is indeed related to the presenter's topic because it also concerns the creation of a new calculation method for CH in 2 and 3 dimensions.

Another paper related to the subject presented by Carlos Seara is the one written by Vladimir Kovalevsky and Henrik Schulz entitled *Convex Hulls in a 3-dimensional Space* [6]. In this paper, the authors present a different method to compute the CH of a set of points in three dimensions. In addition to the fact that the method is different from the two discussed earlier, the specificity of this method lies in the fact that the CH is represented in the form of a new data structure, namely a list of cells.

Note that, although these two papers also deal with CH, a difference persists in the fact that they do not focus on the rectilinear hull convex unlike the seminar presenter but on a more general shape.

5 Other groups

Among the numerous research groups concerned by the subject of Convex Hull and more generally of computational geometry, we can notably quote, among the private research groups, the following groups: IBM Research Laboratory or NASA Langley Research Center (GEOLAB). Concerning the university research groups, we can of course mention the Algorithms Research Group of the ULB or the UPC Research Group on Discrete, Combinatorial and Computational Geometry, DCCG of the Polytechnic University of Catalonia to which the presenter of this seminar contributes.

6 Open science

It seems that the author of the seminar supports open science. Indeed, all publications written or co-authored by the seminar presenter are freely downloadable via his personal website. Moreover, all his articles published on ResearchGate are either available for free download or can be requested. However, as a computer scientist and being interested in open source, I have concerns about the accessibility of the various source codes related to the articles written by the presenter. From my research I have not been able to find any of the source files, however some of his articles include pseudo code.

7 Two scientific questions

The first question that came to my mind during the presentation is the following: **Why did the authors confine themselves precisely to the rectilinear case of CH?** Indeed, this point has not been clarified during the seminar however it seems to be an essential point of the presentation, is it because this kind of research has never been done before or is it simply arbitrary? From this first question follows directly my second question: **Are there concrete application cases for the rectilinear CH as it is for example the case for the general version presented in the paper of Ben Kenwright [5]?** This question seems to me important because in computational geometry, the advances made are often useful for real cases, for example 3D rendering engines of video games, optimization of 3D objects, lattice structures, etc.

8 Criticisms about the scientific

The only criticism that could be addressed to this presentation is the fact that, although the presenter showed interactive widgets for the $RCH(P)$ and $RCH_\theta(P)$ cases in 2 dimensions, he did not show any for the 3-dimensional case, which I find questionable because it is after all the main topic of the presentation. Maybe it was due to lack of time but it raises the question about the application of the proposed algorithm for practical cases.

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