

Lab 4: Beginner Stats

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1 Load packages

```
library(tidyverse)
library(see)
library(car)
library(patchwork)
library(ggsci)
library(ggthemes)
library(performance)
library(Hmisc) #for correlation matrix
library(corrplot)#to visualize correlation matrices
library(car) #contains some statistical tests we need to assess assumptions
```

2 A note on statistics and experimental design

Statistics is a complex field with a long history. We could spend an entire course or even an entire career focusing on the intricate details of statistical decisions and ideas. We've already spent some time on this! I want you to have the statistical grounding necessary to plan your experiments and analyze your data. For biologists, statistics are a tool we can leverage to perform the best possible experiments and test our hypotheses. The T-test is the start of our stats journey. It's a simple test and one that you may not use often, but the theory behind it sets the stage for what is to come!

3 1.) Correlation between numerical variables

3.1 Correlation Coefficients

A **correlation coefficient** (r) tells us the relationship (strength and direction) between two variables. These coefficients can be positive or negative and will range from 0 to 1 (or negative 1). Values nearer to 1 (or negative 1) indicate stronger correlations and values closer to 0 indicate weaker correlations

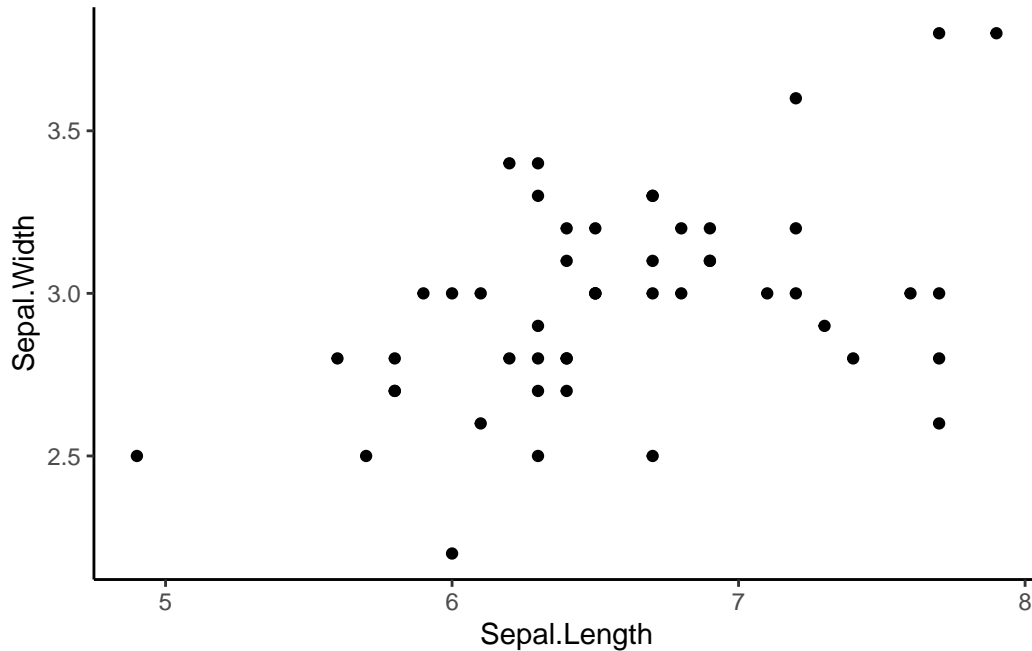
Let's try out some correlations using the iris data.

Is there a correlation between sepal length and sepal width? Let's test each species separately for now.

make a scatterplot to visualize

```
#filter down to a single species
virg<-iris %>%
  filter(Species=='virginica')

#make a plot
ggplot(virg, aes(x=Sepal.Length, y=Sepal.Width))+
  geom_point()+
  theme_classic()
```



3.2 Calculate a correlation coefficient (r)

```
cor(virg$Sepal.Length, virg$Sepal.Width)
```

```
[1] 0.4572278
```

This value ($r=0.45$) positive and middle of the road/strong. This tells us that some correlation likely exists.

3.3 Spearman's Test

A hypothesis test on the correlation H0: The correlation between these two variables is 0

Ha: The correlation \neq 0

```
cor.test(virg$Sepal.Length, virg$Sepal.Width, method="spearman")
```

```
Warning in cor.test.default(virg$Sepal.Length, virg$Sepal.Width, method =  
"spearman"): Cannot compute exact p-value with ties
```

Spearman's rank correlation rho

```
data: virg$Sepal.Length and virg$Sepal.Width  
S = 11943, p-value = 0.002011  
alternative hypothesis: true rho is not equal to 0  
sample estimates:  
rho  
0.4265165
```

The above output gives us the r value (cor=0.457) AND a p-value for a hypothesis test that the two correlations do not differ. If $p < 0.05$ we can reject our H0 and say that the correlation differs from 0. Here, $p = 0.0008$ so we can reject H0 and suggest that we have a significant positive correlation! Rho is similar to r and in this case our correlation coefficient (0.42). It is slightly lower than the r we calculated above.

3.4 Multiple Correlations

```
iris2<-iris[,c(1:4)] #filter iris so we only have the numerical columns!  
  
iris_cor<-cor(iris2, method="spearman")  
  
iris_cor
```

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
Sepal.Length	1.0000000	-0.1667777	0.8818981	0.8342888
Sepal.Width	-0.1667777	1.0000000	-0.3096351	-0.2890317
Petal.Length	0.8818981	-0.3096351	1.0000000	0.9376668
Petal.Width	0.8342888	-0.2890317	0.9376668	1.0000000

The above correlation matrix shows r (correlation coefficient) not p values!

Getting r and p values

```
mydata.rcorr = rcorr(as.matrix(iris2))  
mydata.rcorr #top matrix = r, bottom matrix = p
```

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
Sepal.Length	1.00	-0.12	0.87	0.82
Sepal.Width	-0.12	1.00	-0.43	-0.37
Petal.Length	0.87	-0.43	1.00	0.96
Petal.Width	0.82	-0.37	0.96	1.00

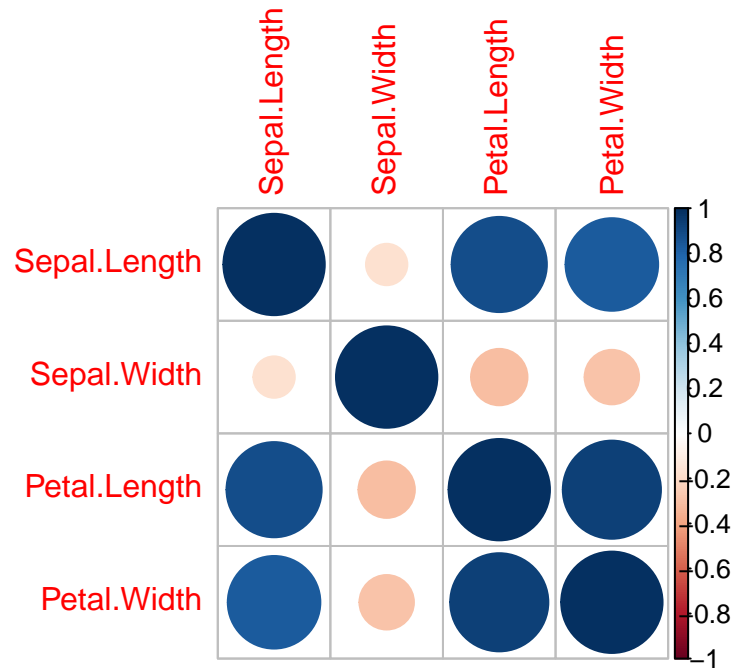
n= 150

P

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
Sepal.Length		0.1519	0.0000	0.0000
Sepal.Width	0.1519		0.0000	0.0000
Petal.Length	0.0000	0.0000		0.0000
Petal.Width	0.0000	0.0000	0.0000	

Plotting our correlations

```
corrplot(iris_cor)
```



3.5 Chi-Square

Chi-Square is for categorical correlations

A Chi-square test is a statistical test used to determine if two categorical variables have a significant correlation between them. These two variables should be selected from the same population. An example - Is the color of a thing red or green? Is the answer to a simple question yes or no?

Data format Technically, a chi-square test is done on data that are in a contingency table (contains columns (variables) in which numbers represent counts. For example, here is a contingency table of household chore data (exciting)

```
chore <- read.delim("http://www.sthda.com/sthda/RDoc/data/housetasks.txt", row.names=1)
chore
```

	Wife	Alternating	Husband	Jointly
Laundry	156	14	2	4
Main_meal	124	20	5	4
Dinner	77	11	7	13
Breakfast	82	36	15	7

Tidying	53	11	1	57
Dishes	32	24	4	53
Shopping	33	23	9	55
Official	12	46	23	15
Driving	10	51	75	3
Finances	13	13	21	66
Insurance	8	1	53	77
Repairs	0	3	160	2
Holidays	0	1	6	153

H₀ = The row and column data of the contingency table are independent (no relationship)

H_a= Row and column variables are dependent (there is a relationship between them)

The test

```
chorechi<-chisq.test(chore)
chorechi
```

Pearson's Chi-squared test

data: chore

X-squared = 1944.5, df = 36, p-value < 2.2e-16

This result demonstrates that there is a significant association between the columns and rows in the data (they are dependent).

A second example

Let's try to assess correlation between two categorical variables in a dataframe we know! We will use mtcars

```
head(mtcars)
```

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

```
#make a contingency table
cartab<-table(mtcars$carb, mtcars$cyl)

chisq.test(cartab)
```

Warning in chisq.test(cartab): Chi-squared approximation may be incorrect

Pearson's Chi-squared test

```
data:  cartab
X-squared = 24.389, df = 10, p-value = 0.006632
```

```
#note that we don't NEED to make the table. We can just do this
chisq.test(mtcars$carb, mtcars$cyl)
```

Warning in chisq.test(mtcars\$carb, mtcars\$cyl): Chi-squared approximation may be incorrect

Pearson's Chi-squared test

```
data:  mtcars$carb and mtcars$cyl
X-squared = 24.389, df = 10, p-value = 0.006632
```

Both tests above are the same (just two options for you). We see that $p < 0.05$, thus we have evidence to reject H_0 and suggest that carb and cyl are dependent / correlated.

4 2.) Simple Linear Regression

A linear regression essentially compares the correlation of one variable with another. The closer the relationship is to 1:1 (a diagonal line at 45 degrees from the x and y axis) the more correlated the two variables are. Does correlation imply causation? NO, it does not. But this type of analysis driven by hypotheses can help us seek causation/ mechanisms and statistically assess relationships.

Let's take a look at a simple linear regression. To do this, we will use the `lm()` function in R. The syntax should always be response variable ~ explanatory variable We will do this with the iris data.


```
lm1<-lm(Sepal.Length ~ Petal.Length, data=iris)
summary(lm1)
```

Call:

```
lm(formula = Sepal.Length ~ Petal.Length, data = iris)
```

Residuals:

Min	1Q	Median	3Q	Max
-1.24675	-0.29657	-0.01515	0.27676	1.00269

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.30660	0.07839	54.94	<2e-16 ***
Petal.Length	0.40892	0.01889	21.65	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4071 on 148 degrees of freedom

Multiple R-squared: 0.76, Adjusted R-squared: 0.7583

F-statistic: 468.6 on 1 and 148 DF, p-value: < 2.2e-16

The above table produces estimates for the slope and intercept of the line.

At the bottom we see R² values (multiple and adjusted. We usually use adjusted Rsquared). We also see an overall p-value for our linear regression model (H₀= slope of our regression line = 0).

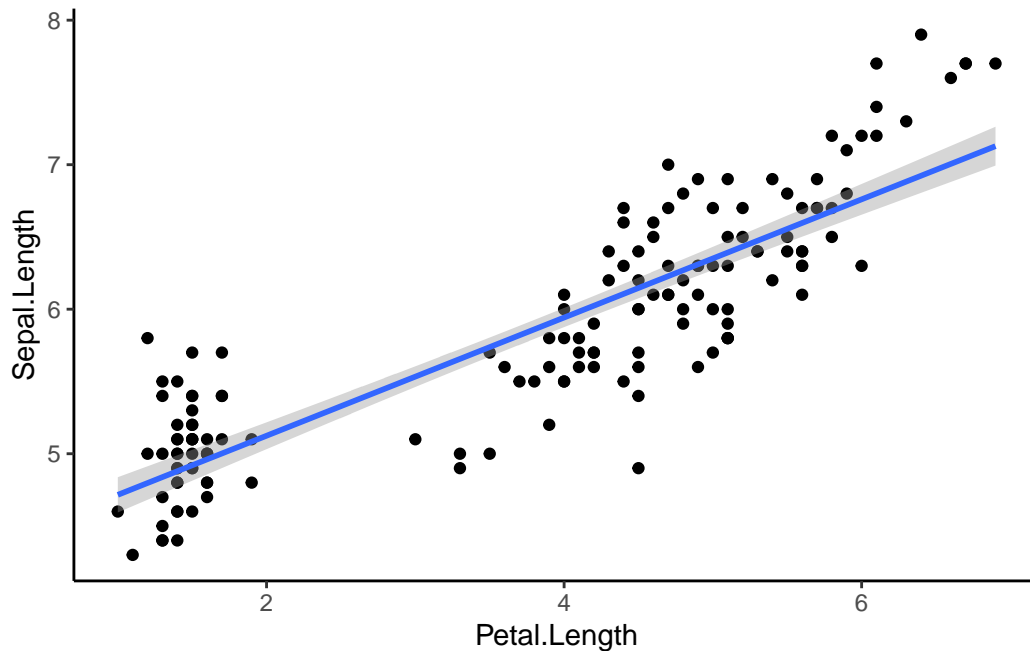
:::panel-tabset

4.1 plotting a regression line

It is very easy to make a regression line in ggplot. We can plot our scatterplot as we normally would and then we add the regression line using the `geom_smooth()` argument.

```
ggplot(iris, aes(x=Petal.Length, y=Sepal.Length))+
  geom_point()+
  geom_smooth(method='lm')+
  theme_classic()
```

`geom_smooth()` using formula = 'y ~ x'



The blue line represents our regression line ($y \sim x$). The gray around the line is the SE. We can add `SE=FALSE` to our `geom_smooth()` to turn that off:

```
geom_smooth(method='lm', SE=FALSE)
```

4.2 Assumptions

Linear regressions have 4 assumptions:

- 1.) Linearity of the data: We assume the relationship between predictor (x) and outcome/dependent variable (y) is approx. linear. At each value of X there is a population of possible Y-values whose mean lies on the regression line.
- 2.) Normality of residuals: The residual error are assumed to be normally distributed. In other words: at each value of X, the distribution of possible Y values is normal
- 3.) Homogeneity of residual variance (homoscedasticity): We assume residual variance is approx. constant. In other words: the variance of Y values is the same at all values of X
- 4.) Independence of residual error terms: At each value of X, the Y-measurements represent a random sample from the population of possible Y values.

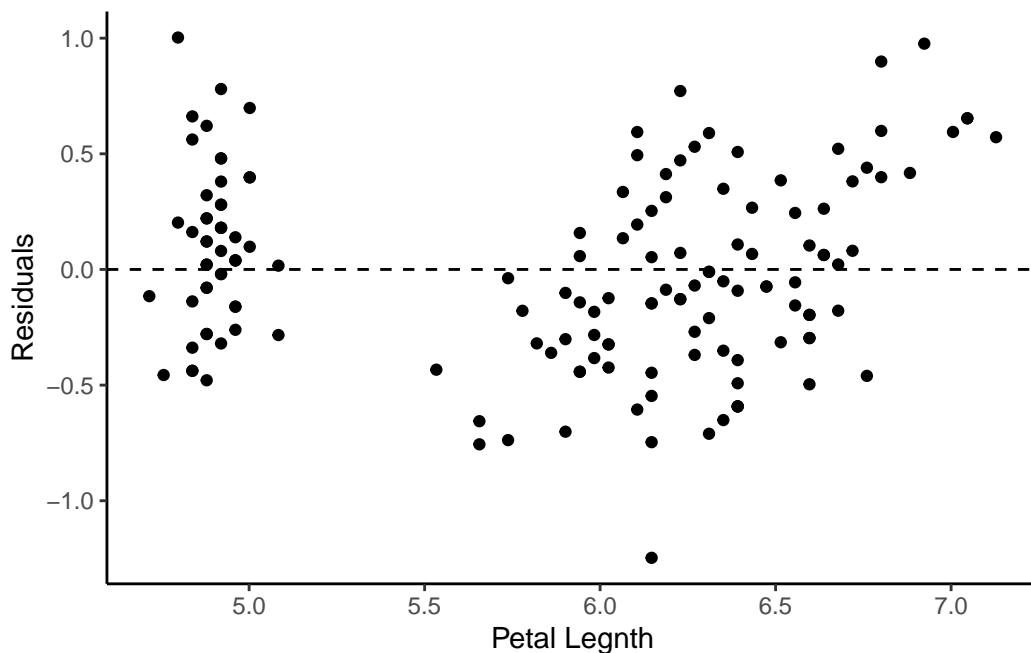
We can also make a residual plot to check some of our assumptions. **Residuals** measure the scatter of points above or below the least-squares regression line. When we calculate the residuals for a linear regression and plot them, $y=0$ is the least squares line. Residuals essentially represent the distance between each point and the linear regression line we see in our regression graph.

```
residuals(lm1)
```

1	2	3	4	5	6
0.22090540	0.02090540	-0.13820238	-0.31998683	0.12090540	0.39822871
7	8	9	10	11	12
-0.27909460	0.08001317	-0.47909460	-0.01998683	0.48001317	-0.16087906
13	14	15	16	17	18
-0.07909460	-0.45641792	1.00268985	0.78001317	0.56179762	0.22090540
19	20	21	22	23	24
0.69822871	0.18001317	0.39822871	0.18001317	-0.11552569	0.09822871
25	26	27	28	29	30
-0.28355574	0.03912094	0.03912094	0.28001317	0.32090540	-0.26087906
31	32	33	34	35	36
-0.16087906	0.48001317	0.28001317	0.62090540	-0.01998683	0.20268985
37	38	39	40	41	42
0.66179762	0.02090540	-0.43820238	0.18001317	0.16179762	-0.33820238
43	44	45	46	47	48
-0.43820238	0.03912094	0.01644426	-0.07909460	0.13912094	-0.27909460
49	50	51	52	53	54
0.38001317	0.12090540	0.77146188	0.25324634	0.58967743	-0.44229252
55	56	57	58	59	60
0.31235411	-0.44675366	0.07146188	-0.75604693	0.41235411	-0.70140030
61	62	63	64	65	66
-0.73783139	-0.12407698	0.05770748	-0.12853812	-0.17872361	0.59413856
67	68	69	70	71	72
-0.54675366	-0.18318475	0.05324634	-0.30140030	-0.36943035	0.15770748
73	74	75	76	77	78
-0.01032257	-0.12853812	0.33503079	0.49413856	0.53056965	0.34878520
79	80	81	82	83	84
-0.14675366	-0.03783139	-0.36050807	-0.31961584	-0.10140030	-0.39210703
85	86	87	88	89	90
-0.74675366	-0.14675366	0.47146188	0.19413856	-0.38318475	-0.44229252
91	92	93	94	95	96
-0.60586144	-0.08764589	-0.14229252	-0.65604693	-0.42407698	-0.32407698
97	98	99	100	101	102
-0.32407698	0.13503079	-0.43337025	-0.28318475	-0.46013708	-0.59210703

103	104	105	106	107	108
0.38075515	-0.29656817	-0.17835262	0.59450955	-1.24675366	0.41718624
109	110	111	112	113	114
0.02164738	0.39897069	0.10789297	-0.07389149	0.24432406	-0.65121480
115	116	117	118	119	120
-0.59210703	-0.07389149	-0.05567594	0.65361733	0.57183287	-0.35121480
121	122	123	124	125	126
0.26253960	-0.71032257	0.65361733	-0.01032257	0.06253960	0.43986292
127	128	129	130	131	132
-0.06943035	-0.21032257	-0.19656817	0.52164738	0.59897069	0.97629401
133	134	135	136	137	138
-0.19656817	-0.09210703	-0.49656817	0.89897069	-0.29656817	-0.15567594
139	140	141	142	143	144
-0.26943035	0.38521629	0.10343183	0.50789297	-0.59210703	0.08075515
145	146	147	148	149	150
0.06253960	0.26700074	-0.05121480	0.06700074	-0.31478371	-0.49210703

```
ggplot(lm1, aes(x=.fitted, y=.resid))+
  geom_point()+
  geom_hline(yintercept=0, linetype='dashed')+
  labs(x='Petal Legnth', y='Residuals')+
  theme_classic()
```



If assumptions of normality and equal variance are met, a residual plot should have: - A roughly symmetric cloud of points above and below the horizontal line at 0 with a higher density of points close to the line

- Little noticeable curvature as we move from left to right
- Approx. equal variance of points above and below the line at all values of X

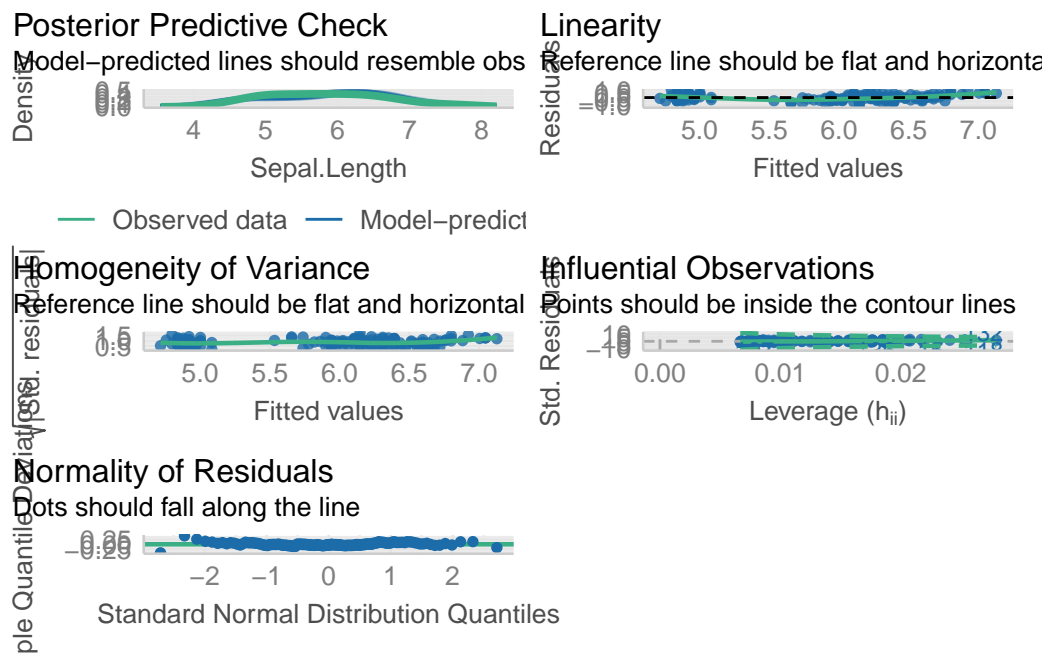
The residual plot above shows meets all assumptions, though this analysis is somewhat subjective.

4.3 An alternative assumption check

I think it is easier to do a more comprehensive visual check with the performance package in R. We can easily visually check the first 3 assumptions using `check_model()`. Assumption 4 requires us to think about experimental design.

```
lm1<-lm(Sepal.Length ~ Petal.Length, data=iris)

check_model(lm1)
```



Using the plots above, we can check 3 / 4 of our assumptions and look for influential observations/outliers. The plots even tell us what to look for on them! This is a bit simpler than trying to analyze the residual plot.

As with the residual plot, this analysis of assumptions is somewhat subjective. That is ok.

4.4 when data are not linear

Sometimes the relationship between two variables is not linear! There are many types of common relationships including logarithmic and exponential. We can often visualize these relationships and **Transform** our data to make them linear with some simple math.

Let's look at an example:

```
head(Loblolly)
```

Grouped Data: height ~ age | Seed

	height	age	Seed
1	4.51	3	301
15	10.89	5	301
29	28.72	10	301
43	41.74	15	301
57	52.70	20	301
71	60.92	25	301

```
p1<-ggplot(Loblolly, aes(x=age, y=height))+  
  geom_point()+  
  geom_smooth()+  
  geom_smooth(method='lm', linetype='dashed', color='firebrick')+  
  theme_classic()+  
  labs(title='original')  
#this is roughly logarithmic in shape
```

```
lob<-Loblolly  
lob$age2<-log(lob$age)
```

```
p2<-ggplot(lob, aes(x=age2, y=height))+  
  geom_point()+  
  geom_smooth()+  
  geom_smooth(method='lm', linetype='dashed', color='firebrick')+  
  theme_classic()+
```

```

labs(title='log transformed')

lob$age3=(lob$age2)^2
p3<-ggplot(lob, aes(x=age3, y=height))+
  geom_point()+
  geom_smooth()+
  geom_smooth(method='lm', linetype='dashed', color='firebrick')+
  theme_classic()+
  labs(title='squared')

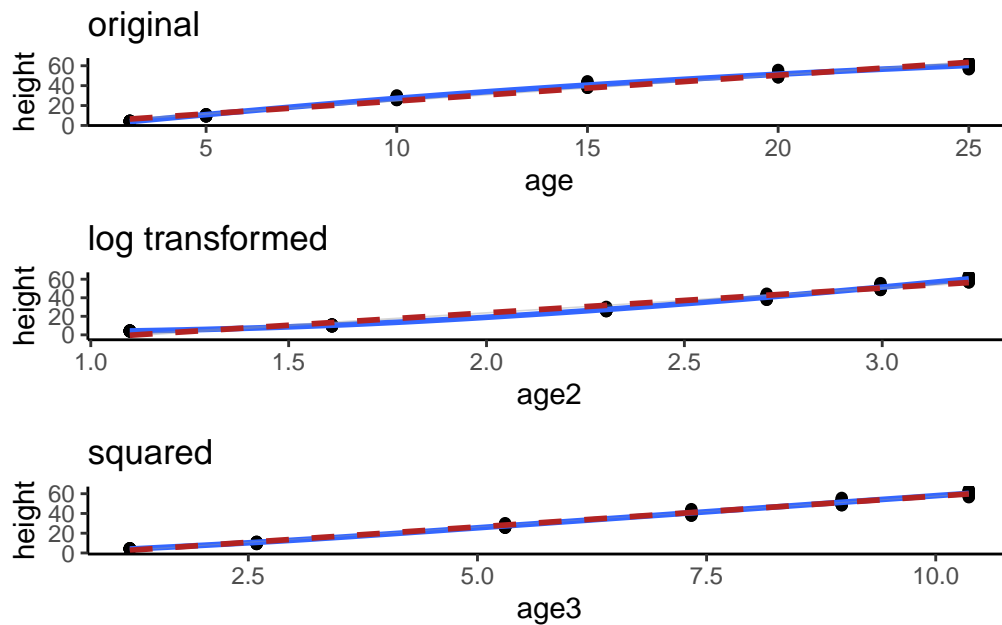
p1/p2/p3

```

```

`geom_smooth()` using method = 'loess' and formula = 'y ~ x'
`geom_smooth()` using formula = 'y ~ x'
`geom_smooth()` using method = 'loess' and formula = 'y ~ x'
`geom_smooth()` using formula = 'y ~ x'
`geom_smooth()` using method = 'loess' and formula = 'y ~ x'
`geom_smooth()` using formula = 'y ~ x'

```



Here we can see that the transformation was fairly trivial (the data were close enough to a straight line already). BUT, technically, the first plot shows a logarithmic trend. We can

transform one of the variables to generate a more linear trend. We can guess a transformation and check it with graphs or we can use our knowledge of mathematical relationships to understand how we might make our relationship more linear.

4.5 Linear Regression with categorical variables

We can look at mtcars this time...

```
head(mtcars)
```

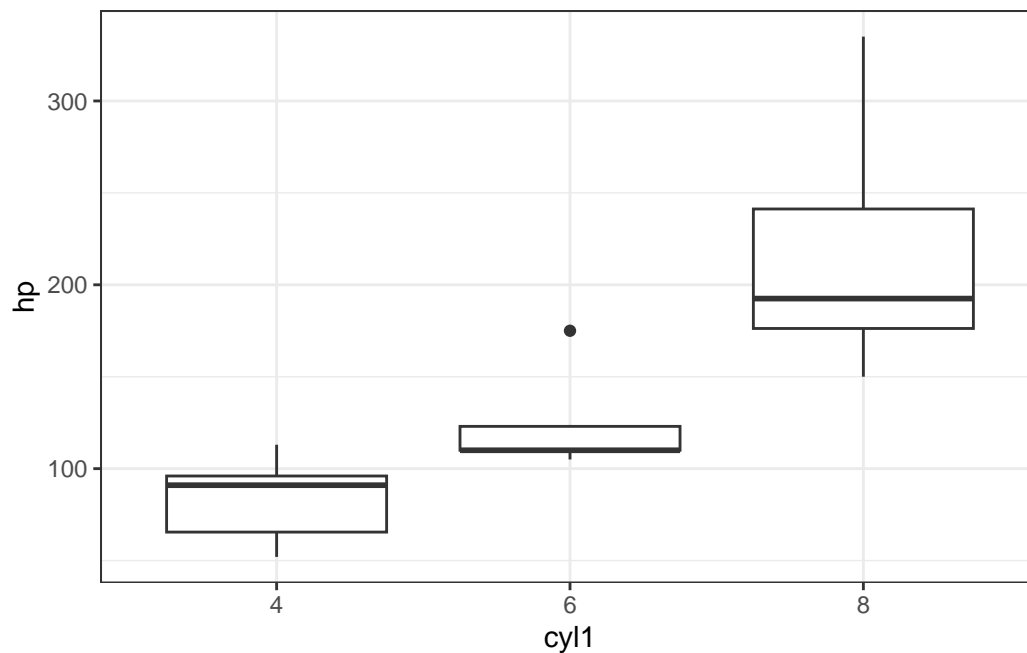
	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

Now, I want to hypothesize that there will be no effect of cylinder on horsepower (this is called a “null hypothesis”). We’ve seen similar hypothesis before in our ANOVA.

First, let’s make cylinder a factor and plot a boxplot so we can see whether there may be a trend here...

```
mtcars$cyl1=as.factor(mtcars$cyl)

ggplot(mtcars, aes(x=cyl1, y=hp))+
  geom_boxplot()+
  theme_bw()
```

I think it is safe to say we see what we might suspect to be a linear(ish) relationship between cyl and hp, where hp increases as cyl increases. What do you think?

Now, let's do some stats on this.

Run the lm

```
lmhp<-lm(hp~cyl1, data = mtcars)
summary(lmhp)
```

Call:

```
lm(formula = hp ~ cyl1, data = mtcars)
```

Residuals:

Min	1Q	Median	3Q	Max
-59.21	-22.78	-8.25	15.97	125.79

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	82.64	11.43	7.228	5.86e-08	***
cyl16	39.65	18.33	2.163	0.0389	*
cyl18	126.58	15.28	8.285	3.92e-09	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.92 on 29 degrees of freedom

Multiple R-squared: 0.7139, Adjusted R-squared: 0.6941

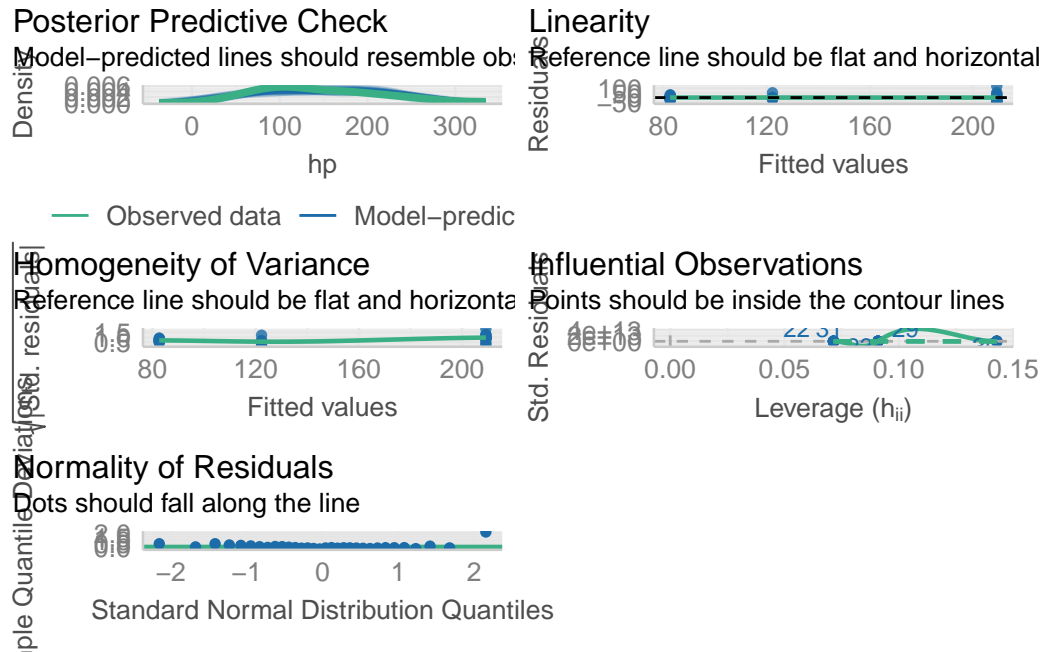
F-statistic: 36.18 on 2 and 29 DF, p-value: 1.319e-08

This time we used a categorical x variable, which makes things a little more interesting. In the coefficients table this time we see `cyl = 6` and `cyl = 8` represented as well as “intercept.” R takes the categorical variables and places them in alpha numeric order in these tables. So “intercept” is actually `cyl=4`. The “estimate” tells us the effect size of each category relative to “intercept.” SO, the mean of `cyl=4` should be 82.64 (check the boxplot above to confirm). The mean of `cyl=6` is not 39.65, but is actually 39.65 higher than mean of `cyl=4` ($82.64 + 39.65 = 132.29$, which checks out). The p-values associated with each of the coefficients test the null hypothesis that each coefficient has no effect. A $p < 0.05$ indicates that the coefficient is likely to be meaningful in the model (changes in the predictor’s value are related to changes in the response value).

Further down, we see an R-squared of nearly 0.70, which is very good evidence of a linear relationship (70% of the variance in y can be explained by x!). The p-value is very nearly 0.00, which indicates a significant linear correlation.

Check assumptions!

```
check_model(lmhp)
```



Here we see some concern about Homoscedasticity and homogeneity of variance. We can probably still assume our model is reliable, but we may want to be careful. We learned ways to numerically assess this last week, but again, with high enough sample size, this won't be an issue. Here, I would suggest that n is too small, so if this were a real statistical test we would have limitations to discuss.

Remember our hypothesis (null) was: "There will be no effect of cylinder on horsepower." We are able to reject this null hypothesis and suggest that indeed horsepower increases as cylinder increases. We might also add caveats that homoscedasticity was not confirmed due to low sample size, but the result seems clear enough that this likely doesn't matter.

3.) T-test

The t-test (or students' t-test) is a basic statistical test used to assess whether or not the means of two groups are different from one another. In this test, the null hypothesis is that the two means are equal (or that there is no difference between the two means).

A t-test should only be used if the following assumptions are met:

- 1.) the two distributions whose means we are comparing must be **normally distributed**
- 2.) The variances of the two groups must be **equal**

Generate example data

```
iris2<-iris %>%
  filter(Species != 'setosa') %>%
  droplevels() #removes the empty levels so when we check levels below we only get the one

#check levels to make sure we only have 2 species!
head(iris2)
```

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
1	7.0	3.2	4.7	1.4	versicolor
2	6.4	3.2	4.5	1.5	versicolor
3	6.9	3.1	4.9	1.5	versicolor
4	5.5	2.3	4.0	1.3	versicolor
5	6.5	2.8	4.6	1.5	versicolor
6	5.7	2.8	4.5	1.3	versicolor

```
levels(iris2$Species)
```

```
[1] "versicolor" "virginica"
```

We will use these data for our examples today. T-test requires *only* 2 groups/populations. We will assess the alternative hypothesis that one of our numerical variables (sepal length, sepal width, petal length, or petal width) differs by species.

But first, we must **test our assumptions**

4.6 Assumption 1.) Assessing normality

Method 1: the Shapiro-Wilk Test If $p < 0.05$ then the distribution is significantly different from normal.

Step 1: we need to create separate data frames for each species to assess normality of each variable by species!

```
versi<-iris2 %>%
  filter(Species=='versicolor') %>%
  droplevels()

virg<-iris2 %>%
```

```
filter(Species=='virginica') %>%
droplevels()
```

Step 2: We can run our shapiro-wilk tests on each variable if we'd like

```
shapiro.test(versi$Petal.Length) #this is normally distributed
```

Shapiro-Wilk normality test

```
data:  versi$Petal.Length
W = 0.966, p-value = 0.1585
```

```
shapiro.test(versi$Petal.Width) # this is not
```

Shapiro-Wilk normality test

```
data:  versi$Petal.Width
W = 0.94763, p-value = 0.02728
```

```
shapiro.test(versi$Sepal.Length) #normal
```

Shapiro-Wilk normality test

```
data:  versi$Sepal.Length
W = 0.97784, p-value = 0.4647
```

```
shapiro.test(versi$Sepal.Width) #normal
```

Shapiro-Wilk normality test

```
data:  versi$Sepal.Width
W = 0.97413, p-value = 0.338
```

```
shapiro.test(virg$Petal.Length) #normal
```

Shapiro-Wilk normality test

```
data:  virg$Petal.Length
W = 0.96219, p-value = 0.1098
```

```
shapiro.test(virg$Petal.Width) #normal
```

Shapiro-Wilk normality test

```
data:  virg$Petal.Width
W = 0.95977, p-value = 0.08695
```

```
shapiro.test(virg$Sepal.Length) #normal
```

Shapiro-Wilk normality test

```
data:  virg$Sepal.Length
W = 0.97118, p-value = 0.2583
```

```
shapiro.test(virg$Sepal.Width) #normal
```

Shapiro-Wilk normality test

```
data:  virg$Sepal.Width
W = 0.96739, p-value = 0.1809
```

Method 2: Visualization

Explore the following visualizations. Do you see clear evidence of normality?

```
a1<-ggplot(data=iris2, aes(Petal.Length, fill=Species))+
  geom_histogram(binwidth = 0.3)+
  facet_wrap(~Species)+
  theme_classic()+
```

```

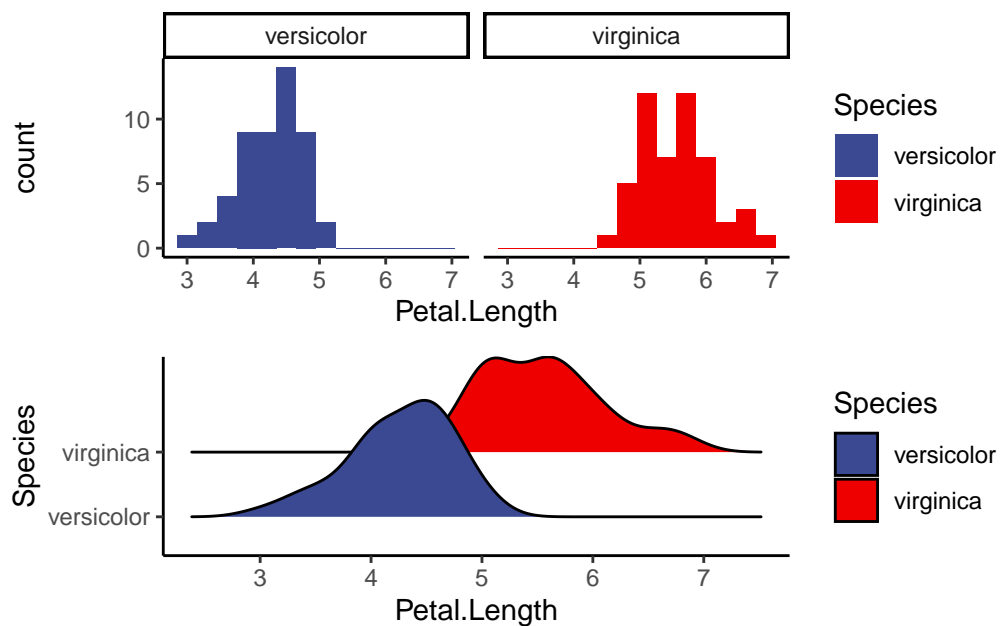
scale_fill_aaas()

a2<-ggplot(data=iris2, aes(x=Petal.Length, y=Species, fill=Species))+
  geom_density_ridges()+ #makes a smooth density curve instead of a histogram!
  theme_classic()+
  scale_fill_aaas()

a1/a2 #compare the visualizations (they are of the same data)- do we see normality here?

```

Picking joint bandwidth of 0.206



```

b1<-ggplot(data=iris2, aes(Petal.Width, fill=Species))+
  geom_histogram(binwidth = 0.3)+
  facet_wrap(~Species)+
  theme_classic()+
  scale_fill_aaas()

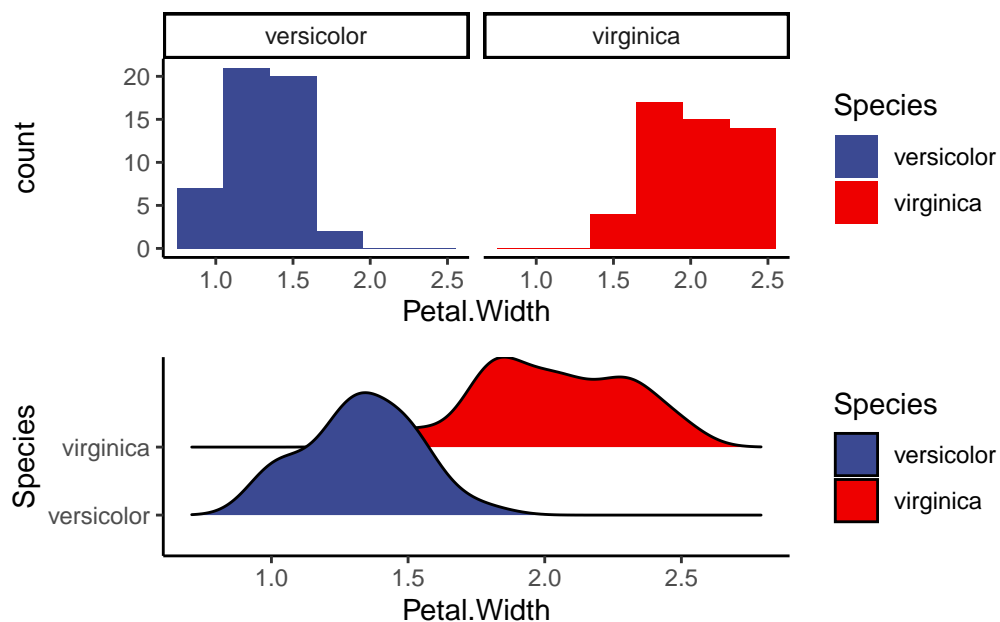
b2<-ggplot(data=iris2, aes(x=Petal.Width, y=Species, fill=Species))+
  geom_density_ridges()+ #makes a smooth density curve instead of a histogram!
  theme_classic()+

```

```
scale_fill_aaas()
```

b1/b2 #compare the visualizations (they are of the same data)- do we see normality here?

Picking joint bandwidth of 0.0972

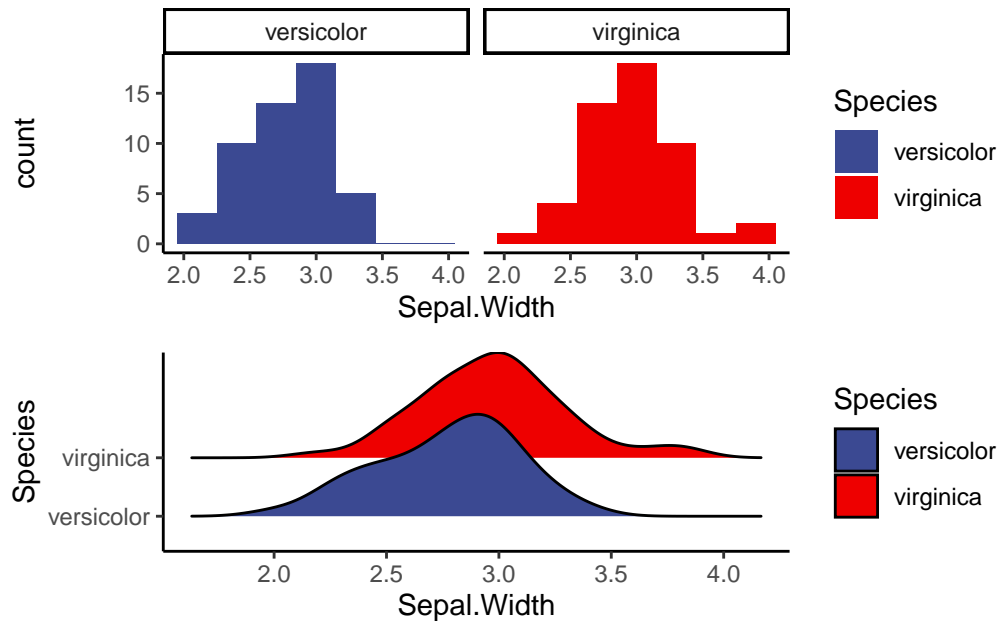


```
c1<-ggplot(data=iris2, aes(Sepal.Width, fill=Species))+
  geom_histogram(binwidth = 0.3)+
  facet_wrap(~Species)+
  theme_classic()+
  scale_fill_aaas()
```

```
c2<-ggplot(data=iris2, aes(x=Sepal.Width, y=Species, fill=Species))+
  geom_density_ridges()+ #makes a smooth density curve instead of a histogram!
  theme_classic()+
  scale_fill_aaas()
```

c1/c2 #compare the visualizations (they are of the same data)- do we see normality here?

Picking joint bandwidth of 0.122

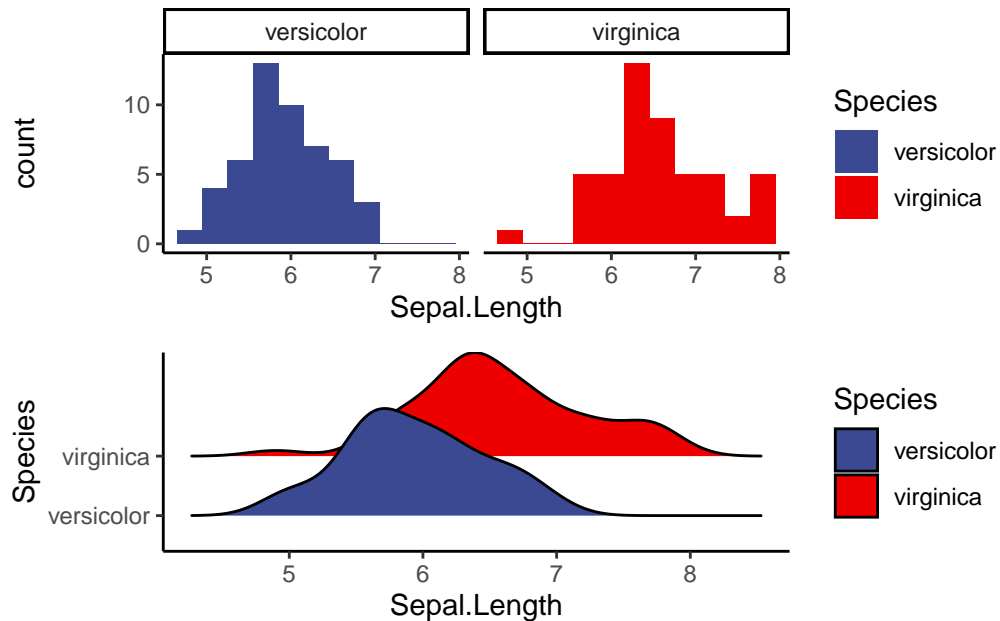


```
d1<-ggplot(data=iris2, aes(Sepal.Length, fill=Species))+
  geom_histogram(binwidth = 0.3)+
  facet_wrap(~Species)+
  theme_classic()+
  scale_fill_aaas()
```

```
d2<-ggplot(data=iris2, aes(x=Sepal.Length, y=Species, fill=Species))+
  geom_density_ridges()+ #makes a smooth density curve instead of a histogram!
  theme_classic()+
  scale_fill_aaas()
```

d1/d2 #compare the visualizations (they are of the same data)- do we see normality here?

Picking joint bandwidth of 0.21



4.7 Assumption 2.) Assessing equal variance

AKA homogeneity of variance

Methods 1: F-test We will use the **F-Test** to compare the variance of two populations. This can only be used with 2 populations and is thus only useful when we run a t-test.

H0 for an F-test is: The variances of the two groups are equal.

Ha: The variances are different

$p < 0.05$ allows us to reject the null (H0) and suggests that the variances are different

note: The F-test assumes our data are already normal! You should not run it on non-normal data

```
#we use var.test to run an F-test
f1<- var.test(Petal.Length ~ Species, data=iris2)
f1 # p>0.05, so we fail to reject H0 (the variances are likely equal)
```

F test to compare two variances

data: Petal.Length by Species

```
F = 0.72497, num df = 49, denom df = 49, p-value = 0.2637
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.411402 1.277530
sample estimates:
ratio of variances
 0.7249678
```

```
f2<- var.test(Petal.Width ~ Species, data=iris2)
f2 # p<0.05, so we reject H0 (variances are likely different)
```

F test to compare two variances

```
data: Petal.Width by Species
F = 0.51842, num df = 49, denom df = 49, p-value = 0.02335
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.2941935 0.9135614
sample estimates:
ratio of variances
 0.5184243
```

```
f3<- var.test(Sepal.Length ~ Species, data=iris2)
f3 # p>0.05, so we fail to reject H0 (the variances are likely equal)
```

F test to compare two variances

```
data: Sepal.Length by Species
F = 0.65893, num df = 49, denom df = 49, p-value = 0.1478
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.3739257 1.1611546
sample estimates:
ratio of variances
 0.6589276
```

```
f4<- var.test(Sepal.Width ~ Species, data=iris2)
f4 # p>0.05, so we fail to reject H0 (the variances are likely equal)
```

F test to compare two variances

```
data: Sepal.Width by Species
F = 0.94678, num df = 49, denom df = 49, p-value = 0.849
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.5372773 1.6684117
sample estimates:
ratio of variances
      0.9467839
```

Method 2: Levene Test

A more flexible test of homogeneity of variance is the Levene Test. It can be used to compare the variance of many populations (not just 2) and is more flexible than the F-test, so it can be used even if the normality assumption is violated.

this is the most commonly used test for homogeneity of variance

leveneTest() is in the car package in R!

N0: Variances of all populations are equal

$p < 0.05$ allows us to reject H_0

```
l1<- leveneTest(Petal.Length ~ Species, data=iris2)
l1 # p>0.05, so we fail to reject H0 (the variances are likely equal)
```

Levene's Test for Homogeneity of Variance (center = median)

```
      Df F value Pr(>F)
group 1  1.0674 0.3041
      98
```

```
l2<- leveneTest(Petal.Width ~ Species, data=iris2)
l2 # p<0.05, so we reject H0 (variances are likely different)
```

Levene's Test for Homogeneity of Variance (center = median)

```
      Df F value  Pr(>F)
group 1  6.5455 0.01205 *
      98
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

13<- leveneTest(Sepal.Length ~ Species, data=iris2)
13 # p>0.05, so we fail to reject H0 (the variances are likely equal)

```

Levene's Test for Homogeneity of Variance (center = median)

```

      Df F value Pr(>F)
group  1  1.0245 0.3139
      98

```

```

14<- leveneTest(Sepal.Width ~ Species, data=iris2)
14 # p>0.05, so we fail to reject H0 (the variances are likely equal)

```

Levene's Test for Homogeneity of Variance (center = median)

```

      Df F value Pr(>F)
group  1  0.0873 0.7683
      98

```

Method 3: Visualization

Since p-values are more like guidelines, we also want to visualize our data to assess homogeneity of variance. We can do that in several ways. You might already have some ideas about this! In general, it seems smart to display the raw data as points and as boxplots. Let's start there!

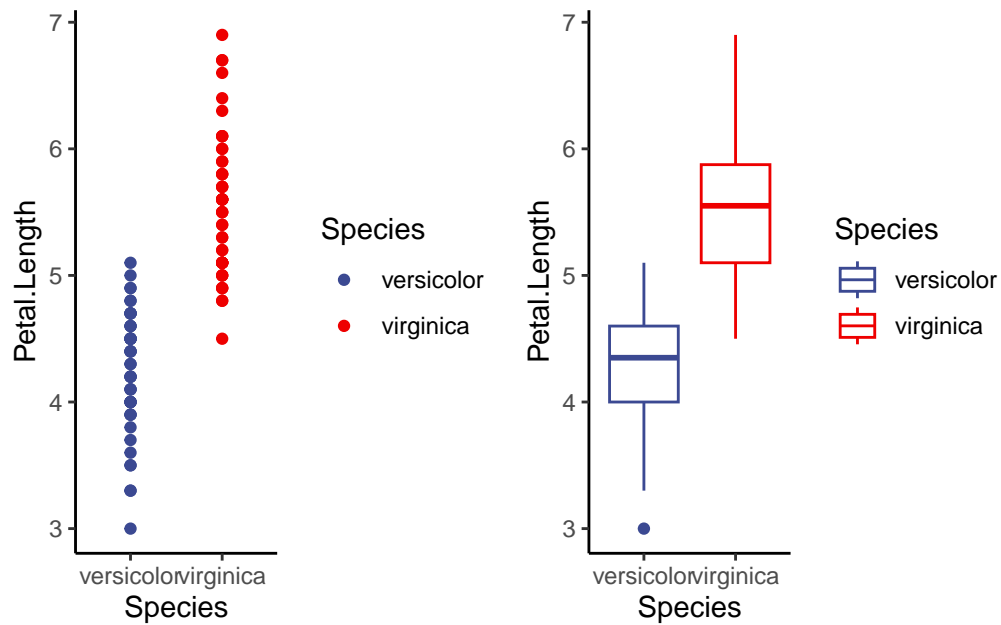
```

v1.1<-ggplot(data=iris2, aes(x=Species, y=Petal.Length, color=Species))+
  geom_point()+
  theme_classic()+
  scale_color_aaas()

v1.2<-ggplot(data=iris2, aes(x=Species, y=Petal.Length, color=Species))+
  geom_boxplot()+
  theme_classic()+
  scale_color_aaas()

v1.1+v1.2

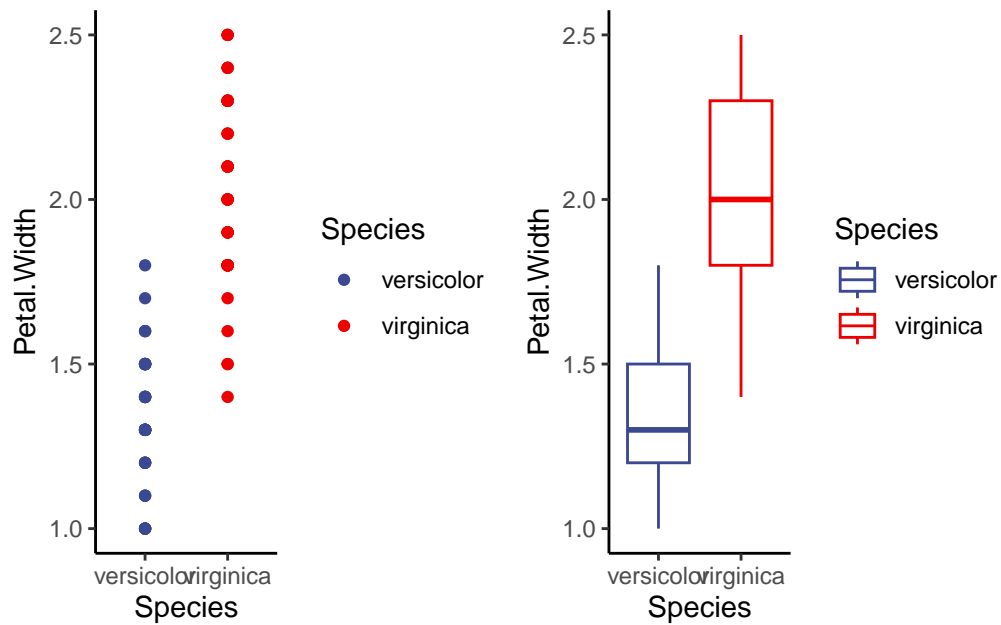
```



```
v2.1<-ggplot(data=iris2, aes(x=Species, y=Petal.Width, color=Species))+
  geom_point()+
  theme_classic()+
  scale_color_aaas()
```

```
v2.2<-ggplot(data=iris2, aes(x=Species, y=Petal.Width, color=Species))+
  geom_boxplot()+
  theme_classic()+
  scale_color_aaas()
```

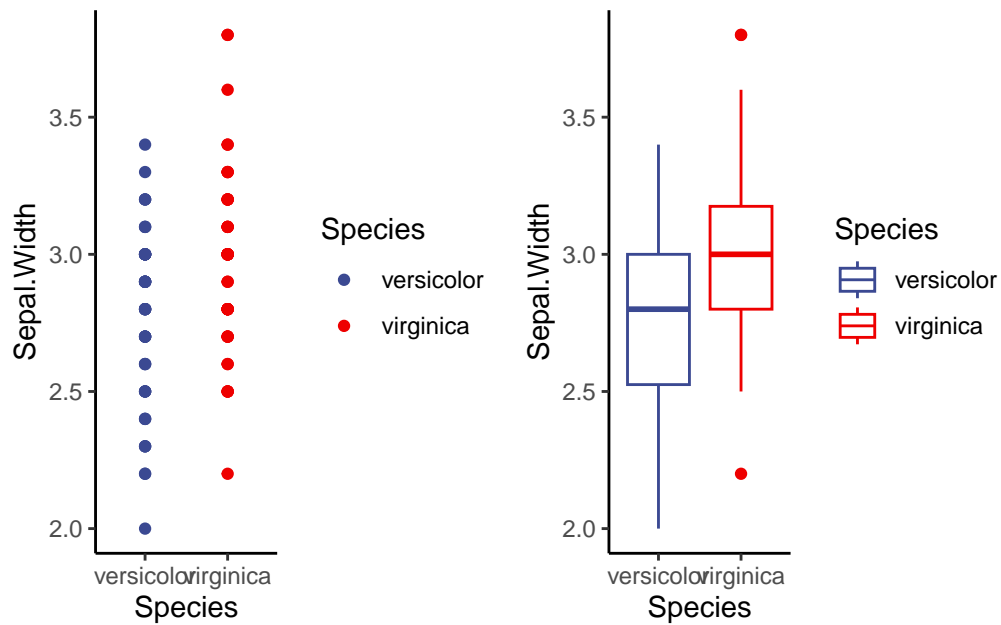
```
v2.1+v2.2
```



```
v3.1<-ggplot(data=iris2, aes(x=Species, y=Sepal.Width, color=Species))+
  geom_point()+
  theme_classic()+
  scale_color_aaas()
```

```
v3.2<-ggplot(data=iris2, aes(x=Species, y=Sepal.Width, color=Species))+
  geom_boxplot()+
  theme_classic()+
  scale_color_aaas()
```

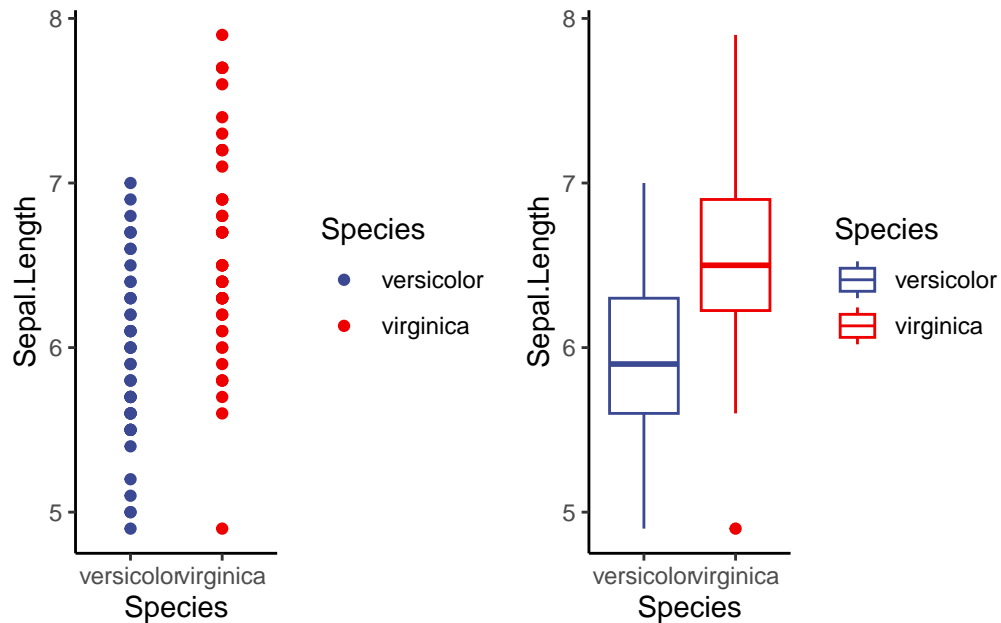
```
v3.1+v3.2
```



```
v4.1<-ggplot(data=iris2, aes(x=Species, y=Sepal.Length, color=Species))+
  geom_point()+
  theme_classic()+
  scale_color_aaas()
```

```
v4.2<-ggplot(data=iris2, aes(x=Species, y=Sepal.Length, color=Species))+
  geom_boxplot()+
  theme_classic()+
  scale_color_aaas()
```

```
v4.1+v4.2
```

4.8 When can we ignore assumptions?

We can if our sample sizes are large. If n is small, we should not ignore this assumption. There are alternatives to dealing with normality that we can discuss in the ANOVA section (such as transforming the data)

[For more info on that](#)

We can also ignore the equal variance requirement if we use the Welch t-test (default in R)

4.9 A basic T-test in R

Finally, let's do some T-tests!

H_0 : No difference between the means of the 2 populations $p < 0.05$ allows us to reject this H_0 (indicating a likely difference)

Step 1: Calculate means and error and plot!

```
meaniris<-iris2 %>%
  group_by(Species) %>%
  dplyr::summarise(meanpl=mean(Petal.Length), sdpl=sd(Petal.Length), n=n(), sepl=sdpl/sqrt(n))

meaniris
```

```
# A tibble: 2 x 14
  Species    meanpl  sdpl      n  sepl meanpw  sdpw  sepw meansl  sds1  ses1
<fct>      <dbl> <dbl> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
1 versicolor  4.26 0.470    50 0.0665  1.33 0.198 0.0280  5.94 0.516 0.0665
2 virginica   5.55 0.552    50 0.0780  2.03 0.275 0.0388  6.59 0.636 0.0780
# i 3 more variables: meansw <dbl>, sds1 <dbl>, ses1 <dbl>
```

```
p1<-ggplot(meaniris, aes(x=Species, y=meanpl, color=Species))+
  geom_point()+
  geom_errorbar(aes(x=Species, ymin=meanpl-sepl, ymax=meanpl+sepl), width=0.2)+
  scale_color_aas()+
  theme_classic()+
  labs(title='Petal Length')
```

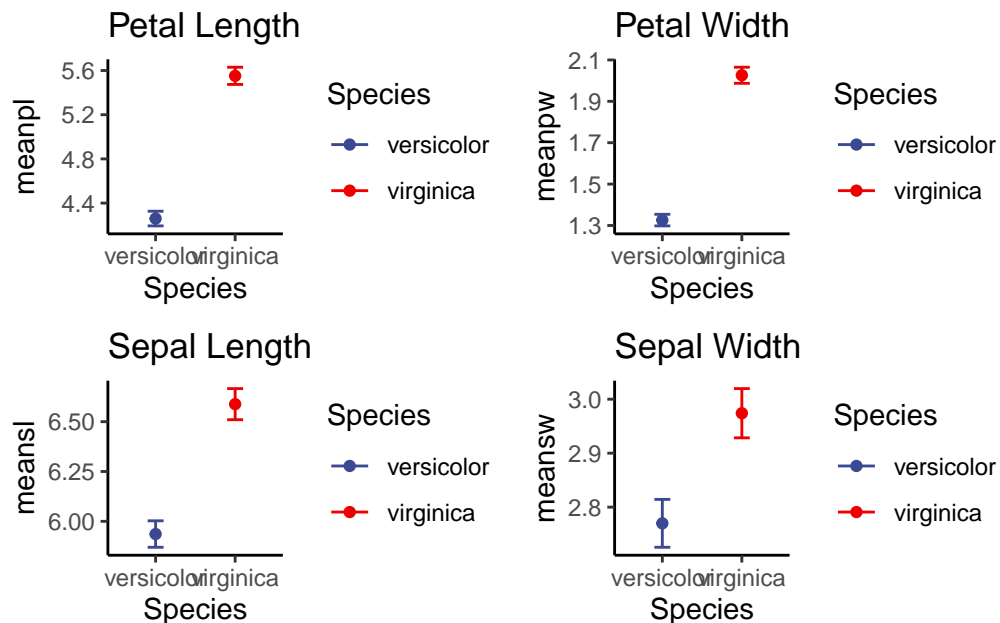
```
p2<-ggplot(meaniris, aes(x=Species, y=meanpw, color=Species))+
  geom_point()+
  geom_errorbar(aes(x=Species, ymin=meanpw-sepw, ymax=meanpw+sepw), width=0.2)+
  scale_color_aas()+
  theme_classic()+
  labs(title='Petal Width')
```

```
p3<-ggplot(meaniris, aes(x=Species, y=meansl, color=Species))+
  geom_point()+
  geom_errorbar(aes(x=Species, ymin=meansl-ses1, ymax=meansl+ses1), width=0.2)+
  scale_color_aas()+
  theme_classic()+
  labs(title='Sepal Length')
```

```
p4<-ggplot(meaniris, aes(x=Species, y=meansw, color=Species))+
  geom_point()+
  geom_errorbar(aes(x=Species, ymin=meansw-sesw, ymax=meansw+sesw), width=0.2)+
  scale_color_aas()+
  theme_classic()+
  labs(title='Sepal Width')
```

```
theme_classic()+
labs(title='Sepal Width')
```

```
(p1+p2)/(p3+p4)
```



4.10 Some examples

Does Petal Length differ by species?

```
t1<-t.test(data=iris2, Petal.Length~Species, alternative='two.sided', var.equal=FALSE) #two
t1 #p<0.05 suggests that there is a significant difference in petal length between species
```

Welch Two Sample t-test

```
data: Petal.Length by Species
t = -12.604, df = 95.57, p-value < 2.2e-16
alternative hypothesis: true difference in means between group versicolor and group virginica
95 percent confidence interval:
-1.49549 -1.08851
```

sample estimates:

mean in group versicolor	mean in group virginica
4.260	5.552

Our $p < 0.05$ suggests that there is a significant effect of species on petal length (petal length differs by species). BUT, do we get a clear explanation of which group is higher or lower? Look at the Welch T-test output and you can see the means! You can also use the graph we made to visualize this!

Does Petal Width differ by species?

```
t2<-t.test(data=iris2, Petal.Width~Species, alternative='two.sided', var.equal=FALSE) #two
t2
```

Welch Two Sample t-test

data: Petal.Width by Species

t = -14.625, df = 89.043, p-value < 2.2e-16

alternative hypothesis: true difference in means between group versicolor and group virginica

95 percent confidence interval:

-0.7951002 -0.6048998

sample estimates:

mean in group versicolor	mean in group virginica
1.326	2.026

Does Sepal Width differ between species?

```
t3<-t.test(data=iris2, Sepal.Width~Species, alternative='two.sided', var.equal=FALSE) #two
t3
```

Welch Two Sample t-test

data: Sepal.Width by Species

t = -3.2058, df = 97.927, p-value = 0.001819

alternative hypothesis: true difference in means between group versicolor and group virginica

95 percent confidence interval:

-0.33028364 -0.07771636

```
sample estimates:
mean in group versicolor  mean in group virginica
                2.770                2.974
```

Does Sepal Length differ between species?

```
t4<-t.test(data=iris2, Sepal.Length~Species, alternative='two.sided', var.equal=FALSE) #tw
t4
```

Welch Two Sample t-test

```
data: Sepal.Length by Species
t = -5.6292, df = 94.025, p-value = 1.866e-07
alternative hypothesis: true difference in means between group versicolor and group virginica
95 percent confidence interval:
 -0.8819731 -0.4220269
sample estimates:
mean in group versicolor  mean in group virginica
                5.936                6.588
```

SO, when is a t-test actually useful and when isn't it? We use a T-test **ONLY** when we want to compare two means / two populations. If we have more than 2 groups, a T-test is not appropriate! Instead, we need to use an analysis of variance (ANOVA) or possibly something more complex!

5 4: Lab 4 Assignment

Remember: When you run a statistical test, it is important to check assumptions. So, when you are asked to run a test below, do not forget to assess assumptions (when possible)

- 1.) Using the penguins data (make a data frame :)), test for correlation between flipper length and bill length. Make a graph, run the appropriate correlation test, and interpret the results.
- 2.) Run an analysis for multiple correlations between all of the numerical variables in penguins. Make a nice looking table of the statistical outputs (r and p) using kable (we have not learned

this– this is a test of your ability to use your new R problem solving skills). Interpret your statistical results (what is and what isn't correlated, is the correlation strong, weak, positive, negative). A multiple correlation plot may help.

3.) Filter the penguins data so there is only 1 species. Test the hypothesis “There is no relationship between bill length and bill depth” for this species. Make a graph, use the appropriate statistical test, and assess whether the hypothesis is rejected or whether we fail to reject it (and why). Repeat for each of the 3 species.

4.) Filter out chinstrap penguins so we only have 2 species. Now, assess the following hypothesis: “There is no difference in flipper length between gentoo and adelia penguins.” Make a graph that is a good visual hypothesis test, run the appropriate stats, report the results, and use those results to assess the hypothesis.

5.) Render your document and submit on moodle.