PyFLOWGO FOR DUMMIES

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Abstract

Lava flow advance may be modeled through tracking the evolution of the lava's thermo-rheological properties, which are defined by viscosity and yield strength. These rheological properties evolve, in turn, with cooling and crystallization. Such model was conceived by Harris and Rowland (2001) who developed a 1-D model, FLOWGO, in which velocity of a control volume flowing down a channel depends on rheological properties computed following the lava cooling and crystallization path estimated via a heat balance box model. We provide here an updated version of FLOWGO written in a modern and flexible language, Python, that is open-source and compatible with any operating system. Python also provides useful libraries, and its object-oriented approach allows a great flexibility.

The Software is freely available at: `https://github.com/pyflowgo/pyflowgo`

The present document is an user's guide for PyFLOWGO. Further information about software architecture, numerical solution and code validation are presented in an article: Chevrel et al. (2017): Chevrel M.O., Labroquère J., Harris A. and Rowland S. (2017) PyFLOWGO: an open-source platform for simulation of channelized lava thermo-rheological properties. In prep.

1 Installation and requirements

PyFLOWGO (https://github.com/pyflowgo/pyflowgo) has been written in Python^{TM} 3.3, therefore it might not be compatible with earlier versions. If you are not familiar with Python, it is suggested to download the IDE PyCharm here: https://www.jetbrains.com/pycharm/ that is a friendly way to use/develop in Python and run this software.

PyFLOWGO requires some mathematical modules; including "numpy", "scipy", "matplotlib". We therefore recommend to download anaconda environment from Continuum with Python 3 which is availlable at: https://www.continuum.io/downloads. This includes many packages useful for PyFLOWGO and allows to avoid conflicts with other versions of the packages. With anaconda, you need to install the environment named py3k containing Python 3 and the necessary packages, run in a shell, to activate the environment:

```
$ conda create -n py3k python=3 anaconda
$ conda source py3k
```

For more expert, if you don't want to use anaconda, you can still use pip to install the necessary packages under Python 3:

```
$ pip install -r requirements.txt
```

2 Content of the software

PyFLOWGO contains:

- `main_flowgo.py`: the main script that needs to be run in order to calculate the properties of the lava control volume down flow.
- folder named `pyflowgo` which contains all models used by PyFLOWGO (this folder should not be modified unless new models are added).
- folder named `test` with all the tests of the models (do not modify unless to make a test for your new model).
- folder named `resource` containing:
  - `template.json` and `template_2.json` that are two examples of input parameters ;
  - `example_slope_profile.txt` that is a x, y text file where x is distance down flow and y is the slope (in °) at that point and is an example of the line-of-steepest-descent down which the control volume is moved.
  - folder named `field_data` containing three CVS file of ground-truth data.
- `plot_results.py`: an example to plot the main results.
- `plot_results_field_data.py`: an example to plot the main results and the ground-truth data.
- `pyflowgo_for_dummies.pdf`: this document.

3 Input parameters in json file

The json file is a special format which is human readable and can easily be read in any Python script. Use `template.json` in `resource` to enter the characteristics of your lava flow. This file is composed of 3 main parts, and must be completed as described in the next sections.

3.1 Lava name, slope profile and step size

1. Give the lava name you want, this name will be used to name the output file.
2. Write the path to the line-of-steepest-descent file in "`slope_file`" (line 3, Fig.1). The slope file must be in format `.txt` containing in the first column the distance from the vent in meters and in the second column the slope in degrees.
3. Enter the down flow step size in meters (see section 5 in Chevrel et al. 2017 for convergence analysis0.

Figure 1: Lava name and path to the slope file.

```
{
  "lava_name": "template",
  "slope_file" : "resource/example_slope_profile.txt",
  "step_size": 10 ,
```

3.2 Models

The next 15 lines of the json file are dedicated to choose the models (Fig.2). For the `heat_budget_models` you must simply write `"yes"` or `"no"` if you want to consider the flux or not. So far, there are seven other models that must be defined: `"crystallization_rate_model"`; `"melt_viscosity_model"`; `"relative_viscosity_model"`; `"yield_strength_model"`; `"crust_temperature_model"`; `"effective_cover_crust_model"`; `"vesicle_fraction_model"`. To know what is the name of the model you have to write see Table 1, and for more details about the model see Annex 2 (section 6) or Chevrel et al. (2017).

Figure 2: Example of models choice in json file

```
  "heat_budget_models":{
   "radiation" : "yes",
   "conduction" : "yes",
   "convection" : "yes",
   "rain" : "no",
   "viscous_heating" : "no"
  },

  "models":{
  "crystallization_rate_model":"basic",
  "melt_viscosity_model":"vft",
  "relative_viscosity_model":"er",
  "yield_strength_model":"basic",
  "crust_temperature_model":"constant",
  "effective_cover_crust_model": "basic",
  "vesicle_fraction_model":"constant"
  },
```

Table 1: Models' choice

Model's name	Symbol	What to write in the json file
heat_budget_models		
radiation	Q_{rad}	"yes" / "no"
conduction	Q_{cond}	"yes" / "no"
convection	Q_{conv}	"yes" / "no"
rain	Q_{rain}	"yes" / "no"
viscous_heating	Q_{visc}	"yes" / "no"
models		
crystallization_rate_model	$\partial\phi/\partial T$	"basic" / "bimodal" / "bimodal_f_temp" / "melts"
melt_viscosity_model	η_{melt}	"basic" / "dragoni" / "shaw" / "vft"
relative_viscosity_model	η_r	"er" / "mp" / "kd" / "costa1" / "costa2" / "ptp1" / "ptp2" / "ptp3"
yield_strength_model	τ_0	"basic" / "dragoni"
crust_temperature_model	T_{crust}	"basic" / "hon" / "bimodal"
effective_cover_crust_model	f_{crust}	"basic" / "bimodal"
vesicle_fraction_model	ϕ_b	"constant" / "bimodal"

3.3 Input values

In order to correctly write the input values in the json file use the description of the parameters given in table 2.

Table 2: Description of input parameters

Input parameters in json file	Symbol	Definition	Unit	Constant for Earth
lava_name		name of the lava flow	n.a	
slope_file		file containing distance (m) and slope (°)		
step_size		step size for lava advance	m	
terrain_conditions				
width	w	channel's width	m	
depth	d	channel's depth	m	
gravity	g	gravity of the planet	m/s^2	9.81
max_channel_length	L_{max}	maximum flow length*	m	
eruption_conditions				
eruption_temperature	T_{erupt}	temperature of the eruption	K	
viscosity_eruption	η_{erupt}	viscosity of the lava at T_{erupt}, only for "basic" melt viscosity model	Pa s	
lava_state				
position	x	distance from the vent at which the iteration starts	m	
critical_distance	x_{critic}	distance when the bimodal models change, only when "bimodal" models"	m	
time	t	time at which the iteration starts, only for "hon" and "honbimodal" crust temperature model	s	
crystal_fraction	ϕ	initial crystal fraction	n.a	
density_dre	ρ_{DRE}	dense rock equivalent density	kg/m^3	
vesicle_fraction	ϕ_b	initial fraction of vesicle at the vent	n.a	
liquidus_temperature	L_0	temperature of the liquidus , only for "dragoni" melt viscosity model	K	
radiation_parameters				
stefan-boltzmann_sigma	σ	stefan-boltzmann constant	W/m^{2}K^4	5.669E-8
emissivity_epsilon	ε	emissivity	n.a.	0.98
conduction_parameters				
basal_temperature	T_{base}	temperature at the base of the flow	K	
core_base_distance	H_b	percentage of base layer over flow depth	%	
rain_parameters				
rainfall_rate	$\partial R/\partial t$	rainfall rate	m/s	
density_water	ρ_{H_2O}	density of the water	kg/m^3	958
latent_heat_vaporization	L_{H_2O}	latent heat of vaporisation of the water	J/kg	2800000
convection_parameters				
wind_speed	U	wind speed	m/s	
ch_air	C_H	value from Greeley and Iverson (1987)**	n.a.	3.599E-3

air_temperature	T_{atmo}	temperature of the air	K	
air_density	ρ_{atmo}	density of the air	kg/m^3	0.4411
air_specific_heat_capacity	Cp_{atmo}	heat capacity of the air	J/kg K	1099
thermal_parameters				
buffer	$buffer$	difference between T_{core} and T_{hot}	K	
crust_cover_fraction	f_{init}	initial crust cover fraction	n.a.	
alpha	α	coefficient for velocity dependence of the crust cover	n.a	
initial_crust_temperature	T_{init}	chilled crust temperature	K	
melt_viscosity_parameters				
shaw_slope	s	coefficient calculated from melt chemical composition only for "shaw" melt viscosity model		
a_vft	A	coefficient calculated from melt chemical composition, only for "vft" melt viscosity model	Pa.s	
b_vft	B	coefficient calculated from melt chemical composition, only for "vft" melt viscosity model	J/mol	
c_vft	C	coefficient calculated from melt chemical composition, only for "vft" melt viscosity model	K	
crystals_parameters				
crystals_grown_during_cooling	ϕ_{grown}	fraction of crystal grown during emplacement, only for "basic" crystallization rate model	n.a	
solid_temperature	T_{solid}	temperature at which the lava cannot flow, only for "basic" crystallization rate model	K	
crystallization_rate_1	C_1	crystallization rate, only for "bimodal" crystallization rate model	crystals/°C	
crystallization_rate_2	C_2	crystallization rate, only for "bimodal" crystallization rate model	crystals/°C	
latent_heat_of_crystallization	L	latent heat of crystallization	J/kg	350000
relative_viscosity_parameters				
max_packing	ϕ_m	maximum fraction of crystals that allows flowing, only for "kd" and "mp" relative viscosity models	n.a.	
einstein_coef	b	Einstein coefficient or intrinsic viscosity, only for "kd" relative viscosity models	Pa.s	
strain_rate		strain rate, only for "costa1" and "costa2" relative viscosity models	s^{-1}	0.0001 or 1

*used in case the limiting conditions ($v_{mean} = 0$ or $\phi = \phi_{max}$ or $T_{core} = T_{solid}$) are not reached.

** $C_H = (U'/U)^2$ where U' is the fraction of wind speed according to Keszthelyi and Denlinger (1996).

4 Run PyFLOWGO with PyCharm

1. Enter the json path to the script parameters in the configuration of `main_flowgo.py`, In PyCharm open "edit configuration" by clicking on the arrow next to main_flowgo (Fig.3), and write the path of the json file in "script parameters" (Fig.4).

Figure 3: main_flowgo

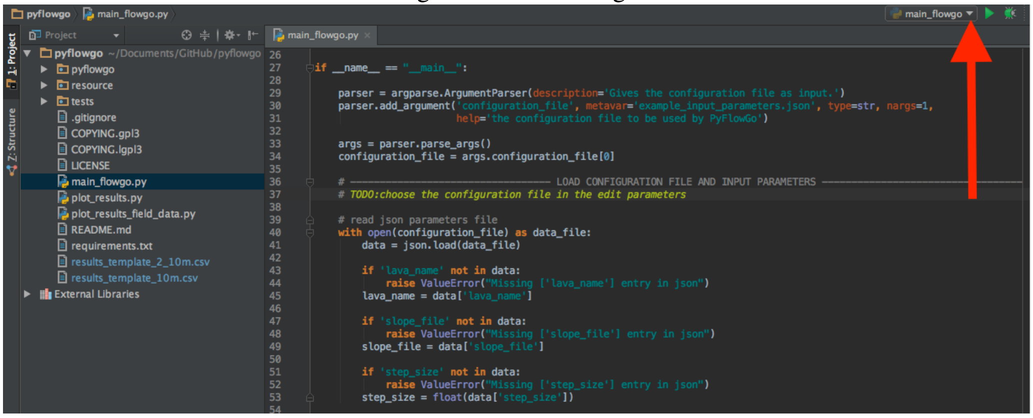
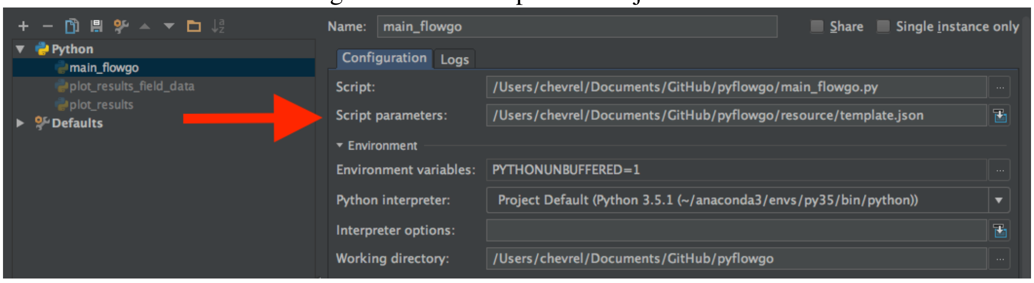


Figure 4: Enter the path to the json file.



This step basically configures PyCharm to run the following command:
$ `python3 main_flowgo.py./resource/template.json`

2. Remember that the path to the slope_file needs to have been correctly written in the json file.

3. Run `main_flowgo.py` using the green arrow.

5 Results

The output contain most of the calculated properties as function of distance for the vent. A `.csv` file will be created and saved with the lava name (as entered in the json configuration file) and ending with the step_size
Examples: `results_template_10m.csv`

6 Results visualization

Results can be plotted using `plot_results.py`. This script reads the `.csv` file you want (or an array of `.csv` files) and create plots of the main properties as function of distance from the vent. The script `plot_field_data.py` plots the ground-truth data on the same plot as the model results. Both scripts create four figures:

- figure 1: core temperature, mean velocity, bulk viscosity, yield strength, channel width, crystal fraction
- figure 2: heat fluxes
- figure 3: crustal and surface conditions
- figure 4: slope profile

[Note that these two scripts are just for visualizing the results, we invite the user to create her/his own plots.]

Annex 2: Description of the models

6.1 Heat flux models

6.1.1 Radiative heat flux

Heat loss due to radiation from the lava surface to the atmosphere is expressed as:

$$Q_{rad} = \sigma \varepsilon T_{eff}^4 \, w \tag{1}$$

where σ (W/m^2 K^4) is the Stefan − Boltzmann constant, ε is emissivity and T_{eff} (K) is the effective surface temperature, which is calculated using a two-component model for the lava surface (Pieri and Baloga 1986, Crisp and Baloga 1990, Pieri et al. 1990):

$$T_{eff} = \left[f_{crust}(T_{crust}^4 - T_{atmo}^4) + (1 - f_{crust})(T_{hot}^4 - T_{atmo}^4) \right]^{0.25} \tag{2}$$

where T_{atmo} is the temperature of the surrounding atmosphere, f_{crust} is the fraction of crusted lava, T_{crust} is the cool crust temperature, $1 - f_{crust}$ represents the fraction of exposed uncrusted hot lava and T_{hot} is the hot component temperature. The different models used to calculate f_{crust}, T_{crust} are described in sections 6.6, 6.7, respectively.

6.1.2 Forced convection heat flux

Heat loss due to atmospheric convection from the lava surface is calculated via (e.g. Keszthelyi et al. 2003):

$$Q_{conv} = h_{conv}(T_{conv} - T_{atmo}) \, w \tag{3}$$

where h_{conv} is the convective heat transfer (in W/m^2 K) and T_{conv} (K) the characteristic surface temperature. The convective heat transfer depends on atmospheric conditions and can be defined as:

$$h_{conv} = U \, C_H \, \rho_{atmo} \, Cp_{atmo} \tag{4}$$

where U is wind speed (m/s), C_H the wind friction factor as defined by Greeley and Iversen (1987), ρ_{atmo} (kg/m^3) is atmospheric density and Cp_{atmo} the heat capacity of the air (J/kg K) in contact with the lava surface. The characteristic surface temperature is calculated via :

$$T_{conv} = \left[f_{crust} T_{crust}^{1.33} + (1 - f_{crust}) \left(T_{hot}^{1.33} \right) \right]^{0.75} \tag{5}$$

6.1.3 Heat flux due to rain

The heat flux due to vaporization of rainwater falling onto the lava surface is expressed by:

$$Q_{rain} = \frac{\partial R}{\partial t} \rho_{H_2O} \, L_{H_2O} \, w \tag{6}$$

where $\partial R / \partial t$ (m/s) is the rainfall rate and ρ_{H_2O} (kg/m^3) and L_{H_2O} (J/kg) are, respectively, the density and latent of vaporisation of water.

6.1.4 Conductive heat flux

The heat flux through the base and the levées of the flow is driven by conduction (after Keszthelyi 1995) and expressed as:

$$Q_{cond} = \kappa_{lava} \frac{T_{core} - T_{base}}{h_{base}} \, w \tag{7}$$

where κ_{lava} (W/mK) is the thermal conductivity of the lava, T_{core} (K) the lava core temperature, T_{base} (K) the temperature of the basal layer and h_{base} (m) the thickness of that basal layer that is defined between the underlaying surface and the thermal boundary between the interior of the flow (at T_{core}) and the level at which T_{base} is reached. It is usually calculated via

$$h_{base} = \frac{H_b}{100} d \tag{8}$$

where H_b is the proportion occupied by the basal layer in respect to the entire flow thickness (d, in m).

6.1.5 Viscous heating

Viscous heating in the lava channel is expressed here, for a channel that is wider than it is deep ($w > d$) following Costa and Macedonio (2003):

$$Q_{visc} = \eta_{bulk} (V_{mean}/d)^2 \, w \tag{9}$$

where η_{bulk} (Pa·s) is the bulk viscosity of the molten lava as calculated in section 6.3 and V_{mean} is the mean velocity of the lava.

6.2 Crystallization rate models

The crystallization rate per degree of cooling ($\partial\phi/\partial T_{cool}$) may be calculated via different models. Here we provide three models that the user is free to chose to run PyFLOWGO.

6.2.1 "basic"

The basic model proposed by Harris and Rowland (2001) takes into account the amount of crystallization grown during the eruption (ϕ_{grown}), that occurred between the eruption temperature (T_{erupt}) and the solid temperature (T_{solid}) defined as the temperature at which the material cannot flow anymore:

$$\frac{\partial\phi}{\partial T_{cool}} = \frac{\phi_{grown}}{T_{erupt} - T_{solid}} \tag{10}$$

6.2.2 "bimodal"

This bimodal model was proposed by Robert et al. (2014) and Harris and Rowland (2015) and allows the crystallization rate to be changed after a given distance (x_{critic}).

$$\begin{aligned} &\text{If } x \leqslant x_{critic}\text{: } (\partial\phi/\partial T_{cool}) = \mathrm{C}_1 \\ &\text{If } x > x_{critic}\text{: } (\partial\phi/\partial T_{cool}) = \mathrm{C}_2 \end{aligned} \tag{11}$$

where x_{critic}, the constant C_1 and C_2 are of user's choice.

6.2.3 "bimodal_f_temp"

This bimodal model allows changing the crystallization rate after a given temperature (T_{critic}).

$$\begin{aligned} &\text{If } T_{core} \geqslant T_{critic}\text{: } (\partial\phi/\delta T_{cool}) = \mathrm{C}_1 \\ &\text{If } T_{core} < T_{critic}\text{: } (\partial\phi/\delta T_{cool}) = \mathrm{C}_2 \end{aligned} \tag{12}$$

where T_{critic}, the constant C_1 and C_2 are of the user's choice.

6.2.4 "melts"

The MELTS model allows to extract the crystallization rate per degree of cooling from MELTS-based look-up table as suggested by Harris and Rowland (2001),Harris and Rowland (2015) and Riker et al. (2009).

The look-up table is a CSV file containing the amount of crystals (in fraction) as a function of temperature (in °C) that must be previously build by the user from MELTS software (Ghiorso and Sack 1995) at the degree step of user's choice. A linear interpolation of this data is computed and gives a function ϕ_{interp} representing the fraction of crystal in function of the temperature. The fraction of crystal variation per degree of cooling is then computed using finite differences on this interpolated function:

$$\frac{\partial\phi}{\partial T_{cool}} \approx -\frac{\phi_{interp}(T_{core} + \Delta T) - \phi_{interp}(T_{core} - \Delta T)}{2\Delta T} \tag{13}$$

with ΔT the finite difference step. In practice, ΔT is chosen small enough: $\Delta T = 1e^{-10}$.

Note here that the degree of cooling is conventionally the inverse of the usual derivate, explaining the minus sign in the finite difference equation.

6.3 Viscosity models

Magma and lava are complex systems composed of polydispersed particle mixture of crystals and bubbles of various shapes and sizes in a liquid phase (the silicate melt), the viscosity of this mixture may therefore be defined as:

$$\eta_{bulk} = \eta_{melt}\eta_r \tag{14}$$

where the viscosity of the interstitial melt, η_{melt} (Pa.s) is Newtonian and depends on temperature and composition, and the relative viscosity, η_r (n.a.), that depends on the volumetric abundance and aspect ratio of the particles (bubbles and crystals) as well as on the strain-rate of the flow.

Here, PyFLOWGO offers the possibility to the user to calculate the bulk viscosity of the lava using one of the following melt viscosity model in pair with a relative viscosity model of his choice.

6.3.1 Melt viscosity models, η_{melt}

"dragoni"

This model calculates the viscosity of the melt at the lava temperature (T_{core}) using the relation proposed by Dragoni (1989):

$$\eta_{melt} = \eta_0 \, exp(0.04(T_0 - T_{core})) \tag{15}$$

where η_0 (Pa.s) is the viscosity of the lava at the liquidus temperature T_0.

"basic"

This model proposed in the original FLOWGO version is derived from Dragoni (1989) where instead of liquidus viscosity and temperature, it is the eruption conditions that are used:

$$\eta_{melt} = \eta_{erupt} \exp\left(0.04(T_{erupt} - T_{core})\right) \tag{16}$$

where η_{erupt} is the viscosity of the lava at eruption temperature, T_{erupt}.

"shaw"

This model calculates the melt viscosity according to the Arrhenian relationship proposed by Shaw (1972) and reformulated here as:

$$log(\eta_{melt}) = (s\frac{10^4}{T_{core}} - 1.5s - 6.4)/2.303) - 1 \tag{17}$$

where η_{melt} is here in Pa.s and T_{core} in K and s is the characteristic slope previously computed from the melt chemical composition using Shaw (1972).

"vft"

This model is based on the so called Vogel-Fulcher-Tammann equation (Vogel 1921, Fulcher 1925, Tammann and Hesse 1926) that describes the non-Arrhenian behavior of the melt viscosity calculated via:

$$log(\eta_{melt}) = A + \frac{B}{C - T_{core}} \tag{18}$$

where A (Pa.s), B (J/mol) and C (K) are fitting parameters depending on chemical composition that must be previously estimated either from viscometry at high and low temperature or from the melt chemical composition generally using the model proposed by Giordano et al. (2008).

6.3.2 Relative viscosity models,η_r, crystals

The relative viscosity models offered here take only into account the crystal cargo except for one model that considers both crystals and bubbles (section 6.3.3). More complex formulation may take into account bimodal particle size distribution and shape (e.g. Castruccio et al. 2010 and Cimarelli et al. 2011) and bubbles content as function of their ability to deform (Llewellin and Manga 2005; Pal 2003) but are not presented here.

"er"

This Einstein-Roscoe model calculates the effect of crystal cargo on viscosity according to the Einstein-Roscoe relationship (as first introduced by Shaw 1965 as originally used in the first FLOWGO version:

$$\eta_r = (1 - R\phi)^{-2.5} \tag{19}$$

where R = 1.51 for spherical solid particles as suggested by Pinkerton and Stevenson (1992).

"kd"

This model calculates the effect of crystal cargo on viscosity according to the Krieger-Dougherty relationship (Krieger 1972, Krieger and Dougherty 1959, Pabst 2004):

$$\eta_r = \left(1 - \frac{\phi}{\phi_m}\right)^{-b\phi_m} \tag{20}$$

where b is the Einstein coefficient (also called intrinsic viscosity) and ϕ_m is the crystal maximum packing and both are fitting parameters depending on the particles shape (see for examples Mueller et al. 2010, Cimarelli et al. 2011, Mader et al. 2013).

"mp"

This model calculates the effect of crystal cargo on viscosity according to Maron and Pierce (1956):

$$\eta_r = \left(1 - \frac{\phi}{\phi_m}\right)^{-2} \tag{21}$$

where ϕ_m is the only fitting parameter depending on the particles shape (e.g. Mueller et al. 2010 and Mader et al. 2013).

"costa1 - 2"

This model allows to calculate the effect of high crystal fraction (above maximum packing) taking into account an applied deformation (strain rate) according to Costa et al. (2009):

$$\begin{gathered} \eta_r = \frac{1 + \left(\frac{\phi}{\phi_*}\right)^{\delta}}{(1-F)^{b\phi_*}} \\ \text{with:} \\ F = (1-\xi)\mathrm{erf}\left[\frac{\sqrt{\pi}}{2(1-\xi)}\frac{\phi}{\phi_*}\left(1 + \left(\frac{\phi}{\phi_*}\right)^{\gamma}\right)\right] \end{gathered} \tag{22}$$

where ϕ_* represents the critical solid fraction at the onset of the exponential increase of η_r; γ is a measure of the rapidity of η_r increase with crystal fraction as ϕ approaches ϕ_* and δ controls the increase of η_r at $\phi > \phi_*$. ξ, γ and δ are all empirical parameters depending on the particles shape and applied strain rate. For examples and values the reader is oriented toward the following articles: Costa et al. (2009), Cimarelli et al. (2011) and Chevrel et al. (2013).

By default the model offered in PyFLOWGO is either for spherical particles or elongated particles applying respectively the model *costa1* or *costa2*; and for strain rate set at $1s^{-1}$ or $10^{-4}s^{-1}$.

6.3.3 Relative viscosity model, η_r crystals and bubbles

"ptp1-2-3"

We offer here only one model allowing the treatment for the viscosity of a three - phase mixture (including the melt) comprising a suspension of rigid spherical particles (ϕ) and bubbles (ϕ_b) following Phan-Thien and Pham (1997). The user can chose one of the three following cases:

Model *ptp1*, crystals are smaller than bubbles:

$$\eta_r = \left(1 - \frac{\phi}{1-\phi_b}\right)^{-5/2} (1-\phi_b)^{-1} \tag{23}$$

Model *ptp2*, crystals and bubbles are of the same size:

$$\eta_r = (1 - \phi - \phi_b)^{\frac{5\phi - 2\phi_b}{2\phi - \phi_b}} \tag{24}$$

Model *ptp3*, crystals are larger than bubbles:

$$\eta_r = \left(1 - \frac{\phi_b}{1 - \phi}\right)^{-1} (1 - \phi)^{-5/2} \tag{25}$$

6.4 Vesicle fraction models, ϕ_b

6.4.1 "constant"

This simple model implies that the vesicles fraction is constant down flow and equals to the input value.

6.4.2 "bimodal"

This bimodal model, proposed by Harris and Rowland 2015, allows the vesicle fraction to be changed after a given distance; x_{critic} (simulating the degassing):

$$\begin{aligned} &\text{If } x \leqslant x_{critic}\text{: } \phi_b = \phi_{b1} \\ &\text{If } x > x_{critic}\text{: } \phi_b = \phi_{b2} \end{aligned} \tag{26}$$

where x_{critic}, ϕ_{b1} and ϕ_{b2} are of the users' choice.

6.5 Yield strength models

The velocity model depends the yield strength of the lava (τ_0) and the yield strength of the flow as a whole also called the basal shear stress (τ_b). Lava yield strength can be calculated as function of temperature and crystallinity following Dragoni (1989) and Pinkerton and Stevenson (1992) as proposed in the first FLOWGO version. [Note that this method is only applicable for basaltic lava.]

The difference between the two proposed models is whether the temperature of the liquidus or the temperature of the eruption is considered.

6.5.1 "dragoni"

The Dragoni (1989) model considers the liquidus temperature:

$$\tau_0 = 0.01(exp^{0.08(T_0 - T_{core})} - 1) + 6500\phi^{2.85} \tag{27}$$

6.5.2 "basic"

The basic model is the same as the Dragoni model (Eq. 27) but considers the temperature of the eruption (T_{erupt}) instead of the temperature of the liquidus as suggested in Harris and Rowland (2001).

6.6 Effective crust cover fraction models, f_{crust}

The upper surface of the flow is partially cover by a cooler crust named the effective crust cover fraction that directly affects the effective temperature (Eq. 2 for the radiative heat flux) and the characteristic surface temperature (Eq. 5 for the forced convection heat flux) and may be calculated using one of the following models.

6.6.1 "basic"

The basic model given by FLOWGO assumes that the effective crust cover fraction f_{crust} varies down flow as a function of velocity (V_{mean}):

$$f_{crust} = f_{init} \exp^{\alpha V_{mean}} \tag{28}$$

where f_{init} is the initial crust fraction (at vent) and α is a coefficient determined by field observation; f_{crust} can be set between zero (crust free, poorly insulated) and unity (complete crust coverage, well insulated). Harris and Rowland (2001) give f_{crust} = 0.9 and α = -0.16, for a poorly insulated flow, and f_{crust} = 1.0 and α = -0.00756 for a more heavily crusted flow.

Alternatively, f_{crust} can be set to be constant and equals to f_{init} if $\alpha = 0$.

6.6.2 "bimodal"

This bimodal model, proposed by Harris and Rowland (2015), allows to change the velocity dependence after a given distance:

$$\begin{aligned}&\text{If } x \leqslant x_{critic}\text{: } f_{crust} = f_{init} \exp^{\alpha_1 V_{mean}}\\&\text{If } x > x_{critic}\text{: } f_{crust} = f_{init} \exp^{\alpha_2 V_{mean}}\end{aligned} \tag{29}$$

where x_{critic}, α_1 and α_2 are set by the user.

6.7 Crust temperature models, T_{crust}

T_{crust} can either be set constant or varies down flow, or be a combination of the two as suggested in Harris and Rowland (2015).

6.7.1 "constant"

This model simply applies a constant value equals to the initial input crust temperature.

6.7.2 "hon"

This model was suggested in the original FLOWGO where T_{crust} variation down flow follows the model of Hon et al. (1994):

$$T_{crust} = -140 \log\left(\frac{time}{3600}\right) + 303 + 273.15 \tag{30}$$

where the *time* is in second and is calculated via:

$$time = \partial x / V_{mean} \tag{31}$$

This equation implies implicitly that the initial crust temperature is 1070°C. [Warning, this model should not be used for other cases than Hawaiian cases as the Hon model is an empirical relationship determined from Hawaiian flows.]

6.7.3 "bimodal"

As suggested by Harris and Rowland (2015), this bimodal model calculates T_{crust} as a function of time according to Hon et al. (1994) from the vent until a critical distance and then it is set constant:

$$\begin{aligned}&\text{If } x \leqslant x_{critic}\text{: } T_{crust} = -140 \log\left(\frac{time}{3600}\right) + 303 + 273.15\\&\text{If } x > x_{critic}\text{: } T_{crust} = T_{init}\end{aligned} \tag{32}$$

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