

Hofs Theory
COMP105 Fall 2015

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Problem M

Using calculational proof, prove that:

$$(o ((\text{curry map}) f) ((\text{curry map}) g)) == ((\text{curry map}) (o f g))$$

Given:

$$\begin{aligned} ((o f g) x) &== (f (g x)) && \text{; apply-compose law} \\ (((\text{curry } f) x) y) &== (f x y) && \text{; apply-curried law} \end{aligned}$$

If they are equal they will return the same thing when called on a list ys.

Proof

$$\begin{aligned} &((o ((\text{curry map}) f) ((\text{curry map}) g)) ys) \\ &= \{\text{apply-compose law}\} \\ &(((\text{curry map}) f) (((\text{curry map}) g) ys)) \\ &= \{\text{apply-curried law}\} \\ &(\text{map } f ((\text{curry map}) g) ys) \\ &= \{\text{apply-curried law}\} \\ &(\text{map } f (\text{map } g ys)) \\ &= \{\text{apply-compose law}\} \\ &(\text{map } (o f g) ys) \\ &= \{\text{apply-curried law}\} \\ &(((\text{curry map}) (o f g)) ys) \end{aligned}$$

Conclusion

This is the same as the right side when called on ys so:

$$(o ((\text{curry map}) f) ((\text{curry map}) g)) == ((\text{curry map}) (o f g))$$