4) a) $T[x(n)] = g(n)\,x(n)$

i) $T[a(n) + b(n)] = g(n)[a(n) + b(n)]$

$= g(n)a(n) + g(n)b(n)$

$= T[a(n)] + T[b(n)]$

$T[\lambda a(n)] = g(n)\lambda a(n)$ Es lineal!

$= \lambda T[a(n)]$

ii)

$T \circ D_k[x(n)] = T[D_k[x(n)]]$

$= T[y(n)]$ $\quad y(n) = x(n-k)$

$= g(n)y(n)$

$= g(n)x(n-k)$

$D_k \circ T[x(n)] = D_k[T[x(n)]]$

$= D_k[g(n)x(n)]$

$= g(n-k)x(n-k)$

$T \circ D_k[x(n)] \neq D_k \circ T[x(n)]$

iii) $x(n) = \delta(n-k)$

$T[x(n)] = ?$ "Respuesta al impulso"

$T[x(n)] = g(n)x(n) = g(n)\delta(n-k)$

b) $T[x(n)] = \sum_{k=0}^{n} x(k)$

i) $T[a(n) + b(n)] = \sum_{k=0}^{n}[a(k) + b(k)]$

$= \sum_{k=0}^{n} a(k) + \sum_{k=0}^{n} b(k)$

$= T[a(n)] + T[b(n)]$

$$= T[a(n)] + T[b(n)]$$

$$T[\lambda x(n)] = \sum_{k=0}^{n} \lambda x(k)$$

$$= \lambda T[x(n)]$$

Es lineal! :)

ii) $T \circ D_j [x(n)] = \sum_{k=0}^{n} x(k-j)$

$$D_j \circ T[x(n)] = \sum_{k=0}^{n-j} x(k)$$

$$T[D_j[x(n)]] = T[y(n)] \qquad y(n) = D_j[x(n)] = x(n-j)$$

$$= \sum_{k=0}^{n} y(k)$$

$$= \sum_{k=0}^{n} x(k-j)$$

$$D_j[T[x(n)]] = D_j[y(n)] \qquad y(n) = \sum_{k=0}^{n} x(k)$$

$$= y(n-j)$$

$$= \sum_{k=0}^{n-j} x(k)$$

No es invariante por traslación.

iii) $T[\delta_j(n)] = \sum_{k=0}^{n} \delta_j(k) = \sum_{k=0}^{n} \delta(k-j)$

$$= \begin{cases} 1 & \text{si } 0 \leq j \leq n \\ 0 & \text{cc} \end{cases}$$

c) i) $T[x(n)] = x(n-n_0)$

$$T[a(n)+b(n)] = a(n-n_0) + b(n-n_0)$$

$$= T[a(n)] + T[b(n)]$$

$$T[\lambda x(n)] = \lambda x(n-n_0) \qquad \text{Es lineal}$$

$$= \lambda T[x(n)]$$

ii) $D_k \circ T[x(n)] = D_k[x(n-n_0)]$

$$= x(n-k-n_0)$$

$$T \circ D_k [x(n)] = T[x(n-k)]$$

$$= x(n-n_0-k)$$

Es invariante!

iii) $T[\delta_j(n)] = \delta_j(n-n_0)$

$$= \begin{cases} 1 & \text{si } n-n_0 = j \\ 0 & \text{cc} \end{cases}$$

$$= \begin{cases} 1 & \text{si } n = n_0+j \\ 0 & \text{cc} \end{cases}$$

$$= \delta_{n_0+j}(n)$$

d) $T[x(n)] = \sum_{k=n-n_0}^{n+n_0} x(k)$

i) $T[a(n)+b(n)] = \sum_{k=n-n_0}^{n+n_0} a(k) + \sum_{k=n-n_0}^{n+n_0} b(k)$

$$= T[a(n)] + T[b(n)]$$

$$T[\lambda x(n)] = \lambda \sum_{k=n-n_0}^{n+n_0} x(k) = \lambda T[n]$$

<u>Es lineal!</u>

ii) $T \circ D_j [x(n)] = \sum_{k=n-j-n_0}^{n-j+n_0} x(k)$

$$D_j \circ T[x(n)] = D_j\left[\sum_{k=n-n_0}^{n+n_0} x(k)\right]$$

$$= \sum_{k=n-j-n_0}^{n-j+n_0} x(k)$$

<u>Es invariante!</u>

iii) $T[\delta_j(n)] = \sum_{k=n-n_0}^{n+n_0} \delta_j(k)$

$$= \begin{cases} 1 & \text{si } -n_0 \leq j-n \leq n_0 \\ 0 & \text{cc.} \end{cases}$$

e) $T[x(n)] = e^{-|x(n)|}$

i) $T[a(n)+b(n)] = e^{-|a(n)+b(n)|}$

No es lineal! el módulo no se distribuye,

y $e^{a+b} = e^a e^b \neq e^a + e^b$

ii) $T \circ D_j[x(n)] = T[x(n-j)]$

$$= e^{-|x(n-j)|}$$

$D_j \circ T[x(n)] = D_j[e^{-|x(n)|}]$

$$= e^{-|x(n-j)|}$$

Es invariante!

iii) $T[\delta_j(n)] = e^{-|\delta_j(n)|} = \begin{cases} e^{-1} & \text{si } n = j \\ 1 & \text{ce} \end{cases}$

f) $T[x(n)] = a x(n) + b$

i) $T[a(n) + b(n)] = a\{a(n) + b(n)\} + b$

$$= a\, a(n) + a\, b(n) + b$$

No es lineal!

ii) $D_j \circ T[x(n)] = D_j[a x(n) + b]$

$$= a x(n-j) + b$$

$T \circ D_j[x(n)] = T[x(n-j)] = a x(n-j) + b$

Es invariante!

iii) $T[\delta_j(n)] = a \delta_j(n) + b = \begin{cases} a+b & \text{si } j = n \\ b & \text{ce} \end{cases}$

g) i) $T[a(n) + b(n)] = n^2 [a(n) + b(n)]$

$$= n^2 a(n) + n^2 b(n) = T[a(n)] + T[b(n)]$$

$T[\lambda a(n)] = \lambda n^2 a(n) = \lambda T[a(n)]$ //

Es lineal!

ii) $D_j \circ T[x(n)] = D_j[n^2 x(n)] = (n-j)^2 x(n-j)$

$$T \circ D_j[x(n)] = T[x(n-j)] = n^2 x(n-j)$$

¡No es invariante!

iii) $T[\delta_j(n)] = n^2 \delta_j(n) = \begin{cases} j^2 & \text{si } n=j \\ 0 & cc \end{cases}$

h) $T[x(n)] = \sum_{k=-\infty}^{n} x(k)$ Asumo $\sum_{k=-\infty} x(k)$ converge $\forall n$.

i) $T[a(n)+b(n)] = \sum_{k=-\infty}^{n} \{a(k)+b(k)\}$

$$= \sum_{k=-\infty}^{n} a(k) + \sum_{k=-\infty}^{n} b(k)$$

$$= T[a(n)] + T[b(n)]$$

$$T[\lambda x(n)] = \lambda \sum_{k=-\infty}^{n} x(k) = \lambda T[x(n)]$$

Es lineal!

ii) $T \circ D_j[x(n)] = T[x(n-j)]$

$$= \sum_{k=-\infty}^{n} x(k-j) \overset{cv}{=} \sum_{k=-\infty}^{n-j} x(k)$$

$$D_j \circ T[x(n)] = D_j\left[\sum_{k=-\infty}^{n} x(k)\right]$$

$$= \sum_{k=-\infty}^{n-j} x(k)$$

Es invariante!

iii) $T[\delta_j(n)] = \sum_{k=-\infty}^{n} \delta_j(k) = \begin{cases} 1 & \text{si } j \leq n \\ 0 & c \end{cases}$

i) i) $T[a(n)+b(n)] = \sum_{k=0}^{N-1} (a(k)+b(k)) e^{-\frac{2\pi i n k}{N}}$

$$= \sum_{k=0}^{N-1} a(k) e^{-\frac{2\pi i n k}{N}} + \sum_{k=0}^{N-1} b(k) e^{-\frac{2\pi i n k}{N}}$$

$$= T[a(n)] + T[b(n)]$$

$$T[\lambda a(n)] = \sum_{k=0}^{M-1} \lambda a(k) e^{\frac{-2\pi i n k}{M}}$$

$$= \lambda T[a(n)]$$

Es lineal!

ii) $T \circ D_j [x(n)] = T(x(n-j))$

$$= \sum_{k=0}^{M-1} x(k-j) e^{\frac{-2\pi i n k}{M}}$$

$$D_j \circ T[x(n)] = D_j\left[\sum_{k=0}^{M-1} x(k) e^{\frac{-2\pi i n k}{M}}\right]$$

$$= \sum_{k=0}^{M-1} x(k) e^{\frac{-2\pi i (n-j) k}{M}}$$

$$= \sum_{k=0}^{M-1} x(k) e^{\frac{-2\pi i n k}{M} + \frac{-2\pi i (-j) k}{M}}$$

No es invariante!

iii) $T[\delta_j(n)] = \sum_{k=0}^{M-1} \delta_j(k) e^{\frac{-2\pi i n k}{M}}$

$$= \begin{cases} e^{\frac{-2\pi i n j}{M}} & \text{si } 0 \leq j \leq M-1 \\ 0 & cc \end{cases}$$

6) b) $M_1 \times N_1$ y $M_2 \times N_2$

$x(n_1, n_1)$ $\quad h(n_2, n_2)$

Queremos ver cuál es el soporte de $x(n_1, n_1) * h(n_2, n_2)$

$$x(n_1, n_1) * h(n_2, n_2) = \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} x(k_1, k_2) h(n_2 - k_1, n_2 - k_2) = (1)$$

Como $x(n_1, n_1)$ tiene soporte en $M_1 \times N_1$, $0 \leq k_1 \leq M_1$ y $0 \leq k_2 \leq N_1$

Como $x(m_1, m_1)$ tiene soporte en $M_1 \times N_1$, $0 \leq K_1 \leq M_1$ y $0 \leq K_2 \leq N_1$

$$(1) = \sum_{K_1=0}^{M_1} \sum_{K_2=0}^{N_1} x(K_1, K_2)\, h(m_2 - K_1, m_2 - K_2)$$

Quiero ver en qué valores deben estar m_2 y m_2 para que $\underbrace{0 \leq m_2 - K_1 \leq M_2}_{(2)}$ y $\underbrace{0 \leq m_2 - K_2 \leq N_2}_{(3)}$

(2): $K_1 \leq m_2 \leq M_2 + K_1$

Pero $0 \leq K_1 \leq M_1$, es decir:

$0 \leq m_2 \leq M_1 + M_2$

(3): $K_2 \leq m_2 \leq N_2 + K_2$

Pero $0 \leq K_2 \leq N_1$, entonces:

$0 \leq m_2 \leq N_1 + N_2$

Es decir, el soporte de $x * h$ es

$$(M_1 + M_2) \times (N_1 + N_2)$$

7) $T[x(m)] = K\, x(m)$ ← Esto vale para algún K

T LSI, $x(m) = e^{iwm}$ $w \in \mathbb{R}, m \in \mathbb{Z}$

↑

lineal e invariante

$$h(m) = \sum_{-\infty}^{+\infty} h(K)\, \delta(K - m) = h * \delta$$

$k = -\infty$

$$T[x(n)] = \sum_{k=-\infty}^{+\infty} h(k)\, T[\delta(k-n)]$$

$$T[e^{iwn}] = \sum_{k=-\infty}^{+\infty} e^{iwk}\, T[\delta(k-n)]$$

?

8. Demostrar las siguientes propiedades de la convolución discreta:

   a) $h(n) * u(n) = u(n) * h(n)$ (conmutativa)
   b) $h(n) * [a_1 u_1(n) + a_2 u_2(n)] = a_1 h(n) * u_1(n) + a_2 h(n) * u_2(n)$ (distributiva)
   c) $h(n) * u(n - n_0) = h(n - n_0) * u(n)$ (shift invariant)
   d) $h(n) * [u_1(n) * u_2(n)] = [h(n) * u_1(n)] * u_2(n)$ (asociativa)
   e) $h(\cancel{n}) * \delta(n) = h(\cancel{n}) h(n) * \delta(n - n_0) = h(n - n_0)$ ← Enunciado corregido

$n - n_0$

8) a) $h(n) * u(n) = \sum_{k=-\infty}^{+\infty} h(k)\, u(n-k)$

CV: $j = n - k$

$$= \sum_{j=-\infty}^{+\infty} h(n-j)\, u(j) \quad //$$

b) $h(n) * (\lambda_1 u_1(n) + \lambda_2 u_2(n))$

$$= \sum_{k=-\infty}^{+\infty} h(k) [\lambda_1 u_1(n-k) + \lambda_2 u_2(n-k)]$$

$$= \sum_{k=-\infty}^{+\infty} \lambda_1 h(k) u_1(n-k) + \sum_{k=-\infty}^{+\infty} \lambda_2 h(k) u_2(n-k)$$

$$= \lambda_1 h(n) * u_1(n) + \lambda_2 h(n) * u_2(n)$$

c) $h(n) * u(n-n_0)$

$$= \sum_{k=-\infty}^{+\infty} h(k)\, u(n+n_0-k)$$

CV: $j = n+n_0-k$

$$= \sum_{k=-\infty}^{+\infty} h(n+n_0-j)\, u(j)$$

$$= h(n-n_0) * u(n)$$

d) $\left(h(n) * u_1(n)\right) * u_2(n)$

$$= \sum_{k_2=-\infty}^{+\infty} \left(\sum_{k_1=-\infty}^{+\infty} h(k_1) u_1(n-k_1) \right)(k_2)\; u_2(n-k_2)$$

$$= \sum_{k_2=-\infty}^{+\infty} \sum_{k_1=-\infty}^{+\infty} h(k_1) u_1(k_2-k_1)\, u_2(n-k_2)$$

$$= \sum_{k_1=-\infty}^{+\infty} h(k_1) \left\{ \sum_{k_2=-\infty}^{+\infty} u_1(k_2-k_1)\, u_2(n-k_2) \right\}$$

CV: $r = k_2 - k_1$ $\qquad n-r-k_1 = n-k_2$

$$= \sum_{k_1=-\infty}^{+\infty} h(k_1) \sum_{r=-\infty}^{+\infty} u_1(r)\; u_2((n-k_1)-r)$$

$$= \sum_{k_1=-\infty}^{+\infty} h(k_1) \left\{ \sum_{r=-\infty}^{+\infty} u_1(r)\, u_2(n-r) \right\}(n-k_1)$$

$$= h(n) * \left(u_1(n) * u_2(n)\right)$$

e) $\underset{(1)}{h(n-n_0) * \delta(n)} = \underset{(2)}{h(n) * \delta(n-n_0)} = \underset{(3)}{h(n-n_0)}$

(1): $\displaystyle\sum_{k=-\infty}^{+\infty} h(k-n_0)\, \underbrace{\delta(k-n)}_{1 \text{ si } k=n} = h(n-n_0)$

(2): $\displaystyle\sum_{k=-\infty}^{+\infty} h(k)\, \delta(k-n-n_0) = h(n-n_0)$ //