

AREC 615 Problem Set: Linear Programming

This problem set covers the material from the introduction to linear programming through the simplex method, duality, and sensitivity analysis. Questions vary in difficulty and format. **Show your work and justify your answers.** Where appropriate, use R or Excel to solve the problems and include outputs as appendices.

1 LP-by-hand and Interpretation (On Paper)

A small farm grows **heirloom tomatoes** and **bell peppers**.

- Each acre of tomatoes yields a **profit of \$60**, while each acre of peppers yields **\$40**.
- The farm has **8 acres** of suitable land available in total.
- Additionally, tomatoes require **twice as much irrigation** as peppers. Due to water availability, the total irrigation capacity is equivalent to **10 acre-units** of pepper irrigation.

Let:

- x_1 be the number of acres planted with tomatoes
- x_2 be the number of acres planted with peppers

Answer the following:

- a. Formulate the linear program to maximize profit subject to the land and irrigation constraints. Define the decision variables, objective function, and constraints clearly.
- b. Graph and indicate the feasible region and plot one of the isoprofit lines. Identify the optimal solution graphically.
- c. Determine the optimal solution algebraically. What is the optimal objective value?
- d. Identify the **binding** constraints at the optimum.
- e. Write out the tableau for the initial simplex method iteration.
- f. Perform one pivot operation (select entering and leaving variables). Show the updated tableau. Continue iterating until the optimal solution is reached. Identify the optimal basis and solution values.
- g. Compute the **shadow prices** (dual values) associated with the constraints. Interpret the shadow prices in economic terms.

2 Distribution Optimization (Use Computer)

You are managing logistics for a food distribution network. There are **2 warehouses** (W1 and W2) and **5 cities** (C1 to C5) that need weekly shipments of food. Each warehouse can ship to each city — a total of **10 possible routes**. You want to meet the cities' demands at minimum cost while respecting warehouse capacities and various restrictions.

The **shipping cost per unit** from each warehouse to each city is shown below:

	C1	C2	C3	C4	C5
W1	4	6	8	7	5
W2	5	4	7	6	6

Let x_{ij} be the number of units shipped from warehouse i to city j .

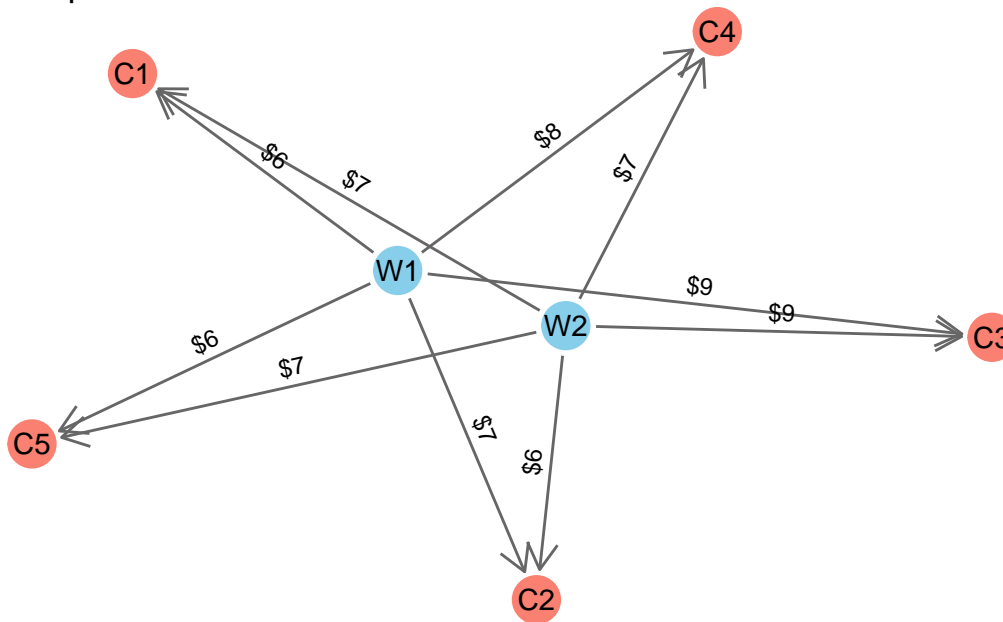
Each **city** has a **minimum demand** that must be met (in units):

- C1: 80, C2: 70, C3: 60, C4: 90, C5: 50

Each **warehouse** has a **maximum capacity** (in units):

- W1: 200, W2: 180

Transportation Network



a. **Formulate** this as a linear program:

- Define all **decision variables**.
- Write out the **objective function**.
- Specify all **7 constraints** clearly. Don't forget the nonnegativity constraints.

b. **Solve** the model using R (e.g., `lpSolve`, `ompr`) or Excel Solver.

- Report the optimal shipment plan and total cost.
- Conduct a sensitivity analysis including ranging analysis on the objective coefficients and constraints.
- Insert a table summarizing the sensitivity analysis results.
- Provide your code or spreadsheet setup in an appendix.

c. **Interpret** the dual values (shadow prices) for the city demand constraints.

- Generate a table showing the shadow prices for all constraints.
- What do the shadow price tell you? Imagine explaining to a non-technical manager.
- If you could increase the capacity of one warehouse by 10 units, which one would you choose and why?

d. Suppose you could build a new warehouse with a capacity of 100 units at a fixed cost.

- Which cities should it serve to minimize total costs?

- Modify the LP to include this new warehouse and solve. You can make up new shipping costs.
 - Re-solve and explain how the solution changes.
 - Can you use your results to provide guidance on the amount the company should be willing to pay for the new warehouse?
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3 Dual Formulation and Interpretation

Consider the following primal LP:

$$\begin{aligned}
 \max \quad & 100x_1 + 150x_2 \\
 \text{s.t.} \quad & x_1 + 2x_2 \leq 10 \\
 & 3x_1 + x_2 \leq 12 \\
 & x_1, x_2 \geq 0
 \end{aligned}$$

- Formulate the **dual** problem.
- Solve the **dual** using R or Excel. What are the optimal dual variable values?
- Verify **strong duality**: does the optimal value of the dual match the primal?
- Check **complementary slackness**. Which constraints are binding, and how does this relate to the dual?