Introduction to Mathematical Programming

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1 Introductions

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- Who am I?
- Student introductions: What year and what field? Research interests?
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2 Course Orientation & Motivation

- Why optimization in economics?
  - Many economic problems are decision problems under constraints where some agent is pursuing an objective and faces constraints.
  - Examples:
    * A farmer allocates land and water across crops.
    * A firm chooses output facing costs and regulations.
    * A planner designs climate policy subject to resource limits and production technologies.
    * A consumer maximizes utility subject to income.
- Analytical solutions not always possible → need **numerical/computational methods**. Closed form solutions do not exist.
  - We will learn to optimize both analytically and numerically.
- Mathematical programming provides a structured, computer-implementable way to model these decisions.

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3 What is a Mathematical Program?

- **General Form**:

$$\max_x f(x) \quad \text{s.t. } g_i(x) \in S_1,\ x \in S_2$$

- Components:
  - **Decision variables**: choices (e.g., acres planted, production levels).
  - **Objective function**: what we optimize (profit, cost, utility).
  - **Constraints**: resource limits, technology, budgets.
    * These constraints can be that functions of x lie within some boundary or that the individual x's lie in some boundary

Example (crop mix):

- Variables: acres in wheat (x_1), corn (x_2).
- Objective: maximize profit.
- Constraints: land, labor, water.

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4 Uses of Mathematical Programming (McCarl §1.3)

`What does McCarl say are the uses of math programming?`

1. **Problem Insight Construction**
   - State problem carefully and really understand the moving parts
   - Clarify objectives, constraints, and trade-offs.
   - Example: Writing down a water allocation model forces us to quantify resource limits. It also forces one to write down the functional relationships between variables, which may be a simplification or approximation of a physical or natural phenomenon: hydrology, population growth functions, atmospheric models.
2. **Numerical Applications**
   - **Prescription of Solutions**
     - Goal: Recommend an **optimal plan** given data, objectives, and constraints.
     - Example: Farm crop mix — how many acres of wheat, corn, soy to maximize profit?
   - **Prediction of Consequences**
     - Goal: Forecast **outcomes under scenarios**.
     - Example: If fertilizer availability drops 20%, what happens to optimal output and profit?
   - **Demonstration of Sensitivity**
     - Goal: Show how solutions **shift with parameters**.
     - Example: How does optimal irrigation use change as water prices rise?
3. **Algorithm Development**
   - solution algorithms… not that important for most of us.

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5 Preview: How Optimization Methods are Used in Research

- **Agricultural & Resource Economics**
  - Crop mix and land allocation models under climate or policy changes.
  - Water allocation across sectors or regions.
  - Forestry and rangeland management under biological growth constraints.
- **Environmental Economics**
  - Policy design problems: cap-and-trade, carbon taxes, renewable energy portfolios.
  - Nonpoint pollution control, groundwater management.
  - Climate-economy integrated assessment models.
- **Industrial Organization & Production**
  - Firm cost minimization and production planning.
  - Transportation and logistics networks.
  - Supply chain optimization.
- **Public Policy & Development**
  - Food aid allocation and trade policy.
  - Health policy interventions under budget constraints.

- Energy access and electrification planning.

Takeaway: *Mathematical programming is not just a classroom exercise; it's a core toolkit for applied economic research.* The methods we learn will connect directly to how we ask and answer research questions in environmental, agricultural, and resource economics.