# Problem Set: Nonlinear Programming

**Instructions**

In this problem set, you will implement three numerical methods for solving nonlinear equations:

- Bisection Method
- Fixed Point Iteration
- Newton's Method

For each problem:

- Write your own implementation from scratch (no use of built-in solvers like `uniroot()` or `nleqslv()`).
- Use a convergence criterion of $|x_{k+1} - x_k| < 10^{-6}$.
- Include a plot of the function over an appropriate interval.
- Report the final root estimate, number of iterations, and whether convergence was successful.

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## 1 Bisection Method

Let

$$f(x) = \cos(x) - x$$

Use the bisection method to solve $f(x) = 0$ on the interval $[0, 1]$.

### 1.1 Tasks

a. Confirm that $f(0) \cdot f(1) < 0$.
b. Implement the bisection method.
c. Plot $f(x)$ over $[0, 1]$ and mark the root.
d. Report: number of iterations, final estimate, and value of $f(x)$ at the root.

   **Comment**: Explain why is convergence guaranteed given the function?

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## 2 Fixed Point Iteration

Consider the function:

$$x = g(x) = \sqrt{1 + x}$$

## 2.1 Tasks

a. Define the iteration $x_{n+1} = g(x_n)$.
b. Use $x_0 = 1$ as the starting value.
c. Plot $g(x)$ and the 45-degree line $y = x$ over $[0, 3]$.
d. Determine whether the iteration converges.
e. Try other initial values (e.g., $x_0 = 0.5$, $x_0 = 3$) and compare behavior.
f. Check the derivative $g'(x)$ at the fixed point. What does this tell you about convergence?

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# 3 Problem 3: Newton's Method

Let

$$f(x) = x^3 - 2x + 2$$

## 3.1 Tasks

a. Implement Newton's method.
b. Use $x_0 = 0$ and $x_0 = -1.5$ as two different initial values.
c. Plot $f(x)$ and its derivative over $[-3, 3]$.
d. Report the outcome of the iteration for each starting point.
e. Compare the convergence (or divergence) behavior. What role does the choice of initial guess play?
f. Why does the method fail to move when starting at $x_0 = 0$? explain.