AREC 615 Problem Set: Linear Programming

This problem set covers the material from the introduction to linear programming through the simplex method, duality, and sensitivity analysis. Questions vary in difficulty and format. **Show your work and justify your answers.** Where appropriate, use R or Excel to solve the problems and include outputs as appendices.

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1 LP-by-hand and Interpretation (On Paper)

A small farm grows **heirloom tomatoes** and **bell peppers**.

- Each acre of tomatoes yields a **profit of $60**, while each acre of peppers yields **$40**.
- The farm has **8 acres** of suitable land available in total.
- Additionally, tomatoes require **twice as much irrigation** as peppers. Due to water availability, the total irrigation capacity is equivalent to **10 acre-units** of pepper irrigation.

Let:

- x_1 be the number of acres planted with tomatoes
- x_2 be the number of acres planted with peppers

Answer the following:

a. Formulate the linear program to maximize profit subject to the land and irrigation constraints. Define the decision variables, objective function, and constraints clearly.
b. Graph and indicate the feasible region and plot one of the isoprofit lines. Identify the optimal solution graphically.
c. Determine the optimal solution algebraically. What is the optimal objective value?
d. Identify the **binding** constraints at the optimum.
e. Write out the tableau for the initial simplex method iteration.
f. Perform one pivot operation (select entering and leaving variables). Show the updated tableau. Continue iterating until the optimal solution is reached. Identify the optimal basis and solution values.
g. Compute the **shadow prices** (dual values) associated with the constraints. Interpret the shadow prices in economic terms.

2 Distribution Optimization (Use Computer)

You are managing logistics for a food distribution network. There are **2 warehouses** (W1 and W2) and **5 cities** (C1 to C5) that need weekly shipments of food. Each warehouse can ship to each city — a total of **10 possible routes**.

The **shipping cost per unit** from each warehouse to each city is shown below:

	C1	C2	C3	C4	C5
W1	4	6	8	7	5
W2	5	4	7	6	6

	C1	C2	C3	C4	C5

Let x_{ij} be the number of units shipped from warehouse i to city j.

Each **city has a minimum demand** that must be met (in units):

- C1: 80, C2: 70, C3: 60, C4: 90, C5: 50

Each **warehouse has a maximum capacity** (in units):

- W1: 200, W2: 180

Due to storage and delivery constraints, the following **additional restrictions** apply:

1. No more than 40 units can be shipped from W1 to C3.
2. At least 20 units must be shipped from W2 to C5.
3. W1 must supply at least 50 units to C1 and C2 combined.
4. W2 must supply at least 100 units total.
5. Total shipment to C4 must not exceed 100 units.
6. No more than 60 units from W2 to C2.
7. Shipments from W2 to C1 must be at least 10.
8. W1 must ship at least 25 units to C5.
9. Shipments from W1 to C2 must not exceed 30.
10. Total shipments from W1 to all cities must be at least 150.
11. Total shipments to C3 must be at least 70.
12. No more than 100 units may be shipped to C5 in total.
13. Combined shipments from both warehouses to C1 must not exceed 90.
14. W2 must ship at least 50 units to C4 and C5 combined.
15. W1 cannot ship to C1 and C4 simultaneously (use a big-M binary constraint).
16. Either W1 or W2 ships to C3, not both (use binary variables).
17. At most one of the warehouses can send more than 60 units to C2.
18. At least one of the warehouses must send at least 30 units to C5.
19. If W1 ships more than 50 units to C2, W2 must ship less than 40 to C4.
20. Total cost must not exceed $2,300.

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a. **Formulate** this as a linear program:

- Define all **decision variables**.
- Write out the **objective function**.
- Specify all **20 constraints** clearly.

b. **Solve** the model using R (e.g., `lpSolve`, `ompr`) or Excel Solver.

- Report the optimal shipment plan and total cost.
- Provide your code or spreadsheet setup in an appendix.

c. **Interpret** the dual values (shadow prices) for the city demand constraints.

- What does the shadow price tell you for C4 and C5?

d. Suppose the total budget is reduced to $2,200.

- Re-solve and explain how the solution changes.
- What happens to the binding constraints?

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3 Dual Formulation and Interpretation

Consider the following primal LP:

$$\begin{aligned} \max \quad & 100x_1 + 150x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \leq 10 \\ & 3x_1 + x_2 \leq 12 \\ & x_1, x_2 \geq 0 \end{aligned}$$

3.0.1 (a)

Formulate the **dual** problem.

3.0.2 (b)

Solve the **dual** using R or Excel. What are the optimal dual variable values?

3.0.3 (c)

Verify **strong duality**: does the optimal value of the dual match the primal?

3.0.4 (d)

Check **complementary slackness**. Which constraints are binding, and how does this relate to the dual?