Ph.D. Research Proposal

**Doctoral Program in Civil Engineering**

**Influential Parameters Leading to Large Dynamic Amplification of Bridges**

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****Abstract****

Brief summary of research.

Keywords

Keywords

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# Introduction

There are over 600,000 bridges in the United States, traversed by hundreds of millions of vehicles every day. The manner in which the bridges react to those vehicles is not well understood, and although decades of research have helped to reduce that knowledge gap, the most apparent conclusion has been that a bridge’s response to moving vehicles is complex and dependent on numerous and interdependent factors. Despite this, design and evaluation specifications provide the means to account for the dynamics of moving vehicles by amplifying vehicle loads by a prescribed factor during analysis. These factors may have been sufficient in the past, but as design codes become less conservative and new loading scenarios are made possible by burgeoning technologies, the current approach may no longer prove adequate.

Vehicle-bridge interaction (VBI) is the name often given to the dynamic behavior of bridge and vehicle and is used in reference to the principles that govern it. The interaction can be described generally as a process whereby the kinetic energy of the forward motion of the vehicle is directed into the bridge. As a vehicle moves over a road surface that is not perfectly smooth, the vehicle must move up and down and may even begin to “bounce” as the vehicle’s natural modes of vibration begin to resonate. The vertical motion of the vehicle is resisted by the bridge and thus imparts varied forces with peak amplitudes in excess of its weight. Furthermore, the persistent and varied loading gives momentum to the mass of the bridge, thereby exciting the bridge’s natural frequencies.

Therefore, in addition to the weight of the vehicle, the bridge also experiences force due to the “bounce” of the vehicle as well as force due to the movement of the bridge mass. While much study has been conducted on the factors that affect the overall amplification, this research will seek to first understand the mechanisms behind both of those amplification components before identifying conditions that maximize each component. Further investigation will examine which component contributes most to the bridge response, realizing that it may likely change, depending on the structure, loading conditions and location of response. This knowledge will provide the means of identifying structures that are more likely to experience unusually high dynamic amplifications and those characteristics that should be avoided or corrected, as well as vehicle types and traffic conditions that exacerbate amplifications. Special consideration will be paid to repeated vehicles (i.e. platooning) and their effect on a bridge’s dynamic response.

# State of the Art

Structural analysis of traffic loading on bridges is performed primarily for the purposes of design or evaluation. The methods of analysis and technological tools historically available have limited the analyses to evaluate the traffic as static loads (i.e. location and magnitude of load remain constant with time). The industry accounts for this disparity by applying a factor to the static load or response, thereby attempting to increase the calculated response to that which the bridge would actually experience from moving vehicular loads.

In this paper, the factor may be referred to as an impact factor (IM) or dynamic amplification factor (DAF). The factors can be defined as follows:

Therefore, the IM is just . The total live load response can be computed by the following:

or

Where Rsta is the static load effect which is amplified by (1 + IM) or the DAF.

The magnitude of this factor (IM or DAF) differs depending on the reference code being used, but in many cases, has been developed from extensive surveying of existing structures and the amplifications they experience.

## Dynamic Amplification in Bridge Codes

The various design and evaluation codes handle dynamic amplification in slightly different ways, but they all specify a factor that is to be applied to the loading model. In the current AASHTO () *LRFD Bridge Design Specification* the impact factor is dependent on road surface roughness, ranging from 0.1 to 0.3. The AASHTO () *Manual for Bridge Evaluation* specifies an impact factor of 0.33.

The factors specified by the Ontario and Canadian Highway design codes are dependent on the number of vehicle axles, with lower factors for more axles. A factor of 0.25 is specified for vehicles with 3 or more axles. Similarly, the Australian code specifies factors based on load type. An impact factor of 0.4 is specified for wheel and axle loads, and 0.35 for triaxle truck and lane load. The British code includes an IM of 0.25 in its two design load models.

The Chinese code’s factors are a function of bridge’s first natural frequency, with a lower bound of 0.05 and an upper bound of 0.45. A bridge with a first natural frequency below 5 Hz would have a specified impact factor less than 0.25.

The New Zealand, European, and Japanese codes all specify factors as a function of bridge span length with the factors decreasing with increased span length. The New Zealand code has a maximum IM of 0.30 for span lengths less than 12 meters. The equation for IM specified by the Japanese code is different for different bridge types. A steel or RC bridge with a span length of 40 feet would be assigned an IM equal to 0.32. The equation specified by the European code differs dependent on the number of lanes. For a single-lane bridge, the impact factor may range from 0.4 to 0.7, while that for a two-lane bridge ranges from 0.1 to 0.3.

## Experimental Evaluation of Amplification Factors

A large number of field tests were carried out in various countries over the years, in-part, for the development of bridge design codes. In the 1950’s, AASHTO sponsored a major investigation, whereby 18 bridges were constructed for the purpose of testing and determining the dynamic effects of moving vehicles on the bridges. This study concluded that the dynamic amplification generally increases with increased vehicle speed, is sensitive to vehicle suspension performance, and initial oscillations of the vehicle produce a large amount of uncertainty in the dynamic response of the bridge. The maximum dynamic amplification factor recorded from these tests was 1.63 for displacements and 1.41 for strains, however, 95% of the measured amplification factors fell below the value specified by the current code at the time. (Paultre, Chaallal, and Proulx 1992)

In 1956 and 1957, field tests were completed on 52 bridges in Canada, with specific attention paid to dynamic amplification. Amplification factors were observed as high as 1.75, while values near 1.30 were more typically observed. These tests concluded that the amplification factors were higher for flexible bridges and were greatly affected by the road surface roughness and irregularities on the bridge as well as approach. In fact, Write and Green recommended that irregularities in the road profile should be eliminated, as this was more influential on dynamic amplification than the many other structural parameters considered (Paultre, Chaallal, and Proulx 1992). Additional tests were carried out in Canada in the 1970s and 1980s. Amplifications factors were obtained as high as 1.85 with higher factors obtained from bridges with first fundamental frequencies of 2 to 5 Hz. Similar results were obtained from field tests of more than 200 bridges in Switzerland, with amplification factors as high as 1.7 for bridges with a first fundamental frequency between 2 and 4 Hz. (Deng et al. 2015)

A more recent field test was completed in Florida of a 3-span prestressed multi-girder bridge. Amplification factors were determined experimentally by measuring the bridge response from loaded five-axle trucks. The amplification factors for one truck was found to be 1.82 while that for two trucks was found to be 1.50 and were observed at higher speeds (80 km/h). The authors were able to reasonably reproduce the bridge response recorded in the field by using FE simulation and incorporating the measured road profile of the approach. By comparing responses with and without road profile, the authors were able to show that road surface irregularities have a significant impact on dynamic amplification. (Kwasniewski et al. 2006)

## Influential Parameters for Dynamic Amplification

Many analytical and experimental studies throughout the past several decades have investigated which parameters are influential to dynamic amplification. These parameters have included: road surface condition, bridge span length and natural frequency, bridge type, vehicle speed, and vehicle weight and suspension characteristics. Of these, research has shown that road surface condition, vehicle speed, and vehicle weight and suspension type have the most effect on dynamic amplification. Although some codes include span length or first natural frequency in their calculations of IM, studies have shown poor correlation between either of these parameters and dynamic amplification. Furthermore, although bridge type may have a significant impact of the dynamic behavior of a bridge, there is a wide variety of bridge types and even more varied structural characteristics within each type. (Deng et al. 2015)

Vehicle speed has been shown to have a significant effect on dynamic amplification. Generally higher amplification factors result from higher speeds. In some studies, it has been shown that there is a critical speed for which the amplification factor is a maximum. However, although the various studies generally agree that vehicle speed is an important factor, the proposed relationships between vehicle speed and amplification factors is inconsistent, suggesting that this relationship is a complicated one and dependent on other factors (Deng et al. 2015). Similarly, vehicle mass and suspension characteristics have been shown to effect dynamic amplification, but, again, no definitive relationship has been established. However, studies generally agree that the amplification factors decrease with increased vehicle weight and soft suspension with high damping (Green, Cebon, and Cole 1995; Nassif and Nowak 1995).

## Influence of Road Surface Roughness on Dynamic Amplification

Previous studies have examined the impact that the road surface has on impact factors. Many analytical studies have shown that a rough road surface may result in higher dynamic amplification. However, the studies do not agree how much road surface effects dynamic amplification, which is likely due to variety of bridge types and geometry, road profiles, and model types employed.

Most analytical studies have used a single-line beam model in which the width of the bridge is reduced to a single beam with appropriate mass and stiffness. Some of the earliest simulation work was done by Aramraks at Purdue University. In his research, single span, 2-span and 3-span bridges were represented as single-line beam models. Surface roughness was idealized as a number of half sine waves, resulting in beam accelerations as much as 10 times those obtained with a smooth road surface (Aramraks 1975).

As computing technology evolved, more complex simulations have been carried out. A simply supported box girder was modeled with a moving mass over a rough road surface that was simulated by using power spectral density functions to produce a more realistic road profile, and obtained dynamic amplification factors (DAF) as high as 3 (Inbanathan and Wieland 1987). Simulations of a 3-span continuous box-girder bridge (modeled as a beam) for a rough road surface generated a maximum DAF of 2.3 (Law and Zhu 2005). Chatterjee et al. used a single line girder model of a continuous bridge, and showed that for certain combinations of speed and frequency ratios between the vehicle and structure, the DAF could exceed 4 (Chatterjee, Datta, and Surana 1994). The effect of long term deflections in addition to road surface roughness was investigated for a simple span and a 3-span prestressed concrete bridge. The study concluded that the long-term deflections had negligible effect on amplification factors, but the road surface roughness could cause amplification factors in excess of 2 (Au, Cheng, and Cheung 2001).

The effect of road surface on bridge responses was further investigated with 3-dimensional FE models. Kou and DeWolf modeled a 4-span continuous plate-girder bridge, and examined the effect of smooth road surface versus 0.5 inch and 1 inch amplitude roughness and concluded that the road roughness had negligible effect on bridge deflections (Kou and DeWolf 1997). In contrast, 3-D FE simulations of a 3-span non-continuous bridge by Li et al. showed that the surface roughness had a large effect on dynamic amplification, especially with increased speed. Based on simulation efforts, a maximum dynamic amplification of nearly 3 was reported for poor road condition and at 70 mph, while a DAF of only 1.2 was recorded from field measurements of the bridge (H. Li, Wekezer, and Kwasniewski 2008).

Indeed, the dynamic amplification factors obtained from field measurements are consistently lower than the factors suggested by analytical research and have similarly wide variation from bridge to bridge. Cooper instrumented two bridges in England and recorded a maximum DAF of 1.42. Cooper also created a probabilistic model of DAF based on road roughness and span length that suggests a maximum mean DAF of 1.27 (Cooper 1997). Park et al. examined the effect of road roughness on dynamic amplification by testing 25 highway bridges in South Korea. None of the bridges exhibited amplification factors greater than 1.25, but their results clearly showed that the amplification factors increased with the International Roughness Index (IRI) (Park, Shin, and Chung 2005). However, further research suggests that no single measure of road roughness can accurately predict DAF because of the many other influential parameters that contribute to DAF (i.e. bridge geometry, mass and stiffness; vehicle dynamic properties; vehicle speed; etc.) (OBrien, Li, and González 2006; Y. Li, OBrien, and González 2006).

## Modeling Vehicle-Bridge Interaction

## Bridges with Excessive Vibrations

The dynamic effect of traffic on bridges is not just of concern for amplification factors or strength limit states (i.e. rating factor). Excessive vibrations may result in reduced fatigue life, as the vibrations can result in more stress cycles per loading event. Furthermore, structural vibrations should be limited so as not to upset the bridge users.

Past studies have looked at what constitutes objectionable vibrations and the characteristics that influence a human’s perception of those vibrations. In general, these studies concluded that as the displacement and frequency increase, the vibration is considered more intolerable to users. However, there is no single parameter that can predict a human’s perception of a given vibration (Gaunt and Sutton 1981). Some of the earliest work on this topic was carried out by Reiher and Meister. They developed sensitivity curves by subjecting people to vertical harmonic vibration (Reiher and Meister 1931). These sensitivity curves still provide an acceptable measure of human perception to vibrations.

AASHTO bridge specifications state that bridges should be designed to avoid psychological effects and that acceleration is the primary factor for human sensitivity to bridge deformations, but fails to provide any specific limits for vibrations (AASHTO 1998). Instead they placed limits on span-to-depth ratios and live load deflections in hopes that this would prevent unsatisfactory dynamic performance. The Ontario Highway Bridge Design Code of 1983 introduced a new serviceability limit state that was meant to control vibrations that would be objectionable to pedestrians by restricting deflections based on the first frequency of the structure (Csagoly and Dorton 1978). The Eurocode also advises bridge designers to limit vibrations to avoid the discomfort to users, but again, fails to provide guidance as to how that may be accomplished (BS 2006).

# Research Objectives and Approach

The research will fall into three broad categories:

1. Understand dynamic amplification and the mechanisms that govern it
2. Identify any shortcomings with current design and assessment methodologies
3. Examine the impact of special loading scenarios on dynamic amplification

The research will take an inductive approach, whereby efforts will first focus on understanding the phenomenon of vehicle-bridge interaction by investigating a specific structure with unusually large dynamic amplifications. The resulting knowledge and tools will be leveraged to generate generalized conclusions about dynamic amplification for a bridge population.

## Understanding dynamic amplification

Structural health monitoring along with finite element simulation will be used to explore the principles and mechanisms behind dynamic amplification. The process of structural identification will be leveraged to incorporate sensing data from a real structure into an FE model. That process will first be conducted to obtain a model that is an accurate representation of the bridge. It will be conducted a second time to ensure the model can accurately simulate a moving vehicle and its interaction with the bridge.

A variety of sensors will be utilized to capture the acceleration and strain of the structure under operating conditions. Care must be taken to sample at rates high enough to capture the modes of interest and avoid aliasing. From the gathered data and through modal processing methods the shapes and frequencies of the natural modes of vibration of the structure will be calculated. Strain readings will be analyzed to determine the operational stress of the bridge and estimate the dynamic amplification.

Characterization of a test vehicle moving over the bridge will occur by instrumenting a fully loaded truck. The accelerations of the test vehicle will provide information on the vehicle-bridge interaction as well as be used to calculate the truck’s first mode of vibration which can be used to characterize the vehicle’s suspension.

The FE model will be constructed based on construction documents that detail the geometry and material of the various bridge components. Uncertainties in the model (e.g. material properties, connections, support conditions, etc.) will be addressed by “calibrating” the model with the test results, whereby model properties will be altered until the simulation results align with the test results. In this way we can be confident that the FE model is truly representative of the actual structure.

Once a representative model has been created, the testing scenario can be simulated by performing… vehicle model run across the model …

The goal is to produce responses similar to those witnessed during the test. It is likely that the model will not readily produce matching responses and will have to be adjusted.

## Parametric Study

Features of the model will be changed or removed in an effort to find out to which parameters large dynamic amplifications are sensitive. The hope is to find out what types of bridges are vulnerable to this phenomenon and how to prevent it or modify existing structures to eliminate the phenomenon. These parameters will consist of characteristics of the structure, vehicle/traffic, and road surface.

## Methods of Simulating Vehicle-Bridge Interaction

Through the efforts of simulating the behavior of the bridge under traffic loading, it is expected that different model types and methods will be employed. As a result, it may become evident that certain methods and model types are better able to simulate this interaction. Further investigation will be undertaken to see if there are simpler methods of modeling this interaction, that produce result comparable to the more complex methods already employed. It is hoped, that for certain scenarios, an acceptable modelling method can be used that the “typical” engineer could understand and implement.

## Implications of Dynamic Amplification

Scenarios will be posed that demonstrate ways in which dynamic amplifications result in problems for the structure. Many of the same methods used in identifying the behavior of the viaduct will again be employed for these scenarios, thus showing how the mechanisms involved with the problem can be identified, the degree of the problem in relation to a limit state quantified, and possible solutions hypothesized and tested.

# Work Plan

## Task 1 – Characterization of structure by field testing

To understand the structure, it is necessary to gather data from the structure. To do this a variety of sensors will be placed on the bridge to record operational strains and accelerations. These baseline responses will give us an idea of the structure’s responses to typical traffic loading and will show if this bridge is indeed experiencing excessive vibrations as reported by motorists. The measurements will also be used for finite element (FE) model validation and calibration.

1. Perform preliminary field test of viaduct to determine which regions experience the largest vibrations. Accelerometers will be installed on the bridge at various locations along its entire length. The magnitude of these responses, relative to one another, will be used to determine which portions of the structure will receive greater scrutiny in future investigations.
2. Design and perform field test of selected portion/portions of viaduct. Instrumentation plan will be designed to capture maximum response while providing adequate spatial distribution to characterize the shape of natural modes of vibration (mode shape) and nature of interaction between superstructure and substructure. Both acceleration and strain will be monitored.
3. Process and Interpret data acquired from field test. Characterization of vibrations will include computation of mode shapes and corresponding frequencies, examination of the frequency content, and evaluation of strength, service and comfort limit states. Dynamic amplification will be estimated by utilizing data filtering methods.
4. Create, validate and calibrate FE model/models of the structure. Models with varying level of resolution and complexity may be required for different purposes. Model will be error screened and validated by comparing behavior of model to that which would be expected for the structural form and to those observed in the field. Model calibration will be completed by deliberately adjusting model parameters until sufficient agreement is found between predicted and measured mode shapes and natural frequencies.
5. Use model to assess structural condition and performance. Investigate possible mechanisms for vibration levels. Examine degree of conservatism lost due to amplification factors greater than assumed for design.

## Task 2 – Field testing and simulation to understand vehicle-bridge interaction

Additional testing and simulation will attempt to answer those questions about dynamic amplification that were not adequately answered by Task 1. This will include field testing whereby both the viaduct and a vehicle will be instrumented, and responses from both will be captured synchronously while the vehicle is traversing the viaduct. Additional models and new model types may be required that are better able to simulate the test scenario. The model will be leveraged to identify the manner of loading that contributes to dynamic amplification and the structural parameters that affect it.

1. Identify appropriate FE software and create appropriate model that is capable of simulating moving vehicle loads and resulting structural response and is calibrated with data from previous test.
2. Design and perform field test of the viaduct with known loading. This will require instrumentation of both the structure and a test vehicle with synchronous capture of data. The instrumented vehicle will traverse the viaduct at a variety of speeds and under different traffic conditions.
3. Process and Interpret data acquired from field test. Examine magnitude and frequency content of test vehicle’s motion on and off the viaduct. Examine magnitude and frequency content of the structure’s vibrations when the test vehicle is on and off the bridge, and the nature of the vibrations for different crossings of the test vehicle.
4. Measure the profile of the bridge road surface and that of the approaches. A reputable company will be located that is capable of measuring the roadway with sufficient resolution and accuracy and at an acceptable price and subsequently contracted. As a result, the roadway profile of every lane will be delivered and formatted such that it can be incorporated into FE simulations.
5. Simulate test scenarios and compare model results with experimental responses. Effort will be made to recreate the magnitude and pattern of vehicle and viaduct accelerations.

## Task 3 – Characterizing dynamic amplification due to vehicle-bridge interaction

Once a model is obtained that is reasonably capable of recreating those responses measured in the field, a series of multi-dimensional parametric studies (using the vehicle/structure interaction capabilities of LUSAS) will be performed in an effort to identify the structural and vehicular mechanisms that govern the distinctive behavior observed, thereby understanding if and how the situation may be remedied by altering the structure or restricting traffic.

1. Identify and rank parameters in the model that are most influential to dynamic amplification. These may include: bridge geometry, mass, stiffness and damping; profile of roadway and approaches; weight, suspension stiffness and damping of vehicle; and initial conditions of both the vehicle and the bridge. This effort will expand the scope of scrutiny from a specific bridge to a population that covers a large portion of short and medium span highway bridges.
2. Evaluate level of influence and interaction of influential parameters. Establish “problematic” ranges for parameters that result in dynamic amplification in excess of that accounted for by current practice.
3. Make recommendations for avoiding large dynamic amplifications, and methods of diagnosing and remedying existing bridges with high dynamic amplification. In this way “rules of thumb” will be developed for determining those conditions for which high dynamic amplifications may result.
4. Look into simpler methods of simulation that can produce the same results as the more complex modeling already used. Single-line and grillage models will be examined compared to full 3D FE models. Vehicle acceleration due to roadway profile alone will be solved independently and the resulting accelerations applied as loads to the structure to see if vehicle-bridge interaction needs to be explicitly modeled.

## Task 4 – Implications of traffic induced bridge dynamics

The dynamics of bridges and vehicle-bridge interaction is given little attention in current design and evaluation codes. Most only account for it by applying a factor to the static load effects and by limiting deflections to reduce excessive vibrations. This task will examine how a more thorough consideration of dynamic behavior under traffic loading will provide a better understanding of the structure’s performance.

1. For which structures is 0.33 inadequate. How effects load rating and design. Examine the effect dynamic amplification has on I-76 bridge ratings.
2. Identify special loading scenarios (i.e. platooning) that may result in large vibrations. FE simulation tools will be used to demonstrate how this scenario can be analyzed and the conditions to be avoided so as to limit vibrations.
3. How should IM testing be conducted. Demonstrate how dynamic behavior led to damage of a bridge. A bridge that was previously tested will be reexamined to determine if the vibrations induced by traffic resulted in a crack of the concrete pier.

## Schedule

# Chapter 5 Current Progress

# Chapter 6 Conclusions

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