# Development of state space model

## Summary of assumptions and shortcomings

* Bridge is modeled as simply supported beam
* Bridge deformation is limited to a shape function that corresponds to the first bending mode of vibration
* Bridge damping is not included
* Vehicle is a single DOF sprung mass with damping

Equations of motion and state space matrices can be found in the appendix.

## Benchmarking

Functions were written according the equations of motion.

The beam was reduced to a single degree of freedom using the following shape function:

A harmonic profile was applied that had a wavelength of 30ft, and amplitude of 0.02 ft (±).

The models were run with a vehicle velocity of 55 mph.

The vehicle had the following properties:

* Mass: 100 kip
* Spring Stiffness: 10 kip/ft
* Damping Coefficient: 100%

The beam had the following properties:

* Length: 200 ft
* E x I: 576000 lb/ft2 x 700 ft4
* Mass: 11 kip/ft

The model was compared to FE results.



This state space model matches well with FE results that only include the first mode. Now compared to FE with 4 modes included:







Yet even when the first 4 bending modes are included, the state-space model is in close agreement.

Differences are expected since the state space model assumes that the bridge can only deform as described by the shape function, but the vehicle presents a point load that does not produce deformation as described by the shape function.

### Compare state-space model with FE model for a parameter set that is more representative of actual conditions.

A more realistic profile (EB\_right\_1\_R), that was gathered from an actual bridge was used for this simulation. The profile was broken into lengths equal to the bridge length.

The bridge had the following properties:

* Mass: 2000 slinch
* Natural Frequency: 3 Hz
* Length: 140 ft
* E x I: 34.5908E12 lb-in2

The vehicle had the following properties:

* Mass: 200 slinch
* Natural Frequency: 3.2 Hz
* Spring Stiffness: 80852 lb/in
* Damping Coefficient: 20%
* Velocity: 720 in/sec





|  |  |  |  |
| --- | --- | --- | --- |
|  | SS max | FE max | % Error |
| Bridge Displacement | -0.4708 | -0.4527 | -4.0% |
| Vehicle Acceleration | -310.5 | -241.8 | -28.4% |
| Contact Force | 129753 | 124297 | -4.4% |

From the results we may conclude that the state-space model is not an exact model of bridge or vehicle response, but is sufficient for approximating the contact force and bridge response. Perhaps more importantly, this model may serve to compute a metric that better represents the impact the roadway profile has on bridge performance.

### Another comparison between state-space model and FE SLG model.

FE Model: 140ft\_span SLG. Mode 1 only included. 0% bridge damping

Matlab script: Benchmark\_statespace.m

fnb = 3; % natural frequency of bridge

fnv = 3.2; % natural frequency of vehicle

mb = 2000; % mass of bridge (slinch)

length = 140; % ft

mt = 200; % mass of vehicle (slinch)

dt = 0.2; % vehicle damping ratio

vel = 720; % vehicle velocity (in/sec)

vb\_ssmodel.bridge\_start = 320\*12;

pro\_file.name = 'ISO\_C10-300e-06\_w-2\_1.csv';

pro\_file.path = 'C:\Users\John\Projects\_Git\DAmp\profiles\artificial\model\_validation';



FE Model: 1sp\_40ft\_SLG. Mode 1 only included. 0% bridge damping

Matlab script: Benchmark\_statespace.m

fnb = 9.94; % natural frequency of bridge

fnv = 10.5; % natural frequency of vehicle

length = 40; % ft

kt = 879292; %lb/in

mt = 200; % mass of vehicle (slinch)

ct = 2652.23; % vehicle damping ratio

vel = 720; % vehicle velocity (in/sec)

[mb, EI] = bridge2beam(fnb, 587723, length\*12, 1);

start = 3864;

vb\_ssmodel.bridge\_start = 320\*12;

pro\_file.name = 'ISO\_C10-300e-06\_w-2\_1.csv';

pro\_file.path = 'C:\Users\John\Projects\_Git\DAmp\profiles\artificial\model\_validation';



### Methods of reducing bridge down to single line.

Previous studies have demonstrated that the state-space model is capable of predicting amplification for a sprung mass travelling over a beam. However, there are many ways in which a bridge may be reduced to a single line girder model. The methods must accomplish the following goals: represent the bridge mass that is excited by the vehicle as it traverses the bridge, and represent the stiffness of the bridge to a load located along the path of travel.

The 3D nature of a bridge prohibits an accurate reduction to a single-line model, but for the purposes of estimating dynamic amplification, an adequate procedure may exist. A bridge is unlike a beam in that it may distribute load laterally. This distribution is dependent on individual member stiffnesses, bridge geometry and load location. A load at mid-span will be shared by adjacent girders much more than if the load were placed near a support. A single-line girder model with uniform stiffness will be unable to capture this effect.

If we are to reduce a bridge to a single-line girder model, it should be defined by parameters that can easily be measured or calculated. The first proposed parameter is given below.

Parameters: Stiffness associated with force placed at mid-span and along the path of travel, first natural frequency (first bending). The first parameter is used to calculate an equivalent EI for the beam model. The natural frequency (along with the calculated stiffness) is used to calculate the equivalent mass.

To refine this method let’s perform a study where the calculated parameter values are can be adjusted by a factor. With simple optimization algorithms we can compare the single-line model with the 3D FE model and determine the factors that produce the most accurate model.

Comparison of 3D FE simulation with state-space model predictions for the first model (1-span; 40 ft.) revealed that the single-line state-space model with a stiffness 4 times (4.07) that determined from midspan flexibility methods. If comparing only the displacement when the vehicle is near midspan (± 3% span\_length), the state space model with a stiffness factor of 1.04 also is a successful predictor. It remains to be seen if this relationship continues for other profiles and other models. When comparing the models, it seems a stiffness very near that calculated using the previously described procedures results in a good estimation of amplification (dynamic displacement/static displacement).

In summary for the first profile and for 40ft single-span bridge a state-space single beam model with a stiffness factor of 1.12 is the best predictor for displacement amplification:



Let’s examine the ability of state-space models to predict amplification and displacement with stiffness amplification factors of 1, 1.2 (maybe include 1.4, 1.6, 1.8, 2 later). Pull max amplification (negative displacement amplification) from time history from on-bridge to 5% after midspan.

# Simplification of contact force

## Contact force without bridge response

To obtain vehicle response without consideration of the bridge response, simulation will be performed for a single sprung mass (vehicle) subjected to the profile. The resulting computed contact force will be applied to a state-space model of a bridge only. The resulting bridge response will be compared to those resulting from the vehicle-bridge state-space model.

Vehicle and bridge parameters will be the same as used in the benchmarking:

The bridge had the following properties:

* Mass: 2000 slinch
* Natural Frequency: 3 Hz
* Length: 140 ft
* E x I: 34.5908E12 lb-in2

The vehicle had the following properties:

* Mass: 200 slinch
* Natural Frequency: 3.2 Hz
* Spring Stiffness: 80852 lb/in
* Damping Coefficient: 20%
* Velocity: 720 in/sec

A real profile (measured from I76, span 3) was used in the simulations

Contact force for two state-space models:



Bridge response to different contact forces:



|  |  |  |  |
| --- | --- | --- | --- |
|  | Maximum disp. (in) | Max amplification | %Error |
| Contact force: vehicle only | -0.4743 | 2.15 | -0.75% |
| Contact force: vehicle w/ bridge | -0.4622 | 2.10 | 1.82% |
| Vehicle-bridge model | -0.4708 | 2.13 | - |



For this scenario, the maximum bridge response was reasonably accurately predicted when contact force was acquired by considering only the vehicle. However, the time history shows that the effect of bridge response on vehicle contact force must be included if the true response of the bridge is required.

This is especially the case for multiple vehicle events, for which the bridge initial conditions must be considered (displacement and velocity). For the following study, all model parameters remain the same, but the final states of the bridge are saved, and used as initial conditions for an additional simulation run, effectively modeling a scenario in which a second vehicle enters the bridge just as the first exits. The following figure compares the bridge response to the second vehicle.



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## Conclusions

The maximum bridge response may be able to be conservatively approximated by first computing a vehicle’s response to the profile and then computing the bridge’s response to the vehicle contact force. However, this method should not be used when considering multiple loading events is not accurate and should be used as last resort.

# Profile Smoothing

This section examines profiles and the resulting vehicle and bridge response they produce.

One of the most widely used metrics for road surface roughness is the IRI. However, this metric is accumulated for long sections of roadway and fails to describe roadway conditions on a particular bridge and the character of any defects.

## Straightedge Criteria

Many states specify a smoothness criterion to address local road roughness that may not be addressed by an IRI or similar metric which averages data over a long portion of road. The specific criterion may differ, but they are usually specified by a maximum deviation (1/8” or ¼” ) over a specified straightedge length (10ft or 16 ft).

The plot below shows the bridge response to a truck traversing a real profile, as well as the response with the same profile “smoothed” to meet 1/8” over 10ft criteria.





The smoothed profile performs better, but still results in an amplification of 1.92.

If the criterion is made to just 1/16th inch over 10ft, the following response is resulted and the amplification is reduced to 1.6.



The straightedge length has better success, reducing the amplification to 1.4 with the 1/8” criterion.



Again, with 20ft straightedge.





Amplification is reduced to 1.27

## Targeting spatial frequencies

To better understand the role of different types of road roughness, we will examine the bridge response to a profile with content from various spatial frequencies removed.

The bridge and vehicle models have the same parameter values as used previously:

* Mass: 2000 slinch
* Natural Frequency: 3 Hz
* Length: 140 ft
* E x I: 34.5908E12 lb-in2

The vehicle had the following properties:

* Mass: 200 slinch
* Natural Frequency: 3.2 Hz
* Spring Stiffness: 80852 lb/in
* Damping Coefficient: 20%
* Velocity: 720 in/sec

The model will be run with a profile measured on a real bridge that was exhibiting excessive vibrations.

This profile will then be filtered to remove specified spatial frequencies. At a vehicle speed of 720 in/sec (41 mph), the following profile wavelengths result in the following corresponding forcing frequencies.

|  |  |  |
| --- | --- | --- |
|  | 720 in/sec | 1080 in/sec |
| 50 ft | 1.2 Hz | 1.8 Hz |
| 30 ft | 2 Hz | 3 Hz |
| 20 ft | 3 Hz | 4.5 Hz |
| 10 ft | 6 Hz | 9 Hz |
| 5 ft | 12 Hz | 18 Hz |

A stopband filter is designed with a stopband range of ±20% specified frequency. Therefore the following filters were implemented.

|  |  |  |
| --- | --- | --- |
| 50 ft. | 0.0013 /in. | 0.0020 /in. |
| 30 ft. | 0.0022 /in. | 0.0033 /in. |
| 20 ft. | 0.0033 /in. | 0.0050 /in. |
| 10 ft. | 0.0067 /in. | 0.010 /in. |
| 5 ft. | 0.0133 /in. | 0.020 /in. |





As one would expect, the dynamic amplification of bridge response was minimized when the profile features corresponding to 20 feet were removed, thereby preventing the profile from causing a forcing frequency that matches the bridge frequency. Simulations were repeated at 1080 in/sec (61mph).



Again, we see that dynamic amplification is minimized when the forcing frequency corresponding to the bridge natural frequency is removed from the profile. The simulation was repeated but with a stiffer bridge with a first natural frequency of 4.5 Hz and a vehicle speed of 1080 in/sec. At this speed, a wavelength of 20 ft corresponds to a forcing frequency of 4.5 Hz, and as the plot below indicates, those profile features with a wavelength near 20ft contributes the most to dynamic amplification.



## Summary

* Much of the bridge response is due to profile features that result in forcing frequency matching the bridge natural frequency at vehicle speed.
* Features corresponding to these frequencies (matching frequency at a range of vehicle speeds) may be removed (by grinding) to alleviate bridge resonance and reduce dynamic amplification.
* More stringent straight-edge requirements may also serve to eliminate problematic frequency content and reduce dynamic amplification.
* IRI may also be a targeted metric, as detailed in the following section.

# Profile Characterization

The objective of this study is to find a relationship between the frequency content of the profile and the dynamic amplification.

The challenging part of this is to characterize the frequency content by using only a few parameters.

## Profile Characterization Parameter 1

The simplest parameter would be the power of the profile at the frequency of the bridge; however, the forcing frequency induced by the profile is dependent on vehicle speed. Therefore, for the sake of simplicity, we may consider the spatial frequency that causes a forcing frequency equal to the bridge natural frequency at a speed of 50 mph. The forcing frequency can be described by the following equation:

Where fF is the forcing frequency, v is the vehicle velocity, and ws is the wavelength of the profile feature.

Therefore, the parameter of interest will be the power of the profile features with a wavelength equal to , or 50mph divided by the bridge natural frequency. For a bridge natural frequency equal to 3 Hz, this wavelength is: (24.4 ft or 7.45 meters).

## Profile Characterization: IRI

The International roughness index was established in 1986, and has become a widely used metric for road roughness. It is computed by simulating a quarter-car (Golden car) model at 80 km/hr (50 mph) over the longitudinal profile, and accumulates the vertical motion of the suspension. (Michael W. Sayers 1995)

Since this metric already exists, and is widely used, it makes sense to try to correlate dynamic amplification with IRI, or with a metric similar to IRI (other outputs of the quarter-car simulation).

### Correlation between IRI and dynamic amplification

It may be easier to show that there isn’t correlation by creating a profile with a low IRI that produces large dynamic amplification.

Correlation between IRI based on golden car state-space model and max displacement / static displacement (dyn. Amp.).

A measured profile was considered. This profile was filtered to remove spatial frequency content that created forcing frequency of 3 Hz (at 50 mph). The IRIs of both the original profile and the filtered profile were computed. A scaling factor was applied to the filtered profile to give it an IRI matching that of the original profile. In this way 2 profiles were obtained that had equal IRIs, but contained different spatial frequency content. These 2 profiles were examined with a vehicle-bridge state space model for which the bridge was assigned stiffness values that resulted in a natural frequency of 3 Hz.

The bridge displacement amplification is compared below (bridge displacement/bridge static displacement).



Figure : Bridge Displacement Amplification with "golden-car" (1.295 vs 1.071)



Figure : Bridge Displacement Amplification with "test-truck" (2.067 vs 1.168)

It can be seen from the above graphs that although 2 profiles may have the same IRI, they can cause very different bridge responses, which is even more pronounced with a heavier vehicle with a natural frequency similar to the bridge.

### Profile with good IRI causing large dynamic amplification

However, a profile with a low IRI value is unlikely to cause significant amplification.

A profile was created by using a pass-band filter to filter out everything but the profile content corresponding to a forcing frequency equal to 3 Hz at 720 in/sec. This profile was then scaled such that it’s IRI was less than 72 in/mi (IRI=71). This profile was analyzed with the vehicle-bridge state-space model and found to cause little dynamic amplification (max amplification = 1.168).



It is doubted that any real profile would have such limited frequency content, and therefore, this scenario presents the worst case scenario (profile is almost entirely composed of spatial frequencies that correspond to the bridge’s natural frequency) and it is expected that any real profile with an IRI of less than 72 would have greater diversity in frequency content and therefore result in dynamic amplification less than the 1.168 experienced in the previous simulation.



### Computation of the IRI

A quarter-car model described by the equations of motion was transformed into a state-space model, and assigned parameter values for the “golden-car”. This model was run with a measured profile at 80 km/h. The profile was first smoothed with a moving average over a distance of 250mm. Initial conditions (vehicle initial displacement and velocity) were assigned based on recommendations provided by (M. W. Sayers 1984). The accumulated suspension travel was computed based on simulation output according to the following equation:

The software created to compute the IRI was checked against available commercial software packages as is summarized in the table below (using profile EB L2run120171111\_0801\_Rt 76).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Begin Station | End Station | ProVAL IRI  (in/mi) | Computed IRI  (in/mi) | % diff |
| 0+00 | 528+00 | 157.41 | 157.43 | 0.01% |
| 528+00 | 10+56.0 | 221.18 | 221.24 | 0.03% |
| 10+56.0 | 15+84.0 | 175.51 | 175.51 | 0.00% |
| 15+84.0 | 21+12.0 | 211.9 | 211.82 | -0.04% |
| 21+12.0 | 24+97.7 | 83.61 | 83.92 | 0.37% |

Side note: The accumulated suspension travel is linearly related to the scale of the profile, i.e. IRI(profile)x2=IRI(profilex2). In fact all of the state’s time histories or linear combinations of them also scale directly with profile. I imagine this is a fundamental property of linear state space models. I do not think the combined bridge, vehicle state-space model will have this same property.

## IRI and ISO 8608

Using ISO 8608 standards we may control the frequency content of artificial profiles (with parameter w), we may then scale the profile to achieve a target IRI. In this way we may obtain a library of profiles with varying IRI and frequency content (w). We may also wish to develop several profiles for a given IRI, w combination to examine the effect phase angle (spatial location of features) has on bridge response.

Simulations will be performed with these profiles at various speeds for various bridge and vehicle parameters (bridge natural frequency, bridge mass, vehicle natural frequency, vehicle mass, vehicle damping). The bounds of the parameters are given in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Lower bound | Upper bound | Step size | Units |
| Profile IRI | 50 | 400 | varies | in/mi |
| Profile waviness (w) | 1.5 | 4.0 | 0.5 |  |
| Bridge natural frequency | 1 | 6 | 1 | Hz |
| Bridge mass | 1500 | 2500 | 500 | slinch |
| Vehicle mass | 10 | 300 | 50 | slinch |

Profile waviness parameter (w) range was set after considering the range of values reported by (Kropáč and Múčka 2009) (1.5-3.5). The values of waviness have a large effect on the frequency content of the profile, as shown in the following plot.



### Phase Angle

To examine the effect of spatial distribution of profile frequency content (i.e. phases) several different profiles were generated for a given IRI and waviness with phase angles being randomly generated with the random number generator reseeded for each profile. The following plot examines the bridge response to the different profiles.



Now with constant profile displacement psd (ISO 8608).



The cases for maximum and minimum amplification are plotted below:



Figure : Profiles with varying phase angles



As we can see in the above plots, the phase angle of the profile can have a large effect on bridge response. One option would be to scale a mean response to account for variability (i.e. d\_amp = factor\*mean(response)). Or reject both parameters as acceptable options.

If the bridge response can be greatly affected by the phase angle distribution of the profile feature wavelengths, then the individuality of the bridge requires a unique parameter (like IRI) that represents bridge response (e.g. multiple-degree of freedom state space model, single-span vehicle-bridge model).

Let’s examine the ability of different models/algorithms to serve as predictive parameters of bridge response to profile induced dynamic amplification. These will include a double-sprung mass model and a sprung mass traveling over simply supported beam. These will be compared to a 3D FE model results.

|  |  |
| --- | --- |
|  |  |
| fnb = | 2.08 natural frequency of bridge |
| fnv | 2.8; % natural frequency of vehicle |
| mb = | 2320; % total mass of bridge (slinch) |
| approach\_length = | 100; % length of approach profile (ft) |
| blength = | 140; % ft |
| mt | = 200; % mass of vehicle (slinch) |
| dt = | 0.2; % vehicle damping ratio |

Next: look at the simple models ability compared to an FE of a single span.

Simulations were also performed with a profile that had the same ISO specs as above, but with the starting position varied (i.e. profile shifted on bridge) resulting in large variation in bridge displacement amplification. This further demonstrates the need to analyze individual profiles.



### Summary

The above graphs illustrate the inability of a double sprung mass to predict dynamic amplification. A more complex model (beam model) can accurately predict amplification for single-span bridges, but tend to underestimate amplification for a 2-span structure.

### Misc.

Possible form of equation for dynamic amplification:

### Limited parametric study (perhaps no longer relevant due to the impact of phase)

Higher waviness corresponds to a profile dominated by large wavelength features.

Power of profile (frequency domain) at spatial frequency (n = fnb/vel):

## Summary

* If a measurement of the profile can be obtained, it should be analyzed with a model that is capable of simulating vehicle-bridge interaction to determine the likely level of dynamic amplification.
* If the IRI of the section of roadway over the bridge (and approach) is known, the frequency content may be assumed to be that corresponding to ISO 8608 waviness (w) equal to (highest response).
* The phase angles of frequency content have an effect on bridge response. Therefore, the effect of a profile on a bridge cannot be estimated with frequency content parameters.
* A simplified beam model is capable of estimating the amplification for a single-span structure.

# Appendix

## ISO 8608

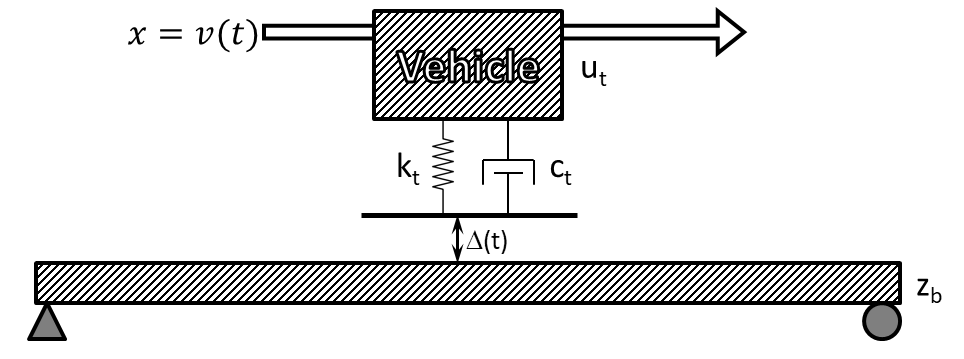
Ln(pxx) = a\*ln(ff)+b pxx = exp(a\*ln(ff)+b) = exp(ln(ff^a))\*exp(b) = exp(b)\*ff^a

Ln(pxx) = -w\*ln(ff)+b pxx = exp(-w\*ln(ff)+b) = exp(ln(ff^(-w))+b) = exp(b)\*ff^(-w)

C = exp(b) = x\*pxx(n) = x\*exp(b)\*n(-w) x =

## Equations of motion for simply-supported bridge with traversing vehicle

The following equations of motion were developed for the idealized dynamic system shown below..



## State-space equations for simply-supported bridge with profile

The state-spaces of the bridge and vehicle ordinates can be described with the following function:

### Assumed deformation shape function

for

## Distributed mass and stiffness

## Force transformation

## Equations of motion for when vehicle is on bridge:

For

## State Definitions

; ; ;

## Input

;

## State Space Equations

;

## State Space matrices for vehicle off bridge: