**Modal Parameter Estimation with Operational Response Data**

From the field measurements, an MxN matrix is obtained, where M is the number of DOF that were instrumented, and N is the length of the record. In order to examine the response for different frequencies, the data may be transformed into the frequency domain with the Fast Fourier Transform (FFT), which decomposes each response history into a sum of sinusoids. The number of discrete frequencies that are tested as part of a Fourier transform is directly proportional to the number of samples in the record (N) and equal to N/2. These frequency bins occur at intervals (∆f) equal to the sample rate of the data (Fs) divided by the number of samples (N).

Because the excitation of the bridge may be thought of as a random process, the response data (vibration) is also random and contains varied frequency content (i.e. vibration occurring at multiple frequencies simultaneously). While FFTs are quite capable at analyzing vibration when there are a finite number of dominant frequency components, power spectral densities (PSD) are better suited to characterize random vibration signals.

The power spectrum of the data (PSD) may be estimated with a periodogram which multiplies the value of each frequency bin in an FFT (*Xf*) by its complex conjugate. The result, which contains only real values, is then normalized by dividing it by the frequency bin width. This normalization effectively eliminates the dependency on bin width so that vibration levels may be compared in signals with differing lengths or sample rates. The periodogram can therefore be described by the following equation.

Because the data set is finite, a modified periodogram is used which serves to reduce spectral leakage by windowing the time-domain signal prior to computing the FFT in order to smooth the edges of the signal.

Welch’s periodogram is an improved estimator of the PSD. Its method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates. The result is Welch's PSD estimate. This method is especially useful for data in which the amplitude of different frequency components varies with time as is exhibited by operational acceleration data. A Hamming window was used for all PSD estimates using Welch’s methods.

Because it is the goal of this analysis to obtain mode shapes of the structure, it is necessary to retain information about the relative phase of the responses at each location which will indicate the direction of deformation (i.e. is a given DOF deflecting upward or downward relative to other DOF). This is achieved by estimating the cross-power spectral density (CPSD). The estimation is performed in same manner as the periodogram described above except the numerator is computed by multiplying the FFT value for one signal (*Xf*) by the complex conjugate of the FFT value for another signal (*Yf*). In this case, both signals are response records.

Since X and Y are complex, the CPSD estimate is also complex. Welch’s methods were again employed to estimate the CPSD for each frequency line and with every signal such that Pxy was computed with *Xf* for each location (*p*) and with each location (*q*) serving as reference (*Yf*), thereby producing a MxMxN matrix (*H*).

Because the response data is from bridge motion, the responses are assumed to be directly related to the modal vectors (ψ) and the frequency response function (FRF) (H) is formally represented by the following equation.

Where *Λ* contains temporal information and *L* is a function of input to the structure.

This representation of the FRF permits principle component analysis by Singular Value Decomposition (SVD). This algorithm decomposes a matrix (A) into three principle components: left singular vector (U), singular values (Σ), and right singular vector (V) according to the following equation.

Therefore, by performing SVD on the FRF matrix at each frequency line, spatial patterns in response may be extracted and provide estimate of modal vectors (shapes). The singular values (Σ), are proportional to the modal scaling of each corresponding mode and are plotted on a log scale as a function of frequency. Each peak of the singular values represents a location of resonance of the structure and the amplitude is directly related to the dominance of the corresponding mode shape at that frequency. At each frequency line, the left singular vector (*U*) is the approximate mode shape (*ψ*) of the response DOF, and the right singular vector (*V*) is the approximate modal participation vector (*L*) and represents the modal coefficients of the input at each DOF. In this case the input is unknown and thus only the left singular vector is used.

This process of identifying mode shapes with SVD is often referred to as the Complex Mode Indicator Function (CMIF). By including multiple columns of the FRF in the SVD (i.e. multiple records used as reference signals), the CMIF can detect and decouple multiple modes within a frequency bandwidth.