# Estimating Dynamic Amplification

As vehicle loading continues to increase and while engineers design bridges more flexible and closer to the margins, the reserve capacities of structures are reduced. The reduction in conservatism can only be justified if the assumptions inherent to our design and evaluation methodologies are made more certain. The structure presented in the first part of this paper indicated that the dynamic amplification assumptions in the current live-load model are inaccurate and can be unconservative. These inadequacies have also been documented in other cases (**cite**).

Therefore, it is necessary to utilize more accurate means of estimating or predicting dynamic amplification. The suitability of a given method is dependent on the application. In the case of existing structures, the in-situ behavior may be recorded, but the as-constructed dimensions and material properties may not be documented. During design, the in-situ behavior or attributes are not known, but the expected form of the structure is fully documented. It is therefore the objective of this part to identify and develop methods of predicting or estimating dynamic amplification that are suitable for the different applications.

Furthermore, these methods should be simple enough that they can be implemented by the typical practicing engineer. The simplicity of a method will undoubtedly come at the expense of accuracy. Therefore, a balance must be struck between accuracy and simplicity that is appropriate for the application. While this decision is ultimately left to the engineer, the following sections will demonstrate the tradeoff by presenting several methods of varying complexity for estimating dynamic amplification.

There are two widely used factors for expressing dynamic amplification. They are referred to as impact factor (IM) and dynamic amplification factor (DAF) and are defined by the following equations:

|  |  |
| --- | --- |
|  | () |
|  | (2) |

Therefore, the IM is just . The total live load response can be computed by the following:

|  |  |
| --- | --- |
| or | () |

Where Rsta is the static load effect which is amplified by (1 + IM) or the DAF.

In this paper the dynamic amplification factor will be computed according to **equation 2**. The responses used in computing the factor may be any structural response, experimentally recorded or obtained though analysis.

## Components of Vehicle Bridge Interaction

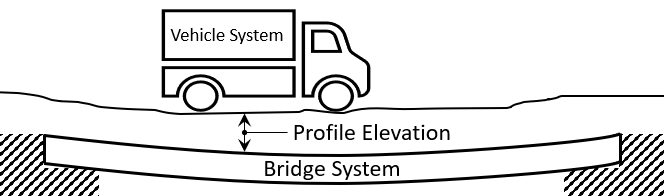


Figure .: Schematic of Combined Vehicle and Bridge Systems

The excitation of the vehicle-bridge system is due to the contact force between the vehicle tire and roadway surface. This contact force is dependent on the difference in vertical position of the bridge roadway surface and the vehicle body which is equal to the deformation of the suspension system assuming the tires remain in contact with the bridge surface. The bridge surface elevation is the combination of bridge motion at the vehicle position and elevation added by the profile. It therefore follows that any model of vehicle-bridge interaction, in order to capture all of the primary mechanisms that give rise to dynamic amplification, should include the following:

* The mass of bridge excited by the vehicle-induced forces
* The stiffness of the bridge
* The mass of the vehicle
* The suspension characteristics of the vehicle
* The vehicle velocity
* The roadway profile accurately positioned on bridge

This last element, an accurately positioned profile, is critical. The effect of a profile feature (bump) is partially dependent on the location of the feature, therefore an accurate model must also include longitudinal (along path of travel) bridge geometry (e.g. span length). This is demonstrated in Chapter 6.

There are many different methods of representing all of these elements in a model, but the success of a model is ultimately judged by its ability to reliably estimate the response of interest. For this study, that response of interest is the amplification of peak bridge responses during live load events (vehicle-crossing). The following sections will present several model types of varying complexity, document their construction, and demonstrate their ability to predict dynamic amplification.

Chapter 2 provides methods of estimating dynamic amplification of existing structures with field testing. Chapter 3 provides an analytical approach to estimating dynamic amplification using finite element analysis. Both approaches are demonstrated in the Part 1 case study. Chapter 4 details the development and performance of a simplified model for estimating dynamic amplification. Chapter 5 demonstrates the shortcomings of common profile roughness metrics in predicting dynamic amplification. Chapter 6 examines the parameters influential to dynamic amplification through simulation studies.

## In-Situ Measurement

There is no substitute for directly measuring a phenomenon. This section provides guidance on methods of directly measuring the dynamic amplification being experienced by a bridge that is in-service.

### Strain vs. Displacement

Within the relevant literature it is not uncommon for experimentally derived dynamic amplification factors to be reported for both displacement and strain (or stress). The amplification factors are almost always greater for displacement than those for stress or strain. If a bridge behaves linearly then its response should be linear (i.e. an increase in load by a factor X will result in an increase in response by the same factor, X) regardless of whether the response in question is global (e.g. displacement) or local (e.g. stress, strain). This is not the case with dynamic amplification factors because of a violation of the key assumption that the measured response is due solely to the applied load.

Dynamic amplification factors assume that the bridge response is due to a vehicle which applies a force equal to its weight increased by a factor to account for its dynamic motion (i.e. an acceleration greater than gravity). If this assumption were true, dynamic amplification factors would be equal for the various response quantities.

However, the bridge response is actually due to a force applied by the vehicle as well as inertial forces that develop due to the acceleration of the mass of the bridge as it is excited by the crossing vehicle (or other excitation sources).

Therefore, the bridge response is the sum of the responses due to these two inputs. To demonstrate this, consider a simply supported beam with a point load placed at midspan. The beam displacement and stress at midspan due to the point load (*P*) at midspan (*L/2*) is as described by the equations provided in the following table, where *E* is the modulus of elasticity of the beam material, *I* is beam section’s moment of inertia and *y* is the distance from the beam section’s neutral axis.

Table .: Displacement and Stress Equations for Point Load and Sinusoidal Distributed Load

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Point Force | | Distributed (Inertial) Force | |
| Loading |  | |  | |
| Displacement |  | () |  | () |
| Stress |  | () |  | () |

A beam oscillating under its first mode deforms according to a sinusoidal shape with a maximum acceleration (*Amax*) at midspan. Therefore, the maximum acceleration at any point (*x*) along the beam length (*L*) can be described by the following equation:

|  |  |
| --- | --- |
|  | () |

If the mass of the beam (*mb*) is uniformly distributed along its length, the equivalent distributed static force (*fs)* induced by the oscillating bridge mass can be described with the following equation.

|  |  |
| --- | --- |
|  | () |

This equation takes the form of the Newton’s second law of motion: .

The beam displacement and stress at midspan due to this distributed force can then be calculated using statics and the relationship between moment and curvature to produce the equations listed in the preceding table.

Therefore, the total response would be the sum of the two components, and the amplification factors can be described with the following equations, where the dynamic vehicle force is the product of the vehicle force amplification (*Av*) and its weight (*P*):

|  |  |
| --- | --- |
|  | () |

|  |  |
| --- | --- |
|  | () |

|  |  |
| --- | --- |
|  | () |

It can be deduced from the above equations that if vehicle amplification factors are to be used, an additional response due to the excitation of the bridge mass must be accounted for by including the second term in the above equations. Furthermore, these equations show that the amplification factor for displacement and stress will be different, and the amplification in excess of the vehicle amplification (second term) is greater for displacement than stress (or strain or moment). The additional amplification due to bridge inertial forces (only first mode considered) is plotted below for different ratios of bridge inertial force (mb\*Amax) to vehicle weight (P).

Figure .: Additional Amplification as a Function of Bridge Inertial Force

The ratio of the additional amplification due to bridge inertial forces is calculated below.

|  |  |
| --- | --- |
|  | () |

The results of equation **13** and the previous plot demonstrate, that regardless of bridge or vehicle, the experimental amplification factor for displacement will always be greater than that for stress or moment as long as the bridge is exhibiting some oscillation. It should be noted that these calculations included only the first mode. If more modes were to be included, the equations for amplification factors would take the following form:

|  |  |
| --- | --- |
|  | () |
|  | () |

The ratio of additional amplification can again be computed.

|  |  |
| --- | --- |
|  | () |

Additionally, these results are limited to midspan responses. Amplification factors at other locations can be computed in a similar manner, using equations for displacement or stress at other locations. This derivation considers only a simply supported beam and a point load, but the results are still representative of the phenomenon occurring in real structures as long as the following conditions remain true:

* The vibration of a bridge takes a shape that can be described by a shape function that is a summation of sines.
* The vehicle force is applied to a small area of the bridge and can be reasonable approximated as a point load.

In summary, amplification factors determined with displacement will be greater than those determined from strain (or stress or moment) due to the distribution of load from the mass loading that is ignored in static analysis. As bridge oscillation increases, the difference between the two factors also increases. Therefore, experimentally determined displacement amplification factors are a more conservative measure of dynamic amplification, but strain amplification factors remain adequate as strain responses more directly measure the stress experienced by the bridge.

### Operational Monitoring

Often operational monitoring, whereby bridge response is recorded during normal operation, is cost-effective (since it is least disturbing to traffic) and provides responses under typical loading conditions. Bridge members should be instrumented at “governing” locations (i.e. expected to experience the largest responses or suspected to have the least reserve capacity). Sensors should be carefully selected based on required response, range, frequency, accuracy, etc. This study is principally interested in material level responses (i.e. stress). Strain is directly related to stress (for linear material) and thus strain gauges are preferred for measuring dynamic amplification. Displacement gauges can also be used but may overestimate amplification when the resulting factors are employed within static analysis that ignores the mass forces associated with the vibrating bridge (as discussed in the previous section). Acceleration gauges may be used to estimate displacement if they remain accurate at frequencies near zero. This requirement is true of any gauge chosen but is more likely to be an issue with AC-coupled, piezoelectric accelerometers.

The process of determining dynamic amplification from operational responses has been already detailed by other researchers. Regardless of the exact method used, the data is filtered to remove high frequency content leaving behind an estimate of the content associated with quasi-static loading. The dynamic amplification is then estimated by computing the ratio of the maximum of the original data to the maximum of the filtered data. Multiple vehicle events should be examined as the degree of dynamic amplification may vary significantly for different vehicles. A demonstration of this process as well as the error that results from substituting filtered response for static response can be found in the case study presented in Part 1.

The filter parameters should be selected such that the pass-band upper limit is less than the first natural frequency but greater than the frequency of loading. In reality, some loading events occur at higher frequencies than the first natural frequency of the structure. In these cases, the filtered response under-estimates static response, subsequently resulting in an over-estimation of amplification. This problem is mitigated by the large mass of the bridge which resists rapid motion but is always an inherent source of error when estimating static response from operational responses. Furthermore, it is unlikely that a “worst-case” scenario will occur during the record interval and thus the estimated amplification can be non-conservative but can be appropriate for operational limit states and is a valuable approximation for assessing in-service performance.

### Load Testing

The static response of the bridge can be measured directly when the load is applied statically during a load test in which the bridge is closed to other traffic. Responses should be recorded for the test-vehicle (loaded truck) motionless as well as travelling over the bridge at speeds corresponding to minimum, typical, and maximum traffic speeds. Dynamic amplification computed from the resulting static and dynamic responses will be accurate for that specific test-vehicle but is not guaranteed to remain conservative for all loading events. A bridge’s performance in design or evaluation is measured by its ability to carry limit-state loads. Test-vehicles should therefore be loaded to a weight similar to the legal load limit. When possible, test-vehicles should also be chosen with a body-bounce natural frequency similar to that of the first-bending mode of vibration of the bridge as this will result in the greatest dynamic amplification (demonstrated in Chapter 6).

The test-vehicle should be placed at locations that produce maximum response or made to “crawl” at speeds low enough to maintain “quasi-static” conditions for the static portion of the load test. The dynamic load test should occur at various speeds and along all paths of travel. The test vehicle must begin a significant distance from the start of the bridge to account for vehicle motion resulting from traversing the approach roadway. The test-vehicle should maintain the set speed from a distance of at least 65 feet (20 meters) (**cite**) away from the beginning of the bridge until it exits the bridge.

### Profile Measurement

In some cases, it becomes necessary to simulate the bridge response to moving vehicles. Any simulations of vehicle-bridge interaction must include bridge deck profile. The roadway profile was shown to be highly influential to bridge response in Part 1 as well as in numerous other studies (**cite**). The profile should contain paired position and elevation information along the entire length of the bridge and approach roadway for every reasonable path of travel. Elevation data may be recorded along a single line or along multiple wheel lines. The spatial resolution should be set small enough to capture all features of interest. Since the bridge is most sensitive to loading near its own natural frequencies, and harmonic profile features induce loading at frequencies equal to the vehicle speed divided by the feature wavelength, the spatial sampling should be such that the distance between samples is no more than half the feature length that induces vibrations at the structure’s highest frequency (*fmax*) that appreciably contributes to bridge deformation when a vehicle is traveling at a minimum (reasonable) speed (*vmin*). Therefore, the following equation may be used to compute the maximum spacing between samples (*Smax*).

|  |  |
| --- | --- |
|  | () |

For a typical highway bridge the maximum contributing natural frequency is likely less than 15 Hz while vehicle speeds can be expected to exceed 20 mph (32 kph). Therefore, according to the above equation, profile measurements should be taken at least every 10 inches (25 cm). Commercial profilographs have sampling intervals on the order of one inch (**cite**) and thus can be expected to produce adequate profile measurements.

## Model-Based Simulation

There are often scenarios in which it is impractical or even impossible to implement certain loading events or measure certain responses. Furthermore, measurements alone generally cannot be used to identify the importance of the various mechanisms. Therefore, when it is the goal to identify the role of bridge components and mechanisms in observed responses or assess the suitability of proposed mitigation strategies (adding damping sources, changing the profile, adding stiffness, etc.) a numerical model is required. The selection and construction of a suitable model for these simulations is critical to reliable predictions.

### Common Structural Model Classes

#### Single-Line Girder Model

This method is the most basic and commonly used approach for the design and performance evaluation of common bridge types within the U.S. This approach approximates structural phenomena through various equations to estimate the equivalent demands a single girder within the structural system will experience. This approach has been shown to under-estimate stiffness, but is generally conservative for the computation of static dead and live load demands (Romano et al., 2017). While this model type is simple and very easy to implement, it is unable to simulate dynamic loading or represent transverse distribution of mass or stiffness.

#### 2D Grid Model

The 2D grid method borrows assumptions from the classical “plane grid” analysis method and is sometimes referred to as a grillage model. The girders and diaphragms are modeled as beam elements having three degrees of freedom (DOF) per node (two rotational and one translational DOF), with no depth information being explicitly represented. The two rotational degrees of freedom capture each girders’ major axis bending and torsional response. The single translational degree of freedom captures the vertical displacements of the girder.

With this method, all of the girders, diaphragms, and bearings are located at the same theoretical elevation in the model. Such models only permit the computation of vertical displacements and rotations within the plane of the bridge model.

#### 2D Frame Model

Similar to the 2D grid model, the 2D frame method of analysis ignores depth information. However, in this approach, the beam elements are equipped with six degrees of freedom at each node, three translational and three rotational. According to White et al. (2012), if there is no coupling between the degrees of freedom for the conventional 2D-grid and the three additional degrees of freedom, 2D-frame models provide no additional information beyond the ordinary 2D-grid solutions. Therefore, for vertical loading of a structure with no appreciable slope, this class of model performs no better than a 2D grid model.

#### Plate Eccentric-Beam Model

Similar to the 2D frame model, the plate eccentric-beam (PEB) model places the model elements in a single plane. However, for a PEB model, the girders are assigned an offset from the deck plane. This provides a more accurate representation of the composite (girder-deck) section and its stiffness. This model type provides better accuracy than 2D-grid or 2D-plane models while maintaining similar levels of simplicity and is suited to prismatic multi-girder bridges.

#### Element-Level Model

This type of model employs both one-dimensional (frame/beam elements) and two-dimensional elements (plate or shell elements) to model girders/diaphragms and the deck, respectively. Beam elements have either 2 or 3 nodes with 6 DOFs each. Plate/shell elements may have 3 (in the case of triangular elements), 4 (in the case of rectangular elements), or up to 9 (in the case of 9-node rectangular shells) nodes with up to 6 DOFs each. In an effort to remain consistent with the three-dimensional geometry of the structure, various link elements (to connect girders to the deck and diaphragm elements to the girders) and constraints (to simulate boundaries) are also employed. This model resolution is commonly termed “element-level” and is the most common class of 3D FE models employed for constructed systems (Çatbaş et al., 2013). This class of model is particularly well suited to structures with complex geometry as the elements can be accurately positioned in the model.

While an element-level FE model can reasonably simulate most bridge responses, it is unable to effectively simulate warping deformation of girders (associated with torsion) or stress concentrations associated with geometric discontinuities. While these shortcomings may be critical in the case of modeling specific construction sequences for complex bridges (White, 2012) and advanced fatigue/fracture assessment, they are not relevant for the global responses induced by VBI and considered for dynamic amplification.

#### Shell Element Method of Analysis

The most significant distinction between element-level and shell element models of multi-girder bridges is that the beams in shell element models are discretized vertically, laterally, and longitudinally using shell elements. This method of modeling girders allows for the accurate simulation of warping of the girders due to torsion. Computation, model construction, and result extraction activities however, are more time consuming and more difficult than with element level models.

### Vehicle Model

While there are many different methods of modeling a vehicle, the simplest method is to reduce the vehicle to a SDOF system, thereby collapsing the spatial distribution of wheel loads to a single point load. This method is conservative for predicting bridge responses and was shown in Part 1 to perform adequately for the purposes of simulating dynamic bridge response and estimating dynamic amplification.

The vehicle can be modeled after a real vehicle by assigning equivalent mass (weight) and by setting suspension characteristics that produce a natural frequency equal to the vehicle’s body-bounce natural frequencies. If there is no reference vehicle, a worst-case vehicle model may be created that has a mass equal to the legal limit, low damping (e.g. 10%), and a suspension stiffness that results in a body-bounce frequency 10-20% greater than the bridge’s first-bending natural frequency (as demonstrated in Chapter 6).

### Model Selection and Implementation for Simulating VBI

Simulation of VBI for predicting dynamic amplification should be performed with a model that is capable of capturing the dynamics of the structure. This requires a model that is geometrically consistent with the real structure such that the mass and stiffness can be accurately modeled and spatially distributed. In most cases, either a 3D element-level model or PEB model can represent all mechanisms and features that are a part of vehicle-bridge interaction and influence dynamic amplification.

It is not the aim of this paper to provide guidance on constructing and validating FE models. The exact methods of model construction and analysis are dependent on the FE software package employed. The selected FE software should be capable of simulating moving sprung masses over a specified profile and bridge model. The model should be constructed using best practices, error-screened, and validated with experimental data whenever possible. Validation with dynamic data (e.g. frequencies and mode shapes) is preferable and ensures the model dynamics match those of the structure. The model should have at least the first natural frequency matching that of the real structure.

Static responses can be simulated with vehicle at a crawl-speed (i.e. <1mph) or with a static linear analysis of the vehicle placed in locations that produce maximum response. Simulated responses should be recorded at governing locations (maximum response or particular vulnerability). Dynamic amplification can be computed for a given location as the ratio of maximum dynamic response to maximum static response.

The case study provided in the first part of this document demonstrates the process of estimating dynamic amplification with FE analysis.

## Simplified Model Formulation

Although 3D FE analysis is capable of accurately simulating vehicle-bridge interaction and estimating dynamic amplification, it is often impractical for current engineering practice due to the required time and expertise or the limitations of available FE software. It is therefore advantageous to develop models that require minimal time and expertise while still providing accurate estimates of dynamic amplification.

### Description

The following pages present a model type that includes all of the mechanisms involved with vehicle and bridge motion as listed in Chapter 1. The model reduces the bridge to a generalized single degree-of-freedom (SDF) system for which its deformation at any point along the bridge’s length is defined by a shape function. The vehicle is also represented as a single sprung mass and is coupled to the bridge degree-of-freedom. The equations of motion for this generalized SDF system are relatively simple and are used to develop state-space equations that define the vertical position of both bridge and vehicle. The following image illustrates the model components.

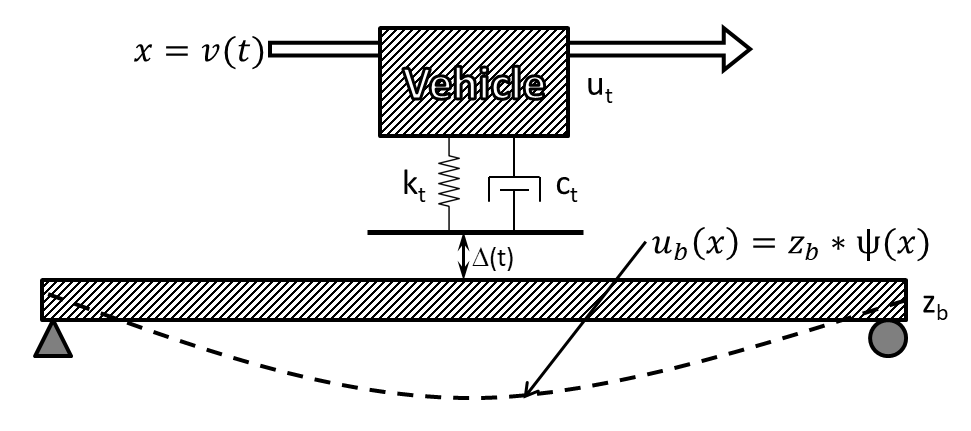


Figure .: Diagram of Simplified 2-DOF Model

Models of this form were developed for single-span bridges and for 2-span continuous bridges with equal span lengths. Although these models include every mechanism that plays a role in dynamic amplification they have several inherent limitations:

1. A half-sine wave () was chosen for the shape function that generalizes the distributed system by defining the shape of the beam deformation. While this shape function accurately describes the deformation associated with the first-bending mode of vibration, it is incorrect for point loading (or any other loading). For a single-span beam subject to a point load at midspan, the error due to this shape function is 1.45%, while, for a two-span continuous beam, the error is 31% (for midspan displacement predictions).
2. The error is further exacerbated when the load is not at midspan as the shape function is symmetric about midspan.
3. The single shape function only accounts for the bridge’s first mode of vibration.
4. By modeling the bridge as a single beam, the lateral distribution of mass and stiffness is neglected, much like the single-line girder model.

While these limitations leave the models much less capable than a full 3D FE model, they prove useful for estimating dynamic amplification and require a fraction of the time investment and computing power.

#### Single Span

This state-space model is developed from the equations of motion for a single sprung mass traveling over a simple-supported beam with distributed mass and stiffness. The beam is reduced to a single degree-of-freedom by generalizing its displacement according to a shape function. A sinusoidal shape function was chosen to capture the excitement of the beam’s first mode of vibration (1st bending). The beam has a uniform stiffness parameter (EI), uniform mass distribution (), and a span length of *L*. Mass proportional damping of the beam is included. The vehicle is reduced to a single point mass (*mt*) with specified spring stiffness (kt), viscous damping coefficient (ct), and traveling at a specified velocity (*v*).

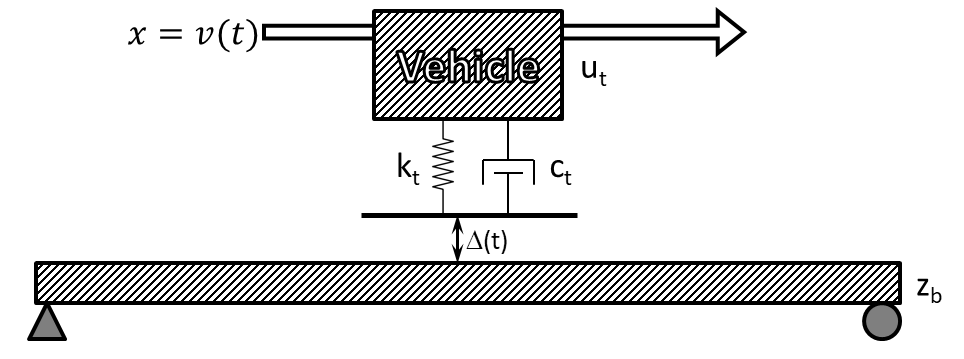


Figure .: Diagram of Single-Span 2-DOF Model

##### Assumed deformation shape function

The beam is assumed to deform with a half sine function (i.e. wavelength is twice the length of the span). That function is described by the following equation.

|  |  |
| --- | --- |
| for | () |

Therefore, the deflection (ub) and velocity (u̇b) of the beam at the vehicle’s location at time (*t*) is related to the generalized coordinate (*zb*) by the following equations.

|  |  |
| --- | --- |
|  | () |
|  | () |

##### Generalized mass, stiffness and damping

The generalized mass () and stiffness () properties for the beam can be calculated as follows.

|  |  |
| --- | --- |
|  | () |
|  | () |

The theoretical midspan displacement (due only to bending) of a single-span beam due to a point load (*P*) at midspan (*Δtheor*) is given by the following equation.

|  |  |
| --- | --- |
|  | () |

Therefore, the error associated with the chosen shape function for a point load (*P*) at midspan may be calculated as follows.

|  |  |
| --- | --- |
|  | () |

The generalized damping property () is defined by the following equation for mass proportional damping.

|  |  |
| --- | --- |
|  | () |

Where *a* is the mass-proportional damping coefficient. The damping ratio is defined by the following equation.

|  |  |
| --- | --- |
|  | () |

Where *ωn* is the radial natural frequency of the beam (first mode). Thus, the damping coefficient (*a*) may be determined based on a specified damping ratio () using the following equation.

|  |  |
| --- | --- |
|  | () |

By substitution, the generalized damping property may be expressed as follows.

|  |  |
| --- | --- |
|  | () |

##### Force transformation

The force applied by the vehicle mass (*p0*) must also be generalized. The force is described as a function of time and position as follows:

|  |  |
| --- | --- |
|  | () |

Where is the Dirac delta function centered at , and . The generalized force () is therefore calculated as follows for .

|  |  |
| --- | --- |
|  | () |

The force (*p0*) applied by the vehicle is calculated based on the relative vertical motion of bridge and vehicle, including profile elevation, as shown below.

|  |  |
| --- | --- |
|  | (31) |

Where *ut* is the vertical position of the vehicle, *ub* is the vertical position of the bridge at the vehicle location and *Δ* is the profile elevation at the vehicle location. Single dot notation indicates the first derivative and therefore *u̇t* is the vertical velocity of the vehicle, *u̇b* is the vertical velocity of the bridge at the vehicle location and *Δ̇* is the rate of change in profile elevation.

##### Equations of motion when the vehicle is on the bridge

The equations of motion of the generalized beam and moving vehicle may therefore be composed as follows for .

|  |  |
| --- | --- |
| Vehicle DOF: | () |
| Beam DOF: | () |

Double-dot notation is used in the preceding equations to indicate the second derivative and therefore *üt* is the vertical acceleration of the vehicle and *z̈b* is the acceleration of the generalized bridge coordinate (*zb*).

##### State Space

The states of this system (*z1, z2, z3* & *z4*) are therefore defined as follows:

|  |  |
| --- | --- |
| ; ; ; | () |

The profile elevation and velocity are assigned to matrix *U* with elements (*u1* & *u2*):

|  |  |
| --- | --- |
| ; | () |

The equations of motion are reorganized in terms of the defined states as follows.

|  |  |
| --- | --- |
| ; | () |
|  | () |
|  | () |
|  | () |
|  | () |

When the vehicle is off the bridge, the bridge experiences free vibration and the vehicle’s motion is independent of the bridge motion. The state-space matrices for this condition are provided as follows.

|  |  |
| --- | --- |
|  | () |
|  | () |
|  | () |

#### Two-span Continuous

This state-space model is developed from the equations of motion for a single sprung mass traveling over a 2-span continuous beam with distributed mass and stiffness. The beam is reduced to a single degree-of-freedom by generalizing its displacement according to a shape function. A sinusoidal shape function was chosen to capture the excitement of the beam’s first mode of vibration (1st bending). The beam has a uniform stiffness parameter (*EI*), uniform mass distribution (), and equal span lengths (*L*). Mass proportional damping of the beam is included. The vehicle is reduced to a single point mass (*m*t) with specified spring stiffness (*kt*), viscous damping coefficient (*ct*), and traveling at a specified velocity (*v*).

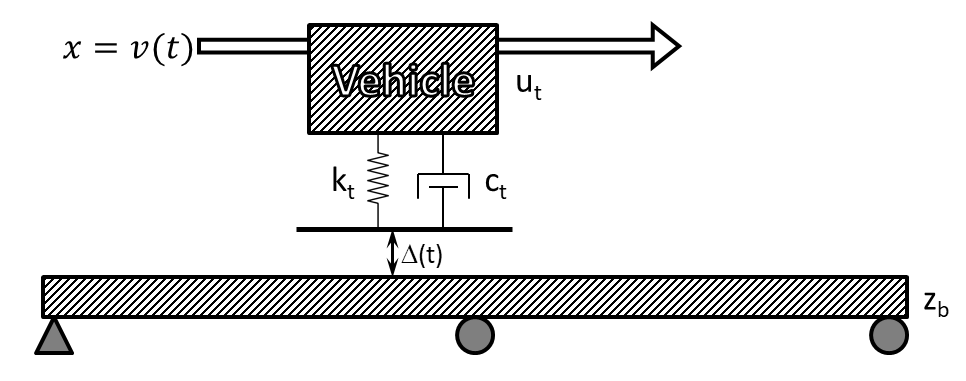


Figure .: Diagram of 2-Span 2-DOF Model

##### Assumed deformation shape function

The beam is assumed to deform with a half sine function (i.e. wavelength is twice the length of a single span). That function is described as follows.

|  |  |
| --- | --- |
| for | () |

Therefore, the deflection (ub) and velocity (u̇b) of the beam at the vehicle’s location at time (*t*) is related to the generalized coordinate (*zb*) by the following equations.

|  |  |
| --- | --- |
|  | () |
|  | () |

##### Generalized mass, stiffness and damping

The generalized mass () and stiffness () properties for the beam are calculated as follows.

|  |  |
| --- | --- |
|  | () |
|  | () |

The theoretical midspan displacement (due only to bending) of a two-span continuous beam due to a point load (*P*) at midspan (*Δtheor*) is given by the following equation.

|  |  |
| --- | --- |
|  | () |

Therefore, the error associated with the chosen shape function for a point load (*P*) at midspan may be calculated as follows.

|  |  |
| --- | --- |
|  | () |

The generalized damping property () is derived in the same manner as was done for the single-span model.

|  |  |
| --- | --- |
|  | () |

##### Force transformation

The force applied by the vehicle mass (*p0*) must also be generalized. The force is described as a function of time and position as follows:

|  |  |
| --- | --- |
|  | () |

Where is the Dirac delta function centered at , and . The generalized force () is therefore calculated as follows for .

|  |  |
| --- | --- |
|  | () |

The force (*p0*) applied by the vehicle is calculated based on the relative vertical motion of bridge and vehicle, including profile elevation, as shown below.

|  |  |
| --- | --- |
|  | () |

Where *ut* is the vertical position of the vehicle, *ub* is the vertical position of the bridge at the vehicle location and *Δ* is the profile elevation at the vehicle location. Single dot notation indicates the first derivative and therefore *u̇t* is the vertical velocity of the vehicle, *u̇b* is the vertical velocity of the bridge at the vehicle location and *Δ̇* is the rate of change in profile elevation.

##### Equations of motion for when vehicle is on bridge

The equations of motion of the generalized beam and moving vehicle may therefore be composed as follows for .

|  |  |
| --- | --- |
| Vehicle DOF: | () |
| Beam DOF: | () |

Double-dot notation is used in the preceding equations to indicate the second derivative and therefore *üt* is the vertical acceleration of the vehicle and *z̈b* is the acceleration of the generalized bridge coordinate (*zb*).

##### State Space

The states of this system (*z1, z2, z3* & *z4*) may therefore be defined as follows:

|  |  |
| --- | --- |
| ; ; ; | () |

The profile elevation and velocity are assigned to matrix *U* with elements (*u1* & *u2*):

|  |  |
| --- | --- |
| ; | () |

The equations of motion are reorganized in terms of the defined states as follows.

|  |  |
| --- | --- |
| ; | () |
|  | () |
|  | () |
|  | () |
|  | () |

When the vehicle is off the bridge, the bridge experiences free vibration and the vehicle’s motion is independent of the bridge motion. The state-space matrices for this condition are provided as follows.

|  |  |
| --- | --- |
|  | () |
|  | () |
|  | () |

### Implementation

The first step in determining the appropriate parameters for defining the state-space model is to define a beam that can approximate bridge response due to a vehicle traveling along specified path of travel. The distributed stiffness (*EI*) can be approximated by first determining the stiffness of the bridge to a point load at midspan along the path of travel. This stiffness value (*Kmid*) may be determined experimentally or with a refined FE model. The appropriate *EI* value is subsequently calculated using the generalized stiffness evaluated for a unit load at midspan. Stiffness is assumed to be uniformly distributed (i.e. *EI* is constant along the length of the beam). The following equations describe that calculation for single-span and two-span models.

|  |  |
| --- | --- |
| Single Span: | () |
| Two-Span: | (68) |

Once the distributed stiffness of the beam is determined, the distributed mass of the beam may be calculated such that the beam has a first-bending natural frequency (*ωn* in rad/sec) equal to that of the bridge. Mass was assumed to be uniformly distributed along the length of the beam for the models presented herein. The total equivalent beam mass (*mb*) may therefore be calculated with the following equation.

|  |  |
| --- | --- |
|  | () |

The vehicle is also reduced to a single degree-of-freedom based on known mass (*mv*) and natural frequency (*ωv*) as described for FE simulations. The suspension spring stiffness (*kv*) may be calculated according to the following equation.

|  |  |
| --- | --- |
|  | () |

The profile should be measured and provided in the form of sequential distance and elevation measurements. The distance values should be monotonically increasing.

With all parameter values obtained and assigned, the scenario may be simulated by stepping through each time increment, solving each “state” in-turn. This is easily accomplished programmatically with a loop. Full instruction on how to implement the state-space model is provided in the appendix and accompanying computer code is available upon request (if not already publicly available).

While the state-space model directly computes bridge displacement, the amplification (and other response quantities) is easily computed and is the quantity reported in many of the supporting figures. The assumed shape functions are unable to accurately predict displacement due to a point load (especially in the case of two-span continuous models) as previously discussed. Furthermore, the simplified models reduce the bridge to a single beam, neglecting the change in transverse load distribution as the load changes position on the bridge. By computing the amplification (normalizing displacement responses by static deflection), the resulting errors, as well as any errors in computing equivalent stiffness and mass, are mitigated.

Therefore, structural responses (e.g. displacement) should not be interpreted directly from these simplified models. Rather these models are intended to only predict the amplification of responses.

### Validation

The models previously described were implemented in MATLAB. The models were error screened by first comparing output to that from FE models of corresponding beams (with only the first mode included). These FE beam models consisted of a single beam with minimal constraints. The FE beam models were assigned parameters matching those implemented in the state-space model. Simulations were performed for which a single sprung mass traversed the beams at 720 in/sec over an artificial profile created using ISO 8608 methods (C10 = 300; w = -2).

The bridge and vehicle models were assigned the parameters as provided in the following table. These bridge and vehicle parameters were roughly based on the case structure presented in Part 1 such that reasonably large responses would be produced. Single span and 2-span models were assigned the same parameter values.

Table .: Beam and Sprung Mass Parameter Values for Validation Simulations

|  |  |  |
| --- | --- | --- |
| Span Length (*L*) | 100 | ft |
| Distributed Mass (*mb/L*) | 4.6 | kip/ft |
| Distributed Stiffness (*EI*) | 7.5x1012 | lb-in2 |
| Vehicle Mass (*mt*) | 100 | slinch |
| Vehicle Suspension Stiffness (*kt*) | 63.1655x103 | lb/in |
| Vehicle Damping Ratio (ζ) | 10% |  |

Simulations were performed with a time-step of 0.0014 sec. This time-step (*Δt*) was assigned based on the 1 in. spatial resolution (*Δd*) of the profile data and the vehicle speed (*v*) according to the following equation.

|  |  |
| --- | --- |
|  | () |

This time-step size was evaluated with FE simulations. The FE simulations with the single-span beam model were performed with a time-step size of 0.0015 seconds and 0.003 seconds. The absolute difference between the midspan displacement predicted by the two simulations is plotted as a percent of the minimum displacement (maximum downward displacement) as predicted by the simulation with 0.0015 time-step.

Figure .: Difference between FEM Predicted Midspan Displacement with Varied Time-Step Size

The maximum absolute difference and mean of the absolute differences between the simulation results for the two different step sizes is summarized in the following table.

Table .: Difference between FEM Predicted Midspan Displacement with Varied Time-Step Size

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Max Diff | Max % Diff | MAE | %MAE |
| FEM Δt=0.0015 vs Δt=0.003 | 0.001056 | 0.22% | 0.000308 | 0.07% |

The difference between the simulated responses illustrated in the preceding plot and table confirms that the chosen time-step size is adequately small, and the solution has converged.

Comparison of state-space model responses are compared to the responses predicted by the FE beam models in following plots. The FE model solutions included only the first mode of vibration to provide a more direct comparison with the state-space model. Some error was expected (and observed) because the state-space models are still an approximate representation of beam behavior. That error was more pronounced for models of two-span continuous beams.

Figure .: Comparison of Single-Span Beam Model Displacement Simulations

Figure .: Comparison of 2-Span Beam Model Displacement Simulations

The errors between the state-space and FEM beam models were quantified by computing the difference between maximum displacements (downward) as well as the mean-absolute-error. These were also expressed as a percent of the maximum displacement (downward) as predicted by the FE model. Those errors are summarized in the following table.

Table .: Differences between FEM and State-Space Displacement Predictions

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | State-Space Max Disp. | FEM Max Disp. | % Diff. | MAE | %MAE |
| Single-Span | -0.54011 | -0.47087 | 14.70% | 0.023058 | 4.90% |
| 2-Span | -0.26863 | -0.24819 | 8.24% | 0.024253 | 9.77% |

The disagreement evident in the preceding plots and table may be contributed to the simplicity of the state-space model compared to the FE model. In reducing the beam to a single degree-of-freedom, the state-space model assumes all deformation occurs according the specified shape function. While this function is accurate for the first mode of vibration, it is less adequate at describing the deformation due to concentrated loading as presented by the moving sprung mass. This assumption also ignores contribution from other modes. These higher modes were not included in the FE simulations presented in the preceding plots to provide a more direct comparison, but they do contribute to responses, especially those modes with low frequencies.

|  |  |
| --- | --- |
| First Bending | Second Bending |

Figure .: First Two Mode Shapes for 2-Span Continuous Beam

Due to the nature of two-span continuous bridges, the second bending mode (illustrated in the above image) is likely to occur at a frequency near to that of the first-bending mode. Thus, simulations that only consider the first mode will suffer greater inaccuracy when performed for two-span continuous bridges. The following plot compares the FE simulation of the two-span beam model with only the first mode included, and with the first five modes included.

Figure .: FEM Displacement Predictions with 1 Mode and 5 Modes for 2-span Continuous Beam

The differences between the FEM simulations with the first 5 modes included and just the first mode included were quantified by computing the difference between maximum displacements (downward) as well as the mean-absolute-error. These were also expressed as a percent of the maximum displacement (downward) as predicted by the simulation with the first 5 modes included. Those differences are summarized in the following table.

Table .: Differences between FEM Displacement Predictions with 1 Mode and 5 Modes

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 Mode Max Disp. | 5 Modes Max Disp. | % Diff. | MAE | %MAE |
| -0.2482 | -0.3048 | 18.58% | 0.0673 | 22.09% |

The above plot illustrates the inadequacy of a two-span model that includes only the first mode of vibration. However, the purpose of the simplified model proposed herein is to estimate dynamic amplification rather than accurately predict displacement. The ability of the model to perform in this regard is next investigated.

### Performance Assessment

It is always preferable to measure a model against ground truth values, which in this case would be the dynamic amplification as recorded on an actual structure. However, the number of sample structures that have been instrumented for the determination of dynamic amplification and have also had their profile measured is very limited. There are simply too few samples from real structures to adequately assess the performance of the state-space models. As a result, the performance of the state-space models was evaluated by comparing the dynamic amplification predicted by the model to that predicted by a 3D FE model.

#### Bridge Models

A total of six test-case models were created with varied geometry and stiffness while remaining representative of real structures. This was accomplished by basing the models on existing bridges. These bridges have varying length, width and skew. Furthermore, the dynamics of the bridges had been determined from previous field tests and have first natural frequencies ranging from 2 to 10 Hz. A single-span model and a two-span continuous model were created based on each of the three case structures. The plate eccentric-beam model type was employed for these FE models as described in Chapter 3 and in Part 1.

Road profiles were assigned to a line that defined the vehicle path of travel. The deck width of some models was great enough to accommodate multiple lanes and therefore multiple paths of travel were defined. Each vehicle path included an approach length of 320 feet. This approach length is more than sufficient to account for the vehicle’s initial conditions (demonstrated in Chapter 6).

Linear transient dynamic analyses of the moving mass were performed using LUSAS’ IMDPlus. This product option features several Interactive Modal Dynamics techniques; the relevant portion is described below.

An IMDPlus analysis uses conventional eigenvalue analyses to obtain the undamped modes of vibration for a structure over the frequency range of interest. The modal response in the form of frequencies, participation factors and eigenvectors, together with the moving mass vehicle loads, enable IMDPlus to compute the dynamic response for each mode of vibration. The assumption of linear structural behavior allows the IMDPlus facility to utilize linear superposition methods to calculate the total response of the structure from each of the contributing frequencies.

The simulation assumptions and limitations inherent to this solver are as follows.

* The system is linear in terms of geometry, material properties and boundary conditions.
* There is no cross-coupling of modes caused by the damping matrix. This is reasonable for all but the most highly damped structures or applications.
* Critical Damping ratios of 100% or more are not permitted due to the solution of the time domain response of the structure using either the Hilber–Hughes-Taylor (HHT) method or Duhamel’s Integral.
* There is no loss of contact between the unsprung masses (wheels) of the spring-mass systems and the structure at any time during the analysis
* Only vertical motion of the spring-mass systems is considered in a moving mass analysis.
* Mass of the spring-mass systems are not included in the eigenvalue solution

##### 140ft. Bridge

The 140 ft span models were based on the geometry of the bridge presented in Part 1—a multi-girder highway bridge with steel plate girders. Simple beam elements were used in place of the cross-frame diaphragms that existed on the actual bridge. These beam elements were assigned equivalent stiffness as detailed in Part 1.

The first natural frequency of the models was 2.08 Hz. The path of travel was defined over the first interior girder. The midspan displacements of that girder due to twin point loads totaling 1 lb., spaced 6’ apart at midspan on the vehicle path are given below. These values were used to compute equivalent EI values for implementing the state-space models as described previously.

Table .: Midspan Stiffness Values for 140 ft. Bridge Models

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1-span | | | 2-span | | |
|  | in/lb | lb/in |  | in/lb | lb/in |
| Path 1, Girder 2 | -4.49887E-6 | 222278 | Path 1, Girder 2 | -4.04871E-6 | 246992 |

##### 100ft. Bridge

This set of bridges was created based on a 2-lane bridge in Maryland with structure number [80053010](http://bridgereports.com/1240581): a continuous bridge with two spans with a length of 100 feet and with (5) AASHTO Type IV girders spaced at 99 in. Cast-in-place concrete diaphragms are located at the center of each span and at the ends of the spans. They extended to the bottom of the girder webs and were modeled with a rectangular section (37”x9”). The deck is 9.5” thick; no sidewalks are present; 4’ tall by 2’ wide concrete barriers are placed along either side.

The first natural frequency of the models is 3.99 Hz. Three paths were defined on this model for simulations. Their locations are at 4’, 10.5’ and 16.5’ from the exterior girder. The midspan displacements of the girder closest to each path due to twin point loads totaling 1 lb., spaced 6’ apart at midspan on the vehicle path are given below.

Table .: Midspan Stiffness Values for 100 ft. Bridge Models

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1-span | | | 2-span | | |
|  | in/lb | lb/in |  | in/lb | lb/in |
| Path 1, Girder 1 | -2.00994E-6 | 497527 | Path 1, Girder 2 | -1.49518E-6 | 668815 |
| Path 2, Girder 2 | -1.56329E-6 | 639676 | Path 2, Girder 3 | -1.21648E-6 | 822043 |
| Path 3, Girder 3 | -1.49866E-6 | 667262 | Path 3, Girder 3 | -1.19393E-6 | 837570 |

##### 40 ft. Span

This set of bridges was based on a 2-span simply supported bridge located in Maryland with structure number: [70042010](http://bridgereports.com/1240510). The bridge features a 15-degree skew, (7) wide-flange (rolled) steel girders (W30X108) spaced at 84 in., channel diaphragms (15C33.9) and concrete barriers modeled with a rectangular section (32”x19”). The deck is 8.5” thick and the barriers are 32” tall by 19” wide; there is no sidewalk. The skew of the models was increased to 16 degrees for ease of modeling.

The first natural frequency of the models is 9.95 Hz. The simulations were performed with 10 global modes included. Path 1 was located over the first interior girder and path 2 was located 16’ (transversely) from the exterior girder. The midspan displacements of the girder closest to each path due to twin point loads totaling 1 lb., spaced 6’ apart at midspan on the vehicle path are given below.

Table .: Midspan Stiffness Values for 40 ft. Bridge Models

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1-span | | | 2-span | | |
|  | in/lb | lb/in |  | in/lb | lb/in |
| Path 1, Girder 2 | -1.70148E-6 | 587723 | Path 1, Girder 2 | -1.38695E-6 | 721006 |
| Path 2, Girder 3 | -1.90733E-6 | 524293 | Path 2, Girder 3 | -1.51232E-6 | 661235 |

##### State-Space Parameters

The 2-DOF state-space models were implemented in the manner described in earlier in this chapter. The model input parameters included:

* Span length
* Number of Spans
* Bridge Mass
* EI
* Bridge Damping Ratio
* Vehicle Mass
* Vehicle Spring Stiffness
* Vehicle Damping Coefficient
* Vehicle Velocity

The bridge related parameters were defined that corresponded to each FE model and vehicle path totaling 12 cases. The EI value was calculated based on the midspan stiffness values determined from the FE models (**equation xx**). The mass values were then calculated to achieve a natural frequency that matched the FE models (**equation xx**). A structural damping ratio of 1% was assigned for all models. The state-space beam model parameters are summarized in the following table.

Table .: State-Space Beam Parameters

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Span Length (ft) | Number of Spans | 1st Nat. Freq. (Hz) | Path | Kmid (lb/in) | EI (lb-in2) | Mass per Span (slinch) |
| 1 | 40 | 1 | 9.95 | 1 | 587723 | 1.354E+12 | 305.16 |
| 2 | 2 | 524293 | 1.208E+12 | 272.22 |
| 3 | 2 | 9.98 | 1 | 721006 | 8.186E+11 | 183.37 |
| 4 | 2 | 661235 | 7.507E+11 | 168.16 |
| 5 | 100 | 1 | 3.99 | 1 | 497527 | 1.791E+13 | 1606.46 |
| 6 | 2 | 639676 | 2.303E+13 | 2065.44 |
| 7 | 3 | 667262 | 2.402E+13 | 2154.51 |
| 8 | 2 | 3.99 | 1 | 668815 | 1.186E+13 | 1064.14 |
| 9 | 2 | 822043 | 1.458E+13 | 1307.94 |
| 10 | 3 | 837570 | 1.486E+13 | 1332.65 |
| 11 | 140 | 1 | 2.08 | 1 | 222278 | 2.196E+13 | 2641.00 |
| 12 | 2 | 2.08 | 1 | 246992 | 1.202E+13 | 1446.09 |

The vehicle parameter values matched those assigned in the FE models.

#### Vehicle Models

For each model, a corresponding vehicle model was created with a natural frequency just slightly above the first natural frequency of the bridge. Vehicle models consisted of a single sprung mass with viscous damping (10%). The following table details the parameters for each vehicle model.

Table .: SDOF Vehicle Model Parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 40 ft span | 100 ft span | 140 ft span |  |
| Mass | 200.00 | 200.00 | 200.00 | slinch |
| Spring Stiffness | 8.7929E+05 | 1.6150E+05 | 4.9846E+04 | lb/in |
| Damping Coefficient | 2652.23 (10%) | 1136.67 (10%) | 631.48 (10%) | lb-s/in |
| Damped Natural Frequency | 10.5 | 4.5 | 2.5 | Hz |

A vehicle model was also created that was used in analyses across all models. This vehicle model was a single sprung mass with a natural frequency of 2.8 Hz. The parameters were as listed below.

Table .: 2.8 Hz SDOF Vehicle Model Parameters

|  |  |  |
| --- | --- | --- |
| Mass | 200 | slinch |
| Spring Stiffness | 61902.2 | lb/in |
| Damping Coefficient | 1407.43 (20%) | lb-s/in |

#### Profiles

A total of 15 profiles were evaluated. Three of the profiles were obtained from the profiles recorded from the case study bridge. Another twelve were artificial and generated using the methods defined by the ISO 8608 standards. This standard defines a roughness metric, but also describes the process whereby the profile is defined by frequency content and is generated through the summation of sine functions with amplitudes set according to PSD parameters. These parameters, defined by the standard, include a waviness value (*w*) and the amplitude of the PSD function at a spatial wavelength equal to 10 meters (*C10*). The PSD amplitude of each frequency band is therefore assigned according to the following equation.

|  |  |
| --- | --- |
|  | () |

**A more detailed description of generating artificial profiles according to the ISO 8608 standard can be found in the appendix.** The profiles were created using the following parameter set:

Table .: Profile Parameters for Performance Assessment Study

|  |  |
| --- | --- |
| Waviness | {2, 3, 4} |
| C10 (\*10-6) | {300, 600} |

Full factorial sampling of the above parameter sets yields 6 profiles, however, each profile generation was performed twice with different phase angles (reseeded random number generator for random uniform distribution). Therefore, a total of 15 test case profiles are examined (3 real profiles, 12 artificially generated). Each profile was created with a length such that it could fully cover the longest vehicle path, including approach. Since the longest test-case bridge was 280 feet long with a 320-foot long approach, a minimum profile length of 600 feet was required. The test-case profiles were therefore created with a total length of 650 feet.

#### Simulation Decisions

##### LUSAS FE

The simulation process consisted of following steps.

1. Eigenvalue analysis to obtain undamped natural modes of vibration
2. Incremental unit load analysis along vehicle path of travel
3. Calculation of the equivalent modal forces for each of these distinct locations
4. Definition of moving-mass solver parameters
5. Selection of desired response quantities
6. Moving mass analysis

The decisions made in these steps are as follows.

Table .: Moving Mass Simulation Decisions

|  |  |  |  |
| --- | --- | --- | --- |
| Decision | Selection | Units | Step |
| Number of modes to solve for/include | 15 |  | 1, 4 |
| Incremental distance along load-path | 6 | inches | 2 |
| Time integration scheme | Hilber Hughes Taylor (HHT) |  | 4 |
| Profile interpolation method | Linear |  | 4 |
| Structural damping | 1% |  | 4 |
| Vehicle speed | 720 | in/sec | 4 |
| Solution time-step | 0.0015 | sec | 4 |

Vertical displacement of midspan nodes was selected for analysis output. The amplification for any given run is then computed according to the following equation.

|  |  |
| --- | --- |
|  | () |

Where *δmax* is the maximum downward deflection and *δstatic* is the maximum static deflection reported when the moving mass is analyzed at 5 in/sec.

##### State-Space

The appropriate parameters (as described in a preceding section) were assigned to the state-space model using custom MATLAB scripts and functions. The scripts subsequently looped through all the cases, performing simulation for each. Beam and sprung mass position and velocity time histories were output as well as contact force. The displacement amplification was computed according to the following equation.

|  |  |
| --- | --- |
|  | () |

Where *δmax* is the minimum (maximum negative) displacement of the beam DOF over the time period for which the vehicle was on the span. For 2-span models, the displacement of the second span was reported as the minimum displacement of the beam DOF over the time period for which the vehicle was on the second span multiplied by a factor of -1, which is consistent with the assumed shape function. The static displacement (*δstatic*) is the maximum static deflection according to the generalized stiffness parameter and calculated with the following equations.

|  |  |
| --- | --- |
| Single Span: | () |
| Two-Span: | () |

Where *Pveh* is the static weight of the vehicle.

The IRI of each profile over each span was also computed for comparison. These computations were also performed with a state-space model based on the golden-car model and benchmarked against FHWA’s profile analysis program: ProVAL.

#### Results

The four parameter categories (i.e. bridge, path of travel, vehicle, and profile) were sampled to obtain a total of 239 different scenarios. Each scenario was simulated with a detailed 3D FE model and with a state-space model. The predicted amplification is compared in the plots below.

Figure .: Performance of 2DOF State-Space Models for Predicting Dynamic Amplification

It can be observed from these plots that the state-space models are more conservative for scenarios that result in high levels of amplification, but more accurate at lower amplification levels. This is because the simplified state-space model overestimates the dynamic response due to the manner in which loading and deformation is generalized by the shape functions used in the simplified state-space models.

It is not expected that dynamic amplification will reach such high values on real structures since the values were obtained from simulations with unrealistically rough artificial profiles and assumed the vehicle tires did not lose contact with the roadway surface. While this is not representative of reality, these simulations were intended to examine the performance of the simplified models compared to 3D FE models. It was therefore important to compare the model types across a wide range of response levels. The suitability of a 3D element-level model for predicting dynamic amplification is demonstrated in Part 1.

Therefore, while the simulated responses are not representative of the range of dynamic amplification that would be expected on real structures, they still serve to demonstrate the performance of the state-space models.

##### Vehicle Dynamics and Bridge Vulnerability

In the following plot the dynamic amplification data points are grouped by the type of vehicle used in the analysis where the “matching vehicle” corresponds to the vehicles that have natural frequencies close to the bridge’s first natural frequency and “non-matching vehicle” corresponds to the 2.8 Hz vehicle. The data points associated with the 140 ft bridges have been omitted since the 2.8 Hz vehicle’s natural frequency is close to the natural frequencies (2.08 Hz) of those bridges.

Figure .: Performance of 2DOF State-Space Model Grouped by Vehicle

As can be seen, the bridges with higher first natural frequencies fail to be excited by the non-matching (2.8 Hz) vehicle. Furthermore, it is postulated that fully loaded trucks most often have first natural frequencies less than 4 Hz, and thus bridges with high first natural frequencies have reduced vulnerability to dynamic amplification.

## IRI & Other Vehicle-Only Models

There are several methods already widely used to assess the roughness of roadway profiles. The International Roughness Index (IRI) is the most complex and simulates a specified vehicle (golden quarter-car) traveling over the profile. Other metrics ignore the vehicle and deal only with the profile data. The ISO 8608 parameters, for example, describe the spatial frequency content of the profile. However, all of these roughness metrics fail to consider the bridge or the position of the profile. Studies were performed to examine if these metrics had any ability to predict dynamic amplification.

ISO 8608 parameters describe the spatial frequency content of the profile. Studies presented in Chapter 6 show that the spatial frequency of the profile content does influence dynamic amplification. However, these parameters ignore any transient features and ignore the phase angle distribution and therefore are inadequate for predicting dynamic amplification (also demonstrated in Chapter 6. The inadequacy is further demonstrated by the supplied correlation plot.

Figure .: Performance on ISO 8608 Parameters for Predicting Dynamic Amplification

The IRI includes the vehicle in its model and may be expected to demonstrate better ability to predict dynamic amplification. However, the following correlation plot shows that the IRI cannot reliably predict dynamic amplification. The plot also reveals that while profiles with high IRI may not always result in high dynamic amplification, bridges with high dynamic amplification have high profile IRI values. This provides further evidence to encourage and mandate a smooth deck surface.

Figure .: Performance of IRI for Predicting Dynamic Amplification

Another simple model (quarter-car model) was assessed that included representation of the vehicle but ignored bridge behavior. Position of the profile on the bridge was included by applying a sine window to the vehicle response over the time period for which the vehicle is on the bridge. The maximum of the windowed contact force is reported as a factor of the vehicle self-weight. This contact-force amplification metric is compared to FE predictions in the plot below. The metric correlated relatively well with dynamic amplification, but that correlation coefficient was not consistent between bridges and therefore is not recommended for any amplification predictions.

Figure .: Performance of Quarter-Car Vehicle Model for Predicting Dynamic Amplification

## VBI Mechanisms and Parameters

In several of the simulation studies described in this section the simplified 2-DOF state-space model was employed. This model type was used when appropriate due to the minimal computing power required, permitting a large number of simulations to be automated and performed in a relatively short amount of time. This model (state-space) is described in detail in Chapter 4.

### Bridge and vehicle decoupled

In the interest of further understanding the degree to which the vehicle and bridge interact, the vehicle motion is decoupled from the bridge. Therefore, the contact force is only computed as a function of vehicle motion and profile elevation. This method neglects bridge motion and therefore represents the contact force for a completely rigid bridge. As a result, this method’s accuracy will suffer with increased bridge flexibility and vehicle mass (i.e. as bridge deformation increases).

|  |  |  |
| --- | --- | --- |
|  |  |  |

Figure .: Diagram of Decoupled Model

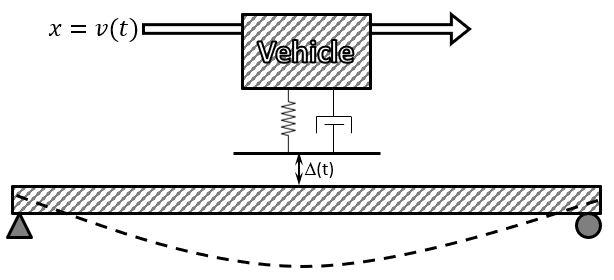


Figure .: Diagram of Coupled 2-DOF Model

The simplified 2-DOF state-space model presented in Chapter 4 was altered to decouple the beam degree of freedom from the vehicle degree of freedom. The force (*p0*) applied by the vehicle is calculated based on the vertical motion of the vehicle, including profile elevation, as shown below.

|  |  |
| --- | --- |
|  | () |

Where *ut* is the vertical position of the vehicle and *Δ* is the profile elevation at the vehicle location. Single dot notation indicates the first derivative and therefore *u̇t* is the vertical velocity of the vehicle and *Δ̇* is the rate of change in profile elevation.

#### Equations of motion when the vehicle is on the bridge

The equations of motion of the generalized beam and moving vehicle may therefore be composed as follows for .

|  |  |
| --- | --- |
| Vehicle DOF: | () |
| Beam DOF: | () |

Double-dot notation is used in the preceding equations to indicate the second derivative and therefore *üt* is the vertical acceleration of the vehicle and *z̈b* is the acceleration of the generalized bridge coordinate (*zb*).

#### State Space

The equations of motion are reorganized in terms of the defined states as follows.

|  |  |
| --- | --- |
| ; | () |
|  | () |
|  | () |
|  | () |
|  | () |

The state-space matrices for when the vehicle is not on the bridge are the same as presented for the coupled state-space model (Chapter 4).

#### Coupled and Decoupled Models Compared

The coupled and decoupled models were compared by performing simulations with each using the same beam and vehicle parameters as summarized in the following table.

Table .: Coupled and Decoupled Model Parameters

|  |  |  |
| --- | --- | --- |
| Span Length (*L*) | 140 | ft |
| Distributed Mass (*mb/L*) | 7283.3 | lb/ft |
| Distributed Stiffness (*EI*) | 2.196E+13 | lb-in2 |
| Vehicle Mass (*mt*) | 200 | slinch |
| Vehicle Suspension Stiffness (*kt*) | 4.935E+04 | lb/in |
| Vehicle Damping Ratio (ζ) | 10% |  |

An artificial profile was created according to ISO 8608 standards with parameters that resulted in a very rough bridge (C10 = 600; w = -2). The vehicle speed was set to 720 in/sec (65.8 kmh). These parameter values were chosen as they were known to result in large dynamic amplification and thus would provide good demonstration of the differences between the two models.

The following plot compares the bridge response from this decoupled analysis to that from the coupled vehicle-bridge model.



Figure .: Displacement Predicted by Coupled Model and Decoupled Model

Figure .: Difference between Displacement from Coupled Model and Uncoupled Model as a Percent of Maximum

Figure .: Difference between Contact Force from Coupled Model and Uncoupled Model as a Percent of Vehicle Weight

The displacement difference plot above was computed by dividing the difference by the maximum displacement predicted by the coupled model. The contact force difference plot above was computed by dividing the difference by the static weight of the vehicle. The difference between the maximum predicted displacements and the mean-absolute-error between the displacements predicted by the coupled and uncouple models are summarized in the following table.

Table .: Difference between Predicted Midspan Displacement for Coupled and Uncoupled Models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Max Coupled Model (in) | Max Uncoupled Model (in) | % Diff. | MAE | %MAE |
| -0.7689 | -1.1944 | 55.34% | 0.2793 | 36.33% |

Despite the simplicity of the model used to obtain the above results, it is still effective at modelling the exchange of energy between the two systems. As can be seen, decoupling the bridge and vehicle in simulations reduces the accuracy of the analysis and tends to overestimate the bridge response, especially at higher levels of amplification. This is because the uncoupled model fails to consider the influence of bridge motion on the vehicle contact force.

The roadway profile may be thought of as having a thickness equal to the difference between the roadway surface elevation and the mean surface elevation. When that profile is on a rigid roadway, the entire thickness imposed by the profile must be accommodated by the vehicle (i.e. only the vehicle can displace vertically). However, when that the profile is on a bridge, a portion of that imposed thickness is accommodated by the bridge, as it is accelerated by the resulting force. Therefore, the total displacement imposed on the vehicle suspension is reduced, thereby reducing the contact force.

In the case of the decoupled model, the contact force is overestimated because the roadway is assumed to be rigid and the reduction neglected. It is therefore recommended that any simulation of bridges under vehicle crossings include the interaction of the two systems if accurate prediction of bridge responses is desired.

### Approach Profile

The vehicle model used in the previous study (10% damping) was again utilized to examine how long of an approach length was necessary when performing simulations of vehicle-bridge interaction. This question can be answered by determining the time required for profile induced motion to damp out.

The maximum displacement experienced by the vehicle in the decoupled model for the simulations performed in the previous section was taken as initial conditions for an additional simulation with the same vehicle model. The vehicle was allowed to free vibrate, essentially simulating a “pull-release”. The resulting contact force is plotted below as a fraction of the static vehicle weight (contact force amplification).



Figure .: Free Decay of Vehicle Contact Force

After 1.8 seconds the peak amplification falls below 10% amplification. At highway speeds of 75 mph (120 kmh) this 1.8 seconds requires 200 feet (60 m) of roadway. While it is unlikely that these extreme conditions (due to the roughness of the profile used in simulations) would be present on real life bridges, this length provides a conservative upper bound for approach roadway length to be used in simulations with measured profiles.

### Influential System Parameters

Many studies on vehicle-bridge interaction will examine the role of parameters separately, and it was the original intention to organize this study in a similar manner. However, as it will soon be made clear, the parameters that effect VBI are interdependent. The following section therefore seeks to demonstrate and characterize the interdependency by first examining roadway profile which the case study showed to be critical to VBI and dynamic amplification.

A given profile is composed of elevation changes over the length of the roadway. This surface may be approximated by a summation of harmonic functions, and thus spectral analysis may be performed in much the same way as was done for acceleration data. The profile can therefore be described by its spatial frequency (cycles per unit distance) content. When a vehicle travels over a harmonic profile, the elevation change experienced at the vehicle’s location is harmonic with a frequency determined by the velocity of the vehicle. A profile with a given spatial frequency will therefore induce a force that acts on the vehicle with a frequency equal to the product of the spatial frequency and vehicle velocity. Therefore, the effect of profile is dependent on the velocity of the vehicle.

Several simulations with a 3D FE model (140ft; single-span) were performed for a sinusoidal profile with 30-foot wavelength and ½ in. amplitude. The bridge model had the first and second natural frequencies between 2.2 and 2.5 Hz, while the first natural frequency of the vehicle model was 2.8 Hz. The plot below shows the peak vehicle and bridge response at different vehicle speeds and the resulting forcing frequencies. It is no surprise that the peak responses occur at speeds that induce a forcing frequency that matches their respective natural frequency.

Figure .: Effect of Vehicle Forcing Frequency on Bridge Displacement and Vehicle Contact Force

The bridge displacement amplification was calculated by dividing the maximum midspan displacement by the static bridge displacement. The vehicle contact force amplification was calculated by dividing the maximum contact force by the vehicle’s static weight.

#### Multi-dimensional parametric study

Additional simulations were performed with the simplified state-space model. Vehicle stiffness was varied to produce vehicle models with natural frequencies ranging from 2 to 5 Hz. Bridge stiffness was also varied to produce bridge models with natural frequencies of the same range. Simulations were performed with the same harmonic profile with a wavelength of 30 ft. Vehicle speeds ranged from 540 to 2400 in/sec (10-136 mph) to produce forcing frequencies between 1.5 and 6.7 Hz.

Table .: Bridge Attributes for Parametric Study

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Bridge 1 | Bridge 2 | Bridge 3 | Bridge 4 |  |
| Span Length | 100 | 100 | 100 | 100 | ft |
| Mass (total) | 2000 | 2000 | 2000 | 2000 | slinch |
| EI | 5.603E+12 | 1.261E+13 | 2.241E+13 | 3.502E+13 | lb-in2 |
| Damping Ratio | 1.0% | 1.0% | 1.0% | 1.0% |  |
| 1st Nat. Freq. | 2 | 3 | 4 | 5 | Hz |

Table .: Vehicle Attributes for Parametric Study

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Truck 1 | Truck 2 | Truck 3 | Truck 4 |  |
| Mass | 200 | 200 | 200 | 200 | slinch |
| spring k | 31582.7 | 71061.2 | 126330.9 | 197392.1 | lb/in |
| Damping Ratio | 10.0% | 10.0% | 10.0% | 10.0% |  |
| Nat. Freq. | 2 | 3 | 4 | 5 | Hz |

The following plots summarize the results of this parametric study.

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

Figure .: Effect of Bridge and Vehicle Frequency on Bridge Displacement and Vehicle Contact Force (1/4)

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

Figure .: Effect of Bridge and Vehicle Frequency on Bridge Displacement and Vehicle Contact Force (2/4)

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

Figure .: Effect of Bridge and Vehicle Frequency on Bridge Displacement and Vehicle Contact Force (3/4)

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

Figure .: Effect of Bridge and Vehicle Frequency on Bridge Displacement and Vehicle Contact Force (4/4)

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

Figure .: Effect of Bridge and Vehicle Frequency on Bridge Displacement and Vehicle Contact Force (Total)

In the preceding figures the amplification of bridge and vehicle responses is plotted. The bridge displacement amplification is calculated by dividing the maximum midspan displacement by the static response ( given by the following (dictated by shape function):

|  |  |
| --- | --- |
|  | () |

Where *P* is the static weight of the vehicle, *L* is the span length, *E* is the beam modulus of elasticity and *I* is the beam’s moment of inertia.

The vehicle contact force amplification is calculated by dividing the maximum contact force by the static force of the vehicle which is merely the vehicle’s weight.

Simulations with the 5 idealized bridge models were again performed with the simplified state-space model and an artificial profile created according to ISO 8608 standards (C10 = 300; w = -2). The vehicle parameters were varied such that their natural frequency was a function of bridge natural frequency. The mass of the vehicles was held constant at 200 slinch (35025 kg), as was the damping ratio at 10%. The vehicle suspension stiffness was assigned such that the vehicles natural frequency varied from 0.8 to 4 times the bridge’s natural frequency. The following plot illustrates the optimum ratio of vehicle frequency to bridge frequency for the 5 bridge models.



Figure .: Effect of Vehicle Frequency as a Fraction of Bridge Natural Frequency on Bridge Displacement Amplification.

The following tables summarizes the frequency factor at which a maximum response as achieved.

Table .: Frequency Factor for Maximum Amplification

|  |  |
| --- | --- |
|  | Freq. Factor |
| Bridge1 | 1.76 |
| Bridge2 | 1.24 |
| Bridge3 | 1.14 |
| Bridge4 | 1.18 |
| Bridge5 | 1.12 |

The above simulations demonstrate the role of the profile as well as other parameters in vehicle-bridge interaction. The profile best excites the vehicle when its forcing frequency matches that of the vehicle, and the bridge is most excited when the profile forcing frequency matches its own natural frequency and the vehicle natural frequency is slightly larger than its own.

#### Profile Effects

The previous section demonstrated that profile frequency content that result in a forcing frequency similar to the bridge’s natural frequency produces maximum bridge response. However even a harmonic profile cannot be entirely described by its frequency content. The distribution of phase angles for the different harmonic components have a large effect on the final form of the profile and how the vehicle-bridge system responds to that profile. The following plot is of two profiles created according to ISO 8608 standards with identical frequency content but different phase angle distribution (C10 = 300; w = -2). These ISO parameter values result in a profile with a great deal of lower frequency (large wavelengths) content (largest included wavelength was 20 meters).

Figure .: Profiles with Matching Frequency Content but Different Phase Angle Distribution

Simulations were performed with these two profiles and the 2.5 Hz vehicle described in the previous section on the 140 ft single-span 3D FE model. The resulting bridge displacement is compared in the following plot.

Figure .: Bridge Displacement for Profiles with Different Phase Distribution

As can be seen, two profiles with identical frequency content but different phase angle distribution will cause different bridge responses. Similarly, the position of the profile can make a large difference in vehicle and bridge response. Simulations were performed with one of the profiles used in the previous simulation using the 2-DOF simplified model (Bridge 1 & Truck 1). The position of the profile was varied by offsetting it in increments of 5 feet.



Figure .: Effect of Profile Position on Bridge Displacement Amplification

As can be seen in the plot above, the bridge response varies greatly by just adjusting the position of the profile on the bridge. This is because the profile features that result in large contact forces will induce greater bridge response when they are located near the more flexible regions of the bridge (i.e. midspan). Furthermore, most real profiles are not harmonic but rather have many transient features. Profiles that contain harmonic content with large wavelengths that result in forcing frequencies similar to vehicles or the bridge should be avoided (e.g 5-50ft.). However, the frequency content of the profile alone has no reliable correlation with dynamic amplification and spatial information must be included in any dynamic amplification analysis.

### Summary

The roadway profile, on and off the bridge, serves to induce vertical oscillation in the vehicle. That oscillation results in an oscillating force at the point of contact between vehicle and roadway. As the vehicle crosses the bridge, the contact force excites the mass of the bridge. The exchange of energy from the vehicle to the bridge serves to reduce the motion of the vehicle and thus the dynamic component of the vehicle contact force.

Vehicle and bridge parameters therefore effect dynamic amplification based on their influence on the dynamics of the system and how those system dynamics relate to the profile characteristics. While parameter effect cannot be quantified independently, they can still be generalized as follows:

* Higher bridge mass, stiffness and damping generally serve to reduce dynamic amplification.
* Higher vehicle mass increases dynamic amplification while vehicle damping helps to reduce amplification.
* Higher vehicle speed leads to increased dynamic amplification.
* Longer bridge length results in a longer period of time for which the vehicle is present on the bridge and may therefore result in greater dynamic amplification.

Anything further than these generalizations requires simulation of vehicle-bridge interaction with the specific profile or direct measurement by field experiment.

## Summary

* The vehicle and bridge comprise a coupled dynamic system that is energized by the vehicle traversing a profile.
* Determining dynamic amplification of in-service bridges may be performed with operational monitoring or a load test. In either case, strain gauges are recommended over displacement gauges or accelerometers.
* Dynamic amplification estimated by filtering operational monitoring data may over-estimate amplification.
* A 3D FE model is capable of simulating vehicle-bridge interaction and is recommended for predicting dynamic amplification for structures with complex geometry or that are otherwise ill-suited to the simplified state-space models.
* A simple model that reduces both the bridge and vehicle to SDF systems has been shown to reliably predict dynamic amplification and is recommended when FE simulation is impractical.
* Any metric that is to be used for predicting dynamic amplification must include a representation of the bridge. Therefore, dynamic amplification should not be predicted by current roughness metrics (e.g. IRI and ISO 8608) that only consider the profile and vehicle.
* It is conservative to compute bridge response with contact forces determined without consideration of bridge motion.
* The effect of bridge, vehicle and profile parameters are interdependent.
* Bridge responses are greatest when the profile induces oscillation in the vehicle close to the bridge’s natural frequency and when the vehicle’s natural frequency is 10-20% greater than that of the bridge.
* The profile position and phase angle distribution of harmonic content has a large influence on bridge response. Therefore, a profile should be accurately positioned in any VBI simulations.