# Part 2: Estimating Dynamic Amplification

There are two widely used factors for expressing dynamic amplification. They are referred to as impact factor (IM) and dynamic amplification factor (DAF) and are defined by the following equations:

|  |  |
| --- | --- |
|  | (4) |
|  | (5) |

Therefore, the IM is just . The total live load response can be computed by the following:

|  |  |
| --- | --- |
| or | (6) |

Where Rsta is the static load effect which is amplified by (1 + IM) or the DAF.

In this paper the dynamic amplification factor will be computed according to **equation 5**. The responses used in computing the factor may be any structural response, experimentally recorded or obtained though analysis.

## Visualizing the Problem

The excitation of the vehicle-bridge system is due to the contact force between the vehicle tire and roadway surface. This contact force is dependent on the difference in vertical position of the bridge surface and the vehicle body. The bridge surface elevation is the combination of bridge motion at the vehicle position and elevation added by the profile. It therefore follows that any model of vehicle-bridge interaction must include the following:

* The mass of bridge excited by the vehicle-induced forces
* The stiffness of the bridge
* The mass of the vehicle
* The suspension characteristics of the vehicle
* The vehicle velocity
* The roadway profile accurately positioned on bridge

This last element, an accurately positioned profile, is critical. The effect of a profile feature (bump) is partially dependent on the location of the feature, therefore an accurate model must also include longitudinal (along path of travel) bridge geometry (e.g. span length).

There are many different methods of representing all of these elements in a model, but the success of a model is ultimately judged by its ability to reliably estimate the response of interest. For this study that response of interest is the amplification of peak responses during live load events (vehicle-crossing). The following sections will present several model types of varying complexity, document their construction and demonstrate their ability to predict dynamic amplification.

## In-Situ Measurement

There is no substitute for directly measuring a phenomenon. This section provides guidance on methods of directly measuring the dynamic amplification being experienced by a bridge that is in-service.

### Strain vs. Displacement

In many pieces of relevant literature experimentally derived dynamic amplification factors are reported for both displacement and stress or strain. The displacement factors are almost always greater than those for stress or strain. If a bridge behaves linearly then its response should be linear (i.e. an increase in load by a factor X will result in an increase in response by the same factor, X) regardless of whether the response in question is global (e.g. displacement) or local (e.g. stress, strain). This is not the case with dynamic amplification factors because of a violation of the key assumption that the measured response is due solely to the applied load.

Dynamic amplification factors assume that the bridge response is due to a vehicle which applies a force equal to its weight increased by a factor to account for its dynamic motion (acceleration > gravity). If this assumption were true, dynamic amplification factors would be equal for the various response quantities.

However, the bridge response is actually due to a force applied by the vehicle as well as a force due to the acceleration of the mass of the bridge as it is excited by the crossing vehicle (or other excitation sources).

Therefore, the bridge response is the sum of the two responses. To demonstrate this, let’s consider a simply supported beam with a point load placed at midspan. The beam displacement and stress at midspan due to the point load at midspan would equal the following:

* Displacement:
* Stress:

We can assume the beam is oscillating with a sinusoidal mode shape with a maximum acceleration Amax, at midspan. Therefore, the maximum acceleration at any point along the beam can be described by the following equation:

If the mass of the beam is uniformly distributed along its length, the equivalent static force induced by the moving bridge mass can be described with the following equation.

This equation takes the form of the Newton’s second law of motion: .

The beam displacement and stress at midspan due to this distributed force can then be calculated using statics and the relationship between moment and curvature to produce the following:

* Displacement:
* Stress:

Therefore, the total response would be the sum of the two components, and the amplification factors would take the following form, where the dynamic vehicle force is the product of the vehicle force amplification and its weight (Av\*P):

It can be deduced from the above equations that if vehicle amplification factors are to be used, an additional response due to the excitation of the bridge mass must be accounted for by including the second term in the above equations. Furthermore, these equations show that the amplification factor for displacement and stress will be different, and the amplification in excess of the vehicle amplification (second term) is greater for displacement than stress (or strain or moment). The ratio of the additional amplification is calculated below.

This shows, that regardless of bridge, or vehicle, the experimental amplification factor for displacement will always be greater than that for stress or moment. It should be noted that these calculations included only the first mode. If more modes were to be included, the equations for amplification factors would take the following form:

The ratio of additional amplification can again be computed.

Additionally, these results are limited to midspan responses. Amplification factors at other locations can be computed in a similar manner, using equations for displacement or stress at other locations. This derivation considers only a simply supported beam and a point load, but the results are still representative of the phenomenon occurring in real structures because the following conditions remain true:

* The vibration of a bridge takes a shape that can be described by a shape function that is a summation of sines.
* The vehicle force is applied to a small area of the bridge and can be reasonable approximated as a point load.

In summary, amplification factors determined with displacement will be greater than those determined from strain (or stress or moment) due to the distribution of load from the mass loading that is ignored in static analysis. Therefore, experimentally determined displacement amplification factors will be a more conservative measure of dynamic amplification, but strain amplification factors remain adequate as strain responses more directly measure the stress experienced by the bridge.

### Operational Monitoring

Often operational monitoring, whereby bridge response is recorded during normal operation, is the cheapest method and least disturbing to traffic and provides responses to typical loading conditions. Members that are expected to experience the largest responses as well as those suspected to have the least reserve capacity should be instrumented. Sensors should be carefully selected based on required response, range, accuracy, etc. This study is principally interested in material level responses (i.e. stress). Strain is directly related to stress and thus strain gauges are preferred for measuring dynamic amplification. Displacement gauges can also be used but may overestimate amplification as discussed previously (Part 1). Acceleration gauges may be used to estimate displacement if they remain accurate at frequencies near zero. This requirement is true of any gauge chosen but is more likely to be an issue with capacitive accelerometers.

The process of determining dynamic amplification from operational responses has been already detailed by other researchers. Regardless of the exact method used, the data is filtered to remove high frequency content leaving behind the content associated with quasi-static loading. The dynamic amplification is then estimated by computing the ratio of the maximum of the original data to the maximum of the filtered data. Multiple vehicle events should be examined as the degree of dynamic amplification may vary significantly for different vehicles. A demonstration of this process can be found in the case study presented in the first part.

The filter parameters should be selected such that the pass-band upper limit is less than the first natural frequency but greater than the frequency of loading. In reality, some loading events occur faster than the first natural frequency of the structure. In these cases, the filtered response under-estimates static response, subsequently resulting in an over-estimation of amplification. This problem is mitigated by the large mass of the bridge which resists rapid motion but is always an inherent source of error when estimating static response from operational responses. Furthermore, it is unlikely that the “worst-case” scenario occurred during the record interval and thus the estimated amplification can be non-conservative but can be appropriate for operational limit states and is a valuable approximation for assessing in-service performance.

### Load Testing

The static response of the bridge can be measured directly when the load is applied statically as in a load test. Responses should be recorded for the test-vehicle (loaded truck) motionless as well as travelling over the bridge at speeds corresponding to minimum, typical, and maximum traffic speeds. Dynamic amplification computed from the resulting static and dynamic responses will be accurate for that specific test-vehicle but is not guaranteed to remain conservative for all loading events. A bridge’s performance in design or evaluation is measured by its ability to carry limit-state loads. Test-vehicles should therefore be loaded to a weight similar to the legal load limit. When possible, test-vehicles should also be chosen with a body-bounce natural frequency similar to that of the first-bending mode of vibration of the bridge as this has been shown to result in the greatest dynamic amplification.

The test-vehicle should be placed at locations that produce maximum response or made to “crawl” at speeds low enough to maintain “quasi-static” conditions for the static portion of the load test. The dynamic load test should occur at various speeds and along all paths of travel. The test vehicle must begin a significant distance from the start of the bridge to account for vehicle motion resulting from traversing the approach roadway. The test-vehicle should maintain the set speed from a distance of at least 65 feet (20 meters) away from the beginning of the bridge until it exits the bridge.

### Profile Measurement

In some cases, it becomes necessary to simulate the bridge response to moving vehicles. Any simulations of vehicle-bridge interaction must include bridge deck profile. The profile should contain paired position and elevation information along the entire length of the bridge and approach roadway for every reasonable path of travel. Elevation data may be recorded along a single line or along multiple wheel lines. The spatial resolution should be set small enough to capture all features of interest. Bridge motion is most effected by profile features with a length of several feet and more. Commercial profilographs have sampling intervals on the order of one inch and thus can be expected to produce adequate profile measurements.

## Finite Element Analysis

There are often scenarios in which it is impractical or even impossible to implement certain loading events or measure certain responses. In such circumstances it becomes necessary to perform simulation of the loading event to predict expected responses. The selection and construction of a suitable model for these simulations is critical to accurate predictions.

A 3D element-level FE model can represent all mechanisms and features that are a part of vehicle-bridge interaction and influence dynamic amplification. By creating a model that is geometrically consistent with the real structure the mass and stiffness can be accurately modeled and spatially distributed. The model should be error-screened and calibrated with experimental data from the real structure and should have at least the first natural frequency matching that of the real structure.

It is not the aim of this paper to provide guidance on constructing and validating FE models. The exact methods of model construction and analysis are dependent on the FE software package employed. The selected FE software should be capable of simulating moving sprung masses over a specified profile and bridge model. The model should be constructed using best practices and should be validated with experimental data whenever possible. Validation with dynamic data (e.g. frequencies and mode shapes) is preferable and ensures the model dynamics match those of the structure.

The vehicle can be modeled after a real vehicle by assigning equivalent mass (weight) and by setting suspension characteristics that produce a natural frequency equal to the vehicle’s body-bounce natural frequencies. If there is no reference vehicle, a worst-case vehicle model may be created that has a mass equal to the legal limit, low damping (e.g. 10%), and a suspension stiffness that results in a body-bounce frequency 10-20% greater than the bridge’s first-bending natural frequency.

Static responses can be simulated with vehicle at a crawl-speed (i.e. <1mph) or with a static linear analysis of the vehicle placed in locations that produce maximum response. Simulated responses should be recorded at locations of maximum response or particular vulnerability. Dynamic amplification can be computed for a given location as the ratio of maximum dynamic response to maximum static response.

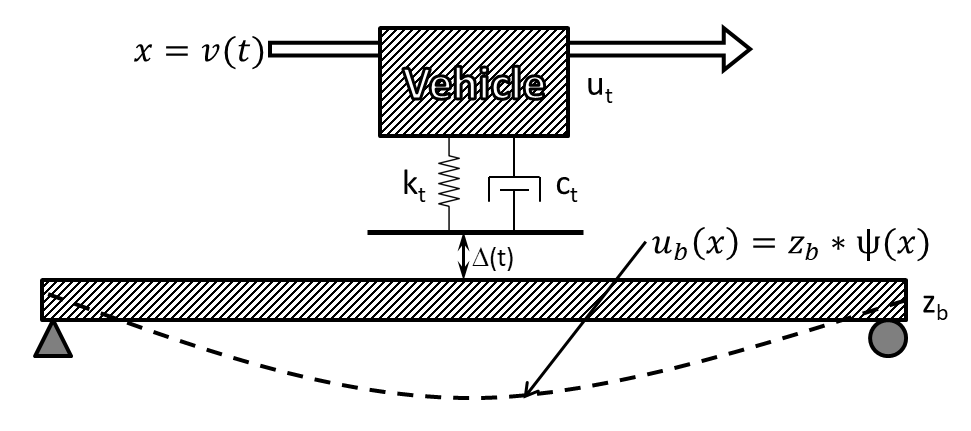
The case study provided in the first part of this document demonstrates the process of estimating dynamic amplification with 3D FE analysis.

## 2D Condensation & State-Space

Although 3D FE analysis is capable of accurately simulating vehicle-bridge interaction and estimating dynamic amplification, it is often impractical for current engineering practice due to the required time and expertise. It is therefore advantageous to develop models that require minimal time and expertise while still providing accurate estimates of dynamic amplification.

### Description

The following pages present a model type that includes all of the mechanisms involved with vehicle bridge motion as listed in the beginning of this part. The model reduces the bridge to a generalized single degree-of-freedom (SDF) system for which its deformation at any point along the bridge’s length is defined by a shape function. The vehicle is also represented as a single sprung mass and is coupled to the bridge degree-of-freedom. The equations of motion for this generalized SDF system are relatively simple and are used to develop state-space equations that define the position of both bridge and vehicle. The following image illustrates the model components.



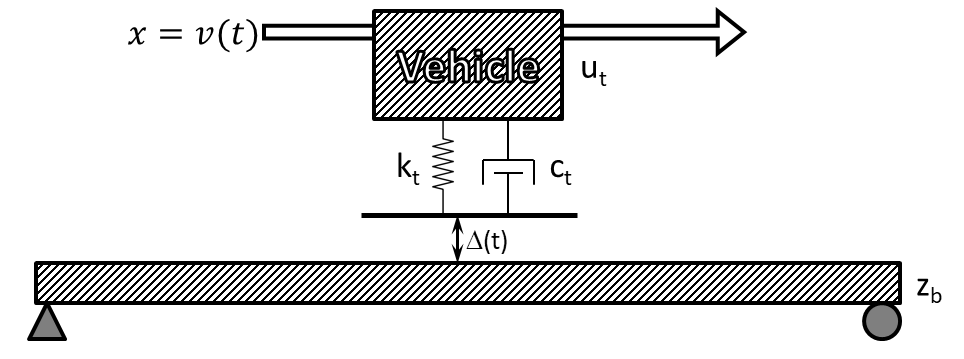
Models of this form were developed for single-span bridges and for 2-span continuous bridges with equal span lengths. Although these models include every mechanism that plays a role in dynamic amplification they have several inherent limitations:

1. A sine wave () was chosen for the shape function that generalizes the distributed system by defining the shape of the beam deformation. While this shape function accurately describes the deformation associated with the first-bending mode of vibration, it is incorrect for point loading (or any other loading).
2. The single shape function cannot account for the excitation of the bridge’s higher modes of vibration.
3. By modeling the bridge as a single beam, the lateral distribution of mass and stiffness is neglected.

While these and other limitations leave the models much less capable than a full 3D FE model, they prove useful for estimating dynamic amplification and require a fraction of the time investment and computing power.

#### Single Span

This state-space model is developed from the equations of motion for a single sprung mass traveling over a simple-supported beam with distributed mass and stiffness. The beam is reduced to a single degree-of-freedom by generalizing its displacement according to a shape function. A sinusoidal shape function was chosen to capture the excitement of the beam’s first mode of vibration (1st bending). The beam has a uniform stiffness parameter (EI), uniform mass distribution (), and a span length of *L*. Mass distributed damping of the beam is included. The vehicle is reduced to a single point mass (*mt*) with specified spring stiffness (kt), viscous damping coefficient (ct), and traveling at a specified velocity (*v*).



##### Assumed deformation shape function

The beam is assumed to deform with a half sine function (i.e. wavelength is twice the length of the span). That function is described by the following equation.

for

Therefore, the deflection and velocity of the beam at the vehicle’s location at time (t) is described by the following equations.

##### Generalized mass, stiffness and damping

The generalized mass and stiffness properties for the beam can be calculated as follows.

The generalized damping property is defined by the following equations for mass distributed damping. The generalized damping is given by the following equation.

Where *a* is the mass-proportional damping coefficient. The damping ratio is defined by the following equation.

Thus, the damping coefficient (a) may be determined based on a specified damping ratio () using the following equation.

By substitution, the generalized damping property may be expressed as follows.

##### Force transformation

The force applied by the vehicle mass (p0) must also be generalized. It’s position and magnitude are mathematically described as follows:

Where is the Dirac delta function centered at , and . The generalized force is therefore calculated as follows for .

The force (p0) applied by the vehicle is calculated based on the relative vertical motion of bridge and vehicle, including profile elevation, as shown below.

##### Equations of motion for when vehicle is on bridge

The equations of motion of the generalized beam and moving vehicle may therefore be composed as follows for .

Vehicle DOF:

Beam DOF:

##### State Space

The states of this system may therefore be defined as follows:

; ; ;

The profile elevation and velocity are assigned to matrix *u*.

;

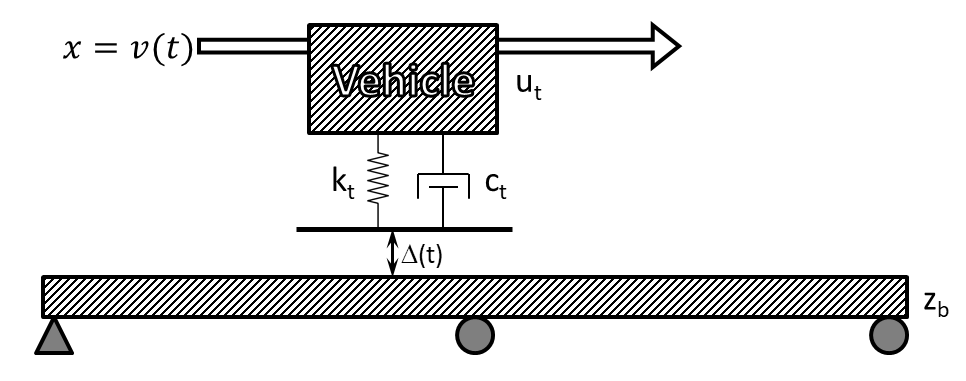
The equations of motion are reorganized in terms of the defined states as follows.

;

When the vehicle is off the bridge, the bridge experiences free vibration and the vehicle’s motion is independent of the bridge motion. The state-space matrices for this condition are provided as follows.

#### Two-span Continuous

This state-space model is developed from the equations of motion for a single sprung mass traveling over a 2-span continuous beam with distributed mass and stiffness. The beam is reduced to a single degree-of-freedom by generalizing its displacement according to a shape function. A sinusoidal shape function was chosen to capture the excitement of the beam’s first mode of vibration (1st bending). The beam has a uniform stiffness parameter (EI), uniform mass distribution (), and equal span lengths (L). Mass distributed damping of the beam is included. The vehicle is reduced to a single point mass (*m*t) with specified spring stiffness (kt), viscous damping coefficient (ct), and traveling at a specified velocity (*v*).



##### Assumed deformation shape function

The beam is assumed to deform with a half sine function (i.e. wavelength is twice the length of a single span). That function is described as follows.

for

Therefore, the deflection and velocity of the beam at the vehicle’s location at time (t) is described by the following equations.

##### Generalized mass, stiffness and damping

The generalized mass and stiffness properties for the beam can be calculated as follows.

The generalized damping property is derived in the same manner as was done for the single-span model. defined by the following equations for mass distributed damping.

##### Force transformation

The force applied by the vehicle mass (p0) must also be generalized. It’s position and magnitude are mathematically described as follows:

Where is the Dirac delta function centered at , and . The generalized force is therefore calculated as follows for .

The force (p0) applied by the vehicle is calculated based on the relative vertical motion of bridge and vehicle, including profile elevation, as shown below.

##### Equations of motion for when vehicle is on bridge

The equations of motion of the generalized beam and moving vehicle may therefore be composed as follows for .

Vehicle DOF:

Beam DOF:

##### State Space

The states of this system may therefore be defined as follows:

; ; ;

The profile elevation and velocity are assigned to matrix *u*.

;

The equations of motion are reorganized in terms of the defined states as follows.

;

When the vehicle is off the bridge, the bridge experiences free vibration and the vehicle’s motion is independent of the bridge motion. The state-space matrices for this condition are provided as follows.

### Implementation

The first step in determining the appropriate parameters for defining the state-space model is to define a beam that can approximate bridge response due to a vehicle traveling along specified path of travel. The distributed stiffness (EI) can be approximated by first determining the stiffness of the bridge to a point load at midspan along the path of travel. This stiffness value (Kmid) may be determined experimentally or with a refined FE model. The appropriate EI value is subsequently calculated using the generalized stiffness evaluated for a unit load at midspan. Stiffness is assumed to be uniformly distributed (i.e. EI is constant along the length of the beam). The following equations describe that calculation for single-span and two-span models.

|  |  |
| --- | --- |
| Single Span: | (7) |
| Two-Span: | (8) |

Once the distributed stiffness of the beam is determined, the distributed mass of the beam may be calculated such that the beam has a first-bending natural frequency equal to that of the bridge. Mass was assumed to be uniformly distributed along the length of the beam for the models presented herein. The total bridge mass may therefore be calculated with the following equation.

|  |  |
| --- | --- |
|  | (9) |

The vehicle is also reduced to a single degree-of-freedom based on known mass and natural frequency as described for FE simulations. Conservative vehicles may be implemented that have mass equal to legal limits and suspension stiffness that results in a body-bounce natural frequency approximately 10-20% greater than the bridge natural frequency. The suspension spring stiffness may be calculated according to the following equation.

|  |  |
| --- | --- |
|  | () |

The profile should be measured and provided in the form of sequential distance and elevation measurements. The distance values should be monotonically increasing.

With all parameter values obtained and assigned, the scenario may be simulated by stepping through each time increment, solving each “state” in-turn. This is easily accomplished programmatically with a loop. Full instruction on how to implement the state-space model is provided in the appendix and accompanying computer code is available upon request (if not already publicly available).

While the state-space model directly computes bridge displacement, the amplification (and other response quantities) is easily computed and is the quantity reported in many of the supporting figures. It became quickly evident that the error of these simplified models was mitigated by computing amplification rather than deflection. This is not surprising as it serves to reduce the effect of bridge stiffness, a parameter which is represented in vastly different ways (3D element-level vs SDF). It should also be noted that structural responses (e.g. displacement) should not be interpreted directly from these simplified models. Rather these models are intended to predict the amplification of responses.

### Validation

The models previously described were implemented in MATLAB. The models were error screened by first comparing output to FE models of corresponding beams. Simulations were performed for which a single sprung mass traversed the beams at 720 in/sec over an artificial profile created using ISO 8608 methods. The bridge and vehicle models were assigned the parameters as provided in the following table. Single span and 2-span models were assigned the same parameter values.

|  |  |  |
| --- | --- | --- |
| Span Length (*L*) | 100 | ft |
| Distributed Mass (*mb/L*) | 4.6 | kip/ft |
| Distributed Stiffness (*EI*) | 7.5x1012 | lb-in2 |
| Vehicle Mass (*mt*) | 100 | lb |
| Vehicle Suspension Stiffness (*kt*) | 63.1655x103 | lb/in |
| Vehicle Damping Ratio (ζ) | 10% |  |

Comparison of FEM results to state-space model results are provided by the following plots. Some error was expected (and observed) because the state-space models are still an approximate representation of beam behavior. That error was more pronounced for models of two-span continuous beams.

Figure 1: Comparison of Single-Span Beam Model Displacement Simulations

Figure 2: Comparison of 2-Span Beam Model Displacement Simulations

The disagreement evident in the preceding plots may be contributed to the simplicity of the state-space model compared to the FE model. In reducing the beam to a single degree-of-freedom, the state-space model assumes all deformation occurs according the specified shape function. While this function is accurate for the first mode of vibration, it is less adequate at describing the deformation due to concentrated loading as presented by the moving sprung mass. This assumption also ignores contribution from other modes. These higher modes were not included in the FE simulations presented in the preceding plots to provide a more direct comparison, but they do contribute to responses, especially those modes with low frequencies.

|  |  |
| --- | --- |
| First Bending | Second Bending |

Figure 3: First Two Mode Shapes for 2-Span Continuous Beam

Due to the nature of two-span continuous bridges, the second bending mode (illustrated in the above image) is likely to occur at a frequency near to that of the first-bending mode. Thus, simulations that only consider the first mode will suffer greater inaccuracy when performed for two-span continuous bridges. The following plot compares the FE simulation of the two-span beam model with only the first mode included, and with the first five modes included.

The above plot illustrates the inadequacy of a two-span model that includes only the first mode of vibration. However, the purpose of the simplified model proposed herein is to estimate dynamic amplification rather than accurately predict displacement. The ability of the model to perform in this regard is next investigated.

### Performance Assessment

It is always preferable to measure a model against ground truth values, which in this case would be the dynamic amplification as recorded on an actual structure. However, the number of sample structures that have been instrumented for the determination of dynamic amplification and have also had their profile measured is very limited. There are simply too few samples from real structures to adequately assess the performance of the state-space models. As a result, the performance of the state-space models was evaluated by comparing the dynamic amplification predicted by the model to that predicted by a 3D FE model.

#### Bridge Models

A total of six test-case models were created with varied geometry and stiffness while remaining representative of real structures. This was accomplished by basing the models on existing bridges. These bridges have varying length, width and skew. Furthermore, the dynamics of the bridges had been determined from previous field tests and have first natural frequencies ranging from 2 to 10 Hz. A single-span model and a two-span continuous model were created based on each of the three case structures. The plate eccentric-beam model type was employed for these FE models as described in the first part of this paper.

Road profiles were assigned to a line that defined the vehicle path of travel. The deck width of some models was great enough to accommodate multiple lanes and therefore multiple paths of travel were defined. Each vehicle path included an approach length of 320 feet. This approach length is more than sufficient to account for the vehicle’s initial conditions (20 meter minimum as recommended by **xx**).

Linear transient dynamic analyses of the moving mass were performed using LUSAS’ IMDPlus. This product option features several Interactive Modal Dynamics techniques; the relevant portion is described below.

An IMDPlus analysis uses conventional eigenvalue analyses to obtain the undamped modes of vibration for a structure over the frequency range of interest. The modal response in the form of frequencies, participation factors and eigenvectors, together with the moving mass vehicle loads, enable IMDPlus to compute the dynamic response for each mode of vibration. The assumption of linear structural behavior allows the IMDPlus facility to utilize linear superposition methods to calculate the total response of the structure from each of the contributing frequencies.

The simulation assumptions and limitations inherent to this solver are as follows.

* The system is linear in terms of geometry, material properties and boundary conditions.
* There is no cross-coupling of modes caused by the damping matrix. This is reasonable for all but the most highly damped structures or applications.
* Critical Damping ratios of 100% or more are not permitted due to the solution of the time domain response of the structure using either the Hilber–Hughes-Taylor (HHT) method or Duhamel’s Integral.
* There is no loss of contact between the unsprung masses (wheels) of the spring-mass systems and the structure at any time during the analysis
* Only vertical motion of the spring-mass systems is considered in a moving mass analysis.
* Mass of the spring-mass systems are not included in the eigenvalue solution

##### 140ft. Bridge

The 140 ft span models were based on the geometry of the bridge presented in the first part of this paper—a multi-girder highway bridge with steel plate girders. Simple beam elements were used in place of the cross-frame diaphragms that existed on the actual bridge, but the beam elements were assigned stiffness values such that butterfly modes matched those for the real structure, thereby assuring the transverse stiffness was adequately represented in the model.

The first natural frequency of the models was 2.08 Hz. The path of travel was defined over the first interior girder. The midspan displacements of that girder due to twin point loads totaling 1 lb., spaced 6’ apart at midspan on the vehicle path are given below. These values were used to compute equivalent EI values for implementing the state-space models as described previously.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1-span | | | 2-span | | |
|  | in/lb | lb/in |  | in/lb | lb/in |
| Path 1, Girder 2 | -4.49887E-6 | 222278 | Path 1, Girder 2 | -4.04871E-6 | 246992 |

##### 100ft. Bridge

This set of bridges was created based on a 2-lane bridge in Maryland with structure number [80053010](http://bridgereports.com/1240581): a continuous bridge with two spans with a length of 100 feet and with (5) AASHTO Type IV girders. Diaphragms are located at the center of each span and at the ends of the spans. The deck is 9.5” thick; no sidewalks are present; 4’ tall by 2’ wide concrete barriers are placed along either side.

The first natural frequency of the models is 3.99 Hz. Three paths were defined on this model for simulations. Their locations are at 4’, 10.5’ and 16.5’ from the exterior girder. The midspan displacements of the girder closest to each path due to twin point loads totaling 1 lb., spaced 6’ apart at midspan on the vehicle path are given below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1-span | | | 2-span | | |
|  | in/lb | lb/in |  | in/lb | lb/in |
| Path 1, Girder 1 | -2.00994E-6 | 497527 | Path 1, Girder 2 | -1.49518E-6 | 668815 |
| Path 2, Girder 2 | -1.56329E-6 | 639676 | Path 2, Girder 3 | -1.21648E-6 | 822043 |
| Path 3, Girder 3 | -1.49866E-6 | 667262 | Path 3, Girder 3 | -1.19393E-6 | 837570 |

##### 40 ft. Span

This set of bridges was based on a 2-span simply supported bridge located in Maryland with structure number: [70042010](http://bridgereports.com/1240510). The bridge features a 15-degree skew, (7) wide-flange (rolled) steel girders, channel diaphragms and concrete barriers. The deck is 8.5” thick and the barriers are 32” tall by 19” wide; there is no sidewalk. The skew of the models was increased to 16 degrees for ease of modeling.

The first natural frequency of the models is 9.95 Hz. The simulations were performed with 10 global modes included. Path 1 was located over the first interior girder and path 2 was located 16’ (transversely) from the exterior girder. The midspan displacements of the girder closest to each path due to twin point loads totaling 1 lb., spaced 6’ apart at midspan on the vehicle path are given below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1-span | | | 2-span | | |
|  | in/lb | lb/in |  | in/lb | lb/in |
| Path 1, Girder 2 | -1.70148E-6 | 587723 | Path 1, Girder 2 | -1.38695E-6 | 721006 |
| Path 2, Girder 3 | -1.90733E-6 | 524293 | Path 2, Girder 3 | -1.51232E-6 | 661235 |

##### State-Space Parameters

The 2-DOF state-space models were implemented in the manner described in a preceding section. The model input parameters included:

* Span length
* Number of Spans
* Bridge Mass
* EI
* Bridge Damping Ratio
* Vehicle Mass
* Vehicle Spring Stiffness
* Vehicle Damping Coefficient
* Vehicle Velocity

The bridge related parameters were defined that corresponded to each FE model and vehicle path totaling 12 cases. The EI value was calculated based on the midspan stiffness values determined from the FE models (**equation xx**). The mass values were then calculated to achieve a natural frequency that matched the FE models (**equation xx**). A structural damping ratio of 1% was assigned for all models. The state-space beam model parameters are summarized in the following table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Span Length (ft) | Number of Spans | 1st Nat. Freq. (Hz) | Path | Kmid (lb/in) | EI (lb-in2) | Mass per Span (slinch) |
| 1 | 40 | 1 | 9.95 | 1 | 587723 | 1.354E+12 | 305.16 |
| 2 | 2 | 524293 | 1.208E+12 | 272.22 |
| 3 | 2 | 9.98 | 1 | 721006 | 8.186E+11 | 183.37 |
| 4 | 2 | 661235 | 7.507E+11 | 168.16 |
| 5 | 100 | 1 | 3.99 | 1 | 497527 | 1.791E+13 | 1606.46 |
| 6 | 2 | 639676 | 2.303E+13 | 2065.44 |
| 7 | 3 | 667262 | 2.402E+13 | 2154.51 |
| 8 | 2 | 3.99 | 1 | 668815 | 1.186E+13 | 1064.14 |
| 9 | 2 | 822043 | 1.458E+13 | 1307.94 |
| 10 | 3 | 837570 | 1.486E+13 | 1332.65 |
| 11 | 140 | 1 | 2.08 | 1 | 222278 | 2.196E+13 | 2641.00 |
| 12 | 2 | 2.08 | 1 | 246992 | 1.202E+13 | 1446.09 |

The vehicle parameter values matched those assigned in the FE models.

#### Vehicle Models

For each model, a corresponding vehicle model was created with a natural frequency just slightly above the first natural frequency of the bridge. Vehicle models consisted of a single sprung mass with viscous damping (10%). The following table details the parameters for each vehicle model.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 40 ft span | 100 ft span | 140 ft span |  |
| Mass | 200.00 | 200.00 | 200.00 | slinch |
| Spring Stiffness | 8.7929E+05 | 1.6150E+05 | 4.9846E+04 | lb/in |
| Damping Coefficient | 2652.23 (10%) | 1136.67 (10%) | 631.48 (10%) | lb-s/in |
| Damped Natural Frequency | 10.5 | 4.5 | 2.5 | Hz |

A vehicle model was also created that was used in analyses across all models. This vehicle model was a single sprung mass with a natural frequency of 2.8 Hz. The parameters were as listed below.

|  |  |  |
| --- | --- | --- |
| Mass | 200 | slinch |
| Spring Stiffness | 61902.2 | lb/in |
| Damping Coefficient | 1407.43 (20%) | lb-s/in |

#### Profiles

A total of 15 profiles were evaluated. Three of the profiles were obtained from the profiles recorded from the case study bridge. Another twelve were artificial and generated using the methods defined by the ISO 8608 standards. This standard defines a roughness metric, but also describes the process whereby the profile is defined by frequency content and is generated through the summation of sine functions with amplitudes set according to PSD parameters. These parameters, defined by the standard, include a waviness value (*w*) and the amplitude of the PSD function at a spatial wavelength equal to 10 meters (*C10*). The PSD amplitude of each frequency band is therefore assigned according to the following equation.

**A more detailed description of generating artificial profiles according to the ISO 8608 standard can be found in the appendix.** The profiles were created using the following parameter set:

|  |  |
| --- | --- |
| Waviness | {2, 3, 4} |
| C10 (\*10-6) | {300, 600} |

Full factorial sampling of the above parameter sets yields 6 profiles, however, each profile generation was performed twice with different phase angles (reseeded random number generator for random uniform distribution). Therefore, a total of 15 test case profiles are examined (3 real profiles, 12 artificially generated). Each profile was created with a length such that it could fully cover the longest vehicle path, including approach. Since the longest test-case bridge was 280 feet long with a 320-foot long approach, a minimum profile length of 600 feet was required. The test-case profiles were therefore created with a total length of 650 feet.

#### Simulation Decisions

##### LUSAS FE

The simulation process consisted of following steps.

1. Eigenvalue analysis to obtain undamped natural modes of vibration
2. Incremental unit load analysis along vehicle path of travel
3. Calculation of the equivalent modal forces for each of these distinct locations
4. Definition of moving-mass solver parameters
5. Selection of desired response quantities
6. Moving mass analysis

For decisions made in these steps are as follows.

|  |  |  |  |
| --- | --- | --- | --- |
| Decision | Selection | Units | Step |
| Number of modes to solve for/include | 15 |  | 1, 4 |
| Incremental distance along load-path | 6 | inches | 2 |
| Time integration scheme | Hilber Hughes Taylor (HHT) |  | 4 |
| Profile interpolation method | Linear |  | 4 |
| Structural damping | 1% |  | 4 |
| Vehicle speed | 720 | in/sec | 4 |
| Solution time-step | 0.0015 | sec | 4 |

Vertical displacement of midspan nodes was selected for analysis output. The amplification for any given run is then computed according to the following equation.

Where *δmax* is the maximum downward deflection and *δstatic* is the maximum static deflection reported when the moving mass is analyzed at 5 in/sec.

##### State-Space

The appropriate parameters (as described in a preceding section) were assigned to the state-space model using custom MATLAB scripts and functions. The scripts subsequently looped through all the cases, performing simulation for each. Beam and sprung mass position and velocity time histories were output as well as contact force. The displacement amplification was computed according to the following equation.

Where *δmax* is the minimum (maximum negative) displacement of the beam DOF over the time period for which the vehicle was on the span. For 2-span models, the displacement of the second span was reported as the minimum displacement of the beam DOF over the time period for which the vehicle was on the second span multiplied by a factor of -1, which is consistent with the assumed shape function. The static displacement (*δstatic*) is the maximum static deflection according to the generalized stiffness parameter and calculated with the following equations.

|  |  |
| --- | --- |
| Single Span: | () |
| Two-Span: | ) |

Where *Pveh* is the static weight of the vehicle.

The IRI of each profile over each span was also computed for comparison. These computations were also performed with a state-space model based on the golden-car model and benchmarked against FHWA’s profile analysis program: ProVAL.

#### Results

The four parameter categories (i.e. bridge, path of travel, vehicle, and profile) were sampled to obtain a total of 239 different scenarios. Each scenario was simulated with a detailed 3D FE model and with a state-space model.

The predicted amplification is compared in the plots below. It can be observed from these plots that the state-space models are more conservative for scenarios that result in high levels of amplification, but more accurate at lower amplification levels. It is not expected that dynamic amplification will reach such high values on real structures. These values were obtained in simulations with unrealistically rough artificial profiles, but still serve to demonstrate the performance of the state-space models.

##### Vehicle Dynamics and Bridge Vulnerability

In the following plot the dynamic amplification data points are grouped by the type of vehicle used in the analysis where the “matching vehicle” corresponds to the vehicles that have natural frequencies close to the bridge’s first natural frequency. The data points associated with the 140 ft bridges have been omitted since the 2.8 Hz vehicle’s natural frequency is close to the natural frequencies of those bridges.

As can be seen, the bridges with higher first natural frequencies fail to be excited by the 2.8 Hz vehicle. Furthermore, it is postulated that fully loaded trucks most often have first natural frequencies less than 4 Hz, and thus bridges with high first natural frequencies have reduced vulnerability to dynamic amplification.

## Simulating VBI

In several of the simulation studies described in this section the simplified 2-DOF state-space model was employed. This model type was used when appropriate due to the minimal computing power required, permitting a large number of simulations to be automated and performed in a relatively short amount of time. This model (state-space) is described in detail in the previous section.

### Bridge and vehicle decoupled

In the interest of further understanding the degree to which the vehicle and bridge interact, the vehicle is analyzed separately from the bridge. The vehicle is first analyzed traveling over just the profile. The contact force obtained as a result of this analysis is then applied to the bridge model. This method neglects bridge motion and therefore represents a completely rigid bridge. Therefore, this method’s accuracy will suffer with increased bridge flexibility and vehicle mass.

|  |  |  |
| --- | --- | --- |
|  |  |  |

Figure 4: Diagram of Decoupled Model

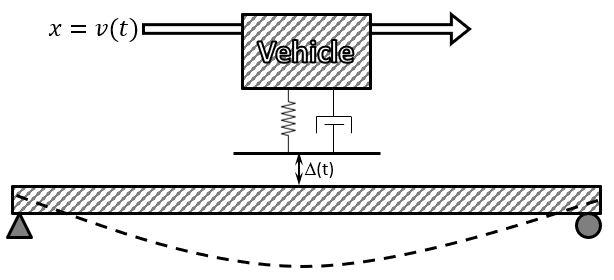


Figure 5: Diagram of Coupled 2-DOF Model

None of the readily available FEM software packages had the capability to apply both time and spatially varying loads, therefore the simplified 2-DOF was used for this demonstration. The following plot compares the bridge response from this uncoupled analysis to that from the coupled vehicle-bridge model.



Despite the simplicity of the model used to obtain the above results, it is still effective at modelling the exchange of energy between the two systems. As can be seen, decoupling the bridge and vehicle in simulations reduces the accuracy of the analysis and tends to overestimate the bridge response, especially at higher levels of amplification. This is because the uncoupled model fails to consider the influence of bridge motion on the vehicle contact force.

The roadway profile may be thought of as having a thickness equal to the difference between the roadway surface elevation and the mean surface elevation. When that profile is on a rigid roadway, the entire thickness imposed by the profile must be accommodated by the vehicle (i.e. only the vehicle can displace vertically). However, when that the profile is on a bridge, a portion of that imposed thickness is accommodated by the bridge, as it is accelerated by the resulting force. Therefore, the total displacement imposed on the vehicle suspension is reduced, thereby reducing the contact force.

In the case of the decoupled model, the contact force is overestimated because the roadway is assumed to be rigid and the reduction neglected. It is therefore recommended that any simulation of bridges under vehicle crossings include the interaction of the two systems if accurate responses are required.

### Influential System Parameters

Many studies on vehicle-bridge interaction will examine the role of parameters separately, and it was the original intention to organize this study in a similar manner. However, as it will soon be made clear, the parameters that effect VBI are interdependent. The following section therefore seeks to demonstrate and characterize the interdependency by first examining roadway profile which the case study showed to be critical to VBI and dynamic amplification.

A given profile is composed of elevation changes over the length of the roadway. This surface may be approximated by a summation of harmonic functions, and thus spectral analysis may be performed in much the same way as was done for acceleration data. The profile can therefore be described by its spatial frequency (cycles per unit distance) content. When a vehicle travels over a harmonic profile, the elevation change experienced at the vehicle’s location is harmonic with a frequency determined by the velocity of the vehicle. A profile with a given spatial frequency will therefore induce a force that acts on the vehicle with a frequency equal to the product of the spatial frequency and vehicle velocity. Therefore, the effect of the is dependent on the velocity of the vehicle.

Several simulations with a 3D FE model (140ft; single-span) were performed for a sinusoidal profile with 30-foot wavelength and ½ in. amplitude. The bridge model had the first and second natural frequencies between 2.2 and 2.5 Hz, while the first natural frequency of the vehicle model was 2.8 Hz. The plot below shows the peak vehicle and bridge response at different vehicle speeds and the resulting forcing frequencies. It is no surprise that the peak responses occur at speeds that induce a forcing frequency that matches their respective natural frequency.

The bridge displacement amplification was calculated by dividing the maximum midspan displacement by the static bridge displacement. The vehicle contact force amplification was calculated by dividing the maximum contact force by the vehicle’s weight.

#### Multi-dimensional parametric study

Additional simulation was performed with the simplified state-space model. Vehicle stiffness was varied to produce vehicle models with natural frequencies ranging from 2 to 5 Hz. Bridge stiffness was also varied to produce bridge models with natural frequencies of the same range. Simulations were performed with the same harmonic profile with a wavelength of 30 ft. Vehicle speeds ranged from 540 to 2400 in/sec (10-136 mph) to produce forcing frequencies between 1.5 and 6.7 Hz.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Bridge 1 | Bridge 2 | Bridge 3 | Bridge 4 |  |
| Span Length | 100 | 100 | 100 | 100 | ft |
| Mass (total) | 2000 | 2000 | 2000 | 2000 | slinch |
| EI | 5.603E+12 | 1.261E+13 | 2.241E+13 | 3.502E+13 | lb-in2 |
| Damping Ratio | 1.0% | 1.0% | 1.0% | 1.0% |  |
| 1st Nat. Freq. | 2 | 3 | 4 | 5 | Hz |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Truck 1 | Truck 2 | Truck 3 | Truck 4 |  |
| Mass | 200 | 200 | 200 | 200 | slinch |
| spring k | 31582.7 | 71061.2 | 126330.9 | 197392.1 | lb/in |
| Damping Ratio | 10.0% | 10.0% | 10.0% | 10.0% |  |
| Nat. Freq. | 2 | 3 | 4 | 5 | Hz |

The following plots summarizes the results of this parametric study.

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

|  |  |  |  |
| --- | --- | --- | --- |
| Bridge Displacement Amplification |  | | Vehicle Contact Force Amplification |
|  | Forcing Frequency (Hz) | Bridge Response  Vehicle Response | |

In the preceding figures the amplification of bridge and vehicle responses is plotted. The bridge displacement amplification is calculated by dividing the maximum midspan displacement by the static response given by the following (dictated by shape function):

|  |  |
| --- | --- |
|  | (3) |

The vehicle contact force amplification is calculated by dividing the maximum contact force by the static force of the vehicle which is merely the vehicle’s weight.

Simulations with the 5 idealized bridge models were again performed with the simplified state-space model and an artificial profile created according to ISO 8608 standards (C10 = 300; w = -2). The vehicle parameters were varied such that their natural frequency was a function of bridge natural frequency. The mass of the vehicles was held constant at 200 slinch (35025 kg), as was the damping ratio at 10%. The vehicle suspension stiffness was assigned such that the vehicles natural frequency varied from 0.8 to 4 times the bridge’s natural frequency. The following plot illustrates the optimum ratio of vehicle frequency to bridge frequency for the 5 bridge models.



The following tables summarizes the frequency factor at which a maximum response as achieved.

Table 8: Frequency Factor for Maximum Amplification

|  |  |
| --- | --- |
|  | Freq. Factor |
| Bridge1 | 1.76 |
| Bridge2 | 1.24 |
| Bridge3 | 1.14 |
| Bridge4 | 1.18 |
| Bridge5 | 1.12 |

The above simulations demonstrate the role of the profile as well as other parameters in vehicle-bridge interaction. The profile best excites the vehicle when its forcing frequency matches that of the vehicle, and the bridge is most excited when the profile forcing frequency matches its own natural frequency and the vehicle natural frequency is slightly larger than its own.

However even a harmonic profile cannot be entirely described by its frequency content alone. The distribution of phase angles for the different harmonic components have a large effect on the final form of the profile and how the vehicle-bridge system responds to that profile. The following plot is of two profiles created according to ISO 8608 standards with identical frequency content but different phase angle distribution (C10 = 300; w = -2).

Figure : Profiles with Matching Frequency Content but Different Phase Angle Distribution

Simulations were performed with these two profiles and the 2.5 Hz vehicle described in the previous section on the 140 ft single-span 3D FE model. The resulting bridge displacement is compared in the following plot.

Figure 7: Bridge displacement for profiles with different phase distribution

As can be seen, two profiles with identical frequency content but different phase angle distribution will cause different bridge responses. Similarly, the position of the profile can make a large difference in vehicle and bridge response. Simulations were performed with one of the profiles used in the previous simulation using the 2-DOF simplified model (Bridge 1 & Truck 1). The position of the profile was varied by offsetting it in increments of 5 feet.



As can be seen in the plot above, the bridge response varies greatly by just adjusting the position of the profile on the bridge. This is because the profile features that result in large contact forces will induce greater bridge response when they are located near the more flexible regions of the bridge (i.e. midspan). Furthermore, most real profiles are not harmonic but rather have many transient features. Profiles that contain features with large wavelengths that result in forcing frequencies similar to vehicles or the bridge should be avoided (e.g 5-50ft.). However, the frequency content of the profile alone has no reliable correlation with dynamic amplification and spatial information must be included in any dynamic amplification analysis.

Vehicle and bridge parameters therefore effect dynamic amplification based on their influence on the dynamics of the system and how those system dynamics relate to the profile characteristics. While parameter effect cannot be quantified they can still be generalized as follows:

* Higher bridge mass, stiffness and damping generally serve to reduce dynamic amplification.
* Higher vehicle mass increases dynamic amplification while vehicle damping helps to reduce amplification.
* Higher vehicle speed leads to increased dynamic amplification.
* Longer bridge length results in a longer period of time for which the vehicle is present on the bridge and may therefore result in greater dynamic amplification.

Anything further than these generalizations requires simulation of vehicle-bridge interaction with the specific profile or direct measurement by field experiment.

## IRI & Other Vehicle-Only Models

There are several methods already widely used to assess the roughness of roadway profiles. The International Roughness Index (IRI) is the most complex and simulates a specified vehicle (golden quarter-car) traveling over the profile. Other metrics ignore the vehicle and deal only with the profile data. The ISO 8608 parameters, for example, describe the spatial frequency content of the profile. However, all of these roughness metrics fail to consider the bridge or the position of the profile. Studies were performed to examine if these metrics had any ability to predict dynamic amplification.

ISO 8608 parameters describe the spatial frequency content of the profile. Studies presented in the first part of this document show that the spatial frequency of the profile content does influence dynamic amplification. However, these parameters ignore any transient features and ignore the phase angle distribution and therefore are inadequate for predicting dynamic amplification, as was evidenced by the plot which compared the bridge response for two profiles with identical ISO parameters but different distribution of phase angles. The inadequacy is further demonstrated by the supplied correlation plot.

The IRI includes the vehicle in its model and may be expected to demonstrate better ability to predict dynamic amplification. However, a correlation plot shows that the IRI cannot reliably predict dynamic amplification. The plot also reveals that while profiles with high IRI may not always result in high dynamic amplification, bridges with high dynamic amplification have high profile IRI values. This provides further encouragement to encourage and mandate a smooth deck surface.

Another simple model was assessed that included representation of the vehicle but ignored bridge behavior. Position of the profile on the bridge was included by applying a sine window to the vehicle response over the time period for which the vehicle is on the bridge. The maximum of the windowed contact force is reported as a factor of the vehicle self-weight. This contact-force amplification metric is compared to FE predictions in the plot below. The metric correlated relatively well with dynamic amplification, but that correlation coefficient was not consistent between bridges and therefore is not recommended for any amplification predictions.

## Summary

* The vehicle and bridge comprise a coupled dynamic system that is energized by the vehicle traversing a profile.
* Dynamic amplification estimated by filtering operational monitoring data may over-estimate amplification.
* Determining dynamic amplification of in-service bridges may be performed with operational monitoring or a load test. In either case, strain gauges are recommended over displacement gauges or accelerometers.
* A 3D FE model is capable of simulating vehicle-bridge interaction and is recommended for predicting dynamic amplification for structures with complex geometry or that are otherwise ill-suited to the state-space models.
* A simple model that reduces both the bridge and vehicle to SDF systems has been shown to reliably predict dynamic amplification, and is recommended if FE simulation is not practical.
* Any metric that is to be used for predicting dynamic amplification must include a representation of the bridge. Therefore dynamic amplification should not be predicted by current roughness metrics (e.g. IRI and ISO 8608) that only consider the profile and vehicle.