Guide to Vehicle Bridge Interaction

This document is meant to detail, describe, and illustrate the mechanisms and behavior associated with vehicle-bridge-interaction (VBI).

Latest objectives/questions

* If the profile is known, how can dynamic amplification be estimated.
  + Simulate bridge-vehicle interaction with beam model and SDOF vehicle.
    - Bridge parameters to match bridge geometry and mass distribution and first vertical mode
    - Vehicle parameters (e.g. mass, natural frequency, damping)
* If the profile is unknown
  + Assume profile
  + Bridge vulnerability
    - What types of bridges are susceptible to large dynamic amplifications
* Show that a profile induces much more dynamic amplification vs. a smooth profile.
  + What are the parameters that effect dynamic amplification
  + Theoretical moving point load (smooth profile & constant contact force) vs profile, etc.
* Can a decent profile cause amplification of 1.33 or greater
  + Create profile with good IRI that causes amplification in excess of 1.33 (varying slopes of psd function)
* How much does frequency content of the profile matter
  + Start with a handful of frequencies
  + Total power within range of frequency (fnb ± x)
  + Power at fnb and slope of log fit to psd
* Is the difference in contact force when considering the bridge, vs vehicle only, due to the change in vehicle displacement, or the addition of bridge displacement (or both)
  + Can the contact force be adequately estimated by accurately simulating vehicle displacement and multiplying by the vehicle suspension spring stiffness
* Can a profile be formed by superimposing a bridge displacement (sin\*sin) with the roadway profile?
  + Record bridge displacement at vehicle location
  + Sum bridge displacement record with roadway profile
  + Run vehicle only model with this new profile and compute contact force
  + Compare contact force with that produced from bridge-vehicle model
  + Try with again with profile with sin\*sin added to roadway profile
* Can initial conditions of the bridge cause an increase in dynamic amplification
  + Run state-space model again, this time set the initial conditions to those that would occur at a time such that the vehicle contact force will be in phase with the bridge oscillations.
* Can initial conditions of the vehicle cause an increase in dynamic amplification
  + Give vehicle initial conditions that puts its motion in phase with the bridge motion (or opposed)

# Conclusions

Dynamic amplification refers to the additional response of a bridge to vehicle when it’s moving versus when it is stationary (static analysis). A profile with a low IRI will not have problematic amplification factor. A profile with a high IRI may contribute to large dynamic amplification, depending on the frequency content.

* The magnitude of the amplification is largely due to the profile of the bridge roadway. (Profile vs. smooth)
* The dynamic amplification (resulting from the profile) can be reduced by enforcing a smooth profile
  + Straightedge requirements: 1/8” over 20ft keeps amplification under 1.33
  + Removing frequencies corresponding to the bridge natural frequency.
  + Profiles with low IRI will have low amplification
  + Profiles with high IRI may have large amplifications based on frequency content (or waviness)
* IRI is a metric capable of describing the amplitude of the profile but the waviness is needed to describe the distribution of features with different wavelengths.
* While profile frequency content has an effect on bridge response amplification, phase angle distribution contributes large variability. Therefore, the specific profile must be analyzed.
* Vehicle-bridge interaction can be estimated with simplified models (i.e. single-line beam model), but should accurately predict first vertical (global) mode and have accurate mass distribution as well as accurately model span continuity.
* The total response of a bridge to dynamic loading can be broken into 2 components:
  + The response due to the vehicle force (amplified due to its vertical acceleration)
  + Response due to induced bridge motion (excitation of modes of vibration)
* Dynamic amplification factors, when computed as the ratio of total dynamic bridge response to bridge response due only to a static vehicle, will be greater when computed for displacements than when computed for moments, stresses, and strains. This is due to the fact that the bridge response due to the vehicle force and the accelerated mass of the bridge, but is forced to be expressed as a function of only the vehicle force
* The vehicle contact force is a result of a coupled dynamic system. The contact force cannot be accurately computed by considering only the vehicle’s interaction with the profile when the profile is harmonic.
* The vehicle contact force (magnitude, maximum) can be reasonably approximated by considering only the vehicle’s response to the profile when that profile is transient in nature as is seen in real profiles.
* Vehicle contact force, computed without consideration of the bridge, cannot be accurately used to predict bridge response, especially when the bridge is subjected to sequential vehicle crossings.
* Vehicle response is greatest when the profile contains frequency content that produces a forcing frequency equal to the vehicle first natural frequency. The vehicle response to a profile has been studied and is extensively covered in this paper: (Camara et al. 2017).
* Bridge response is greatest when the profile has frequency content that produces a forcing frequency just below the bridge’s natural frequency (≈0.9fn), and the vehicle has a natural frequency near and above the bridge’s natural frequency.
* A vehicle traveling over a bridge that is oscillating at a frequency (fn) will experience a transformed bridge motion, whereby the “felt” motion is the sum of two frequency components equal in magnitude. The frequency of these two components can be described by: . Therefore, the vehicle experiences a positive and negative shift in frequency (displacement, velocity, or acceleration) according to the vehicle velocity and the distance between modal points for that mode of vibration.
* Based on 2DOF system simulations, the displacement of a bridge due to dynamic vehicle loads is a function of bridge mass and stiffness, profile acceleration corresponding to frequency content at or near the bridge natural frequency, the mass of the vehicle, and to a lesser extent the damping of the bridge and vehicle. The displacement is directly related to truck mass and profile acceleration, and indirectly related to bridge mass, bridge damping and vehicle damping.

Idea: create simple (1dof) system that has matching 1st natural frequency, and equivalent mass (mass distributed according to the mode shape: ). Apply time varying contact force to this simple model and record resulting acceleration of sprung mass. Compare the acceleration of the single dof system with that of the FE model analyzed with a moving mass over the profile. Correlate the maximum acceleration of the single dof model with that from the FE model. If a correlation can be made between the 2 accelerations, a correlation is likely also possible between the the maximum acceleration of the single dof model and the dynamic amplification (according to model forms inspired by (Equation 1) developed in the later pages.

How can we estimate the following quantities for accurate approximation of bridge response due to dynamic loading:

1. amplification of vehicle weight (due to vertical vehicle acceleration) as well as
2. the expected excitation of bridge vibration due to vehicle force

The degree to which the structure is excited is dependent on the magnitude and character (location, frequency) of the force imparted by the vehicle.

The amplification of the vehicle’s contact force (factor of vehicle’s static weight) is dependent on the distance between the vehicle’s center of mass and the bridge roadway surface. Can this be simplified to be considered only as a function of the vehicle’s motion (bridge motion disregarded.

# Bridge Vulnerability

The following simplified bridge parameters may have an effect on dynamic amplification

* Bridge mass
* Length
* Stiffness (1st mode freq.)

For a given bridge, there are limitless profiles, and therefore we wish to choose a profile that is the worst case for the particular bridge. The profile parameters will be kept within the following range:

IRI < 300; waviness 2.0 – 3.5

Therefore the following bridges will be investigated.

Length will be varied from 30 feet to 200 feet, bridge mass will range from 3 to 6 kips/ft. The stiffness will be set to meet varying dead load

deflection criteria (L/2500 to L/500).

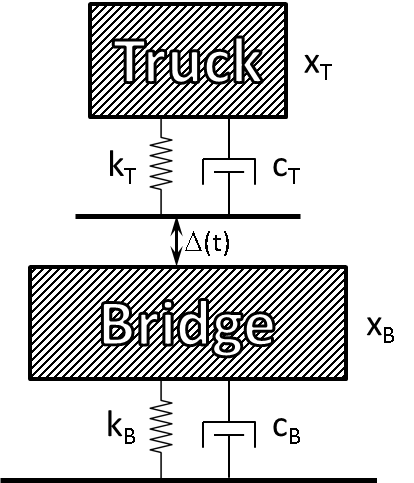
The vehicles will have constant mass and damping and will vary stiffness to achieve a natural frequency equal to 110% of the bridge first natural frequency.

Several profiles will be examined for each structure. The IRI will remain at 250 for all the bridges but waviness values of 2.5, 3 and 3.5 will be used. The following table summarizes the parameters and values.

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Lower Bound | Upper Bound |  |
| Profile waviness | 2.5 | 3.5 |  |
| Bridge Mass per ft | 3 | 6 | Kip/ft |
| Bridge Length | 30 | 200 | Ft |
| DL Deflection | L/2500 | L/500 |  |

# Coupled system: The vehicle response cannot be reasonably approximated by only considering the profile (over rigid roadway).

## 2DOF Representation

To show this, we may simplify the bridge and vehicle systems to a 2-DOF (degree of freedom) system.

For this model the following parameters were set:

|  |  |  |
| --- | --- | --- |
| Bridge Mass | 2000 | Slinch |
| Truck Mass | 200 | Slinch |
| Bridge Damping | 2% |  |
| Truck Damping | 20% |  |
| Natural Frequency of Bridge | 3 | Hz |
| Natural Frequency of Vehicle | 3.6 | Hz |
| Profile Frequency | 2.7 | Hz |
| Profile Amplitude | 0.2 | in |

The stiffness of the bridge was increased by a factor of 1000 to represent a rigid bridge. This approximation is adequate as the contact force and bridge force for the scenarios over “rigid-road” are approximately equal.



We can also compare the force imparted by/to the bridge spring (xB\*kB).



This second plot illustrates that the bridge response is due to not only the amplification of the apparent weight of the vehicle, but also the acceleration of the bridge mass. This is apparent because, otherwise the ratio of bridge force for over-bridge vs over-rigid-road would equal that of contact force.

While for this case, the bridge force and contact force are both greater for the over-bridge scenario, the following example examines a case for which the natural frequency of the bridge and vehicle match the frequency of the profile.

|  |  |  |
| --- | --- | --- |
| Bridge Mass | 2000 | Slinch |
| Truck Mass | 200 | Slinch |
| Bridge Damping | 2% |  |
| Truck Damping | 20% |  |
| Natural Frequency of Bridge | 3.0 | Hz |
| Natural Frequency of Vehicle | 3.0 | Hz |
| Profile Frequency | 3.0 | Hz |
| Profile Amplitude | 0.2 | in |





In this instance, the contact force decreases when over a bridge vs rigid road, however, the bridge force is still greater when over a bridge.

The following table summarizes these simulations

|  |  |  |  |
| --- | --- | --- | --- |
| Case 1 | Over Rigid-Road | Over Bridge | % Increase |
| Max Contact Force (lb) | 24849 | 47207 | 90% |
| Max Bridge Force (lb) | 24867 | 244169 | 882% |
| Case 2 |  |  |  |
| Max Contact Force (lb) | 38288 | 22687 | -41% |
| Max Bridge Force (lb) | 38320 | 141135 | 268% |

In summary, dynamic amplification can be broken into two components: the amplification of apparent vehicle weight as well as the additional response of the bridge due to the acceleration of its mass (and excitation of its natural frequencies). Neither of these components can be adequately captured without including the response of the bridge system.

## FE Model Representation

Similar simulations were performed with FE models.

### Harmonic Profile

The following plots show the contact force on rigid roadway, as well as on a bridge. Two vehicle simulations were run on a series of single-span bridges with a natural frequency of 3 Hz and a 500’ rigid approach. The vehicle parameters were the same as in the state-space simulations previously presented.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Case 1 | Case 2 | Units |
| Truck Mass | 200 | 200 | slinch |
| Bridge Damping | 2% | 2% |  |
| Truck Damping | 20% | 20% |  |
| Natural Frequency of Bridge | 3.0 | 3.0 | Hz |
| Natural Frequency of Vehicle | 3.6 | 3.0 | Hz |
| Profile Frequency | 2.7 | 3.0 | Hz |

### Real Profile (non-harmonic)

The FE model was also run with a more realistic profile (Graph 1), that was gathered from an actual bridge. An FE model of multiple single-span bridges, each with a frequency of 3 Hz and a mass distribution representative of real bridges was analyzed with the realistic profile. Another analysis was completed with an identical FE model, except the bridges in the model were made rigid.

Graph : Real profile used in FE simulation

Figure : Contact Force from FE model with Real Profile (Freq>10Hz filtered out)

These results further support the claim that the vehicle response (and resulting contact force) cannot be accurately predicted unless the bridge system is included in the analysis. Furthermore, the dynamic amplification of contact force does not constitute the entirety of dynamic amplification. The dynamic amplification of bridge response must include the response due to the increased apparent weight of a bouncing vehicle as well as the response associated with the motion of the bridge as its mass is accelerated (and resonating in its natural modes of vibration).

However, it seems that as the roadway profile becomes less harmonic, the force input by the vehicle to the bridge may be reasonable predicted by considering the response of the vehicle over the profile, without consideration of the bridge.

The question remains, whether bridge response can be adequately predicted by analyzing it with the vehicle contact force computed with the profile alone. But regardless, there is no convenient means of analyzing a bridge subjected to a force that varies in both position and magnitude with time. More important is to determine the degree to which the bridge is excited by that contact force.

## Decoupling efforts

In an effort to demonstrate the suitability of a contact force computed without consideration of the bridge we will analyze (2) single dof systems separately. First, a model of the vehicle only will be run, subjected to a given profile (realistic). The contact force will be recorded. A second model of the bridge (idealized as a single sprung mass) will be subjected to the contact force. The resulting bridge response will be compared to that for a 2 dof coupled system subjected to the same profile.

First let’s compare contact force computed using a model of the coupled system and using a model of only the vehicle.



The contact force appears to be similar with and without the bridge considered. The bridge displacement is compared below.



As can be seen in the above plot, the displacement cannot be accurately predicted, and in this instance, is overestimated, if an uncoupled contact force is used. However, this study approximates the structure as a single degree-of-freedom sprung mass, when in reality the force is moving on the bridge thereby applying force at points of varying stiffness and only present for a short period of time.

### Bridge response to contact force

Let’s examine the response of a more realistic structure to the decoupled vehicle contact force.

If the response of a bridge can be determined with only the decoupled contact force then the contact force can be estimated using the profile (or profile parameters), which can in turn be used to estimate the bridge response.

# Frequency or phase shift of moving observer over oscillating mode shape

This study examines the effect of a moving observer on an oscillating mode shape. This scenario represents a vehicle traveling over a vibrating bridge. The vehicle position in the following calculations is for the location on the bridge at the vehicle’s position and thus represents the vehicle contact point position. This scenario is easily described with the following functions.

* Assumed mode shape: for modes: N = 1,2,3…∞
* Assumed motion of bridge: for natural frequency: *fn.* At time: *t*
* Assumed vehicle motion: for vehicle velocity: *v*

Therefore the vertical position of the vehicle at time: t is as follows:

It is clear from the above that the vertical motion of the vehicle (contact point) does not have a frequency equal to natural frequency of the bridge mode. The equation can be written in the following form:

Therefore the vehicle experiences vertical motion with two frequency components equal to . Because of the nature of the integration of trigonometric functions, the frequency shift is the same for vehicle contact point velocity and acceleration.

This motion is illustrated in the following plot of the time history of the vehicle’s (contact point) vertical position (A=1, fn=3Hz, N=1, v=720 in/sec, L=100 ft). The sum of the two harmonic functions results in an apparent beating.



The frequency shift, , is plotted below for varying levels of v/L for the first mode. Alternatively, N/L can be thought of as the distance between nodal points. For a bridge with a span of 100 ft, a vehicle velocity of 65 mph, a frequency shift of ±0.48 Hz can be expected. The acceleration of the vehicle is the second derivative of the position, and therefore, has the same 2 frequency components, but the second component contributes more to the total acceleration response because the component is multiplied by while the first is multiplied by and the second term, , can be assumed to remain positive.

# Stress/strain amplification factors vs displacement amplification factors

In many pieces of relevant literature experimentally derived dynamic amplification factors are reported for both displacement and stress or strain. The displacement factors are almost always greater than those for stress or strain. If a bridge behaves linearly then its response should be linear (i.e. an increase in load by a factor X will result in an increase in response by the same factor, X) regardless of whether the response in question is global (e.g. displacement) or local (e.g. stress, strain). This is not the case with dynamic amplification factors because of a violation of the key assumption that the measured response is due solely to the applied load.

Dynamic amplification factors assume that the bridge response is due to a vehicle which applies a force equal to its weight increased by a factor to account for its dynamic motion (acceleration > gravity). If this assumption were true, dynamic amplification factors would be equal for the various response quantities. The evidence to the contrary indicates that the assumption is false.

I pose the following assumptions are more representative of the phenomenon of dynamic amplification:

The bridge response is due to a force applied by the vehicle as well as a force due to the acceleration of the mass of the bridge as it is excited by the dynamic vehicle force (or other excitation sources).

Therefore, the bridge response is the sum of the two responses. To exemplify this, let’s consider a simply supported beam with a point load placed at midspan. The beam displacement and stress at midpsan due to the point load at midpsan would equal the following:

* Displacement:
* Stress:

We can assume the beam is oscillating with a sinusoidal mode shape with a maximum acceleration Amax, at midspan. Therefore, the acceleration at any point along the beam can be described by the following equation:

Since the mass of the beam is uniformly distributed along its length, we can describe the maximum force at any point along the beam with the following equation for the equivalent static force:

This equation can takes the form of the Newton’s second law of motion: .

The beam displacement and stress at midspan due to this distributed force can then be calculated using statics and the relationship between moment and curvature to produce the following:

* Displacement:
* Stress:

Therefore, the total response would be the sum of the two components, and the amplification factors would take the following form (where the vehicle force is Av\*P):

Equation

It can be deduced from the above equations that if vehicle amplification factors are to be used, an additional response must be accounted for by including the second term in the above equations. Furthermore, these equations show that the amplification factor for displacement and stress will be different, and the amplification in excess of the vehicle amplification is greater for displacement than stress (or strain or moment). The ratio of the additional amplification is calculated below.

This shows, that regardless of bridge, or vehicle, the experimental amplification factor for displacement will always be greater than that for stress or moment. It should be noted that these calculations included only the first mode. If more modes were to be included, the equations for amplification factors would take the following form:

The ratio of additional amplification can again be computed.

Additionally these results are limited to midspan responses. Amplification factors at other locations can be computed in a similar manner, using equations for displacement or stress at other locations. This derivation considers only a simply supported beam and a point load, but the results are still representative of the phenomenon occurring in real structures because the following conditions still remain true:

* The vibration of a bridge takes a shape that can be described by a shape function that is a summation of sines.
* The vehicle force is applied to a small area of the bridge and can be reasonable approximated as a point load.

# Bibliography

Camara, A., V.F. Vázquez, A.M. Ruiz-Teran, and S.E. Paje. 2017. “Influence of the Pavement Surface on the Vibrations Induced by Heavy Traffic in Road Bridges.” *Canadian Journal of Civil Engineering* 44 (12): 1099–1111. https://doi.org/10.1139/cjce-2017-0310.

# Appendix

## Amplification factors for different response quantities

Simply supported beam response to point load at x=L/2

 Displacement at beam mid-span

 Moment at mid-span

 Stress due to moment

 Stress due to point load at mid-span

Response of simply-supported beam to equivalent static load of mode shape (Ψ(x))

 Mode acceleration shape function

 Acceleration of mode shape

 Equivalent static force (F=mass\*accel)

 Maximum equivalent static force

 Shear force along length of beam

 Moment along beam

 Moment at mid-span

 Stress at mid-span due to equivalent static force

 Moment-curvature relationship

 Vertical deflection along the length of the beam (C1=C2=0)

 Deflection at mid-span due to equivalent static force