CONTROL OF SOLAR SAILS

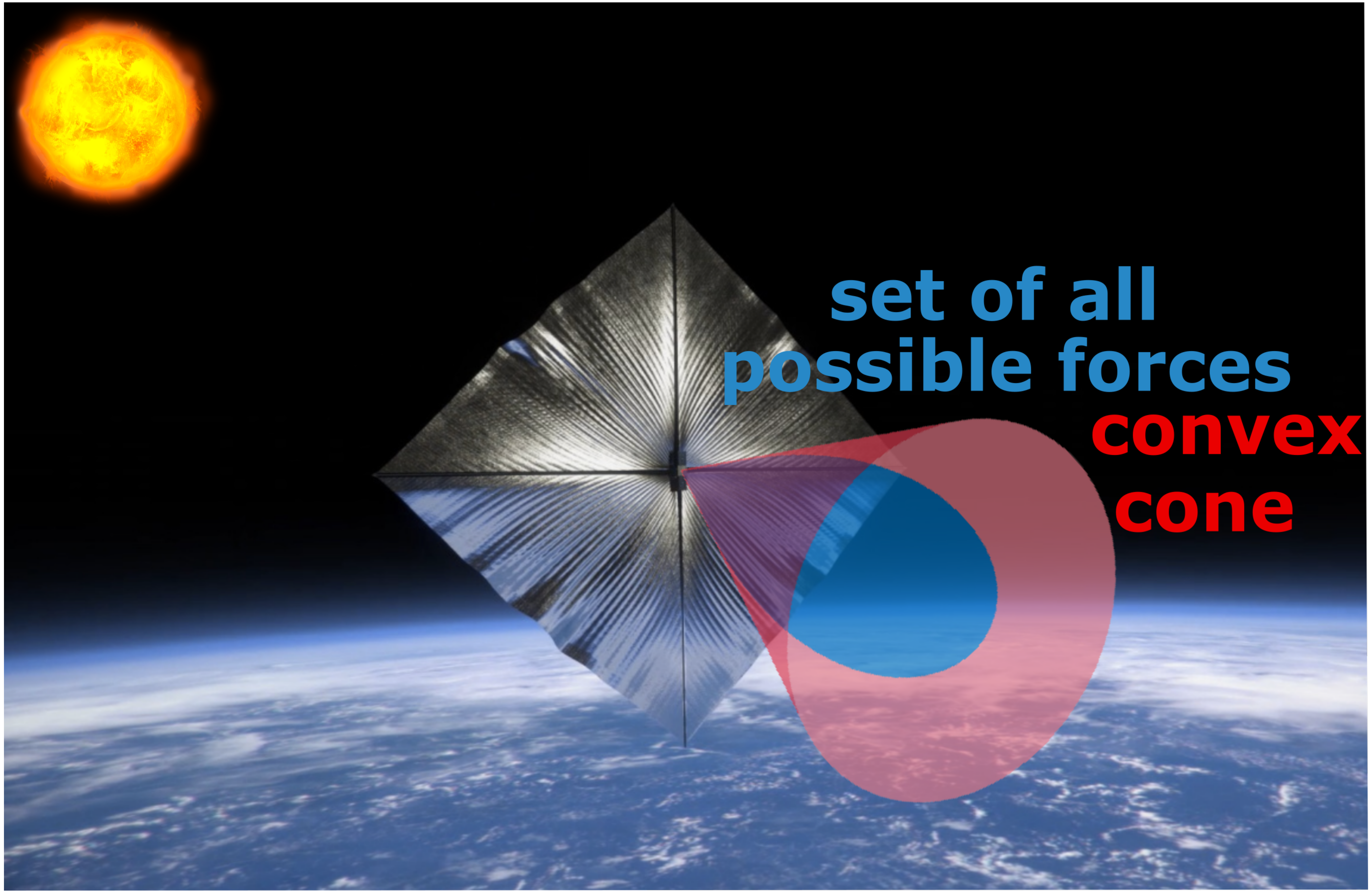
J.-B. Caillau, L. Dell'Elce, A. Herasimenka, J.-B. Pomet

Université Côte d’Azur, CNRS, Inria, LJAD, France

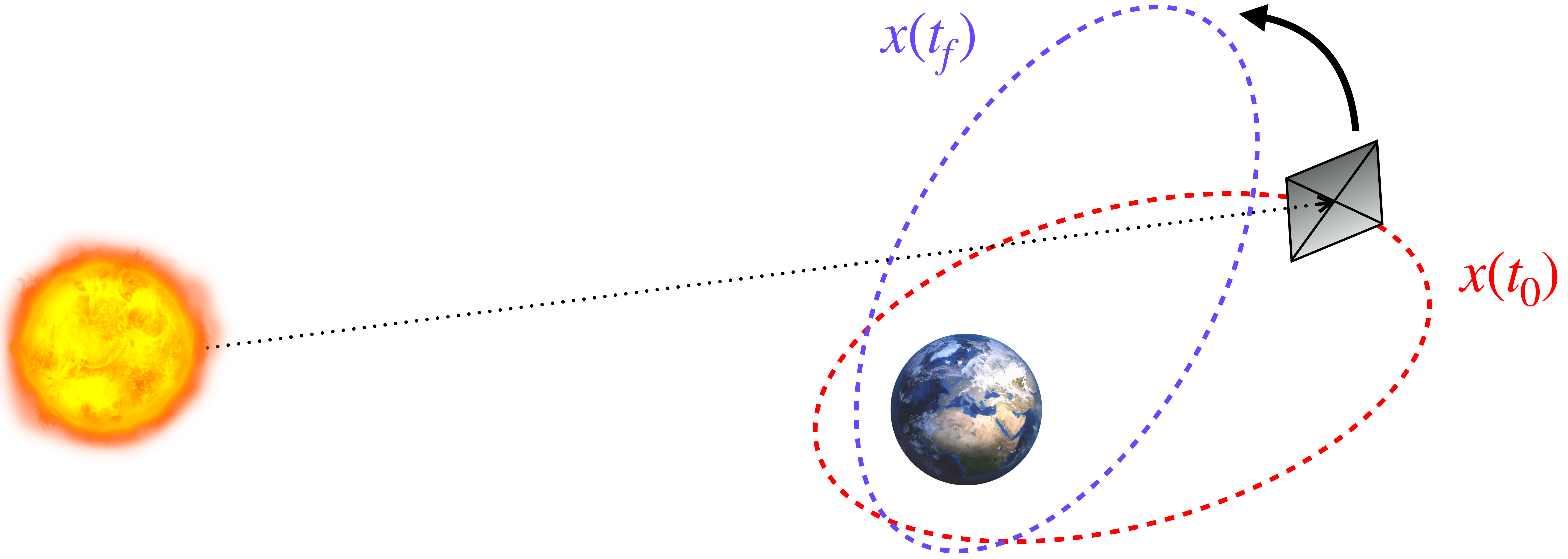
ESA contract no 4000134950/21/NL/GLC/my

Zhejiang U., March 2025

Non-ideal sail: a cone-constrained control problem

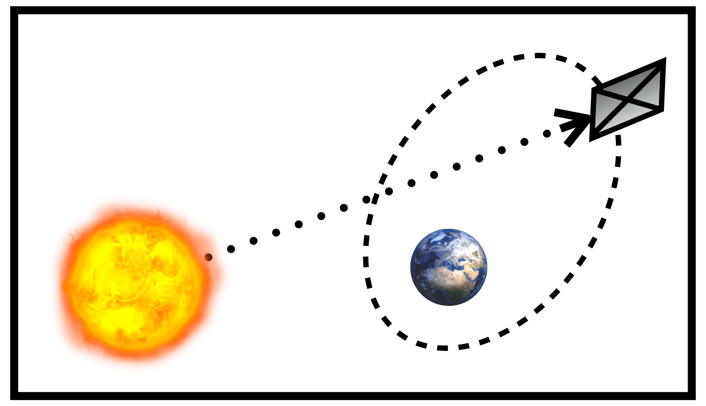


Can non-ideal sails arbitrarily change their orbit?



Is it possible to generate any $x(t_0) \longrightarrow x(t_f)$?

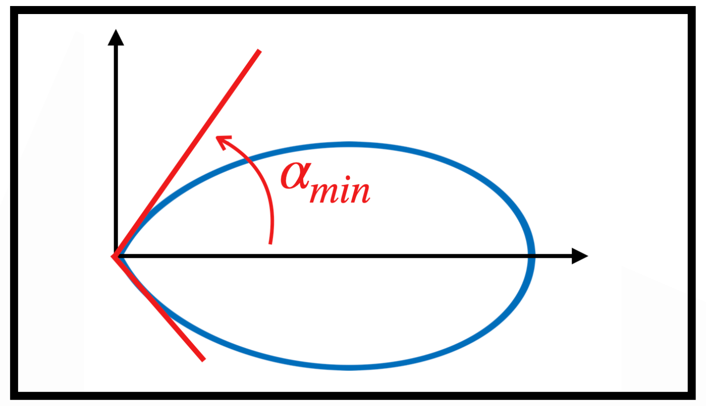
Outline



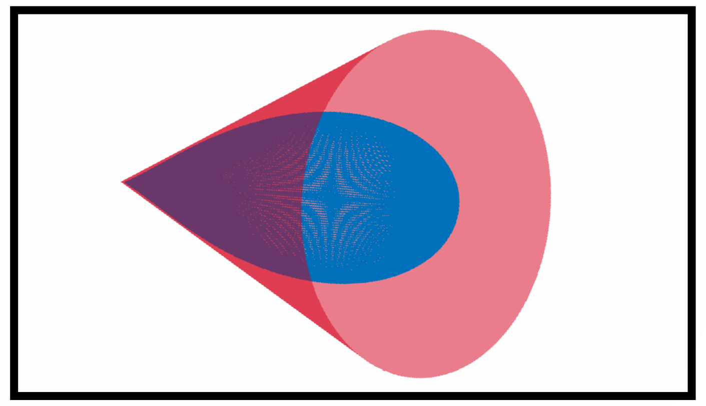
1. Dynamics of the system

$$\left\langle p_I, \frac{dI(f,\boldsymbol{u})}{dt} \right\rangle \geq 0$$

2. Necessary condition for local controllability

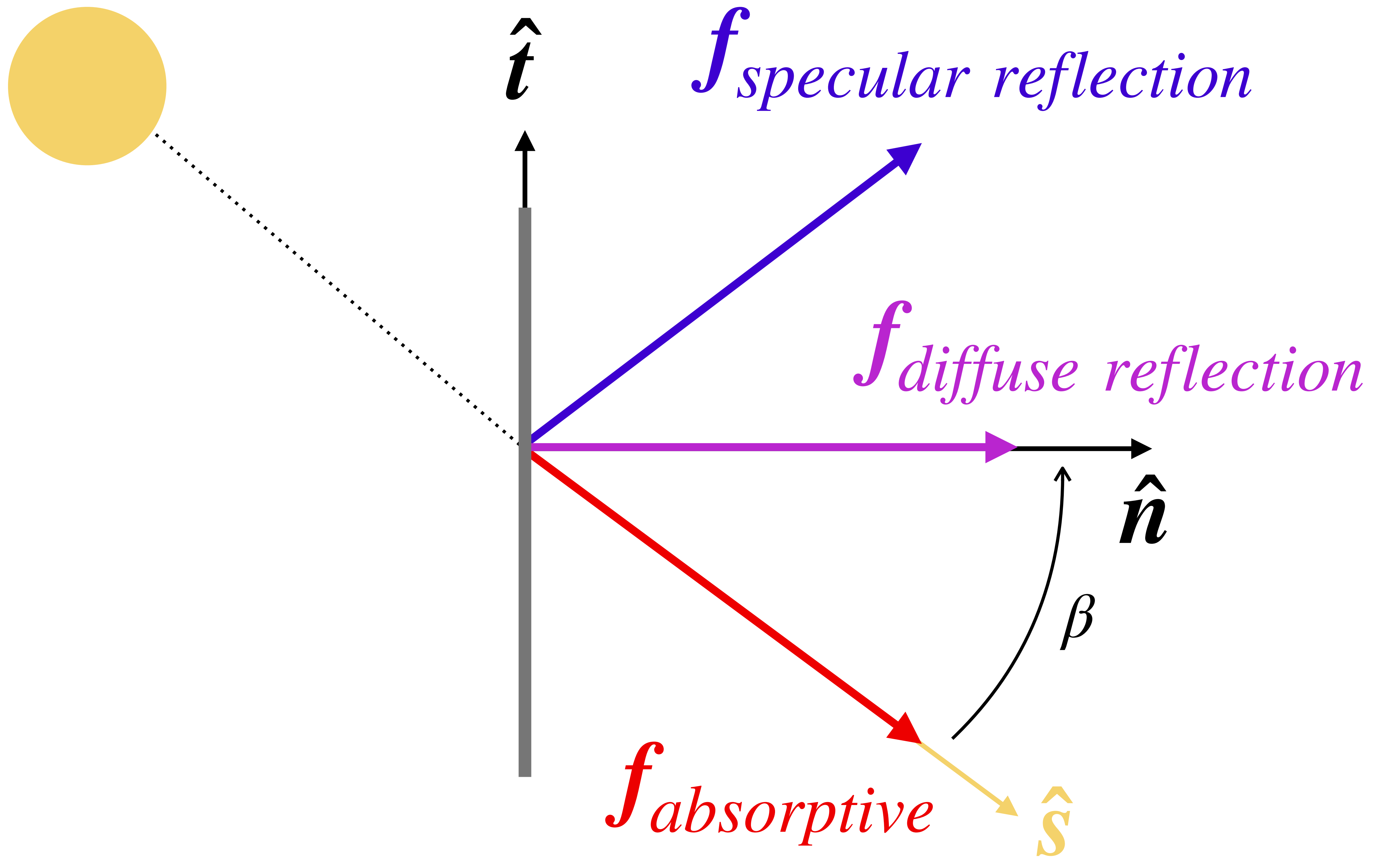


3. Minimum optical requirements



4. Generalization to other problems

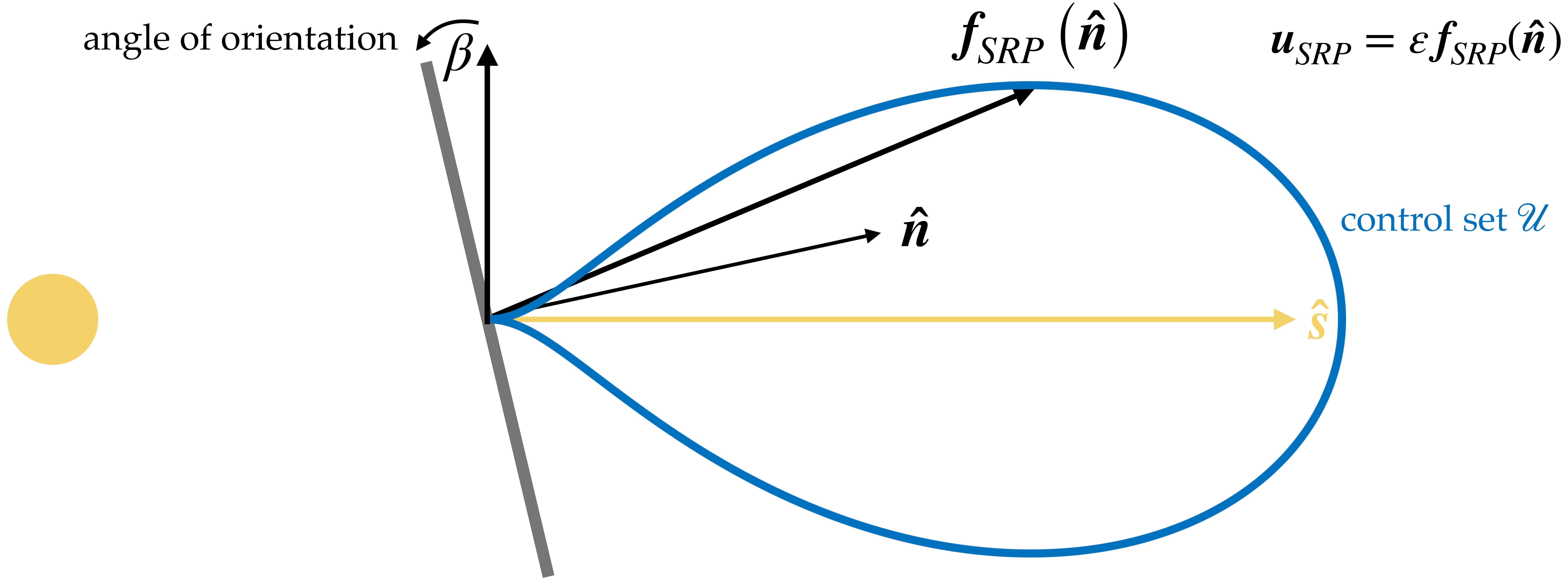
1. Force components of solar sail



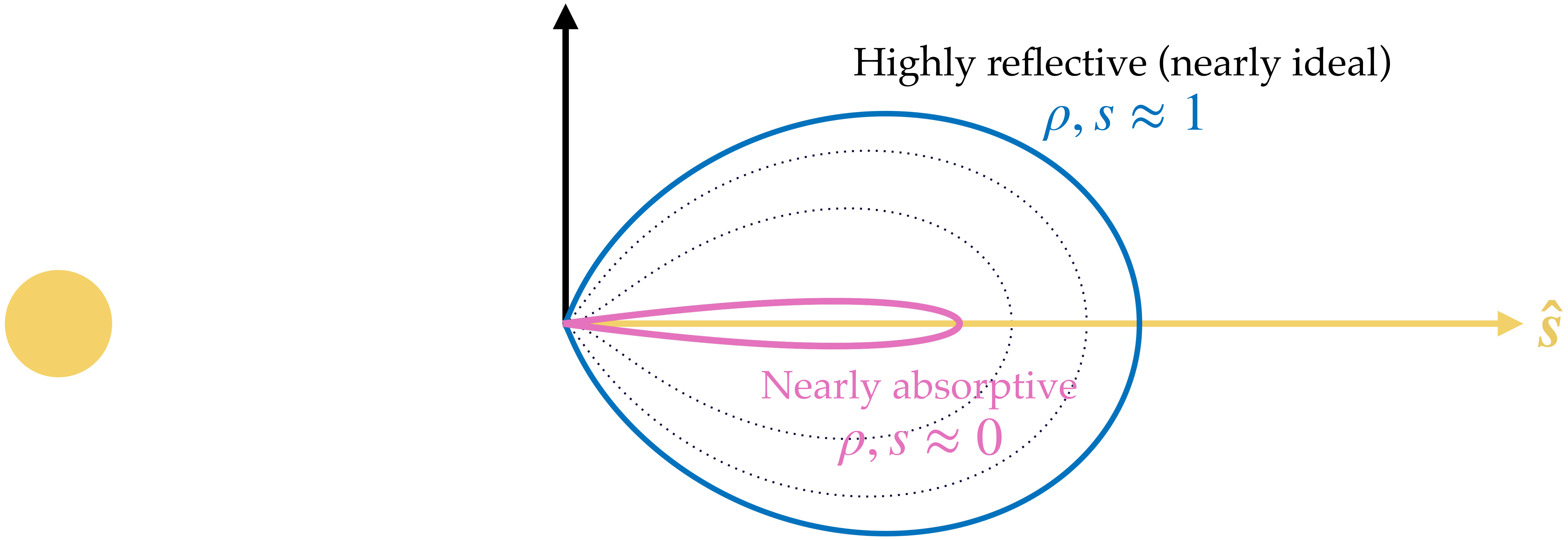
$$f_{SRP} = f_{absorptive} + f_{specular\ reflection} + f_{diffuse\ reflection}$$

1. Control set

$$\dot{x} = F^0(x) + \sum_i u_i F^i(x), \quad u \in \mathscr{U}$$

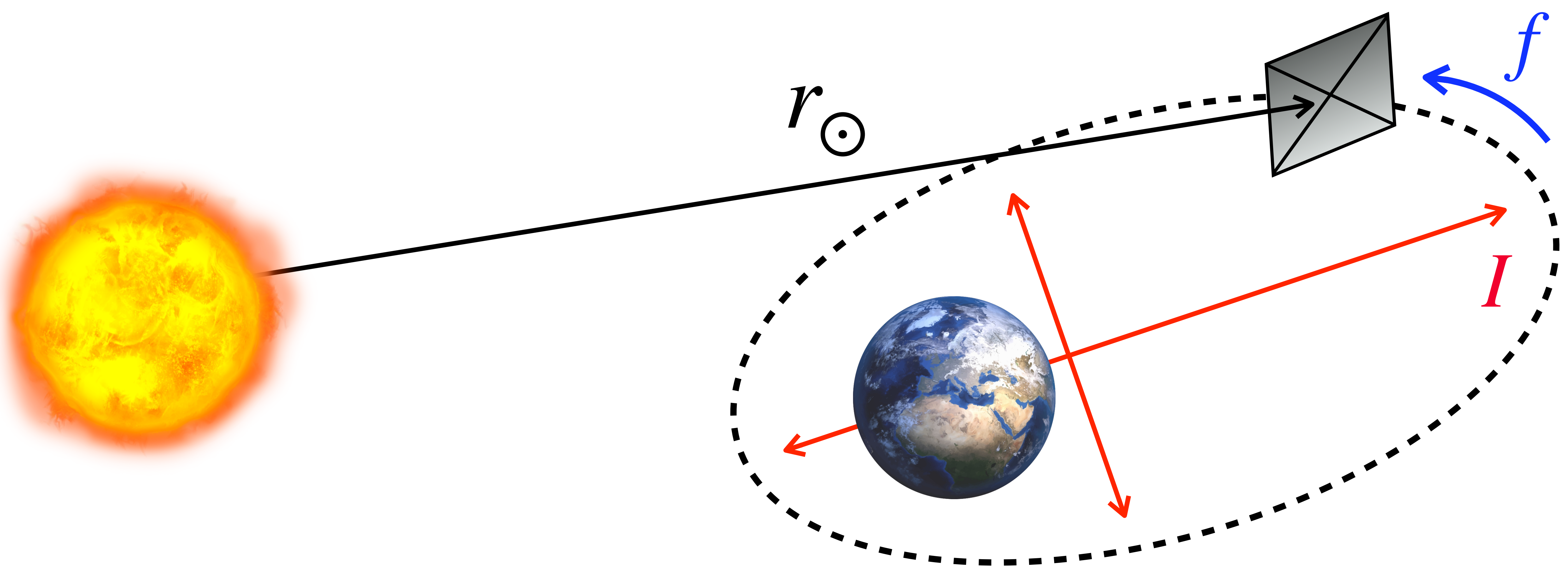


1. Parametrisation of the control set



$\rho, s \in [0,1]$ portion of specular, diffuse reflection

1. Dynamical system



Assumptions:

No eclipses

Sun motion neglected over one orbit

SRP is the only perturbation

$$\dot{x} = F^0(x) + \sum_i u_i F^i(x), \quad u \in \mathcal{U}, \quad i = 1, 2, 3$$

with $x = (I, f)$, $\quad I \in M$, $f \in \mathbb{S}^1$, $\quad F^0, F^i$ given by Gauss variational equations

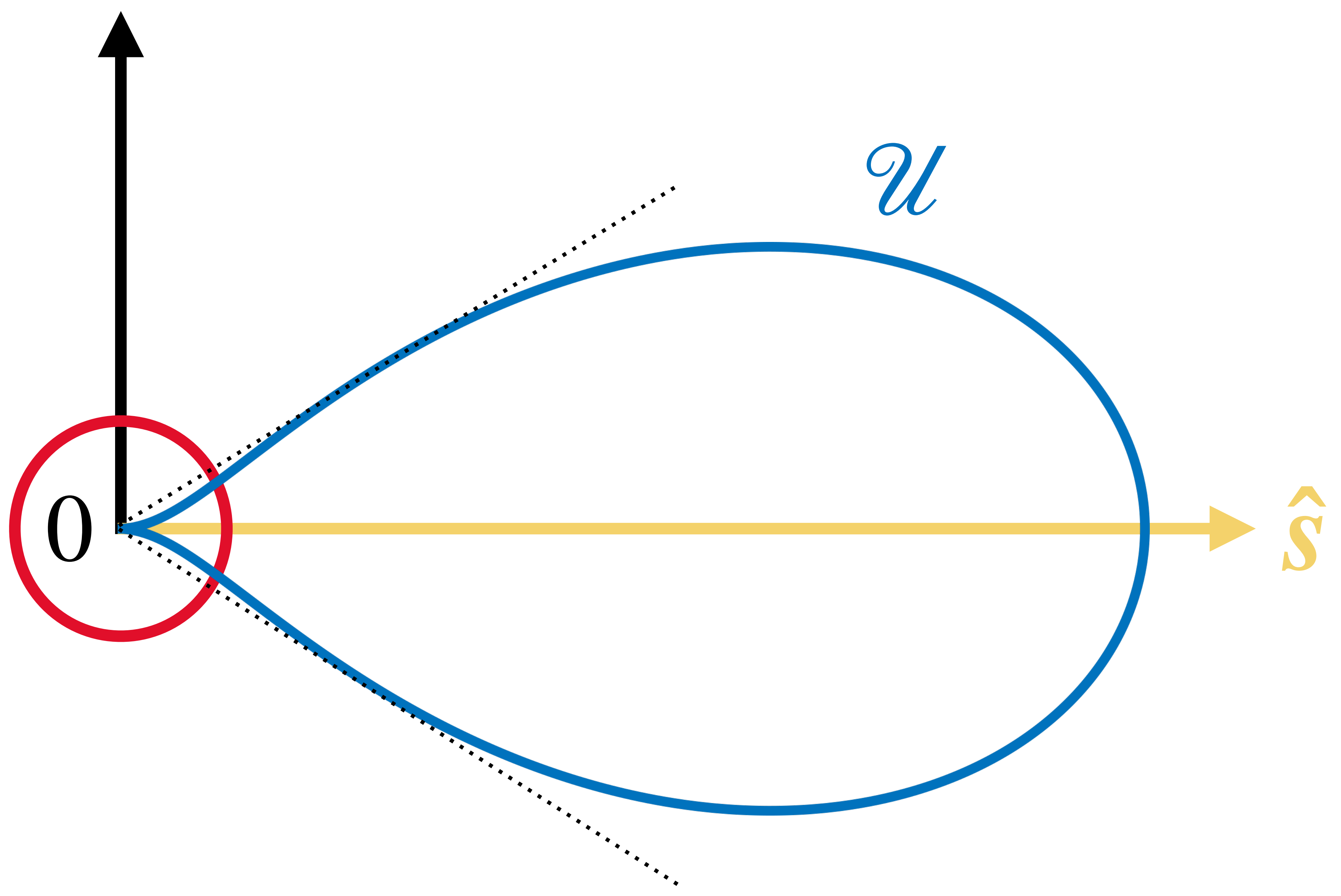
2. Classical approach using Lie brackets

Controllability if [Jurdjevic, 1996]:

i) Periodic drift $\longrightarrow$ **OK**

ii) Bracket generating
OK if $\rho > 0$
first integral if $\rho = 0$

iii) $\mathrm{Conv}\left(\mathscr{U}\right)$ is neighbourhood of the origin $\longrightarrow$ **FAIL**



2. Proposition on controllability

Under the conditions:

(i) system is bracket generating,

(ii) control set $\mathscr{U}$ contains the origin,

(iii) $\forall I \in M, \quad \text{cone}\left\{ \sum_i u_i F^i(I, f),\ u \in \mathscr{U},\ f \in \mathbb{S}^1 \right\} = T_I M,$

the system is controllable* in the following sense: for any $(I_0, f_0),\ (I', f') \in M \times \mathbb{S}^1,$ there is a time T and a integrable control $u \in \mathscr{U}$ that drives (I_0, f_0) to (I', f') at time T.

* The proof is available in [H., Caillau, Dell'Elce, Pomet, 2022]

2. Proposition on controllability

Under the conditions:

(i) system is bracket generating,

(ii) control set $\mathscr{U}$ contains the origin,

How to verify the condition?

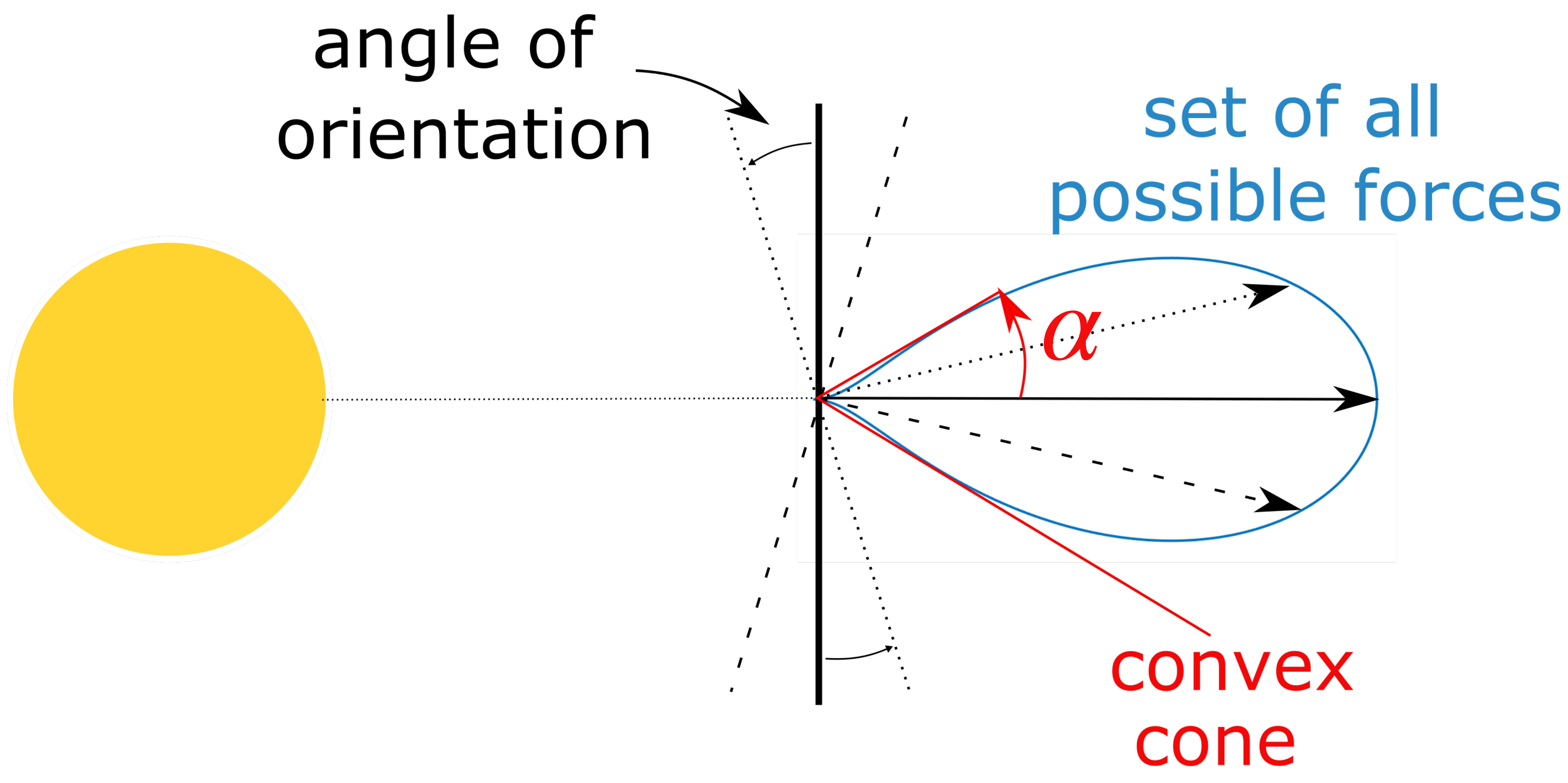
(iii) $\forall I \in M, \quad \text{cone}\left\{ \sum_i u_i F^i(I,f),\ u \in \mathscr{U},\ f \in \mathbb{S}^1 \right\} = T_I M,$

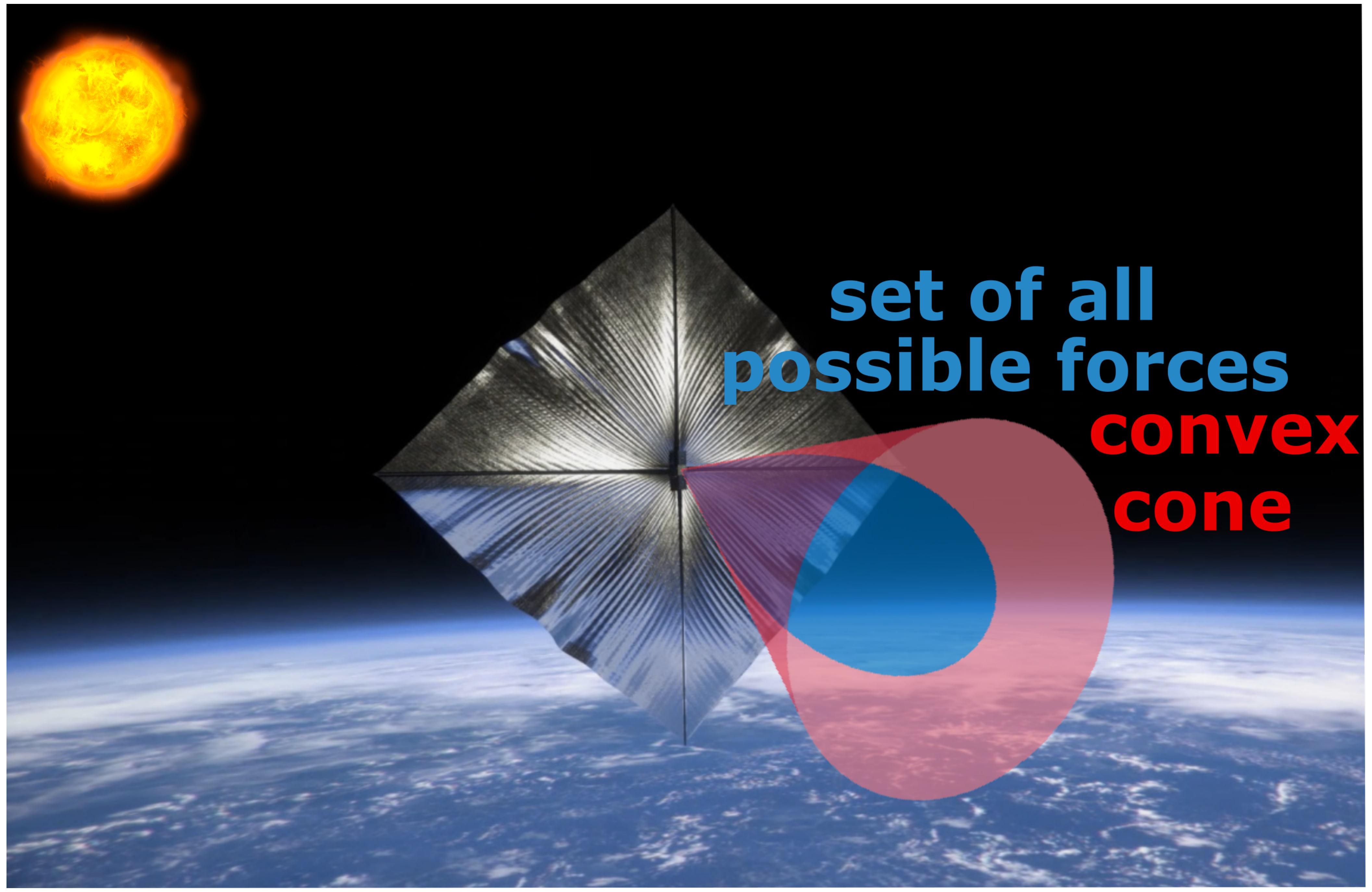
the system is controllable* in the following sense: for any $(I_0,f_0),\ (I',f') \in M \times \mathbb{S}^1,$ there is a time T and a integrable control $u \in \mathscr{U}$ that drives (I_0,f_0) to (I',f') at time T.

* The proof is available in [H., Caillau, Dell'Elce, Pomet, 2022]

2. Convexification of the control set

$$\boldsymbol{u} \in \mathcal{U} \subset K_\alpha := \text{cone}\,(\mathcal{U})$$

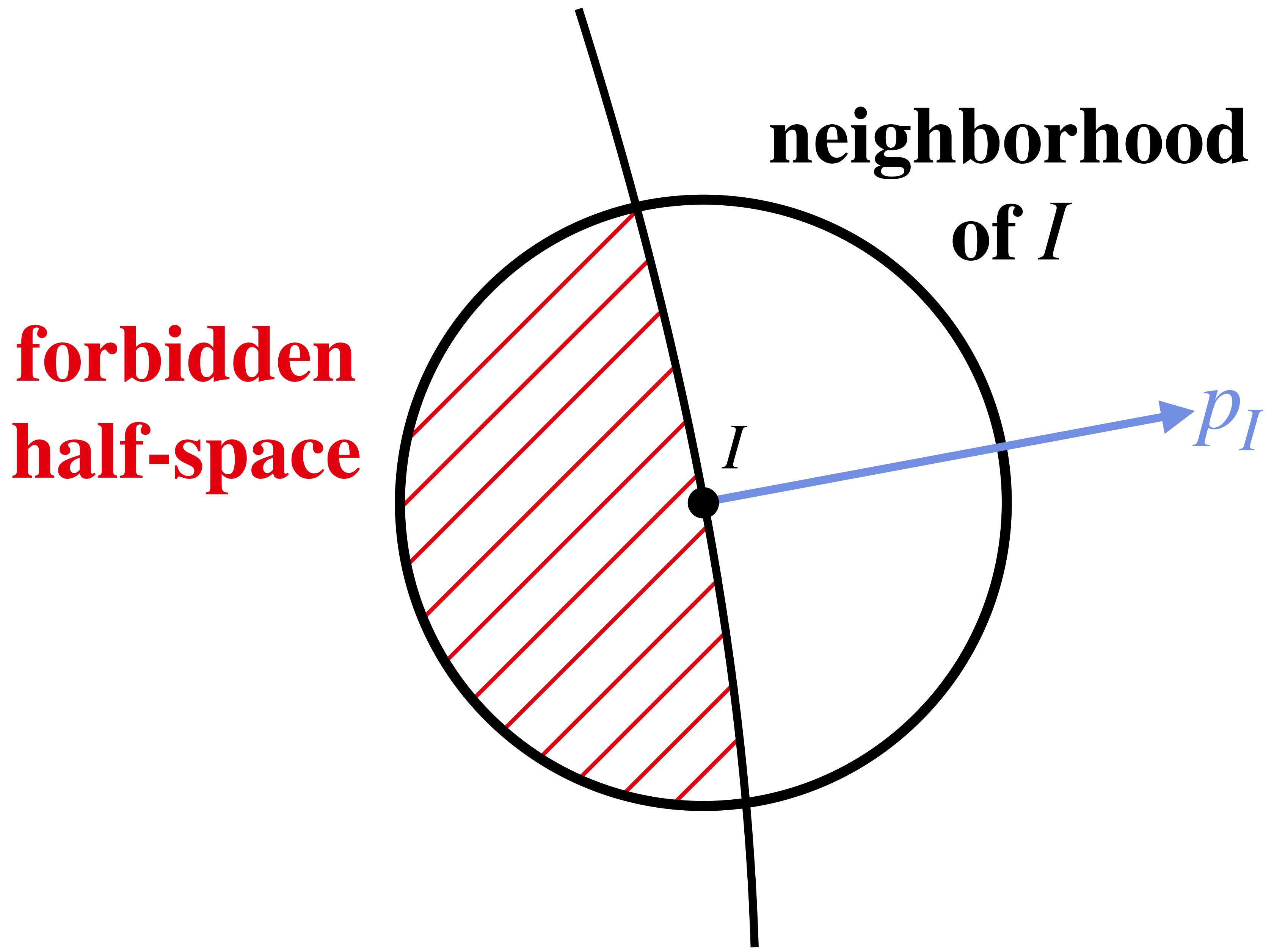




2. How to verify the condition?

Negation: If $\exists$ a one-form $p_I \in T^*M$, s.t.

$$\left\langle p_I \,,\ \frac{dI(f,\boldsymbol{u})}{dt} \right\rangle \geq 0, \quad \forall\, f \in \mathbb{S}^1, \boldsymbol{u} \in \partial K_\alpha \ \longrightarrow\ \text{not locally controllable}$$



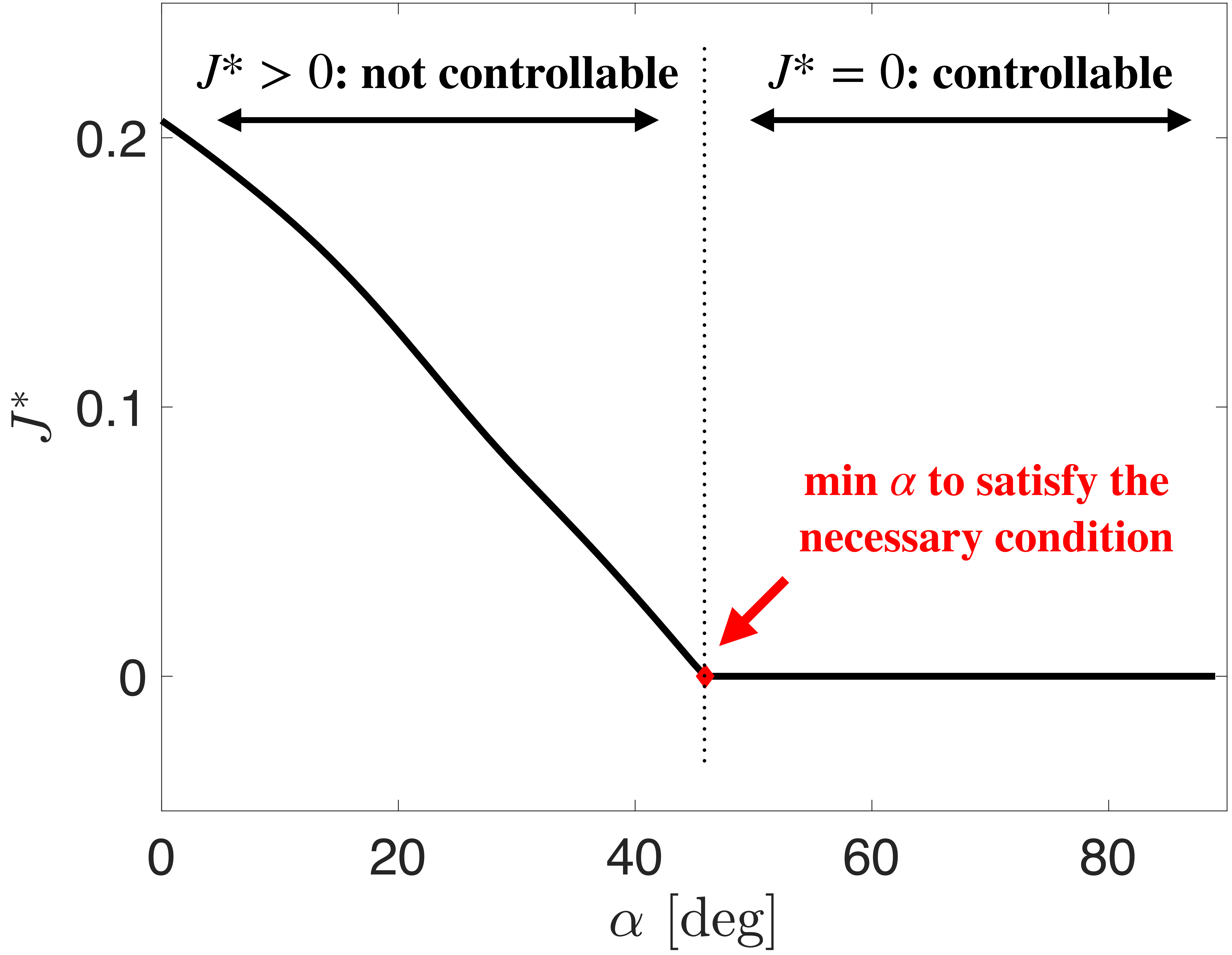
2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\| \leq 1} \quad J \quad \text{s.t.}$$

$$\left\langle p_I \ , \ \frac{dI(f, \boldsymbol{u})}{dt} \right\rangle \geq J, \qquad \forall\, f \in \mathbb{S}^1, \forall\, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

If $J^* > 0 \longrightarrow$ not locally controllable

2. Exploitation: minimum optical requirements



2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\| \leq 1} \quad J \quad \text{s.t.}$$

$$\left\langle p_I\ ,\ \frac{dI(f, \boldsymbol{u})}{dt} \right\rangle \geq J, \qquad\qquad \forall\, f \in \mathbb{S}^1, \forall\, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

Difficulties: $u(f) \in \partial K_\alpha, \quad f \in \mathbb{S}^1$

positivity constraint

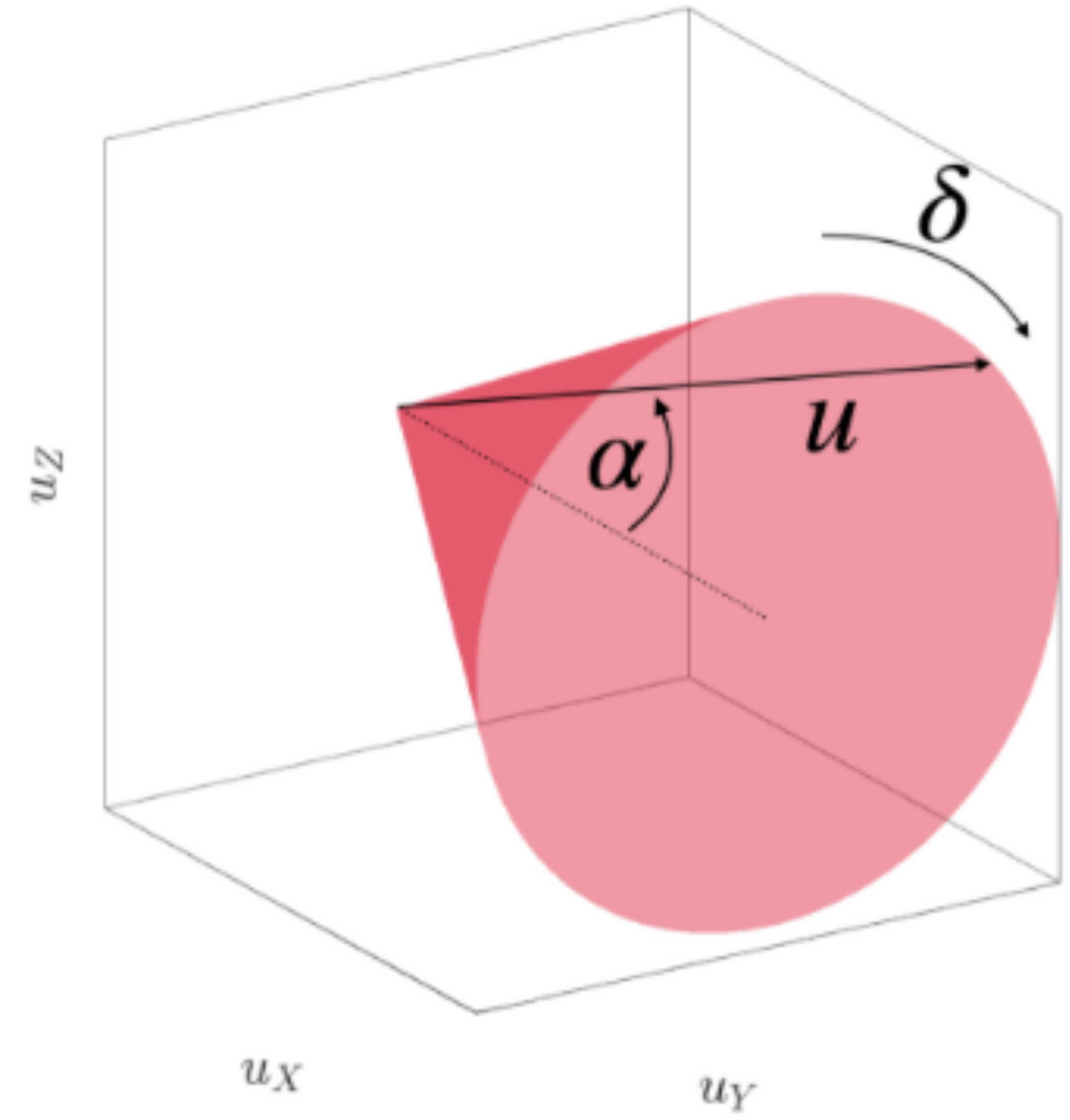
infinite dimension

2. Numerical solution of the semi-infinite problem

Parametrization of the cone

Controls are combinations of generators εu

$$\boldsymbol{u} = \begin{bmatrix} \cos\alpha \\ \cos\delta\ \sin\alpha \\ \sin\delta\ \sin\alpha \end{bmatrix}$$



2. Numerical solution of the semi-infinite problem

Fourier transform (exact) of the dynamics

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \varepsilon\sqrt{\frac{a(1-e^2)}{\mu}}G(I,f)R(I,f)\boldsymbol{u}$$

$$G = \begin{pmatrix} 0 & 0 & \frac{\sin(\gamma_3+f)}{\sin\gamma_2(1+e\cos f)} \\ 0 & 0 & \frac{\cos(\gamma_3+f)}{1+e\cos f} \\ -\frac{\cos f}{e} & \frac{2+e\cos f}{1+e\cos f}\frac{\sin f}{e} & \frac{\cos(\gamma_3+f)}{1+e\cos f} \\ \frac{2ae}{1-e^2}\sin f & \frac{2ae}{1-e^2}(1+e\cos f) & 0 \\ \sin f & \frac{e\cos^2 f+2\cos f+e}{1+e\cos f} & 0 \end{pmatrix}$$

$(1+e\cos f)G(I,f)R(I,f)$ is a trigonometric polynomial of degree 2 in f

2. Numerical solution of the semi-infinite problem

Positive polynomials [Nesterov, 2000; Dumitrescu, 2007]

Leverage on formalism of squared functional systems:

$$\begin{aligned}\Phi(f,\delta) &= \left[1,\, \mathrm{e}^{i\delta}\right]^T \otimes \left[1,\, \mathrm{e}^{if},\, \mathrm{e}^{2if}\right]^T \\ &= \left[1,\, \mathrm{e}^{if},\, \mathrm{e}^{2if},\, \mathrm{e}^{i\delta},\, \mathrm{e}^{if}\,\mathrm{e}^{i\delta},\, \mathrm{e}^{2if}\,\mathrm{e}^{i\delta}\right]^T\end{aligned}$$

$\Lambda_H : \mathbb{C}^N \to \mathbb{C}^{N\times N}$ a linear operator s.t. $\Lambda_H(\Phi(f,\delta)) = \Phi(f,\delta)\Phi^H(f,\delta)$

and $\Lambda_H^* : \mathbb{C}^{N\times N} \to \mathbb{C}^N$ its adjoint operator.

2. Numerical solution of the semi-infinite problem

Positive polynomials [Nesterov, 2000; Dumitrescu, 2007]

Then for any trigonometric bivariate polynomial

$$\left\langle p_I, \widetilde{G}(I,f)\boldsymbol{u} \right\rangle - J \geq 0,\, f \in \mathbb{S}^1, \boldsymbol{u} \in \partial K_\alpha \Leftrightarrow \exists Y \succeq 0$$

$$\text{such that } \widetilde{G}u\, p_I - \boldsymbol{e_1} J = \Lambda_H^*(Y)$$

Fourier coefficients in the basis Φ

defined with Toeplitz matrices

2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\|\leq 1} \quad J \quad \text{s.t.}$$

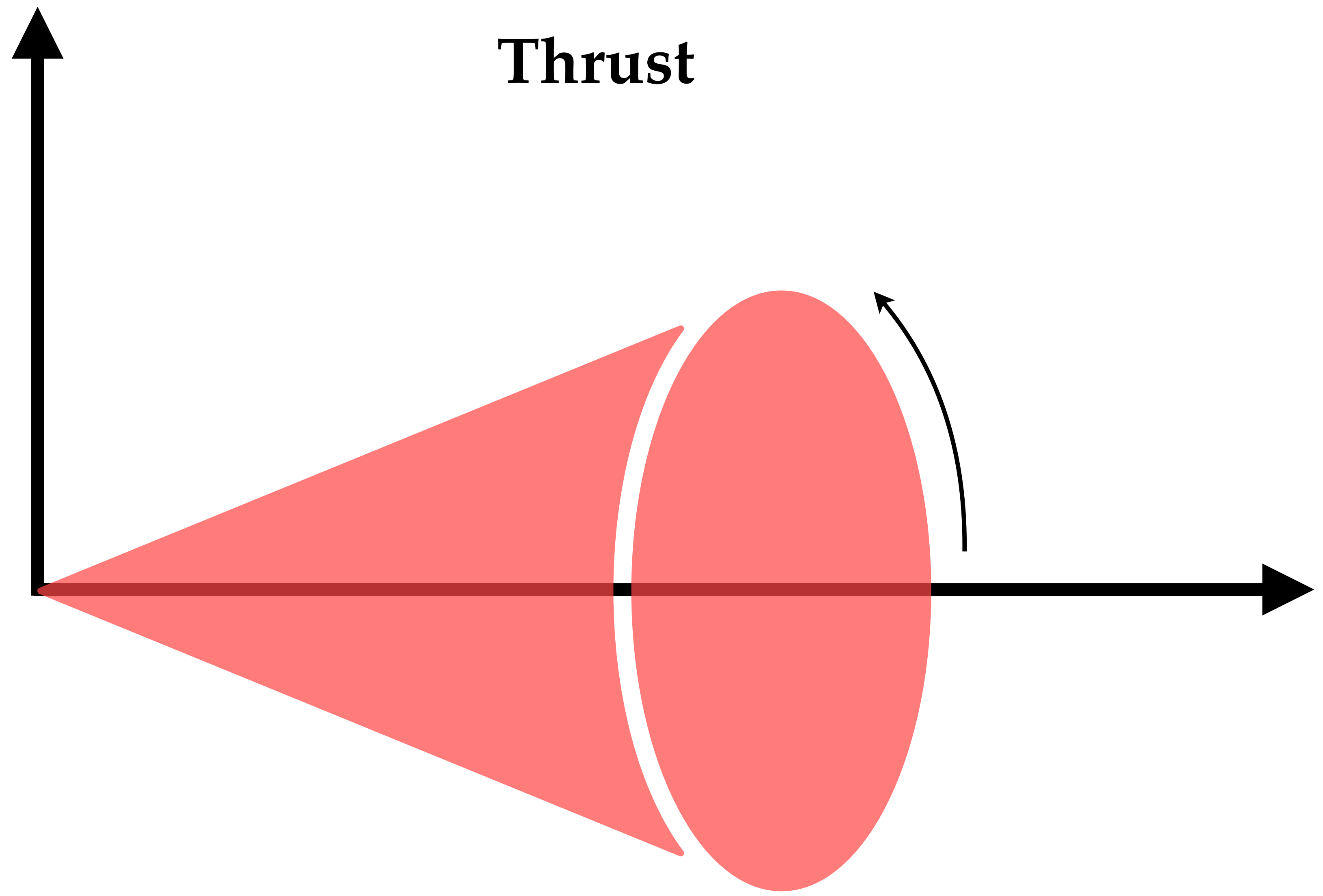
$$\left\langle p_I\ ,\ \frac{dI(f,\boldsymbol{u})}{dt} \right\rangle \geq J, \qquad \forall\, f \in \mathbb{S}^1, \forall\, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

$$\downarrow$$

LMI

2. Effective test of the necessary condition

$$\min_{u \in K_\alpha} \alpha \quad \text{such that the system is controllable}$$



23

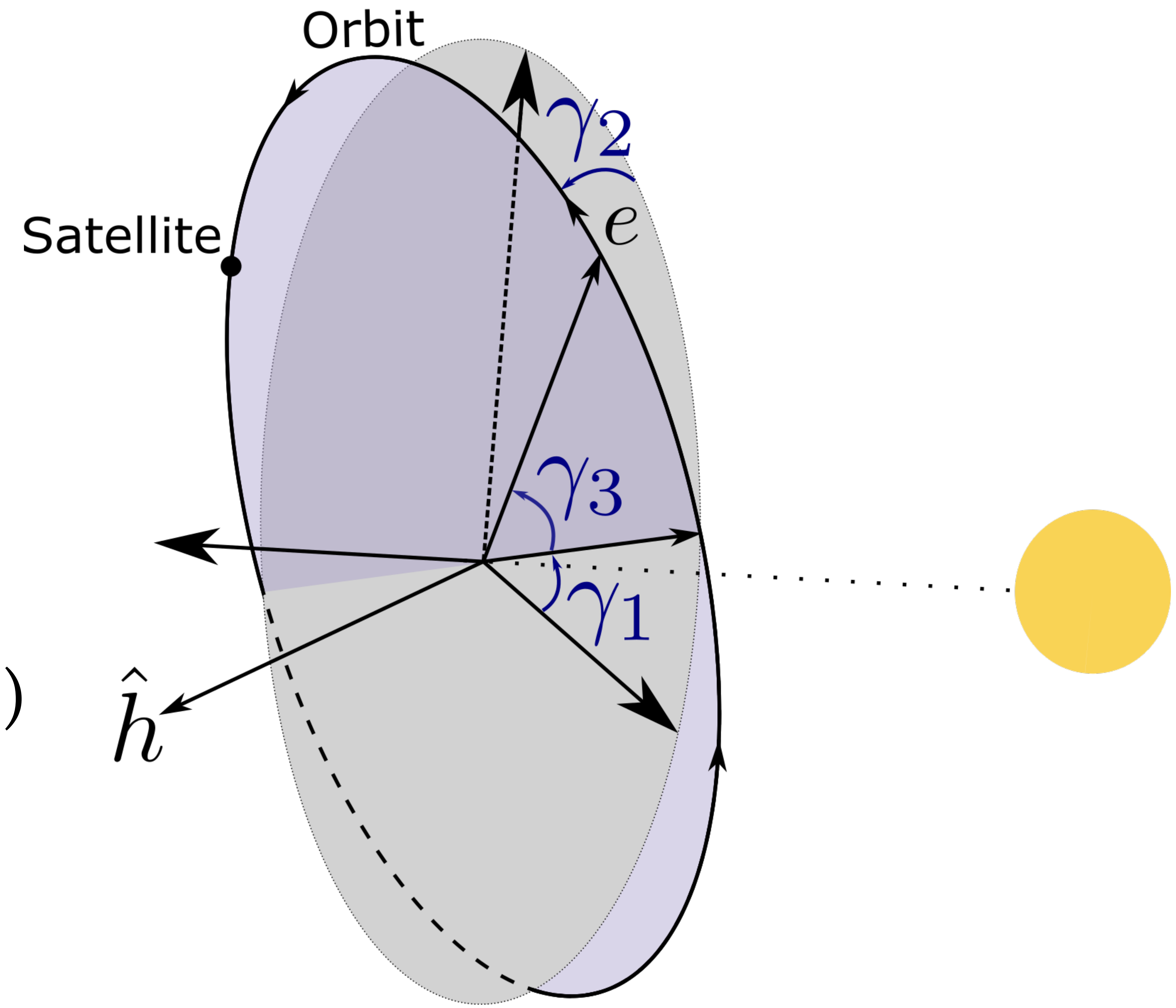
2. Convenient choice of orbital elements

$$I = \left(\gamma_1,\, \gamma_2,\, \gamma_3,\, a,\, e\right)$$

Problem independent of:

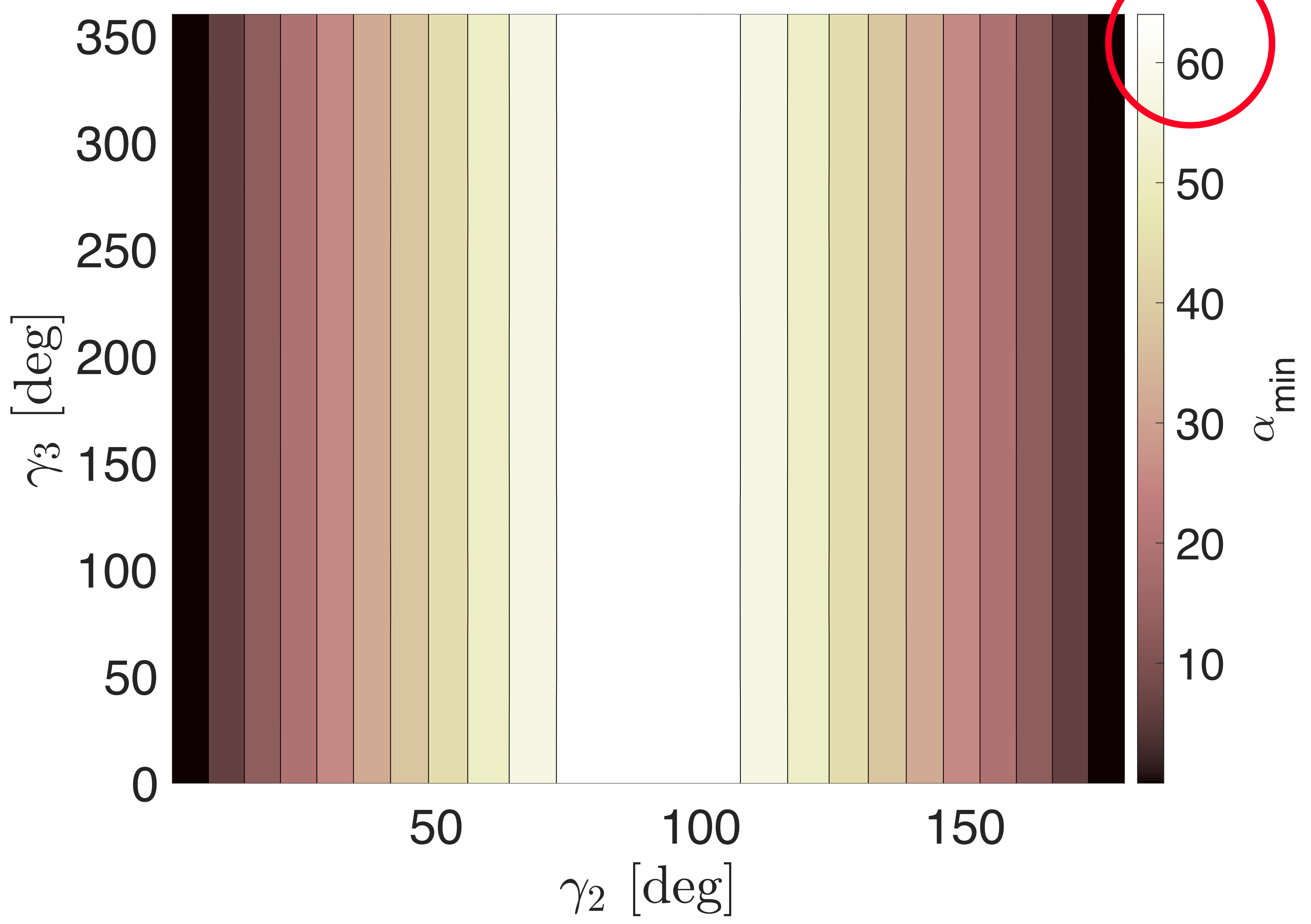
- γ_1 (axial symmetry)

- $a, \mu \rightarrow$ (planet-independent results)

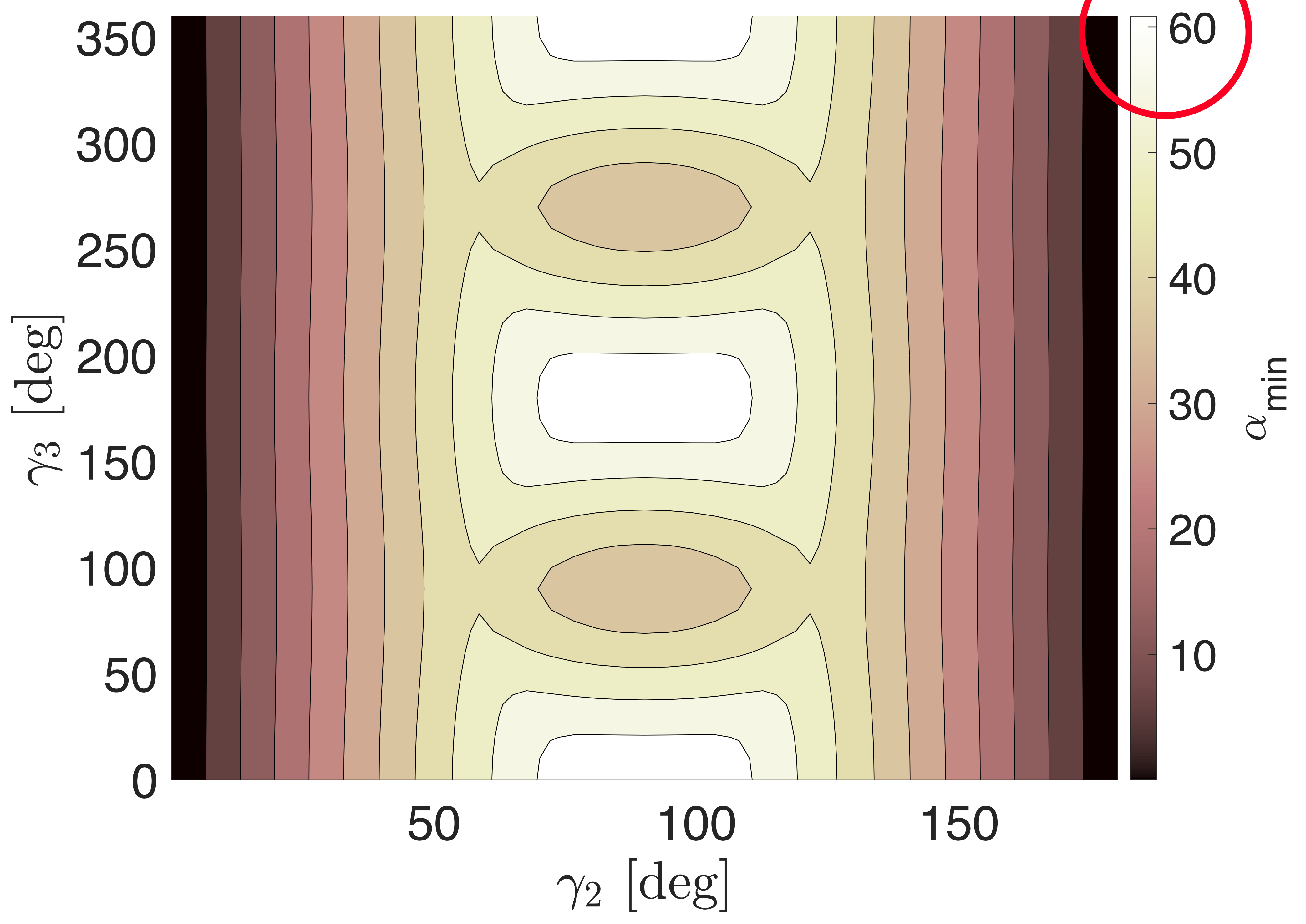


3. Results: minimal requirement

Low eccentricity $e = 0.1$

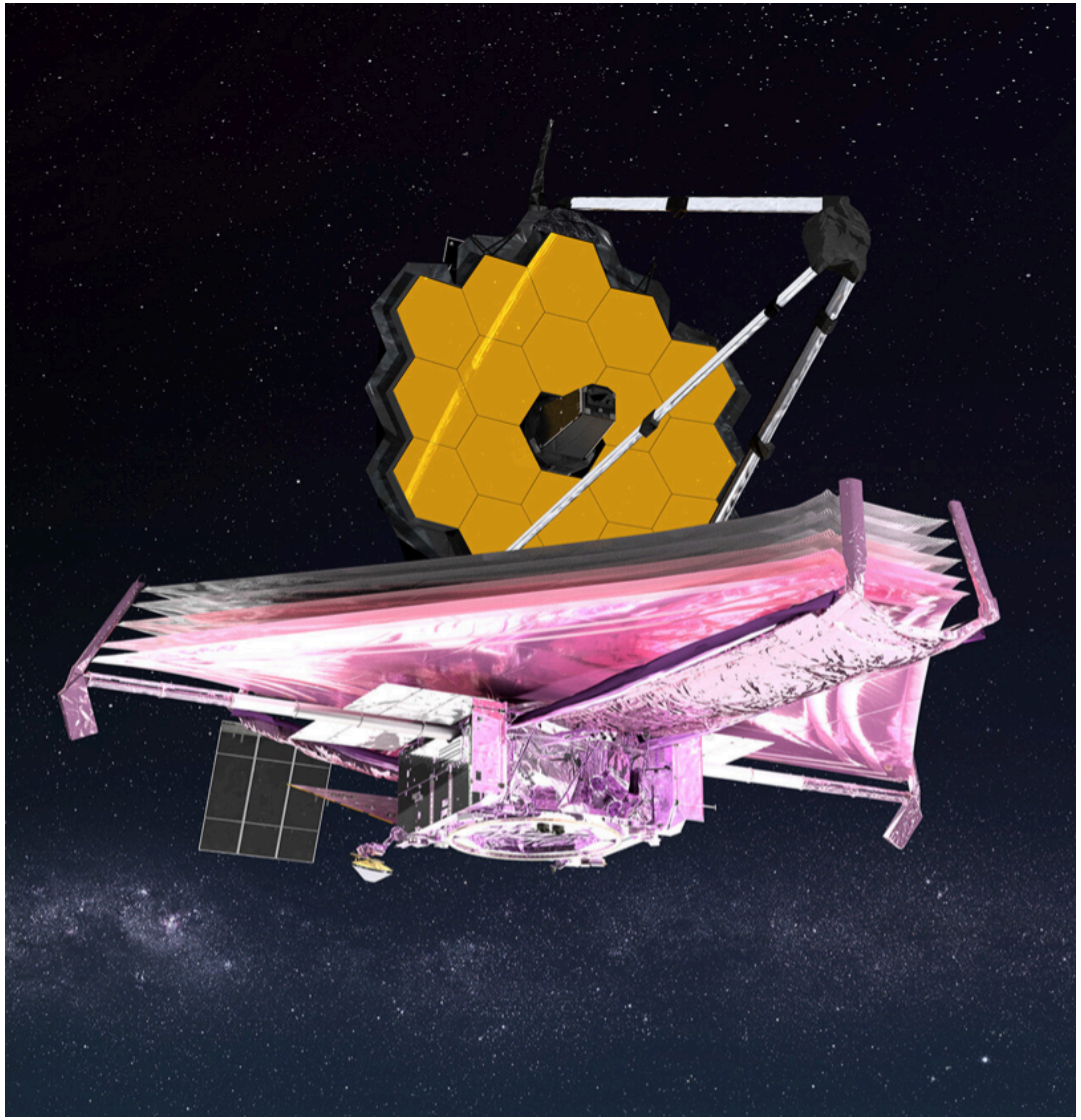


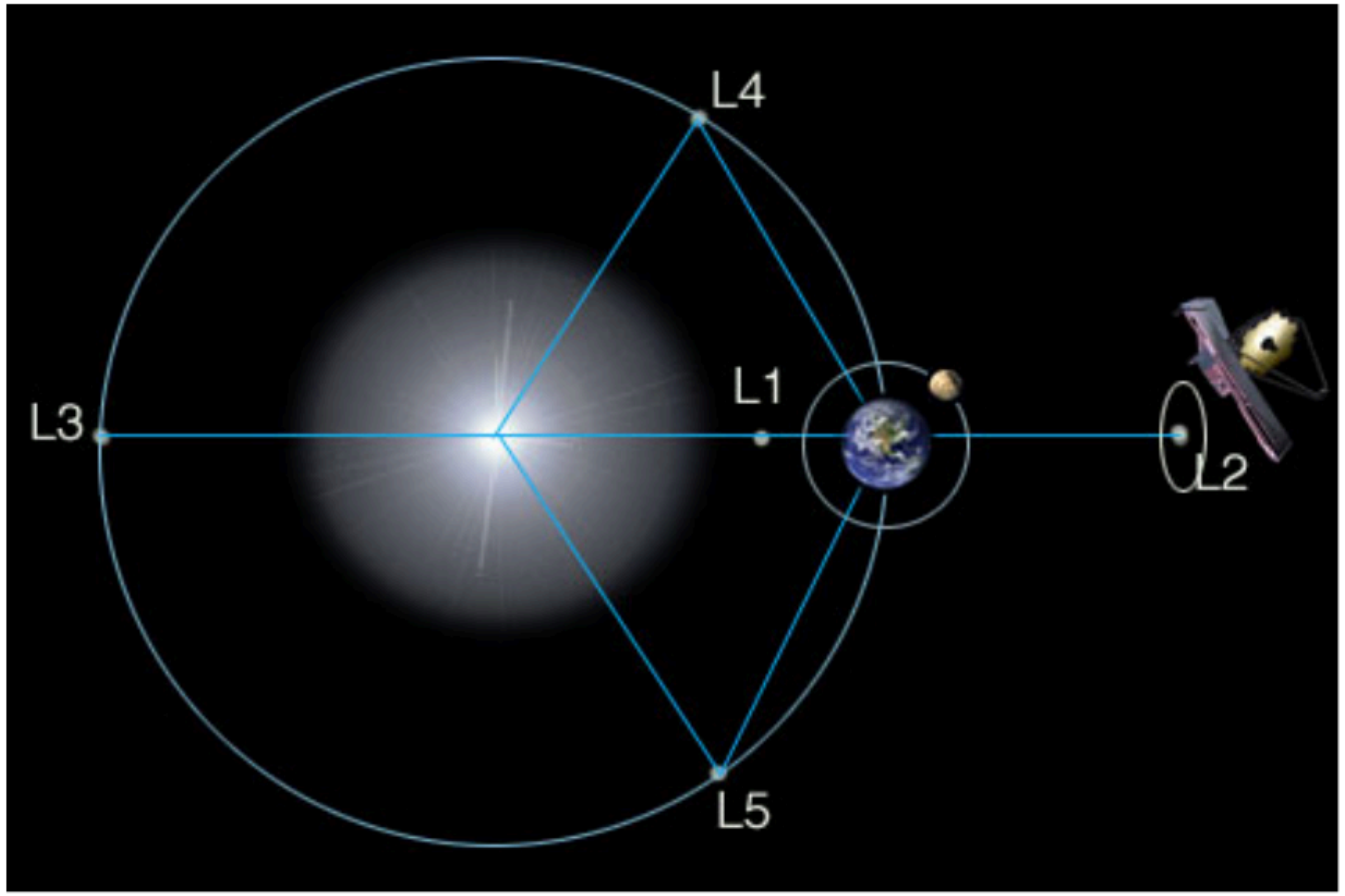
High eccentricity $e = 0.9$



H., Dell'Elce, Caillau, Pomet. Impact of Optical Properties on the Controllability of Solar Sails. *ICATT 2021 - International Conference on Astrodynamics Tools and Techniques*, Jun 2021. ⟨hal-03331300⟩

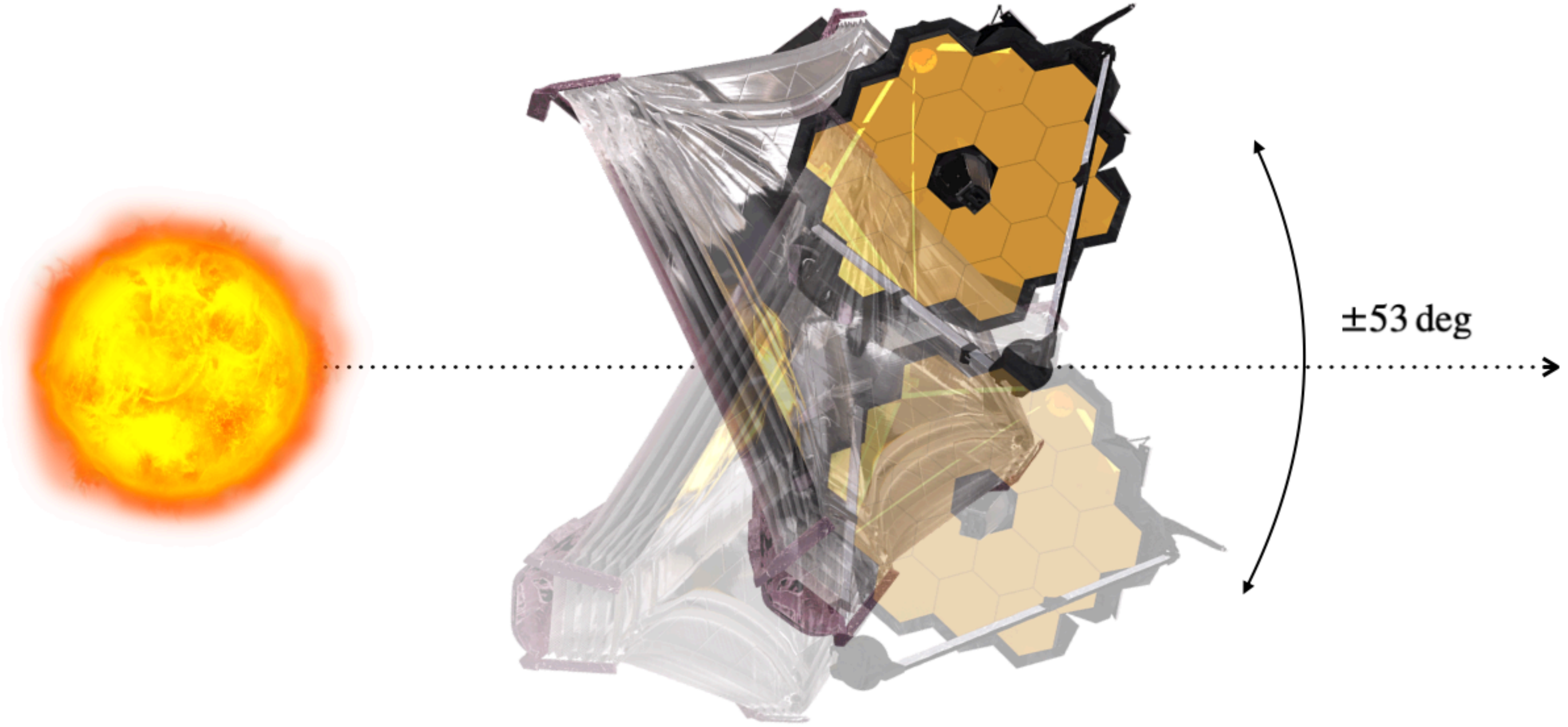
4. Other systems with the same constraint



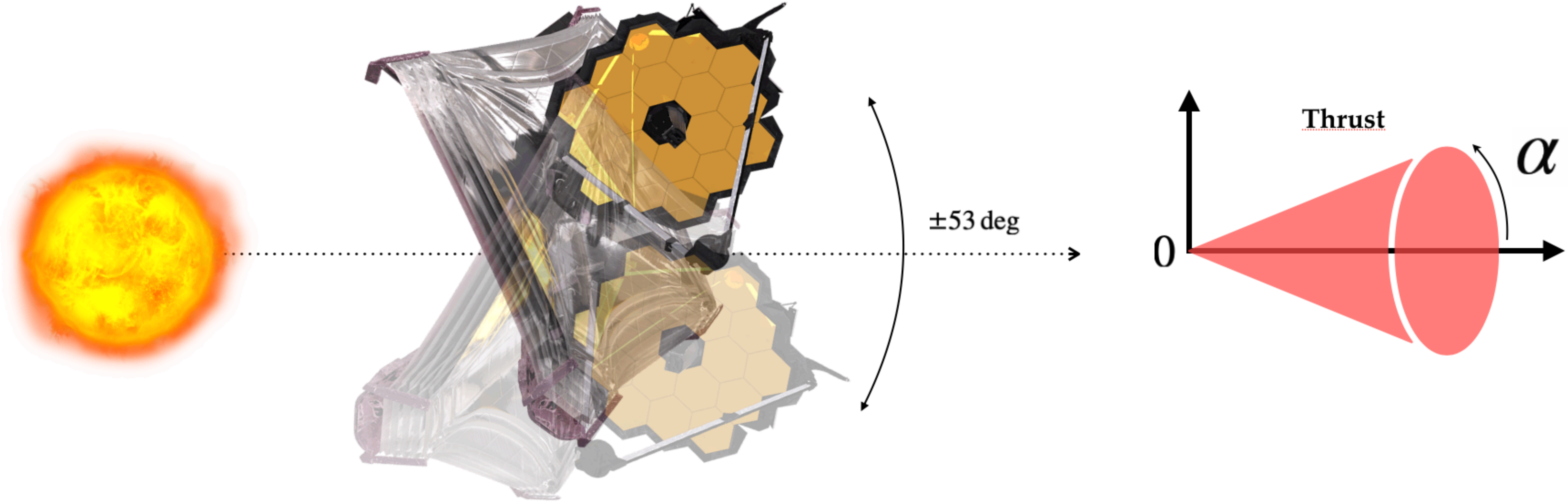


Source: NASA

4. James Webb Space Telescope attitude constraint



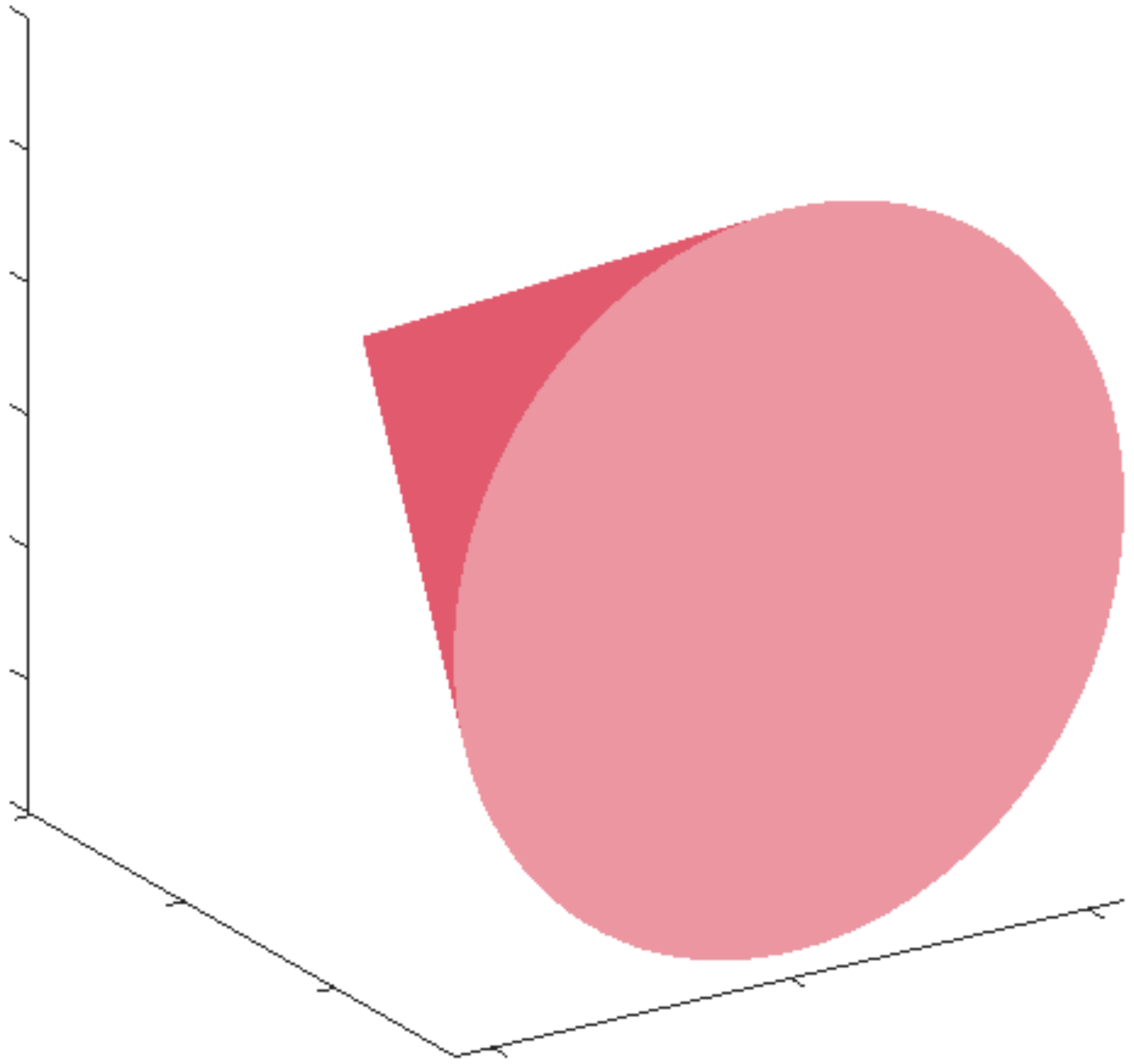
4. James Webb Space Telescope attitude constraint

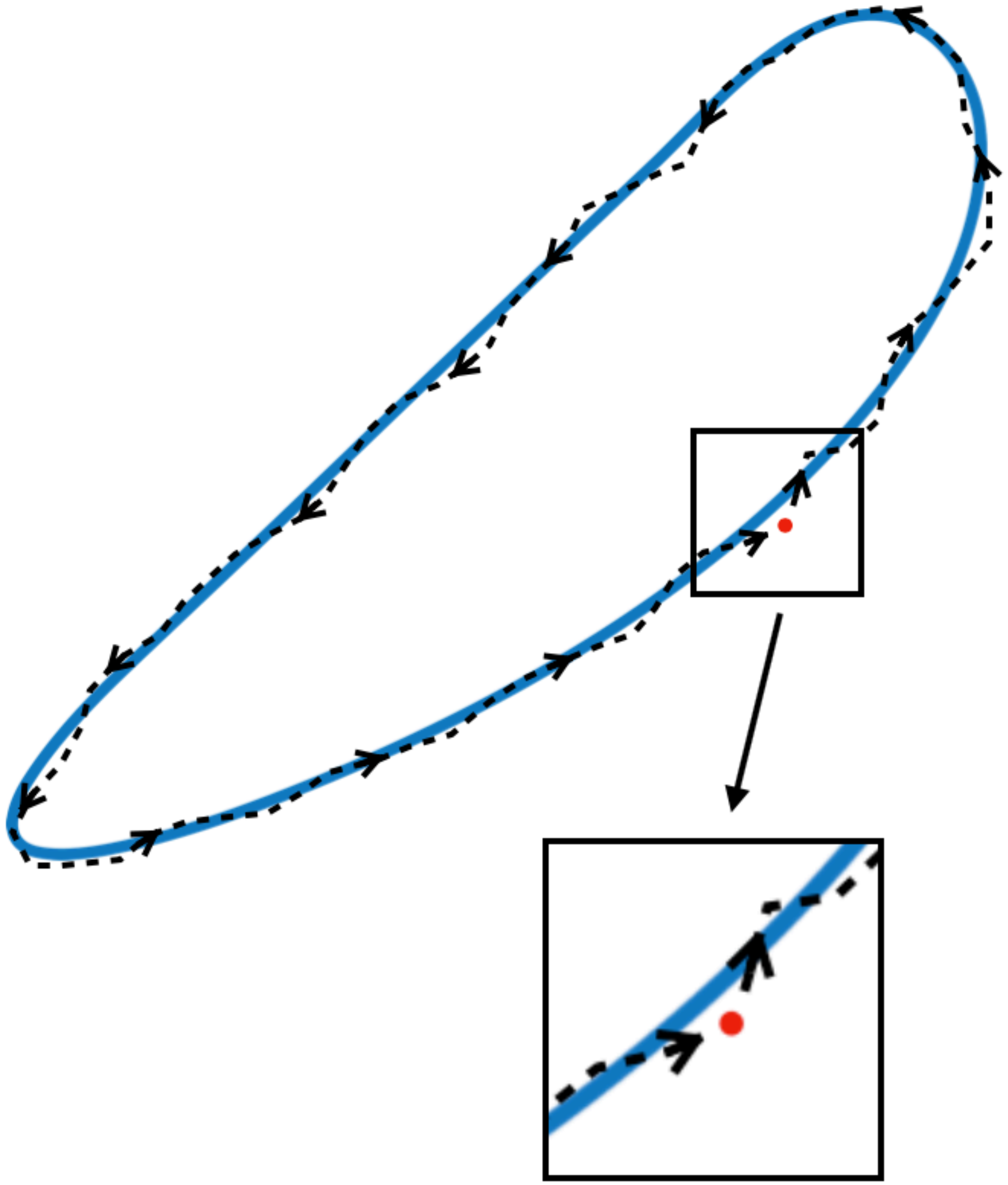


4. Dynamics of any periodical orbit

$$\frac{\mathbf{d}x}{\mathbf{d}t} = f(x) + B(x)\,u, \quad x \in M \subseteq \mathbb{R}^n$$

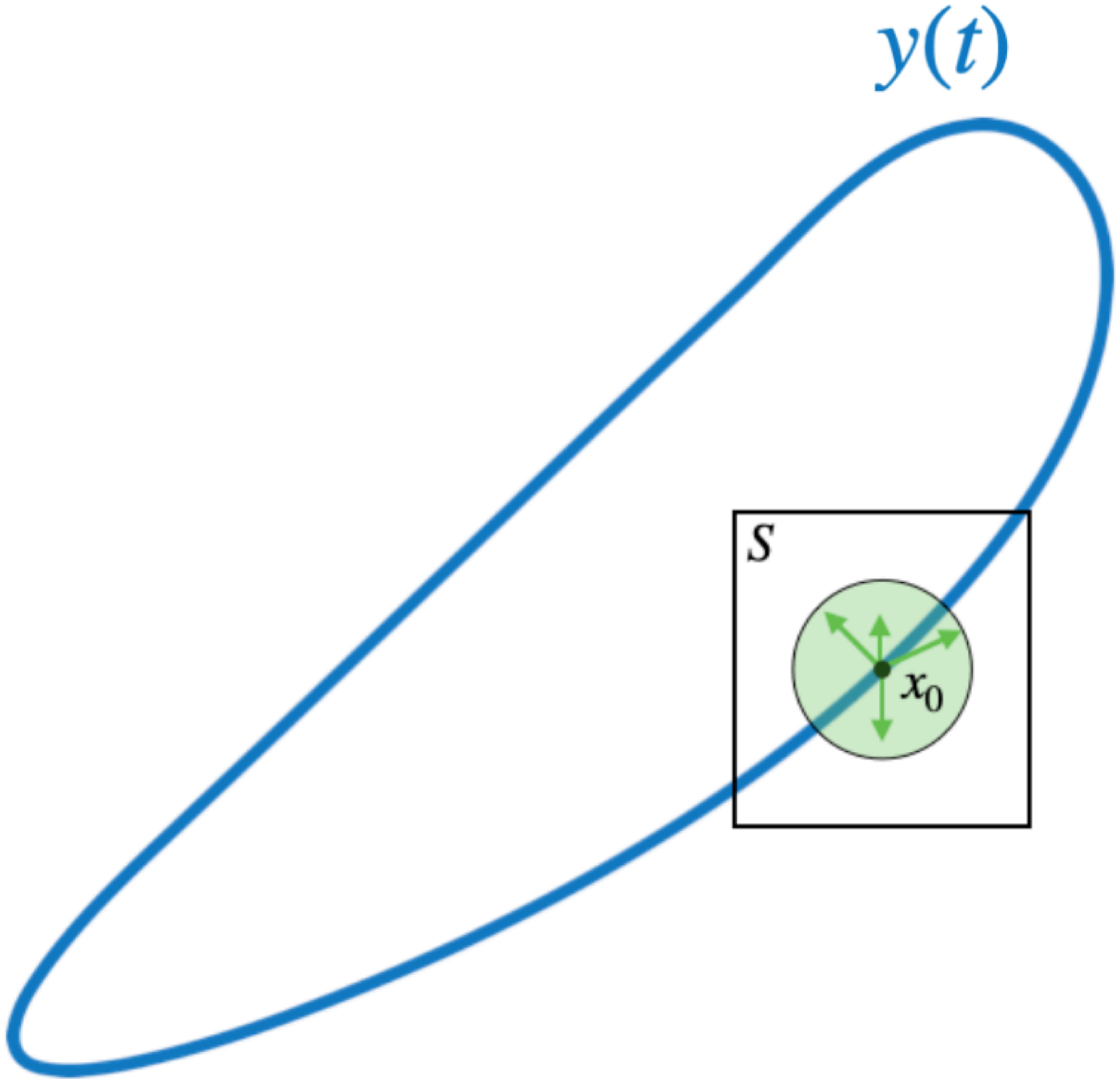
$$u \in K_\alpha \subseteq \mathbb{R}^m,\ \|u\| \le \varepsilon$$





4. Linearization over the periodical trajectory

$$\begin{cases} \dfrac{\mathbf{d}y}{\mathbf{d}t} = f(y) \\ y(0) = y(T) = x_0 \\ S(x_0) = 0 \end{cases}$$

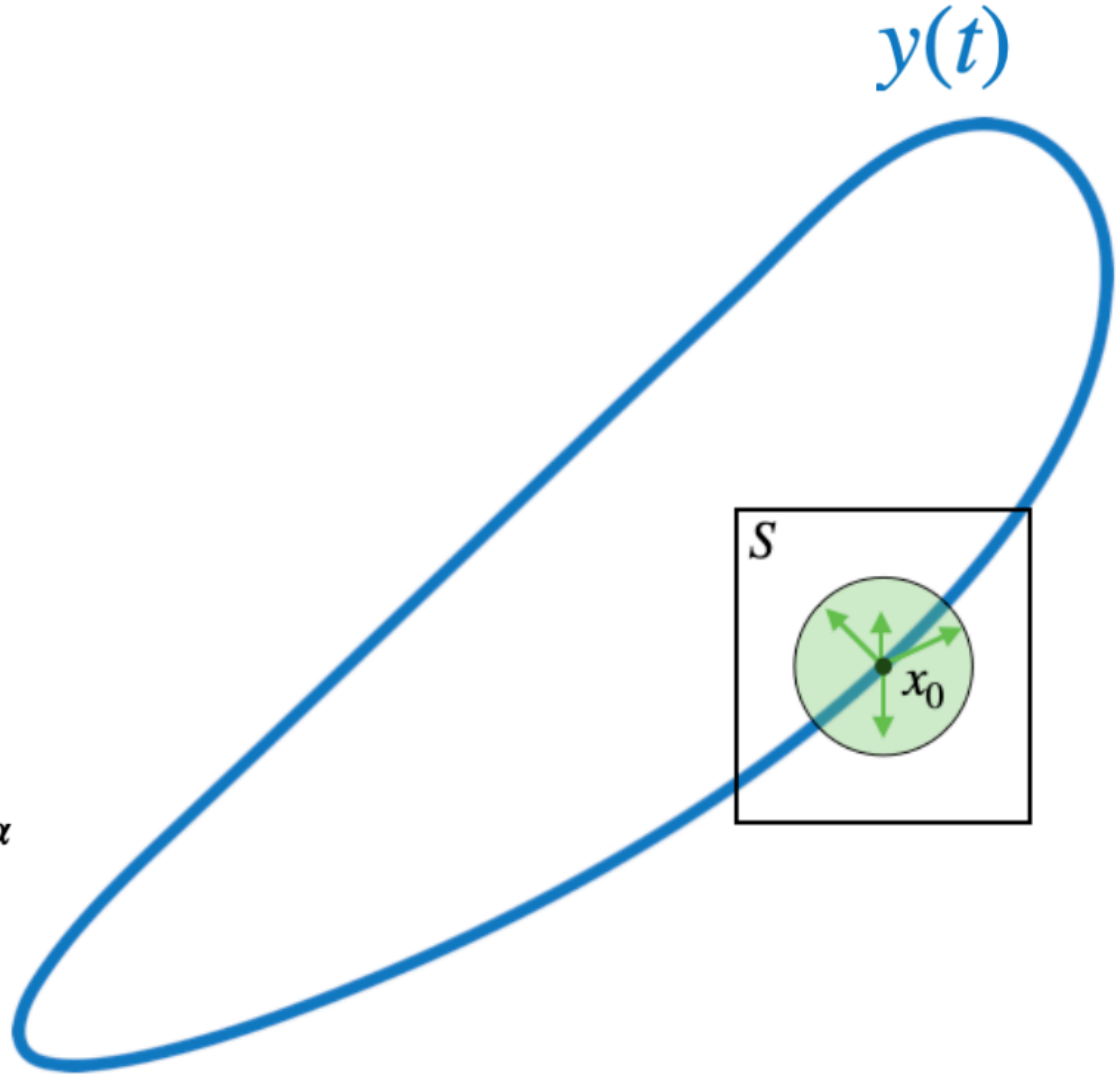


4. Linearization over the periodical trajectory

$$\delta x(t) = \Phi(t, x_0)\, \delta x_0$$

$$\frac{\mathbf{d}\delta x}{\mathbf{d}t} = \left.\frac{\mathbf{d}f}{\mathbf{d}x}\right|_y \delta x + B(y)\, u$$

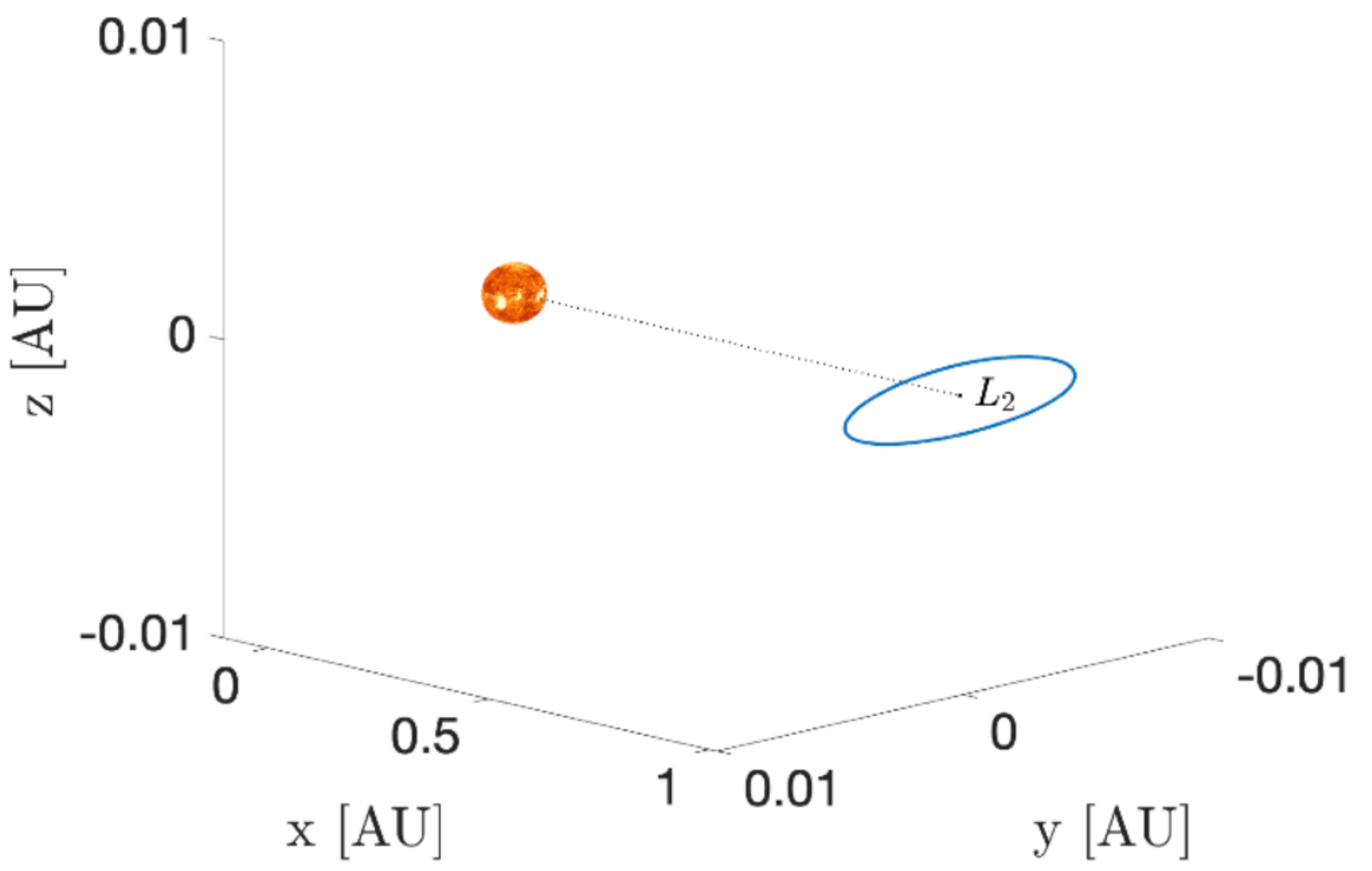
$$\frac{\mathbf{d}\delta x_0}{\mathbf{d}t} = \Phi^{-1}(t, x_0)\, B(y(t))\, u, \quad \delta x_0 \in T_{x_0}S, \quad u \in K_\alpha$$



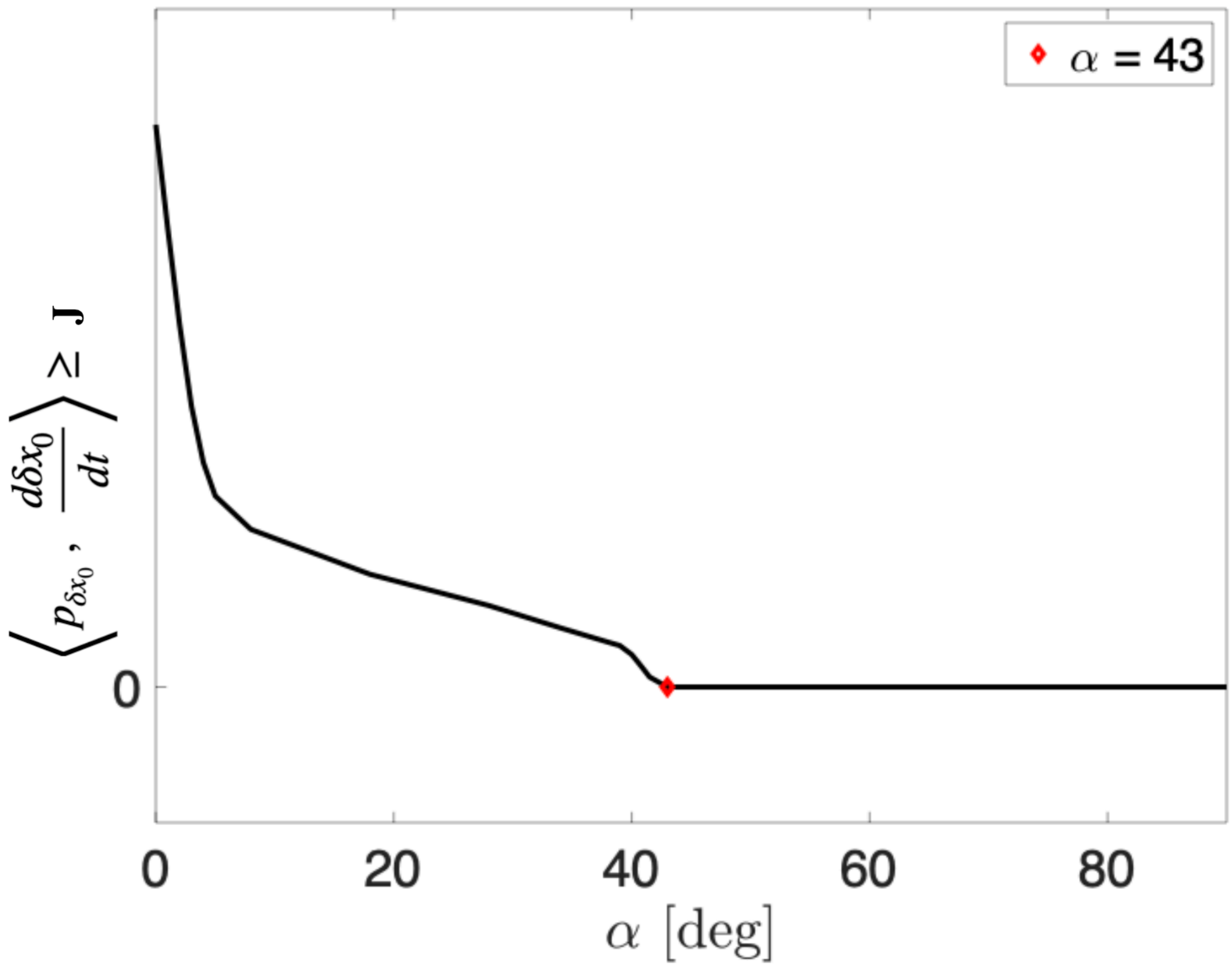
4. Difference with the previous problem

Not exact trigonometrical polynomials $\rightarrow$ truncation of the Fourier series

4. Halo orbit around L2



4. Minimum cone angle for thrust directions



References

Caillau, J.-B.; Dell'Elce, L.; Herasimenka, A.; Pomet, J.-B. On the controllability of nonlinear systems with a periodic drift. *SIAM J. Control Optim*, to appear.

Caillau, J.-B.; Dell'Elce, L.; Herasimenka, A.; Pomet, J.-B. Optimal control of a solar sail. *Optimal Control Appl. Methods*, **45** (2024), 2837-2855.

Herasimenka, A.; Dell'Elce, L.; Caillau, J.-B.; Pomet, J.-B. Controllability Properties of Solar Sails. *J. Guidance Control Dyn*. **46** (2023), no. 5, 900-909.

Herasimenka, A.; Dell'Elce, L.; Farres, A. Controllability of satellites on periodic orbits with cone-constraints on the thrust direction. *Space Flight Mechanics Meeting*, Austin, 2023.