Exercice 2 $\boxed{\text{Geoffroy}}$

$a \geq 1$

$$\lim_{n\to\infty} \int_0^n \left(1-\frac{x}{n}\right)^n e^{ax}\, dx \ ?$$

$$e^{n\ln\left(1-\frac{x}{n}\right)} \xrightarrow[n\to\infty]{} -x$$

$$\ln\left(1-\frac{x}{n}\right) \underset{n\to\infty}{\sim} -\frac{x}{n}$$

$(x \neq 0)$

$$\lim_{n\to\infty} \int_0^\infty \left(1-\frac{x}{n}\right)^n e^{ax} \chi_{[0,n]}(x)\, dx$$

$$f_n(x) \to e^{-x} e^{ax} = e^{x(a-1)}$$

$$\frac{1}{a-1}\left[e^{x(a-1)}\right]_0^\infty$$

$\exists \varphi$ intégrable sur $\mathbb{R}_+$ tq $|f_n(x)| \leq \varphi(x)$ $\forall x$ $\forall n \in \mathbb{N}$

$f_n \geq 0$ $(f_n)_n \nearrow$

$a \geq 1$

Exo 2.

$f : (X, \mathcal{B}) \longrightarrow (Y, \mathcal{C})$ mes.

$\Leftrightarrow (\forall B \in \mathcal{C}) : f^{-1}(B) \in \mathcal{B}$ $\quad x$

$\bigcup_{n \in \mathbb{N}}]-\infty ; 1-\frac{1}{n}]$

$\bigcup$

$1-\frac{1}{n}$

$x < 1$

on suppose χ_A mesurable

$A = \chi_A^{-1}(\{1\})$

$\mathcal{B}_{\mathbb{R}} = \mathcal{B}(\{]-\infty, a], a \in \mathbb{R} \})$

$\underbrace{]-\infty ; 1]}_{\in \mathcal{B}_{\mathbb{R}}} \cap [1 ; +\infty[$

$[1, \infty[= \complement_{\mathbb{R}} \underbrace{]-\infty ; 1[}$

$]-\infty, 1[= ?$

Exo 1.

$\boxed{\text{Saffia}}$

$$\mathcal{B}(A) \overset{?}{=} \mathcal{B}(\tilde{A})$$

$\text{card} = 2^3$

- $\mathcal{B}(A) \subset \mathcal{B}(\tilde{A})$?

$$A \subset \mathcal{B}(\tilde{A}) \Rightarrow \mathcal{B}(A) \subset \mathcal{B}(\tilde{A})$$

$[0,2] \in \mathcal{B}(\tilde{A})$? $[0;2] = [0,1[\cup [1,2]$

- $\mathcal{B}(\tilde{A}) \subset \mathcal{B}(A)$? $\tilde{A} \subset \mathcal{B}(A)$? $[0,1[= [0,2] \setminus \underbrace{[1,2]}_{[0,2] \cap [1,3]}$

Exo 2. $\quad f_n(x) = x\cos\left(\frac{x+1}{n}\right)e^{-x^2}$

. $x = 0$, $f_n(0) = 0$

. $x \neq 0$, $|f_n(x)| = \left|x\cos\left(\frac{x+1}{n}\right)e^{-x^2}\right| \leq |xe^{-x^2}|$

$$\int_{\mathbb{R}_+} |xe^{-x^2}|dx = -\frac{1}{2}\left[e^{-x^2}\right]_0^{+\infty} = +\frac{1}{2}$$

$$\int_{\mathbb{R}_-} |xe^{-x^2}|dx \quad \int_{\mathbb{R}} \qquad = 1 < \infty$$

$= \frac{1}{2}$

Soit $x \in \mathbb{R}$

$a \geq 1$ Luca

$$\lim_{n\to+\infty} \int_0^n \left(1 - \frac{x}{n}\right)^n e^{ax} \, dx$$

$$f_n(x) = \underbrace{\left(1 - \frac{x}{n}\right)^n}_{e^{n\ln(1-\frac{x}{n})}} e^{ax} \chi_{[0,n[}(x) =$$

$$\xrightarrow{n\to+\infty} \underbrace{e^{-x} \quad}_{e^{x(a-1)}} \chi(x)$$

cas $a=1$

$$f_n(x) = \left(1 - \frac{x}{n}\right)^n e^x \chi_{[0,n[}(x) \quad [0,+\infty[$$

$$\xrightarrow{n\to\infty} 1 \cdot \chi_{[0,\infty[}(x)$$

$$1 \in L^1(\mathbb{R}_+)?$$

$$\underbrace{\int_0^\infty 1 \, dx < \infty}_{= \infty}$$

$$\lim_{n\to\infty} \int_{\mathbb{R}_+} f_n \, dx$$

$$= \int f \, dx = \infty$$

$? \; \mathbb{R}_+$

Si f_n mes ≥ 0, $\nearrow$

$$\frac{f_{n+1}(x)}{f_n(x)} \geq 1$$

$$f_n(x) \leq f_{n+1}(x) \quad (\nearrow)$$

$$\frac{f_{n+1}(x)}{f_n(x)} = \frac{\left(1-\frac{x}{n+1}\right)^{n+1}}{\left(1-\frac{x}{n}\right)^n} = e^{(n+1)\ln\left(1-\frac{x}{n+1}\right)-n\ln\left(1-\frac{x}{n}\right)}$$

Ex. 1

$\chi_A : X \to \overline{\mathbb{R}}$

$x \mapsto \begin{cases} 1 & \text{si } x \in A \\ 0 & \text{sinon} \end{cases}$

$(\Leftarrow)$ On suppose χ_A mesurable

$\forall B \in \mathcal{B}_{\overline{\mathbb{R}}}$, $\chi_A^{-1}(B) \in \mathcal{B}$.

$A \subset X$

Soit $B \in \mathcal{B}_{\overline{\mathbb{R}}}$ / $A = \chi_A^{-1}(\{1\}) \in \mathcal{B}$

$\mathcal{B}_{\overline{\mathbb{R}}}$

$\cup$

$X \to y$

$$\chi_A^{-1}(\{1\}) = \{x \in X,\ \underbrace{\chi_A(x) \in \{1\}}_{\Leftrightarrow x \in A}\} = A$$

$\{1\} \in \mathcal{B}_{\mathbb{R}}$?

$\mathcal{B}_{\mathbb{R}} = \mathcal{B}(\{]-\infty, a], a \in \mathbb{R}\})$

$\{1\} =$

$]-\infty, 1] \cap [1, +\infty[$

$[1, \infty[$