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Projecting Crown Measures for Loblolly Pine Trees Using a Generalized Thinning Response Function

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ABSTRACT. Crown size is an important measure of tree vigor. Measures of crown have been used to predict response to silvicultural practices such as thinning and fertilization. This study was aimed toward examining the projection of crown measures and developing a new measure of thinning effect on crown development. Results showed that using a crown height increment model that projects crown size directly or an allometric model, which derives crown ratio from tree diameter and height information, can be an effective means of modeling crown size for individual trees. By incorporating a new thinning response function into the equations, crown dimensions can be projected for thinned and unthinned stand conditions. This new thinning response function is based on biological considerations and is sensitive to thinning intensity, age of thinning, and elapsed time since thinning. FOR SCI. 41(1):43–53.

ADDITIONAL KEY WORDS. *Pinus taeda,* incremental model, allometric model, growth, yield.

CROWN SIZE IS AN IMPORTANT MEASURE of tree vigor (Spurr and Barnes 1980, Smith 1986). Measures of crown have been used to project tree growth, development (Sprinz and Burkhart 1987, Burkhart et al. 1987) and mortality (Avila and Burkhart 1992). Crown measures are also important for evaluating wood quality of tree boles (Kershaw et al. 1990).

Crown length, computed as the difference between the total height and the crown height, is one measure of crown size or vigor. Another commonly used measure of crown size is crown ratio, defined as the crown length divided by the total height. Although they are different measures of crown size, they are closely related and can be derived from each other given that either total tree height or crown height is known.

Studies have shown that reducing stand density through thinning slows the recession of crown height in loblolly pine trees (Short and Burkhart 1992, Ginn et al. 1991) as well as other shade-intolerant conifers (Kramer 1966, Siemon et al. 1976). These results suggest that crowns develop differently for thinned and unthinned stand conditions. With results on differential crown recession in mind, Short and Burkhart (1992) developed a function using the percentage of basal area removed to express thinning effect in a crown height recession model. Their results indicated that including a thinning variable not only improved the fit of the model but also produced a substantial increase in accuracy for predictions from the model. Bailey and Ware (1983) proposed a thinning effect modifier in their basal-area growth model for thinned and unthinned stands. They recommended the ratio of average diameter of trees removed in thinning over average diameter of residual trees after thinning as a suitable thinning response variable. Their thinning variable is as much an indicator of thinning method (below, above, or row thinning) as a measure of thinning intensity. Both thinning variables (Short and Burkhart 1992, Bailey and Ware 1983) are monotonic with regard to time elapsed since thinning.

Two approaches to modeling crown variables have generally been taken. With one approach, crown variables are predicted with allometric methods. That is, crown ratio (as a measure of crown size) is predicted using individual tree variables such as diameter, total height, age, and/or some stand and site variables such as basal area or some competition measure (Dyer and Burkhart 1987, Ward 1964, Wykoff et al. 1982, Dell et al. 1979, Feduccia et al. 1979, Ek and Monserud 1975, Van Deusen and Biging 1985). Another approach is to increment crown height directly over time much like diameter and height are incremented (Short and Burkhart 1992, Maguire and Hann 1990a,b).

The goal of this study was to evaluate allometric and incremental crown models and to derive an improved thinning response function to more accurately predict crown size for thinned and unthinned stand conditions.

DATA

The data analyzed in this study came from a thinning study established in 186 loblolly pine plantations throughout the Piedmont and Coastal Plain physiographic regions in the southeastern United States. Plots were established during the 1980–1982 dormant seasons in 8- to 25-yr-old (mean = 15, standard deviation = 4.2) plantations with site indexes ranging from 11 to 26 m (mean = 18, standard deviation = 2.5). At each location, three plots, comparable in initial site index, number of trees, and basal area/ha, were established: (1) an unthinned control plot, (2) a lightly thinned plot, and (3) a heavily thinned plot. All thinnings were from below. The light thinning removed approximately one-third of the basal area while the heavy thinning removed about one-half of the basal area. The ratio of after to before thinning basal area averaged 0.66 for all thinned plots (standard deviation = 0.10). To date, four measurements have been collected on approximately two-thirds of the plots and five measurements on about one-third of the plots. These measurements include one taken at plot establishment and three or four subsequent remeasurements at a measurement interval of 3 yr. At each measurement, diameter at breast height, total height, and height to the crown base were collected. At establishment, stem maps of each plot were made so that the location of each tree with regard to its neighbors could be quantified. For this study, only trees where all competitors could be identified from a basal area factor sweep of 2.2956 m^2/ha (10 ft^2/ac) were used. Thus, trees which had potential competitors situated outside of the measurement plot were excluded. In all, there were 1991 interior trees from the unthinned control plots, 4265 interior trees from the light-thinned plots, and 3002 interior trees from the heavy-thinned plots available for analyses. Table 1 summarizes the tree and stand characteristics at plot establishment and 12 yr after treatment for the unthinned and the two thinned plots at each location. The total number of 9258 trees for all three plots was randomly divided so that one-half of the data were used for parameter estimation and one-half for verification.

It should be noted that an observation for the incremental model is defined by a measurement interval whereas for the allometric model an observation is defined by one measurement of an individual. Therefore, when fitting the allometric model there were n observations per tree where n is the number of measurements. The incremental model was fitted with $n - 1$ observations per tree.

MODEL SPECIFICATION

Short and Burkhart (1992) developed the incremental crown height model

$$\Delta CH = \beta_0 T^r H^{\beta_1} \exp(\beta_2 CR^{0.5} + \beta_3 CI + \beta_4 A_s) + \epsilon \tag{1a}$$

where

ΔCH = increment of crown height
T = thinning effect function
H = total height
CR = crown ratio
CI = a modification of Heygi competition index (Heygi 1974; Daniels 1976).
A_s = stand age
exp = base of the natural logarithm
$\beta_0 - \beta_4$ = parameters to be estimated
r = rate parameter corresponding to thinning response
ϵ = error term

The thinning effect function, T, is defined as I^{A_t/A_s} where I is the ratio of after thinning basal area to before thinning basal area, A_s is the stand age, and A_t is the stand age at time of thinning. In this study, 3-yr increment, instead of yearly increment, of crown height was modeled in order to avoid using interpolation techniques.

Dyer and Burkhart (1987) proposed an allometric crown ratio model for loblolly pine plantations:

$$CR = 1 - \exp[-(\beta_0 + \beta_1/A_s)D/H] + \epsilon \tag{2}$$

where D is diameter at breast height and all other variables are as previously defined. This function is bounded between 0 and 1, a desirable property for predicting crown ratio. Equation (2) can be easily modified to include the thinning effect measure, T:

$$CR = 1 - T^r \exp[-(\beta_0 + \beta_1 A_s^{-1})D/H] + \epsilon \tag{2a}$$

Equation (2a) is also bounded between 0 and 1 given that T is between 0 and 1.

TABLE 1.

Mean values (standard deviations in parentheses) for tree characteristics by thinning treatment at plot establishment and at the fourth remeasurement (12 yr later).

	Plot establishment			Fourth remeasurement		
Variable[1]	Unthinned	Light thinned	Heavy thinned	Unthinned	Light thinned	Heavy thinned
CR	0.46 (0.14)	0.49 (0.13)	0.50 (0.12)	0.31 (0.09)	0.37 (0.09)	0.40 (0.09)
CI	0.36 (0.20)	0.19 (0.09)	0.14 (0.07)	0.48 (0.20)	0.29 (0.11)	0.22 (0.09)
DBH (cm)	11.9 (3.74)	14.3 (3.96)	14.8 (3.97)	14.5 (3.9)	18.5 (4.2)	19.7 (4.2)
HT (m)	10.0 (3.2)	11.0 (3.5)	11.2 (3.4)	14.1 (2.9)	15.6 (2.9)	15.7 (3.0)
CH (m)	5.6 (2.7)	5.9 (2.9)	5.9 (2.8)	9.8 (2.4)	9.9 (2.5)	9.5 (2.6)
ΔCH (m)	1.5 (0.9)	1.3 (0.9)	1.2 (0.9)	1.4 (1.0)	1.5 (1.0)	1.4 (1.0)

[1] *CR* = crown ratio, *CI* = competition index, *DBH* = diameter breast height, *HT* = total height, *CH* = crown height, ΔCH = increment of crown height.

The thinning effect function, T, found in Equations (1a) and (2a) has certain desirable properties. The first is that when no thinning has occurred, the before-to-after thinning ratio, I, is 1 which means T has no effect on the prediction of crown ratio. The second is that as the ratio A_t/A_s becomes smaller through time, the effect of T becomes smaller. This suggests that over time, the effect of thinning diminishes, and crown development approaches that of an unthinned stand condition. This specification of T, however, ensures a monotonically decreasing response to thinning over time. This implies that the maximum response of crown size to thinning occurs at the time of thinning. Biologically, however, there should be no immediate response at the time of thinning. Instead, response to thinning should begin at zero and increase to some maximum as the crowns of the residual trees respond to the extra growing space and additional sunlight. Then, as the stand again closes, the response should diminish and approach an unthinned condition. With these considerations in mind, a new thinning response function was derived:

$$T = I^{\frac{-(A_s-A_t)^2+k(A_s-A_t)}{A_s^2}} \tag{3}$$

where

T = thinning response

A_s = stand age

A_t = age of stand at time of thinning

I = ratio of after thinning basal area to before thinning basal area

k = duration parameter.

The duration of thinning response (in years) is determined by the value of the duration parameter, k. The first derivative of the exponential part of Equation (3) with respect to $A_s - A_t$, the time elapsed since thinning, indicates that the maximum thinning response will occur at

$$\frac{kA_t}{k + 2A_t}$$

years after the thinning. Thus, age of maximum response depends on age of the stand at time of thinning and k. Using Equation (3), a new allometric crown ratio model was specified:

$$CR = 1 - I^{\frac{r[-(A_s-A_t)^2+k(A_s-A_t)]}{A_s^2}} \exp[-(\beta_0 + \beta_1/A_s)D/H] + \epsilon \tag{2b}$$

where all variables are as previously defined. The rate parameter, r, is dimensionless, and along with I, A_s and A_t define the shape of the response function.

In a similar way, a new crown increment equation can be specified by substituting Equation (3) for the original Short and Burkhart (1992) thinning response function in Equation (1a):

$$\Delta CH = \beta_0 I^{\frac{r[-(A_s-A_t)^2+k(A_s-A_t)]}{A_s^2}} H^{\beta_1} \exp(\beta_2 CR^{0.5} + \beta_3 CI + \beta_4 A_s) + \epsilon \tag{1b}$$

where all variables are as previously defined.

ANALYSES AND RESULTS

Evaluation of Thinning Response Functions

In order to evaluate the two thinning response functions, Equations (1a), (1b), (2a) and (2b) were fitted to the parameter estimation data set. Convergence was achieved for the four models using nonlinear estimation procedures, and all parameter estimates were significant. The parameter estimates and fit statistics are summarized in Table 2. The thinning response rate parameter, r, has a positive sign indicating that trees in plantations under heavy thinning would have smaller crown-height recession [Equations (1a) and (1b)] or larger crown ratio [Equations (2a) and (2b)] than trees in lightly thinned or unthinned stands. The thinning response duration parameter, k, in Equations (1b) and (2b) is also positive in sign, ensuring that thinning impact gradually increases after thinning to some level and then gradually decreases. It can be seen that the fit statistics are similar for the two increment models and for the two allometric models, although Equations (1b) and (2b) with the new thinning response function have slightly lower mean squared errors than their counterpart Equations (1a) and (2a). Figure 1 shows the results of predicting crown height with Equations (1a), (1b), (2a) and (2b) applied to the verification data. Mean residuals are observed minus predicted crown height plotted by thinning intensity and time elapsed since thinning. For predicting crown increment, Equation (1b) with the new thinning response function has a smaller mean residual for the period immediately after thinning than Equation (1a). This reflects the new response function's property of zero response at time of thinning rather than forcing the maximum response to occur at time of thinning as in Equation (1a). For the allometric models, Equation (2b) with the new thinning

TABLE 2.

Comparison of fit statistics for two crown increment models and two allometric models applied to the fitting data set.

	Equation							
	(1a)		(1b)		(2a)		(2b)	
Parameter	Est.	Std. err.	Est.	Std. err.	Est.	Std. err.	Est.	Std. err.
β_0	0.0095	0.00098	0.0104	0.00109	0.183	0.00307	0.153	0.00313
β_1	0.9030	0.02663	0.8727	0.0266	4.361	0.04567	4.634	0.04773
β_2	4.2543	0.08659	4.4169	0.08775				
β_3	1.0663	0.03728	1.0115	0.03767				
β_4	−0.022	0.00197	−0.027	0.00203				
r	0.1001	0.00827	2.8867	0.22855	0.043	0.00603	0.238	0.07765
k			10.668	0.37152			39.75	10.6245
	MSE = 0.6975		MSE = 0.6925		MSE = 0.00607		MSE = 0.00583	

response function has less overall bias by thinning treatment and time since thinning than (2a). From these results, the new thinning response function was judged to be superior.

EVALUATION OF INCREMENT VERSUS ALLOMETRIC MODELS

One important question for model developers is which method of predicting crown size through time should be used to provide the most reliable results. Height to

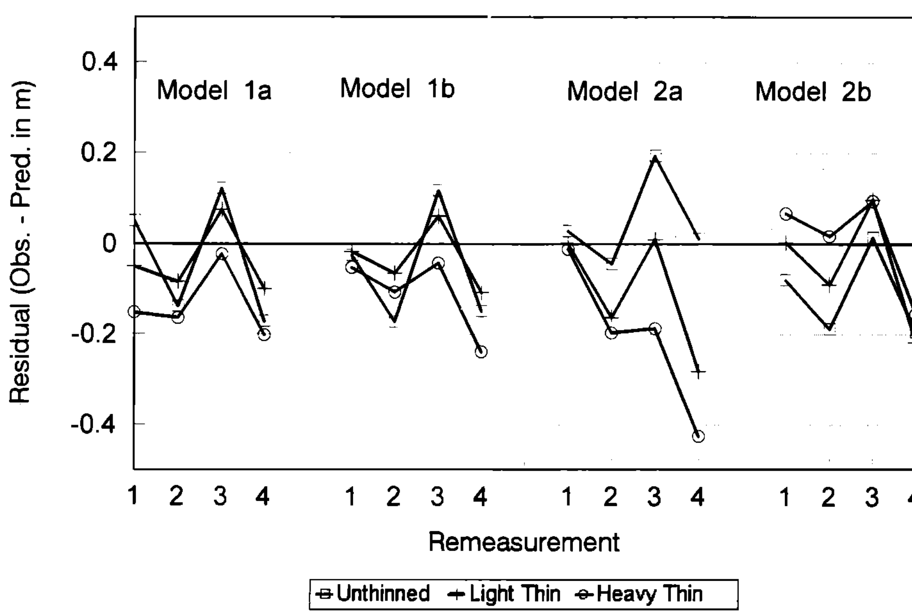

FIGURE 1. Average bias (observed-predicted) of four models [Equations (1a), (1b), (2a), (2b)] used to predict crown height by thinning treatment and time since thinning.

crown could be incremented directly using Equation (1b) along with total height. Then crown ratio can be computed from the accumulated total height and crown height. Alternatively, dbh and total height can be incremented and then crown ratio can be predicted allometrically using Equation (2b) and the accumulated dbh and total height. To test these alternative methods, a simulator was developed where both models could be used to predict crown height at 3, 6, 9, and 12 yr after plot establishment.

In order to develop the simulator, the dbh and height increment models from Burkhart et al. (1987) were fitted to the 3-yr diameter and height growth data in order to obtain 3-yr increment models. Two scenarios were defined. In Scenario 1, Equation (1b) and the total height increment equation were used to increment the crown height and total height of each tree in the verification data for the first growth period after plot establishment. The predicted increments for the period were added to the observed crown height and total height at plot establishment in order to obtain a predicted crown height at 3 yr after establishment. Predicted crown heights at 6, 9, and 12 yr were obtained in the same way except that predicted heights and crown ratios at the previous measurement period were used to predict height and crown increment rather than observed values. Thus total height was incremented along with crown height so that predicted total height at the beginning of each growth period could be used as a predictor variable for the crown increment model.

In Scenario 2, the predicted dbh and total height increment for each growth period were added to the observed dbh and height at plot establishment to obtain predicted dbh and total height at 3, 6, 9, and 12 yr after plot establishment. Then predicted crown ratio was obtained allometrically at each remeasurement using Equation (2b). For comparison with the increment approach, the allometrically obtained crown ratio prediction was converted to crown height.

In order to compare the two scenarios, the following statistic was computed for each scenario by remeasurement and thinning regime:

$$P_{uv} = \frac{\sum_{i=1}^{n}(\widehat{CH}_i - CH_i)^2}{\sum_{i=1}^{n}(CH_i - \overline{CH})^2}$$

where

P_{uv} = percent unexplained variability
CH = crown height.

This statistic provides a relative measure of the efficiency of each method of predicting crown size. Figure 2 shows the P_{uv} for each model by remeasurement and plot. The P_{uv} for each equation increases over time from about 10% at the first remeasurement to about 30% at the four remeasurement. This increase is expected due to the propagative nature of prediction errors. The allometric model, which does not include an explicit measure of competition, has a slightly higher P_{uv} than the increment model at each remeasurement. This is likely due, at least in part, to the greater error propagation associated with Scenario 2. Total

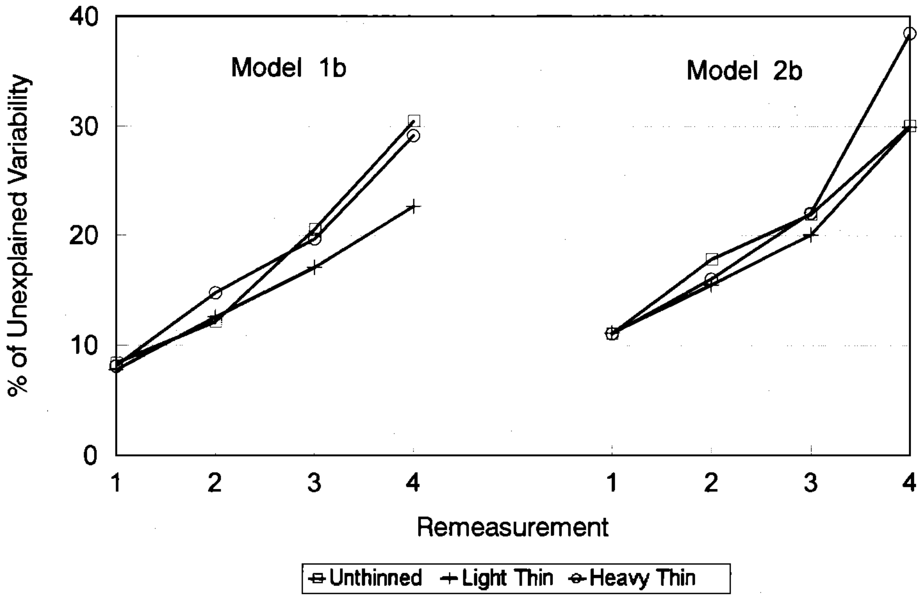

FIGURE 2. Percent of unexplained variability for a crown increment model [Equation (1b)] and an allometric model [Equation (2b)] used to predict crown height by thinning treatment and time since thinning.

height and dbh must be projected to each remeasurement before crown height can be predicted allometrically. For Scenario 1, however, crown increment to the first remeasurement is estimated directly from observed values at plot establishment; for subsequent remeasurements, the increment is estimated from predicted values. Thus, Scenario 1 should have less propagated error than Scenario 2.

Figure 3 is presented to demonstrate the behavior of the new thinning response function. It shows the effect of thinning for four hypothesized plantations. Two were thinned at age 12 with after-to-before thinning basal area ratios of 0.3 and 0.7. The other two were thinned at age 18 with the same 0.3 and 0.7 thinning intensities. The values of r and k are taken from Equation (2b) in Table 2. For the heavy-thinned plots ($I = 0.3$), the response is greater at both ages than for the light-thinned plots ($I = 0.7$). For the younger plantations, the response is greater for both intensities than for the older plantations. Furthermore, the maximum response occurs sooner after thinning for younger plantations than for older plantations. Once the maximum response has been achieved, the effect of thinning gradually diminishes with time. For unthinned plantations and for thinned plantations at the time of thinning, the response function is conditioned to equal one.

One shortcoming of the model is that after thinning it is possible for the response function to exceed one. That is, it is not conditioned to return to one. With large values of k, such as in the allometric model, the thinning response function would seldom exceed one during the normal lifetime of a plantation. For the increment model, it could exceed one within the time span of typical rotations. Therefore, computer programs that implement these equations should set the value of the response to one when it exceeds one.

Statistically, the presence of temporal correlation among the error terms of the dependent variables causes less efficient estimators for regression parameters

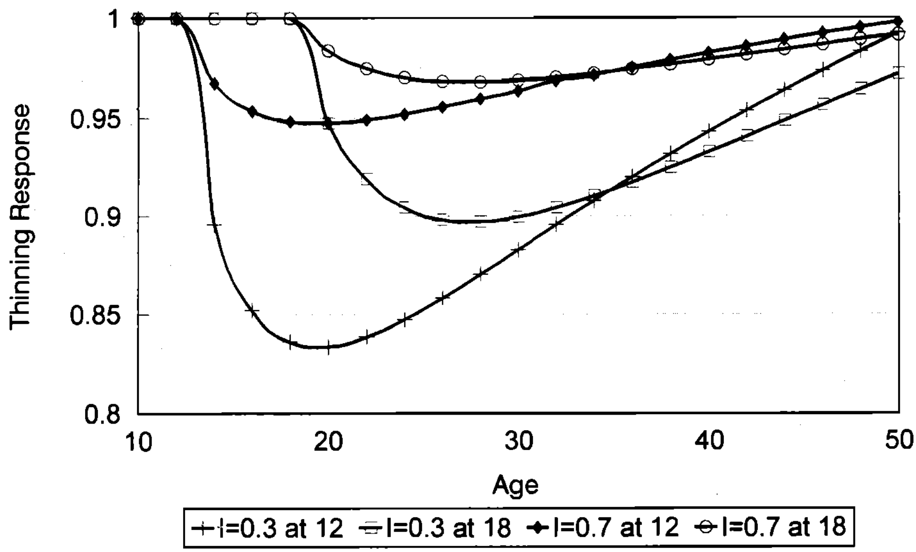

FIGURE 3. Illustration of the effects of thinning intensity (I, defined as the ratio of basal area after to before thinning), time of thinning, and time elapsed since thinning on crown ratio.

and biased estimates of residual variance (Cliff and Ord 1981, Gregoire 1987). In order to test the possibility that temporal correlation might be a problem, we fit Equation (1a) and (1b) to one randomly chosen observation per tree. No substantial difference in bias or precision for either model was found when using one observation per tree versus using all observations per tree to estimate parameters. Therefore, in order to make maximum use of the measurement data available, all observations from all trees (both the fitting and verification data) were combined and a final set of parameter estimates was obtained for both equations. The final equation for predicting crown increment in thinned and unthinned loblolly pine plantations is:

$$\Delta CH = 0.012480 I^{\frac{2.3013[-(A_s-A_t)^2+11.069(A_s-A_t)]}{A_s^2}} H^{0.81933} \exp(4.2678CR^{0.5} + 0.96509CI + -0.02333A_s)$$

$$S_{y.x} = 0.6876$$

The final equation for predicting crown ratio is:

$$CR = 1 - I^{\frac{0.13130[-(A_s-A_t)^2+67.042(A_s-A_t)]}{A_s^2}} \exp[-(0.14780 + 4.7233/A_s)D/H]$$

$$S_{y.x} = 0.07607$$

DISCUSSION AND CONCLUSIONS

In this study, two general types of crown size prediction models were evaluated: an allometric model and an incremental model. The results of this study indicate there is little difference in the predictive ability of the two model types. The increment model, which includes an explicit measure of competition, is slightly more efficient for prediction than the allometric model. However, the allometric model is simpler and avoids the complicated calculation of competition index. Using an allometric model also avoids having to interpolate crown increment when predictions for other than the measurement interval are desired. In any case, for most growth and yield simulators, crown size information is usually not a readily available input. Therefore, an allometric equation is generally needed at least to estimate initial crown size for stand simulation purposes. From that point on, however, crown size could be incremented directly along with other tree dimensions or predicted allometrically from other tree dimensions.

The new thinning response function presented here was found to be useful for predicting the response to thinning for an increment or an allometric crown size model. Actually, it can be viewed as a general response function that could be incorporated as a modifier into a variety of growth and yield models. It is conditioned to have no effect at time of treatment, increase to some maximum effect, and then gradually diminish until there is little or no effect. This pattern of response is typical after stand structure has been altered by thinning. The response function is sensitive to thinning intensity, stand age at time of thinning, and the elapsed time since thinning. It has only two parameters (although there are other ways it could be parameterized), is dimensionless, and thus can be incorporated into existing models without adding undue complexity. Therefore, this response function could be useful as a modifier in other equations such as diameter increment, basal area growth, or survival equations for thinned and unthinned stands (Bailey and Ware 1983). It could also be viewed in a broader sense as a silvicultural response function and applied to modeling other treatments that are known (or hypothesized) to have a similar response pattern. Response to fertilization, for example, might be expected to be zero at time of application, increase to some level and then gradually decrease to the pre-application level.

The duration parameter, k, determines the year of maximum response to treatment, and the length of time during which the response will last. The rate parameter, r, affects the shape of the response curve. Obviously, reliable estimates of r and especially k require data covering the entire response period.

LITERATURE CITED

AVILA, O.B., and H.E. BURKHART. 1992. Modeling survival of loblolly pine trees in thinned and unthinned plantations. Can. J. For. Res. 22:1878–1882.

BAILEY, R.L., and K.D. WARE. 1983. Compatible basal-area growth and yield model for thinned and unthinned stands. Can. J. For. Res. 13:563–571.

BURKHART, H.E., K.D. FARRAR, R.L. AMATEIS, and R.F. DANIELS. 1987. Simulation of individual tree growth and development in loblolly pine plantations on cutover, site-prepared areas. School of For. and Wildl. Resour., Virginia Polytech. Inst. and State Univ. Publ. FWS-1-87.

CLIFF, A.D., and J.K. ORD. 1981. Spatial processes, models & applications. Pion Limited, London.

DANIELS, R.F. 1976. Simple competition indices and their correlation with annual loblolly pine tree growth. For. Sci. 22:454–456.

DELL, T.R., D.P. FEDUCCIA, T.E. CAMPBELL, W.F. MANN JR., and B.H. POLMER. 1979. Yields of unthinned slash pine plantations on cutover sites in the West Gulf Region. USDA For. Serv. Res. Pap. SO–147.

DYER, M.E., and H.E. BURKHART. 1987. Compatible crown ratio and crown height models. Can. J. For. Res. 17:572–574.

EK, A.R., and R.A. MONSERUD. 1975. Methodology for modeling forest stand dynamics. School of Natur. Resour., Staff Pap. Series No. 2. University of Wisconsin, Madison.

FEDUCCIA, D.P., T.R. DELL, W.F. MANN, and B.H. POLMER. 1979. Yields of unthinned loblolly pine plantations on cutover sites in the West Gulf Region. USDA For. Serv. Res. Pap. SO–148.

GINN, S.E., J.R. SEILER, B.H. CAZELL, and R.E. KREH. 1991. Physiological and growth responses of eight-year-old loblolly pine stands to thinning. For. Sci. 37:1030–1040.

GREGOIRE, T.F. 1987. Generalized error structure for forestry yield models. For. Sci. 33:423–444.

HEGYI, F. 1974. A simulation model for managing jack-pine stands. P. 74–90 *in* Growth models for trees and stand simulation, Fries, J. (ed.). Royal Coll. of For., Stockholdm, Sweden.

KERSHAW, J.A., JR., D.A. MAGUIRE, and D.W. HANN. 1990. Longevity and duration Douglas-fir branches. Can. J. For. Res. 20:1690–1695.

KRAMER, H. 1966. Crown development in conifer stands in Scotland as influenced by initial spacing and subsequent thinning treatment. Forestry 39:40–58.

MAGUIRE, D.A., and D.W. HANN. 1990a. Constructing models for direct prediction of 5-year crown recession in southwestern Oregon Douglas-fir. Can. J. For. Res. 20:1044–1052.

MAGUIRE, D.A., and D.W. HANN. 1990b. A sampling strategy for estimating past crown recession on temporary growth plot. For. Sci. 36:549–563.

SHORT, E.A. III, and H.E. BURKHART. 1992. Predicting crown-height increment for thinned and unthinned loblolly pine plantations. For. Sci. 38:594–610.

SIEMON, G.R., G.B. WOOD, and W.G. FORREST. 1976. Effect of thinning on crown structure in radiata pine. N. Z. J. For. Sci. 6:57–66.

SMITH, D.M. 1986. The practice of silviculture. Ed. 8. Wiley, New York. P. 527.

SPRINZ, P.T., and H.E. BURKHART. 1987. Relationships between tree crown, stem, and stand characteristics in unthinned loblolly pine plantations. Can. J. For. Res. 17:534–538.

SPURR, S.H., and B.V. BARNES. 1980. Forest ecology. Ed. 3. Wiley, New York. P. 687.

VAN DEUSEN, P.C., and G.S. BIGING. 1985. STAG-A stand generator for mixed species. Department of For. and Resour. Manage., Res. Note 11. University of California, Berkeley.

WARD, W.W. 1964. Live crown ratio and stand density in young, even-aged, red-oak stands. For. Sci. 10:56–65.

WYKOFF, W.R., N.L. CROOKSTON, and A.R. STAGE. 1982. User's guide to the Stand Prognosis model. USDA For. Serv. Gen. Tech. Rep. INT-133.

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