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Harold E. Burkhart
Virginia Polytechnic Institute and State University

Relationships between tree crown, stem, and stand characteristics in unthinned loblolly pine plantations

PETER T. SPRINZ
Department of Forest Science, Texas Agricultural Experiment Station, Texas A&M University System, College Station, TX, U.S.A. 77843

AND

HAROLD E. BURKHART
Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, VA, U.S.A. 24061

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Empirical and theoretical relationships between tree crown, stem, and stand characteristics for unthinned stands of planted loblolly pine (*Pinus taeda* L.) were investigated. Readily measured crown variables representing the amount of photosynthetic area or distance of the translocation process were identified. Various functions of these variables were defined and evaluated with regard to efficacy in predicting stem and stand attributes. Linear models were used to evaluate the contribution of the crown variables in predicting stem and stand characteristics. The stem attributes modeled included basal area, basal area growth, diameter at breast height, and diameter growth, while the stand attributes modeled were basal area, basal area growth, arithmetic mean diameter, and mean diameter growth. Crown diameter and crown projection area were particularly important in contributing to model fit and prediction of individual stem characteristics, while sum of crown projection areas was found especially important in stand level equations. As these crown measures developed over time so did corresponding stem and stand attributes.

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On a étudié les relations empiriques et théoriques entre la cime et le fût des arbres et les caractéristiques de peuplement dans les plantations non éclaircies de *Pinus taeda* L. On a identifié les variables facilement mesurables de la cime et représentant la surface photosynthétique ou le parcours du processus de translocation. On a défini et évalué diverses fonctions de ces variables quant à leur efficacité pour prédire les caractéristiques des fûts et des peuplements. On a élaboré des modèles linéaires afin d'évaluer la contribution des variables de la cime dans la prédiction des caractéristiques des fûts et des peuplements. Les caractéristiques du fût utilisées incluaient la surface terrière, la croissance en surface terrière ainsi que le diamètre à hauteur de poitrine et la croissance de ce dernier, tandis que les caractéristiques du peuplement étaient la surface terrière, la croissance de celle-ci ainsi que le diamètre moyen arithmétique et la croissance de ce dernier. Le diamètre de la cime et la surface de la projection de la cime se sont révélés d'une importance particulière par leur contribution à l'ajustement du modèle et à la prédiction des caractéristiques des fûts, tandis que la somme des surfaces de projection des cimes s'est révélée particulièrement importante au niveau des équations du peuplement. Au développement des cimes telles que mesurées avec le temps correspondaient des caractéristiques de fût et de peuplement telles qu'évaluées.

[Traduit par la revue]

Introduction

Growth and yield models are widely used in forestry. The basis for these models is quantified relationships of certain stand and stem characteristics at a point in time and over time, and as affected by silvicultural practices. Tree crown dimensions and development have not been thoroughly investigated as a means to improve the predictive ability of individual tree or stand level attributes, however.

In the past, tree crowns have usually not been measured due to difficulties in defining and measuring their changing dimensions and shapes (Spurr 1952; Honer 1972; Honer and Collins 1974). However, recent applications of low-level aerial photographs for measuring crown dimensions (Mitchell 1975*a*, 1980; Burton and Shoulders 1982) suggest that cheaper and more accurate methods are becoming available.

An obvious advantage of using crown characteristics as a basic modeling unit is the growth relationships that exist between the crown and stem. Physiologists have described the internal mechanisms and the cause and effect relationships of how crown development influences stem development. The relationships between crown and stem dimensions and development should also be similar on a stand level, although they have not been well quantified.

The implications of integrating crown information into modeling stem and stand characteristics are numerous. For example, level of stocking, configuration of spacing, pruning, and timing, intensity and method of thinning affect crown dimensions and development, and thus affect stem and stand characteristics. If the effects of various cultural treatments on crown development are quantified, then it is possible to better understand and estimate treatment effects on stem and stand characteristics.

The objective of this work was: (*i*) to identify crown variables that reflect photosynthetic area or translocation distance; (*ii*) to select crown variables that can be measured readily; and (*iii*) to evaluate these crown variables in regard to efficacy in predicting stem and stand attributes for unthinned stands of planted loblolly pine (*Pinus taeda* L.).

Crown and stem relationships

Physiologists since Pressler (1864) have described the relationships between crown and bole development. Open-grown trees develop full, deep crowns generally extending to the ground. In contrast, the crowns of forest-grown trees recede vertically and decrease in growth rate horizontally as competition for light increases (Farrar 1961; Larson 1963). It has also been suggested (Hall 1965) that measurements of crown dimensions at one point in time will not be as indicative of future stem growth as will measurements of crown dimensions at several points in time.

Crown dimensions that have been used in modeling include horizontal measures such as crown diameter (CD) and crown projection area (CPA), and vertical measures such as crown length (CL) and height to live crown (HLC). In this paper, CD is defined as the diameter of the widest horizontal portion of the crown, while CPA is the area of the horizontal projection of the crown calculated from CD. Crown length is the length of the live portion of the crown from the first major branch to the top of the tree, while HLC is the height from the ground to the base of the live crown.

Crown and stem relationships have been used at an individual tree level or as a component of a stand model. CD has been linearly related to the diameter at breast height (dbh) of trees grown in open stands (Krajicek *et al.* 1961; Dawkins 1963; Strub *et al.* 1975) and to diameter and height of trees grown in closed stands (Smith and Bailey 1964; Curtin 1964). Minor (1951, 1960), using tree data from sawlog stands, related dbh to CD, while Bonner (1964), with tree information from unthinned stands, related dbh to CD and height. Vertical measures of CL and HLC have been related to dbh, height, and measures of density (e.g., Curtis and Reukema 1970; Cole and Jensen 1982).

At a stand level, Mitchell (1969, 1975*b*) modeled branch development as a means of simulating horizontal and vertical crown development. Most implementations of crown information in stand modeling have been with CL or a ratio of CL to height (Daniels and Burkhart 1975; Daniels *et al.* 1979; Hahn and Leary 1979).

Certain crown dimensions appear logical for improving the prediction of stem and stand attributes since these dimensions indirectly reflect photosynthetic area or distance that photosynthate must travel to other tree parts. In this study, only crown variables that can be measured readily were investigated.

Individual tree crown attributes

Horizontal crown development can be measured by crown diameter or crown projection area of the widest portion of the live crown. These are indirect measures of photosynthetic area, in that large values of these variables indicate large crown volumes and thus large areas of photosynthate production potential. Age and immediate stocking levels surrounding a tree affect the size and growth of CD and CPA; however, within a density and age class, CD or CPA should be highly related to stem diameter or basal area, respectively. Also, the growth of CD or CPA should be related to the growth of tree diameter or basal area.

Vertical measures of crown dimensions include the effective live crown (ELC) and height to crown diameter (HCD). The former is defined as the height measurement from the top of the tree to CD, while the latter is the height from the ground to CD. If stocking level and age are constant, as ELC increases or HCD decreases, photosynthetic area increases. Within a stocking level and as dominant crown positions develop, ELC and HCD increase, while as stocking levels increase, ELC decreases and HCD increases.

Other measures of crowns include crown volume (CV), crown surface area (CSA), and weighted functions of both. CV represents the total cubic volume of crown available for photosynthetic production and as such, as CV increases, photosynthetic production should increase. However, the entire crown volume does not have the potential for photosynthetic production. This is reflected in CSA, a measure of the surface area of CV excluding the base. In order to weight these indirect measures of photosynthetic potential with the distance photosynthate must travel for lower stem growth, the variables CV and CSA can be weighted by HCD. Thus, given two equal CV (or CSA) measures, the CV (or CSA) closer to dbh would have a larger weighted value on stem development than the CV (or CSA) farther from dbh.

Stand level crown attributes

One measure of horizontal crown development on a per hectare basis is the sum of crown projection area (SCPA). This measure is analogous to basal area per hectare; basal area is the horizontal development of stems, while SCPA is the horizontal development of crowns. While having similar properties and trends as basal area, SCPA also has a unique asymptotic property. As SCPA approaches unit area (i.e., 10 000 m^2/ha for very intolerant and higher levels for more tolerant species), horizontal crown development decreases and vertical crown dimensions decrease. Depending on the stand structure, age, and rate of mortality, stem radial growth will continue after SCPA asymptotes.

Additional stand measures to SCPA are sum of crown volume (SCV) and sum of crown surface area (SCSA). These are variables that respectively measure totals of crown volume and surface area on a hectare basis. These variables represent estimates of total photosynthetic area and should be related to stand development.

Other important measures of crown attributes are mean crown projection area (MCPA) and mean height to crown diameter (MHCD). The MCPA and MHCD are calculated, respectively, as arithmetic means using the sum of crown projection area (SCPA) and sum of heights to crown diameter (HCD), each divided by the number of trees surviving.

Crown, stem, and stand data

Relationships between individual tree and stand level characteristics and crown attributes were explored using data from tree and plot measurements in unthinned loblolly pine plantations measured in 1978 and 1982. The ground measurements included the following: dbh, total height, Cartesian coordinates of individual tree locations, crown class, crown diameter across and within rows, height to live crown, and height to crown diameter. These crown dimensions were selected since they could be easily measured and could be used to calculate the various crown attributes previously mentioned. Since measurements were taken at two points in time, attributes of crown growth were also calculated; they included crown diameter growth, crown projection area growth, height growth to crown diameter, and sum of crown projection area growth.

The data were from research plots located in stands that had received site preparation and hardwood control but no burning. Plot locations included a mixture of old-field and cutover sites in North Carolina, Arkansas, Oklahoma, and Louisiana. With the completion of the second set of measurements, 27 unthinned plots containing 1442 tree observations were available for analysis (Tables 1, 2).

Procedures for determining crown and stem relationships

The basic crown variables that could be measured or calculated readily were one point in time and 4-year change in crown diameter, crown projection area, height to crown diameter, crown volume, and crown surface area, and sums of

TABLE 1. Frequency table and summary information of plots located in sampled unthinned loblolly pine plantations

SI_{25} class (m)	Age-class (years)	Basal area classes (m^2/ha) 5	14	23	32	41	51	Total
18	5	1						1
	10			3				3
	20				1	3	2	6
	30				3	1		4
Total		1		3	4	4	2	14
21	5	9						9
	10		1	2				3
	30				1			1
Total		9	1	2	1			13
Total		10	1	5	5	4	2	27

Variable	Minimum	Mean	Maximum
Age 1 (years)	6	14.3	28
Age 2 (years)	10	18.3	32
Stand arithmetic mean diameter 1[a] (cm)	2.8	12.2	28.4
Stand arithmetic mean diameter 2[b] (cm)	8.4	16.0	29.5
Mean crown projection area 1 (m^2)	4.0	17.8	71.9
Mean crown projection area 2 (m^2)	8.8	28.0	116.7
Stand basal area 1 (m^2/ha)	1.1	21.8	50.0
Stand basal area 2 (m^2/ha)	9.8	30.2	55.0
Sum of crown projection area 1 (m^2/ha)	1436	7200	12694
Sum of crown projection area 2 (m^2/ha)	4603	9900	15268
Trees surviving 1 (trees/ha)	543	1692.9	2831
Site Index 25[c](m)	17	20.0	22

[a]Initial measurement taken at age 1.
[b]Subsequent measurement taken 4 years after age 1.
[c]Devan and Burkhart (1982) with old-field combined Piedmont and Coastal Plain coefficients.

TABLE 2. Frequency table and summary information of trees located in sampled unthinned loblolly pine plantations

SI_{25} class (m)	Age-class (years)	Basal area classes (m^2/ha) 5	14	23	32	41	51	Total
18	5	53						53
	10			224				224
	20				18	131	119	268
	30				79	45		124
Total		53		224	97	176	119	669
21	5	518						518
	10		74	160				234
	30				21			21
Total		518	74	160	21			773
Total		571	74	384	118	176	119	1442

Variable	Minimum	Mean	Maximum
Age 1 (years)	6	12.2	28
Age 2 (years)	10	16.2	32
Tree diameter 1[a] (cm)	0.5	10.9	39.9
Tree diameter 2[b] (cm)	2.5	14.7	41.7
Crown diameter 1 (m)	0.5	2.2	6.8
Crown diameter 2 (m)	0.8	2.7	8.9
Height to crown diameter 1 (m)	0.4	6.1	22.1
Height to crown diameter 2 (m)	1.7	8.5	23.9
Total height 1 (m)	1.4	8.4	23.7
Total height 2 (m)	3.3	11.2	25.2

[a]Initial measurement taken at age 1.
[b]Subsequent measurement taken 4 years after age 1.

crown projection area, crown volume, and crown surface area. The efficacy of these variables was determined by evaluating their contribution in predicting stem and stand attributes. The stem attributes selected were individual tree basal area, diameter at breast height, basal area growth, and diameter growth, while the stand attributes were stand level basal area, basal area growth, arithmetic mean diameter, and diameter growth.

Initially, linear models were used to explore the relationships and associations among the stem, stand, and crown attributes. Specific crown attributes were selected to determine the contribution each made in predicting stem and stand attributes. Linear models were evaluated with transformations of both the dependent and independent variables.

With the range of age and site index values associated with the trees and plots, age and site index or transformations of age and site index were initially included in all models as part of the linear functions. Independent variables consisting of stem, stand, and crown measurements, such as stem basal area or diameter, total height, stand basal area, and crown variables, were included in the variable screening process where appropriate. Retention of independent variables was determined on the basis of the variable's contribution to the model's fit and prediction of the dependent variable. The fit criteria included the standard error of the estimate ($S_{y \cdot x}$) and coefficient of determination (R^2); the prediction criteria included the prediction error sum of squares (PRESS), maximum variance inflation factor (VIF), and variable sign and significance (Montgomery and Peck 1982).

Since stem, stand, and crown variables are interrelated, the PRESS and VIF statistics were used to determine if problems of multicollinearity (linear dependencies among the independent variables) were evolving in the formulation of the models. The PRESS statistic estimates the squared differences between an observed and predicted data point as if the data point came from an independent data set; the lower the PRESS value, the better the model is in predicting the given data. VIF is a statistic that estimates the degree of the linear dependencies among the independent variables in a model. The fit statistics indicate how well a model fits the data set used in its construction, while the prediction statistics indicate how well a model may predict for independent data.

Results of modeling individual tree attributes

A linear combination of age, site index, and a measure of a horizontal crown dimension were necessary for high levels of model fit and predictive abilities for tree basal area (b) and diameter (d) (Table 3). The variables that contributed most to model fit and prediction of stem basal area and diameter were horizontal crown attributes. Crown projection area (CPA) was the horizontal crown measure most important in predicting basal area, while crown diameter (CD) was the horizontal crown measure most important in predicting diameter. In modeling

TABLE 3. Selected models for various stem and stand attributes

Dependent variable	Independent variables	$S_{y \cdot x}$	R^2	Model
Stem attributes				
Basal area at breast height (m^2/tree)	SI, A, CPA	0.0046		$b = e(-4.6309 + 0.0423SI - 16.3300/A + 0.5475\ln CPA)$
Diameter at breast height (cm/tree)	SI, A, B, CD	1.6205	0.95	$d = -9.9223 + 0.3125SI + 0.2819A + 0.1295B + 3.8394CD$
Basal area growth (m^2/tree per 4 years)	SI, A, SCPA, b, CD, CDg	0.0024	0.66	$bg = 0.0332 - 3.5663E\text{-}04SI - 6.1019E\text{-}04A - 6.9501E\text{-}07SCPA + 2.3409E\text{-}04\ln b + 1.8078E\text{-}03CD + 1.6525E\text{-}03CDg$
Diameter growth (cm/tree per 4 years)	SI, A, SCPA, b, CD, CDg	1.1308	0.81	$dg = 9.6086 + 7.3772E\text{-}02SI - 0.2512A - 4.6513E\text{-}04SCPA + 0.4983\ln b + 0.6050CD + 0.7459CDg$
Stand attributes				
Basal area (m^2/ha)	SI, A, SCPA, MCPA	1.8141		$B = e(-6.3464 + 0.0804SI - 18.1737/A + 1.2011\ln SCPA - 0.5307\ln MCPA)$
Basal area growth (m^2/ha per 4 years)	A, SCPA, B	2.2385		$Bg = e(2.5387 + 12.5072/A + 3.0553E\text{-}04SCPA - 7.4244E\text{-}02B)$
Diameter at breast height (cm/tree)	SI, A, MCPA	0.8725	0.99	$D = -13.9734 + 0.3650SI + 0.5469A + 4.3662\ln MCPA$
Diameter at breast height growth (cm/tree per 4 years)	A, SCPA	0.8898	0.88	$Dg = 8.8184 - 0.1521A - 3.8283E\text{-}04SCPA$
	A, SCPA, D	0.7953	0.91	$= 6.2275 - 0.2893A - 7.5839E\text{-}04SCPA + 3.1516\ln D$

NOTE: A, age (years); b, tree basal area at breast height (m^2); bg, tree basal area growth at breast height (m^2/4 years); B, stand basal area (m^2/ha); Bg, stand basal area growth (m^2/ha per 4 years); CD, tree crown diameter (m); CDg, tree crown diameter growth (m/4 years); CPA, tree crown projection area (m^2); d, tree diameter at breast height (cm); dg, tree diameter growth at breast height (cm/4 years); D, stand arithmetic mean diameter at breast height (cm); Dg, stand arithmetic mean diameter growth at breast height (cm/4 years); H, total height (m); MCPA, stand mean crown projection area (m^2); SCPA, sum of crown projection area (m^2/ha); SI, site index base age 25 (m).

tree diameter, a linear model of age, site index, and stand basal area indicated an $S_{y \cdot x}$ of 3.08 and PRESS of 13 735; however, a similar model with CD instead of stand basal area indicated an $S_{y \cdot x}$ of 1.84 and PRESS of 4929. By including both CD and B with site index and age, $S_{y \cdot x}$ was 1.62 and PRESS was 3817.

The important variables in predicting basal area growth were similar to those for predicting diameter growth. In addition to site index and age, important variables in the fit and prediction of radial increment included initial size of the tree, stocking level, and a horizontal crown dimension and its growth. Tree basal area was the expression selected for initial tree size, while the sum of crown projection area (SCPA) was the density measure selected. In both cases, CD and crown diameter growth (CDg) were significant variables in combination with the above in describing radial increment. Also, after the inclusion of site index and age, CDg resulted in the highest level of model fit and predictive ability of radial increment compared with any other single stem or stand variable.

Results of modeling stand level attributes

Adequate models for estimating various stand characteristics of unthinned stands generally required site index, age, and horizontal crown information. For example, stand basal area (B) was best estimated using horizontal crown dimensions of mean CPA and stand CPA in addition to site index and age (Table 3). Several relationships between basal area and other variables were also indicated, such as asymptotic relationship between stand crown projection area and basal area. Before crown closure, development of these two measures was directly related, while after crown closure, stand CPA was relatively constant, but basal area continued to increase.

Basal area growth (Bg) over a 4-year period was related to age, initial stand CPA, and basal area. However, changes in crown structure, as expressed by stand CPA growth, were not related to changes in stand structure as described by basal area. Before crown closure, basal area growth and SCPA growth were related; however, after crown closure, SCPAg approached zero, while Bg continued to increase.

Predictions for average stand attributes, such as arithmetic mean diameter (D) and diameter growth (Dg), were improved when crown characteristics were included. Mean diameter was best estimated using site index, age, and mean CPA. That is, the average size of the horizontal crown area was related to the average size of the stem at breast height. Growth of the average diameter of unthinned stands over the 4-year period was best predicted using age and SCPA or age, initial mean diameter, and stand CPA.

Discussion and summary

Crown, stem and stand attributes are interrelated. Stem and stand growth depend on the amount of photosynthate produced and the distance it must travel to the base of the tree. Increases in crown area can result in increases in photosynthate produced and available for stem growth. However, increases in the translocation distance of photosynthate to the base of the tree reduce amounts of photosynthate available for stem growth. Crown variables representative of the amount of photosynthate produced and distance of the translocation process were selected and evaluated in terms of efficacy in predicting stem and stand attributes. Stem and stand dimensions were found to be related to crown dimensions, while stem growth was found to be related to crown growth.

Tree stem and stand level attributes were better fitted and predicted with models containing crown information than models using only stem or stand information. Models for estimating individual stem attributes of basal area and diameter at breast height required horizontal crown measures of crown projection area and crown diameter, respectively, for adequate predictions. Stem basal area and diameter growth models containing crown diameter, crown diameter growth, and sum of the crown projection area had better fit and prediction statistics than models without measures of horizontal crown area.

Predictions of point in time and change over time stand level atttibutes, such as stand basal area and arithmetic mean diameter, were improved when horizontal crown information of mean crown projection area and sum of the crown projection area were used as independent variables.

The crown attributes that were important in predicting stem and stand attributes are suggestive of the amount of photosynthetic area or distance of the translocation process. Horizontal dimensions of a tree crown, such as crown projection area, crown diameter, and crown diameter growth, can be thought of not only as indirect measures of current photosynthetic area but also of future photosynthetic area. The dimension of the base of a crown may be representative of the amount of photosynthate that is produced by a crown and of the amount that may be produced in the future. Results indicated that as the crown diameter grew, so did basal area and diameter of individual trees. The distance of translocation is related to the distance from the base of the tree to the crown. Factors affecting this distance are age and stand density. As age or stand density increases, competition will intensify, resulting in an increase in translocation distance. The sum of the crown projection areas was particularly important in predicting stem and stand growth.

The results from this research indicate that crown variables are important attributes that can improve the prediction of certain stem and stand characteristics. Since silvicultural practices directly affect crown dimensions, inclusion of crown variables within the modeling process may provide a means to better understand and predict the effects of silvicultural treatments on stem and stand attributes.

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