JOHN BENEDICT C. LABOR

1

a. FALSE

b. FALSE

c. FALSE

d. FALSE

e. FALSE

f. TRUE

g. TRUE

h. FALSE

i TRUE

j. TRUE

k. TRUE

L. FALSE

m. FALSE

2.

FIND ALL VALUES OF a FOR WHICH THE RESULTING LINEAR SYSTEM (a) NO SOLUTION, (b) A UNIQUE SOLUTION, (C) INFINITELY MANY SOLUTION.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 2 & 1 & 3 & 5 \\ -3 & -3 & (a^2-5a) & a-8 \end{array}\right]$$

$r_{12}^{(-2)}: -2R_1 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & -3 \\ -3 & -3 & (a^2-5a) & a-8 \end{array}\right]$$

$r_{13}^{3}: 3R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & -3 & a^2-5a+3 & a+4 \end{array}\right]$$

$r_{23}^{3}: 3R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & a^2-5a+6 & a-5 \end{array}\right]$$

$a^2 - 5a + 6 = 0$

$a^2 - 2a - 3a + 6 = 0$

$a(a-2) - 3(a-2) = 0$

$a = 3 \;,\; a = 2$

$\therefore$ a

NO SOLUTION IF $a = 3$, $a = 2$

$\therefore$ b

A UNIQUE SOLUTION IF $a \neq 3$, $a \neq 2$

$\therefore$ c

INFINITELY MANY SOLUTIONS DOES NOT EXIST.

3.

$$A = \begin{bmatrix} -1 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\lambda I_{2} = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} -\lambda - \cancel{1} & -2-0 \\ -2-0 & 2-\lambda \end{bmatrix}$$

$$\begin{bmatrix} -1-\lambda & -2 \\ -2 & 2-\lambda \end{bmatrix}$$

$$0 = (-1-\lambda)(2-\lambda) - (-2)(-2)$$

$$0 = -(1+\lambda)(2-\lambda) - 4$$

$$0 = -2 + \lambda - 2\lambda + \lambda^2 - 4$$

$$0 = \lambda^2 - \lambda - 6$$

$$0 = (\lambda - 3)(\lambda + 2)$$

$$\boxed{\lambda = 3 \ , \ \lambda = -2}$$

$A^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ SOLVE $(AB)^{-1}$

4a.

$B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & -2 \\ 1 & 1 & -1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$r_1^{\frac{1}{2}} : \frac{1}{2}R_1 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$r_{13}^{-1} : -1R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -2 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & -2 & 0 & 1 & 0 \end{array}\right]$$

$r_2^{(2)} : 2R_2 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -3 & -1 & 0 & 2 \\ 0 & 0 & -2 & 0 & 1 & 0 \end{array}\right]$$

$r_3^{-\frac{1}{2}} : -\frac{1}{2}R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -3 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \end{array}\right]$$

$r_{32}^{(3)} : 3R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & -\frac{3}{2} & 2 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \end{array}\right]$$

$r_{31}^{-\frac{1}{2}} : -\frac{1}{2}R_3 + R_1 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 1 & 0 & -1 & -\frac{3}{2} & 2 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \end{array}\right]$$

$r_{21}^{-\frac{1}{2}} : -\frac{1}{2}R_2 + R_1 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & -\frac{3}{2} & 2 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \end{array}\right]$$

$A^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 4 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & -\frac{3}{2} & 2 \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$, $(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = \cancel{\begin{bmatrix} 1(1) + 1(0) + (-1)(1) & 1(3) + 1(1) - (1)(1) \\ -1(1) + (-\frac{3}{2})(0) + 2(1) & \\ 0(1) + (-\frac{1}{2})(0) + 0(1) & \end{bmatrix}}$

$B^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & -\frac{3}{2} & 2 \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 4 \end{bmatrix}$

$$(AB)^{-1} = \begin{bmatrix} 1(1) + 1(0) - 1(1) & 1(3) + 1(1) - 1(-1) & 1(0) + 1(1) - 1(4) \\ -1(1) - \frac{3}{2}(0) + 2(1) & -1(3) - \frac{3}{2}(1) + 2(-1) & -1(0) - \frac{3}{2}(1) + 2(4) \\ 0(1) - \frac{1}{2}(0) + 0(0) & 0(3) - \frac{1}{2}(1) + 0(-1) & 0(0) - \frac{1}{2}(1) + 0(4) \end{bmatrix}$$

$$\boxed{(AB)^{-1} = \begin{bmatrix} 0 & 5 & -3 \\ 1 & -\frac{13}{2} & \frac{13}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}}$$

4b.

SOLVE $Ax = b$ FOR x

$A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$Ax = b$

$A^{-1}Ax = A^{-1}b$

$Ix = A^{-1}b$

$x = A^{-1}b$

$x = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\frac{3\times 3 \quad 3\times 1}{3\times 1}$

$= \begin{bmatrix} 1(2) + 0(1) - 2(3) \\ 2(2) + 1(1) + 3(3) \\ 4(2) + 2(1) + 5(3) \end{bmatrix}$

$$\boxed{x = \begin{bmatrix} -4 \\ 14 \\ 25 \end{bmatrix}}$$

#5

EVALUATE:

$$\begin{vmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 3 \\ -1 & 2 & -3 & 4 \\ 0 & 5 & 0 & -2 \end{vmatrix}$$

$r_{13}^{(1)}$: $R_1 + R_3 \to R_3$

$$\begin{vmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 3 & -1 & 3 \\ 0 & 5 & 0 & -2 \end{vmatrix}$$

$r_{23}^{(-3)}$: $-3R_2 + R_3 \to R_3$

$$\begin{vmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & -6 \\ 0 & 5 & 0 & -2 \end{vmatrix}$$

$r_{24}^{(-5)}$: $-5R_2 + R_4 \to R_4$

$$\begin{vmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & -17 \end{vmatrix} = 1(1)(-1)(-17)$$

$$\boxed{= 17}$$

6.

FOR WHAT VALUE OF a IS

$$\begin{vmatrix} 2 & 1 & 6 \\ 0 & -1 & 3 \\ 0 & 1 & a \end{vmatrix} + \begin{vmatrix} 0 & a & 1 \\ 1 & 3a & 0 \\ -2 & a & 2 \end{vmatrix} = 14$$

$$|A| + |B| = 14$$

$|A|$

$$\begin{vmatrix} 2 & 1 & 6 \\ 0 & -1 & 3 \\ 0 & 1 & a \end{vmatrix}$$

$r_{23}^{(1)} : 1R_2 + R_3 \to R_3$

$$\begin{vmatrix} 2 & 1 & 6 \\ 0 & -1 & 3 \\ 0 & 0 & 3+a \end{vmatrix}$$

$= 2(-1)(3+a)$

$= -6 - 2a$

$|B|$

$$\begin{vmatrix} 0 & a & 1 \\ 1 & 3a & 0 \\ -2 & a & 2 \end{vmatrix}$$

$R_1 \leftrightarrow R_2$

$$-\begin{vmatrix} 1 & 3a & 0 \\ 0 & a & 1 \\ -2 & a & 2 \end{vmatrix}$$

$r_{13}^{(2)} : 2R_1 + R_3 \to R_3$

$$-\begin{vmatrix} 1 & 3a & 0 \\ 0 & a & 1 \\ 0 & 7a & 2 \end{vmatrix}$$

$r_{23}^{(-7)} : -7R_2 + R_3 \to R_3$

$$-\begin{vmatrix} 1 & 3a & 6 \\ 0 & a & 1 \\ 0 & 0 & -5 \end{vmatrix}$$

$= -(1)(a)(-5)$

$= 5a$

$-6 - 2a + 5a = 14$

$3a = 14 + 6$

$\frac{1}{3}\left[3a = 20\right]\frac{1}{3}$

$\boxed{a = \frac{20}{3}}$

$-6 + \cancel{3}\frac{20}{\cancel{3}} = 14$

$20 - 6 = 14$

$14 = 14$

7\. FIND ALL VALUES OF a FOR WHICH THE MATRIX

$$\begin{bmatrix} a^2 & 0 & 3 \\ 5 & a & 2 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} a^2 & 0 & 3 & a^2 & 0 \\ 5 & a & 2 & 5 & a \\ 3 & 0 & 1 & 3 & 0 \end{array}\right]$$

$$= [a^2 \cdot a \cdot 1 + 0 \cdot 2 \cdot 3 + 3 \cdot 5 \cdot 0] - [3 \cdot a \cdot 3 + 0 \cdot 2 \cdot a^2 + 1 \cdot 5 \cdot 0]$$

$$= [a^3 + 0 + 0] - [9a + 0 + 0]$$

$$= a^3 - 9a$$

* A MATRIX IS SINGULAR WHEN ITS DETERMINANT IS EQUAL TO ZERO

$$a^3 - 9a = 0$$

$$a(a^2 - 9) = 0$$

$$a^2 - 9 = 0$$

$$\sqrt{a^2} = \sqrt{9}$$

$$a = \pm 3$$

$\boxed{\text{THUS THE MATRIX IS SINGULAR WHEN } a = 0,\ a = 3,\ a = -3}$