Pin$^+$ 4D Non-Orientable Dynamics (4D–NOD): A Core Hypothesis with Explicit Sub-Manifolds, Cobordism Gluing, and Gauge-Holonomy Gates

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Abstract

We formulate the 4D–NOD hypothesis on a *Pin*$^+$ baseline in four dimensions. Particles are modeled as compact non-orientable 4-manifolds with S^3 boundary ("quark blocks") that glue along a 4D Y–cobordism into closed objects (hadrons). We construct two explicit blocks: (A) $W_{\mathrm{RP}} := \mathbb{RP}^4 \setminus \mathrm{int}(B^4)$ (non-orientable, Pin$^+$), and (B) a punctured mapping torus W_{rev} of an orientation-reversing S^3-diffeomorphism (also Pin$^+$). With a Y–cobordism Y^4, three blocks glue to a closed Pin$^+$ manifold P once two *bundle gates* hold: an SU(3) holonomy product $g_1 g_2 g_3 = \mathbf{1}$ (color neutrality) and a U(1) holonomy sum $e^{i(\theta_1+\theta_2+\theta_3)} = 1$ (EM neutrality at the glue). We show $[P] = 3[\mathbb{RP}^4]$ in $\Omega_4^{\mathrm{Pin}^+}$, aligning the "16 classes" ledger used in the categorical architecture and providing a precise geometric core for the program.[1] [2] [3] [4] [5] [6]

1 Core axioms and scope

We enforce the following non-negotiable requirements, taken as *model axioms*:

R1. **GR framework**: spacetime is a Lorentzian 4-manifold; all dynamics respect Einstein geometry.

R2. **Spacetime ontology**: spacetime is the sole fundamental entity; matter/forces are geometry.

R3. **Particles as 4D topology**: particles are compact 4D topological structures embedded in spacetime.

R4. **Forces as connections**: interactions are realized as topological connections (handles/cobordisms).

R5. **SM as effective theory**: the Standard Model arises as an effective description of these geometric phenomena.

These axioms are documented in the SKB manuscript and earlier program notes.[7]

[1] GR-first axioms (spacetime as sole entity; particles as 4D structures; forces as topological connections; SM as effective limit) originate from the SKB document. :contentReference[oaicite:0]index=0

[2] Phase-0/1 candidate analysis summary (Pin$^-$ baseline) and tables. :contentReference[oaicite:1]index=1

[3] Research reports (Pin$^-$ emphasis, Phase-0/1 synthesis) to be superseded by the Pin$^+$ baseline adopted here. :contentReference[oaicite:2]index=2 and :contentReference[oaicite:3]index=3

[4] Mathematical Foundations note (Stiefel–Whitney formulas, including $w(T\mathbb{RP}^4) = (1+a)^5$). :contentReference[oaicite:4]index=4

[5] Phase-0/1 analysis script shows manual toggles of w_2 and simplified π_1; we replace these with class-level computations. :contentReference[oaicite:5]index=5

[6] Categorical architecture (pushouts/colimits, TQFT functor, "16 fermion classes" narrative) that we adopt and sharpen in the Pin$^+$ setting. :contentReference[oaicite:6]index=6

[7] See the SKB hypothesis and axioms (Sec. 1.1–1.2; Eqs. (6)–(11)) for the GR-first, geometry-only stance and the flux/holonomy view of charge. :contentReference[oaicite:7]index=7

2 Pin structures and the 4D baseline

Let M be a smooth 4-manifold; write $w_i(M) \in H^i(M;\mathbb{Z}/2)$ for Stiefel–Whitney classes. We use:

$$\begin{aligned} M \text{ admits a } \mathrm{Pin}^+ \text{ structure} &\iff w_2(M) = 0, \\ M \text{ admits a } \mathrm{Pin}^- \text{ structure} &\iff w_2(M) + w_1(M)^2 = 0. \end{aligned}$$

We adopt *Pin*$^+$ as the 4D baseline because the bordism classification used in the categorical layer matches a 16-class ledger in 4D under the program's conventions.[8] [9]

Remark 1 (Programmatics)**.** Pin^+ in 4D aligns with the category-theoretic "16 classes" narrative and avoids ambiguities that arose under Pin^- in Phase-0/1 tables.[10]

3 Canonical non-orientable block W_{RP}

Let $a \in H^1(\mathbb{RP}^4;\mathbb{Z}/2)$ be the generator. Using $w(T\mathbb{RP}^4) = (1+a)^5 = 1 + a + a^4 \pmod 2$, we have $w_1(\mathbb{RP}^4) = a \neq 0$, $w_2(\mathbb{RP}^4) = 0$.[11]

Definition 2 (Punctured projective block)**.** Set $W_{\mathrm{RP}} := \mathbb{RP}^4 \setminus \mathrm{int}(B^4)$. Then $\partial W_{\mathrm{RP}} \cong S^3$.

Proposition 3. *W_{RP} is non-orientable, $\pi_1(W_{\mathrm{RP}}) \cong \mathbb{Z}/2$, and* Pin^+.

Proof. Removing a 4-ball does not change w_i or π_1. From $w_2(\mathbb{RP}^4) = 0$, we get $w_2(W_{\mathrm{RP}}) = 0$. The boundary is S^3 with its unique spin structure. $\square$

4 A mapping-torus block W_{rev} with continuous U(1)

Definition 4 (Orientation-reversing mapping torus)**.** Let $f : S^3 \to S^3$ be a smooth orientation-reverser. The mapping torus $M_f := (S^3 \times [0,1])/(x,0) \sim (f(x),1)$ is a non-orientable 4-manifold fibering over S^1. Define $W_{\mathrm{rev}} := M_f \setminus \mathrm{int}(B^4)$.

Proposition 5. *M_f (hence W_{rev}) is* Pin^+ *and $\pi_1(M_f) \cong \mathbb{Z}$.*

Proof. Use the Serre spectral sequence for $S^3 \hookrightarrow M_f \to S^1$ with local coefficients from f. Since $H^1(S^3;\mathbb{Z}/2) = H^2(S^3;\mathbb{Z}/2) = 0$ and $H^2(S^1;\mathbb{Z}/2) = 0$, we get $H^2(M_f;\mathbb{Z}/2) = 0$, hence $w_2(M_f) = 0$. The long exact homotopy sequence gives $\pi_1(M_f) \cong \pi_1(S^1) = \mathbb{Z}$. $\square$

Remark 6 (U(1) dial)**.** While $\mathrm{Hom}(\mathbb{Z}/2, \mathrm{U}(1)) = \{\pm 1\}$ for W_{RP}, the block W_{rev} yields $\mathrm{Hom}(\mathbb{Z}, \mathrm{U}(1)) \cong \mathrm{U}(1)$, enabling a continuous EM holonomy twist in the glue experiment.

[8] Foundational Stiefel–Whitney formulas, including $w(T\mathbb{RP}^n) = (1+a)^{n+1}$, are summarized in the Mathematical Foundations note. :contentReference[oaicite:8]index=8

[9] The Phase-0/1 reports framed admissibility with Pin^- and highlighted candidate families; our Pin^+ baseline supersedes that filter for the 4D classification role. :contentReference[oaicite:9]index=9 and :contentReference[oaicite:10]index=10

[10] See the categorical architecture memo for the 16-class census, pushouts/holonomy gates, and the TQFT functorial layer. :contentReference[oaicite:11]index=11

[11] Detailed calculation in the Foundations note (Sec. 2.3); cf. the Phase-0/1 discussion of $\mathbb{RP}^4$. :contentReference[oaicite:12]index=12 and :contentReference[oaicite:13]index=13

5 Y–cobordism glue and the proton manifold

Let $Y^4 := B^4 \setminus \bigsqcup_{i=1}^3 \mathrm{int}(B_i^4)$, a 4-ball with three disjoint 4-balls removed. Then

$$\partial Y^4 \cong S^3 \sqcup S^3 \sqcup S^3 \sqcup \overline{S^3},$$

and Y^4 is Pin^+ (as an open subset of spin B^4).

Lemma 7 (Pin^+ compatibility under gluing)**.** *If three* Pin^+ *blocks with* $\partial \cong S^3$ *are glued to the three incoming* S^3 *components of* ∂Y^4 *using the compatible spin structure, and the outgoing* $\overline{S^3}$ *is capped by* B^4*, the resulting closed* 4*-manifold* P *is* Pin^+*.*

Proof. By naturality of characteristic classes and $H^2(S^3; \mathbb{Z}/2) = 0$, the unique class w_2 on the union restricts to zero on each piece, hence $w_2(P) = 0$. □

Theorem 8 (Bordism class of the proton)**.** *If the three blocks are* W_{RP} *(or each caps to* $\mathbb{RP}^4$*), then in* $\Omega_4^{\mathrm{Pin}^+}$ *one has*

$$[P] = 3\,[\mathbb{RP}^4].$$

Proof sketch. Form a Pin^+ cobordism whose boundary is $\bigsqcup_{i=1}^3 \mathbb{RP}^4 \sqcup \overline{P}$ by capping each block with B^4 and adding Y^4. This realizes the stated relation. □

Remark 9 ("16 classes")**.** Assign $[\mathbb{RP}^4] = 1$ in the program's 16-class ledger; then the minimal proton composite lands in class 3.[12]

6 Bundle gates: SU(3) color and U(1) electromagnetism

Let $\rho_q : \pi_1(W_q) \to \mathrm{SU}(3)$ be color holonomies and $\chi_q : \pi_1(W_q) \to \mathrm{U}(1)$ EM holonomies for the three blocks $q \in \{1, 2, 3\}$.

Definition 10 (Glue gates)**.** The Y–merge admits extensions of SU(3) and U(1) bundles *iff*

$$g_1 g_2 g_3 = \mathbf{1} \in \mathrm{SU}(3), \qquad e^{\,i(\theta_1+\theta_2+\theta_3)} = 1 \in \mathrm{U}(1),$$

where $g_q = \rho_q(\gamma_q)$ and $e^{i\theta_q} = \chi_q(\gamma_q)$ for boundary loops γ_q.

Remark 11 (Physics interpretation)**.** Color neutrality and the EM phase-closure are enforced as *existence* of pushouts/colimits in the categorical architecture—no "force" added beyond topology.[13]

7 Computational pipeline (replacing boolean shortcuts)

To avoid Phase-0/1 pitfalls, all invariants are computed at the *class* level:

(a) $H^*(\cdot; \mathbb{Z}/2)$ via Serre spectral sequences (bundles/mapping tori) or Borel methods (quotients).

(b) w_1, w_2 via Wu classes with $\mathrm{Sq}(v) = w$, not by toggles.

(c) π_1 from exact sequences of fibrations, not hard-coded.

Remark 12 (Engineering note)**.** Earlier code set w_2 by fiat to satisfy Pin^- filters and simplified π_1 (e.g. Klein-bottle bases). These patterns must be removed; the new engine exposes cohomology rings and Steenrod action explicitly.[14]

[12]This matches the categorical "fermion class" wheel used for UI and bookkeeping. :contentReference[oaicite:14]index=14

[13]Confinement and interaction admissibility are expressed as colimit existence in the categorical memo. :contentReference[oaicite:15]index=15

[14]See the Phase-0/1 analysis script for the toggle patterns and summary. :contentReference[oaicite:16]index=16 and the Candidate Analysis report for the resulting tables. :contentReference[oaicite:17]index=17

8 Relation to GR metrics and the SKB ansatz (context)

The topological construction above is compatible with GR metric models used for single-particle SKB worldtubes and handle-mediated interactions:

$$ds^2 = -f(r)\,dt^2 + g(r)\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2) + h(r)\,(dt - \alpha\,d\varphi)^2,$$
$$\int p_\mu\,dx^\mu = 2\pi n\,\hbar, \qquad Q = \frac{1}{2\pi}\int_{\partial K} F,$$

which encode periodic mass relations and flux-based charge quantization at the metric/field level.[15]

Remark 13 (Deliberate boundary between layers)**.** The present paper fixes the *topology/cobordism* layer (Pin^+, Y–glue, holonomy gates). Metric choices and stress-energy models live above this layer and can be tuned without changing the gates.

9 Corrections and superseded claims

- **Pin baseline**: replace Pin^- admissibility as the primary 4D classifier by Pin^+ in this program.[16]
- **Quotient identifications**: $S^4/\{\pm 1\} = \mathbb{RP}^4$ should not be double-counted; claims about $\mathbb{C}\mathrm{P}^2$/conjugation being both non-orientable and suitable require re-audit under true class-level computations.[17]

10 Conclusions and next steps

We supplied a Pin^+ core with explicit blocks W_{RP} and W_{rev}, a rigorous Y–cobordism glue, and SU(3)/U(1) holonomy gates. The minimal proton manifold obeys $[P] = 3[\mathbb{RP}^4]$ in $\Omega_4^{\mathrm{Pin}^+}$, aligning with the categorical 16-class ledger. Immediate tasks:

1. Extend the block library (mapping tori, twisted bundles) and compute $w_2 = 0$ cases via Serre/Wu/Steenrod.
2. Implement the class-level invariant engine and refactor Phase-0/1 code accordingly.
3. Couple the topological gates to GR metric models for quantitative overlays (keeping topology fixed).

A Stiefel–Whitney classes on $\mathbb{RP}^4$

Let $a \in H^1(\mathbb{RP}^4; \mathbb{Z}/2)$ be the generator. Using $T\mathbb{RP}^n \oplus \varepsilon^1 \cong (\gamma^1)^{\oplus(n+1)}$ and $w(\gamma^1) = 1 + a$,

$$w(T\mathbb{RP}^4) = (1+a)^5 \equiv 1 + a + a^4 \pmod 2,$$

so $w_1 = a$, $w_2 = 0$, $w_3 = 0$, $w_4 = a^4$.[18]

[15] Metric ansatz, quantization and flux formulas appear in the SKB manuscript (Eqs. (1), (6)–(11), etc.). :contentReference[oaicite:18]index=18

[16] Research reports that center Pin^- are retained for audit but superseded for the 4D classification role by this paper. :contentReference[oaicite:19]index=19 and :contentReference[oaicite:20]index=20

[17] See Phase-0/1 Candidate Analysis; re-audit with the new engine. :contentReference[oaicite:21]index=21

[18] As compiled in the Mathematical Foundations note, with cautionary remarks about literature discrepancies. :contentReference[oaicite:22]index=22

B Mapping torus M_f of S^3

For $S^3 \hookrightarrow M_f \xrightarrow{\pi} S^1$ with orientation-reversing monodromy, $E_2^{p,q} = H^p(S^1; H^q(S^3; \mathbb{Z}/2))$ gives $H^2(M_f; \mathbb{Z}/2) = 0$. Since $TM_f \cong V \oplus \pi^* TS^1$ and $w_2(TS^1) = 0$, we obtain $w_2(M_f) = w_2(V) = 0$.

Acknowledgments

This document supersedes parts of the Phase-0/1 packet that relied on Pin^- admissibility and boolean shortcuts, and formalizes the categorical gates in the Pin^+ baseline.[19]

[19] Candidate tables and script: :contentReference[oaicite:23]index=23 and :contentReference[oaicite:24]index=24; Foundations and Research Reports: :contentReference[oaicite:25]index=25, :contentReference[oaicite:26]index=26, :contentReference[oaicite:27]index=27; Categorical architecture: :contentReference[oaicite:28]index=28.