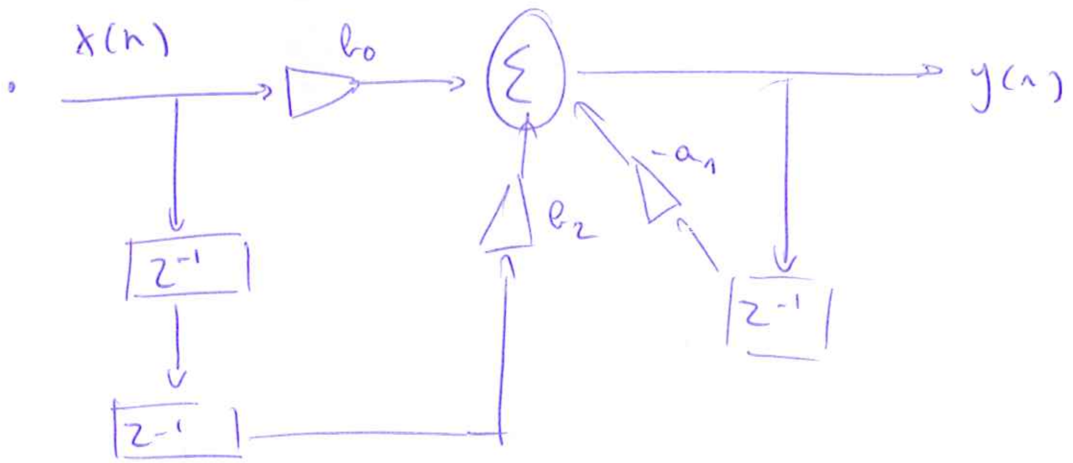
Exercice 1



- Transformée en Z : $Y(z) + a_1 z^{-1} Y(z) = b_0 X(z) + b_2 z^{-2} X(z)$

$$Y(z)(1 + a_1 z^{-1}) = X(z)(b_0 + b_2 z^{-2})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_2 z^{-2}}{1 + a_1 z^{-1}}$$

$$\longrightarrow H(z) = H_1(z) + H_2(z) = \frac{b_0}{1 + a_1 z^{-1}} + \frac{b_2 z^{-2}}{1 + a_1 z^{-1}}$$

$$\longrightarrow h(n) = h_1(n) + h_2(n) =$$

inv Z trans

2^{k} order pole at zero $\longrightarrow B_k \delta(n-k)$

single pole at d_k $\longrightarrow A_k (d_k)^n u(n)$ — step fct

Double pole at d_k $\longrightarrow C_k\, n (d_k)^n u(n)$

Example $2 z^{-2} \longrightarrow 2\delta(n-2)$

$$\frac{1}{1 - 3/4\, z^{-1}} \longrightarrow (3/4)^n u(n)$$

$$\frac{1/2}{(1 + 1/2\, z^{-1})^2} \longrightarrow 1/2\, n(-1/2)^n u(n)$$

$h(n) = h_1(n) + h_2(n) =$

$$b_0 (-a_1)^n u(n) + b_2 (-a_1)^{n-2} u(n-2)$$

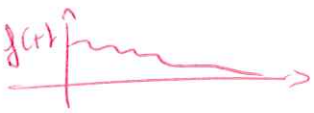
Cond de stabilité

Pour $H(z)$: dénominateur $1 + a_1 z^{-1} \to z + a_1 = 0$

$$z = -a_1$$

$$\to |a_1| < 1$$

stable if $f(t) \to 0$ as $t \to \infty$

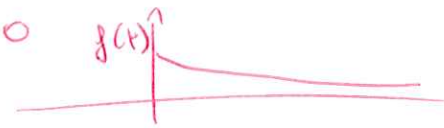


$f(t) = e^{at} u(t)$ ← step func

↓ assume zero initial conditions

$f(t) = e^{at}$

$a < 0$



Exercice 3

Eq aux diff finies : $y(n) = x(n) + 1,5\, x(n-1) + x(n-2) + y(n-1) - 0,5\, y(n-2)$

$$H(z) = \frac{1 + 1,5\, z^{-1} + z^{-2}}{1 - z^{-1} + 0,5\, z^{-2}}$$

$h(0) = 1$

$h(1) = 1 + 1,5 = 2,5$

$h(2) = 1 + 2,5 - 0,5 \cdot 1 = 3$

$h(3) = 3 \cdot 0,5 \cdot 2,5 = 1,75$

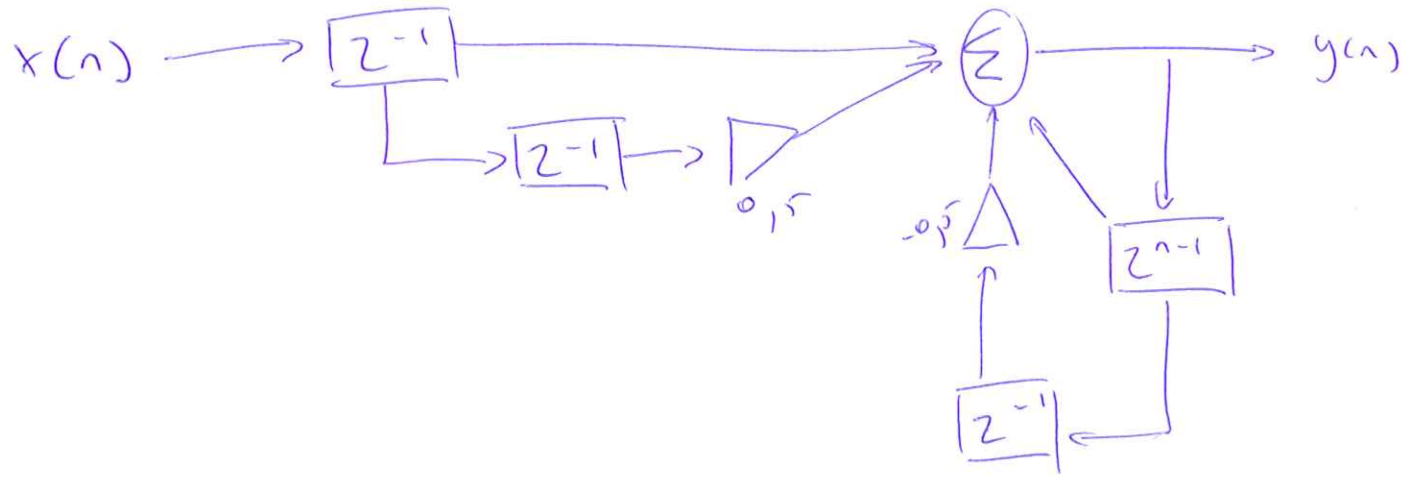
$h(4) = 1,75 - 0,5 \cdot 3 = 0,25$

Exercice 8

$$H(z) = \frac{N(z)}{D(z)}^{*} = \frac{z+0{,}5}{z^2 - z + 0{,}5} = \frac{z^{-1} + 0{,}5\,z^{-2}}{1 - z^{-1} + 0{,}5\,z^{-2}}$$

Eq aux différences $Y(z)\left(1 - z^{-1} + 0{,}5\,z^{-2}\right) = X(z)\left(z^{-1} + 0{,}5\,z^{-2}\right)$

$\rightarrow y(n) - y(n-1) + 0{,}5\,y(n-2) = x(n-1) + 0{,}5\,x(n-2)$



$*\ \left(z - \left(\frac{1}{2} - \frac{1}{2} i\right)\right)\left(z - \left(\frac{1}{2} + \frac{1}{2} i\right)\right)$

$z^2 - z/2 - z/2\,i - z/2 + z/2\,i + \left(\frac{1}{2} - \frac{1}{2} i\right)\left(\frac{1}{2} + \frac{1}{2} i\right)$

$z^2 - z + \frac{1}{4} + \frac{1}{4} = \ldots$

Exercice 12

$$y(n) = x(n) + 2x(n-1) + y(n-1) + 4y(n-2)$$

$$H(z) = Y(z)\left(1 - z^{-1} - 4z^{-2}\right) = X(z)\left(1 + 2z^{-1}\right)$$

$$= \frac{1 + 2z^{-1}}{1 - z^{-1} - 4z^{-2}}$$

Poles $z^2 - z - 4 = 0$

stabilité $\left|\frac{1 \pm \sqrt{17}}{2}\right| < 1$ non instable

donc pas de filtres inverses