Analysis-of-Variance Hypothesis Testing

From Jerrold H. Zar, "Biostatistical analysis," Prentice Hall, 2015 (Appendix D)

D.2 TWO-FACTOR ANALYSIS OF VARIANCE

(a) Factors *A* and *B* Both Fixed (See Example 12.1.)

Source of variation	F	ν_1	ν_2
A	MS_A/MS_e	DF_A	DF_e
B	MS_B/MS_e	DF_B	DF_e
AB	MS_{AB}/MS_e	DF_{AB}	DF_e

(b) Factor *A* Fixed; Factor *B* Random (See Example 12.4.)

Source of variation	F	ν_1	ν_2
A	MS_A/MS_{AB}	DF_A	DF_{AB}
B	MS_B/MS_e	DF_B	DF_e
AB	MS_{AB}/MS_e	DF_{AB}	DF_e

(c) Factors *A* and *B* Both Random

Source of variation	F	ν_1	ν_2
A	MS_A/MS_{AB}	DF_A	DF_{AB}
B	MS_B/MS_{AB}	DF_B	DF_{AB}
AB	MS_{AB}/MS_e	DF_{AB}	DF_e

D.3 THREE-FACTOR ANALYSIS OF VARIANCE

(a) Factors *A*, *B* and *C* All Fixed (See Example 14.1.)

Source of variation	F	ν_1	ν_2
A	MS_A/MS_e	DF_A	DF_e
B	MS_B/MS_e	DF_B	DF_e
C	MS_C/MS_e	DF_C	DF_e
AB	MS_{AB}/MS_e	DF_{AB}	DF_e
AC	MS_{AC}/MS_e	DF_{AC}	DF_e
BC	MS_{BC}/MS_e	DF_{BC}	DF_e
ABC	MS_{ABC}/MS_e	DF_{ABC}	DF_e

(b) Factors *A* and *B* Fixed; Factor *C* Random

Source of variation	F	ν_1	ν_2
A	MS_A/MS_{AC}	DF_A	DF_{AC}
B	MS_B/MS_{BC}	DF_B	DF_{BC}
C	MS_C/MS_e	DF_C	DF_e
AB	MS_{AB}/MS_{ABC}	DF_{AB}	DF_{ABC}
AC	MS_{AC}/MS_e	DF_{AC}	DF_e
BC	MS_{BC}/MS_e	DF_{BC}	DF_e
ABC	MS_{ABC}/MS_e	DF_{ABC}	DF_e

(c) Factor *A* Fixed; Factors *B* and *C* Random

Source of variation	F	ν_1	ν_2
A	$MS_A/(MS_{AB} + MS_{AC} - MS_{ABC})$	DF_A	$\dfrac{(MS_{AB}+MS_{AC}-MS_{ABC})^2}{(MS_{AB})^2/DF_{AB}+(MS_{AC})^2/DF_{AC}+(MS_{ABC})^2/DF_{ABC}}$
B	MS_B/MS_{BC}	DF_B	DF_{BC}
C	MS_C/MS_{BC}	DF_C	DF_{BC}
AB	MS_{AB}/MS_{ABC}	DF_{AB}	DF_{ABC}
AC	MS_{AC}/MS_{ABC}	DF_{AC}	DF_{ABC}
BC	MS_{BC}/MS_e	DF_{BC}	DF_e
ABC	MS_{ABC}/MS_e	DF_{ABC}	DF_e

(d) Factors *A*, *B*, *C* All Random

Source of variation	F	ν_1	ν_2
A	$MS_A/(MS_{AB} + MS_{AC} - MS_{ABC})$	DF_A	$\dfrac{(MS_{AB}+MS_{AC}-MS_{ABC})^2}{(MS_{AB})^2/DF_{AB}+(MS_{AC})^2/DF_{AC}+(MS_{ABC})^2/DF_{ABC}}$
B	$MS_B/(MS_{AB} + MS_{BC} - MS_{ABC})$	DF_B	$\dfrac{(MS_{AB}+MS_{BC}-MS_{ABC})^2}{(MS_{AB})^2/DF_{AB}+(MS_{BC})^2/DF_{BC}+(MS_{ABC})^2/DF_{ABC}}$
C	$MS_C/(MS_{AC} + MS_{BC} - MS_{ABC})$	DF_C	$\dfrac{(MS_{AC}+MS_{BC}-MS_{ABC})^2}{(MS_{AC})^2/DF_{AC}+(MS_{BC})^2/DF_{BC}+(MS_{ABC})^2/DF_{ABC}}$
AB	MS_{AB}/MS_{ABC}	DF_{AB}	DF_{ABC}
AC	MS_{AC}/MS_{ABC}	DF_{AC}	DF_{ABC}
BC	MS_{BC}/MS_{ABC}	DF_{BC}	DF_{ABC}
ABC	MS_{ABC}/MS_e	DF_{ABC}	DF_e

D.4 NESTED ANALYSIS OF VARIANCE

(a) Factor *A* either Fixed or Random; Factor *B* Random and Nested within Factor *A* (See Example 15.1.)

Source of variation	F	ν_1	ν_2
A	MS_A/MS_B	DF_A	DF_B
B	MS_B/MS_e	DF_B	DF_e

(b) Factor *A* Either Fixed or Random; Factor *B* Random and Nested within Factor *A*; Factor *C* Random and Nested within Factor *B*

Source of variation	F	ν_1	ν_2
A	MS_A/MS_B	DF_A	DF_B
B	MS_B/MS_C	DF_B	DF_C
C	MS_C/MS_e	DF_C	DF_e

(c) Factors *A* and *B* Fixed; Factor *C* Random and Nested within Factors *A* and *B* (See Example 15.2.)

Source of variation	F	ν_1	ν_2
A	MS_A/MS_C	DF_A	DF_C
B	MS_B/MS_C	DF_B	DF_C
AB	MS_{AB}/MS_C	DF_{AB}	DF_C
C	MS_C/MS_e	DF_C	DF_e

(d) Factor *A* Fixed; Factor *B* Random; Factor *C* Random and Nested within Factors *A* and *B*

Source of variation	F	ν_1	ν_2
A	MS_A/MS_{AB}	DF_A	DF_{AB}
B	MS_B/MS_C	DF_B	DF_C
AB	MS_{AB}/MS_C	DF_{AB}	DF_C
C	MS_C/MS_e	DF_C	DF_e

(e) Factors *A* and *B* Random; Factor *C* Random and Nested within Factors *A* and *B*

Source of variation	F	ν_1	ν_2
A	MS_A/MS_{AB}	DF_A	DF_{AB}
B	MS_B/MS_{AB}	DF_B	DF_{AB}
AB	MS_{AB}/MS_C	DF_{AB}	DF_C
C	MS_C/MS_e	DF_C	DF_e