Small Signal Compensation of Magnetic Fields Resulting from Aircraft Maneuvers

S.H. BICKEL

Abstract

Tolles and Lawson identified three permanent, five induced, and eight eddy-current fields as sources of magnetic interference associated with airframe maneuvers. Small signal approximations are used here to separate the eddy-current terms and thus decouple the sixteen equations into two sets of eight equations. It was found that a singularity exists in the small signal equations for the permanent and induced terms. This causes an ambiguity amoung three of the coefficients which can be resolved mathematically by resorting to large maneuvers.

Flight test data exhibit a large amount of magnetic hysteresis and the magnetic anomality detector (MAD) equipped aircraft will not remain compensated from one flight to the next. This complicates the problem of resolving the dip angle ambiguity. Furthermore, significant differences exist in the compensation terms as determined from manual pilot maneuvers and the terms when the maneuvers are performed by the automatic pilot. This has been attributed to differences in altitude stability and the frequency content of the maneuvers. It was found that both optimal frequency filtering and altitude compensation can be used to improve the figure of merit (FOM) resulting from pilot maneuvers.

Manuscript received July 14, 1978.

This work was supported in part by the Naval Air Development Center under Contract N62269-75-C-0210.

Author's address: 3325 Russwood Lane, Garland, TX 75042.

0018-9251/79/0700-0518 $00.75 ©1979 IEEE

I. Introduction

In 1950 Tolles and Lawson [1] first reported on the problem of maneuver noise associated with the magnetic field sources of magnetic anomality detector (MAD) equipped aircraft. This early work empolyed the AN/ASQ-8 MAD equipment which was capable of detecting changes in the magnetic field of 0.04 gamma[1] amplitude. However, Tolles and Lawson reported that magnetic noise not associated with the inherent noise level of the equipment made it impossible to achieve such low noise levels in flight.

The current AN/ASQ-81 MAD or magnetometer is an order of magnitude more sensitive than the AN/ASQ-8 equipment when operating in a laboratory environment. Consequently the external noise sources not associated with the inherent noise level of the equipment has become an even more severe problem than it was in 1950. A major source of magnetic noise external to the MAD equipment is noise related to aircraft maneuvers.

Tolles and Lawson identified permanent, induced, and eddy-currect fields as three separate sources of magnetic interference associated with the airframe maneuvers. The field due to the permanent magnetism of the various ferromagnetic structural parts of the aircraft is constant in magnitude and is firmly attached to the aircraft. Since this field turns with the aircraft its relationship with the Earth's field vector changes with maneuver. This change in the magnetic field is regarded as a noise source when it is detected by the magnetometer. To compensate for this noise source one could, in principle, position permanent magnets in the vicinity of the magnetometer so that their fields would cancel the permanent field of the aircraft. To do this it is necessary to have a procedure for either estimating or nulling the three components of the permanent field. However, this is difficult to do in practice, since the permanent noise signals never occur by themselves and the interaction with the induced and eddy-current sources of magnetic interference leads to complex maneuver signals.

Unlike the permanent field, the induced and eddy-current fields are not permanently attached to the aircraft, but have a polarity and magnitude determined by the direction and magnitude of the Earth's field. The induced magnetic moment of the aircraft is proportional to the projection of the Earth's field on the aircraft's ferromagnetic structures. This causes the induced field to turn at twice the angular rate of the aircraft, while the permanent field is firmly attached to the aircraft and turns at the same rate as the aircraft.

The eddy-current fields occur in the skins, ribs, frames, and other structural units of the aircraft which have high electrical conductivity. The eddy-current fields are produced in the same way as are the currents produced in a coil or loop of wire rotating in a uniform magnetic field. Therefore these fields differ from the permanent and induced fields in that they do not depend upon the

[1] 1 gamma = 10^{-5} Oe.

ferromagnetic properties of the aircraft structure but depend rather upon their electrial conduction properties. Since eddy currents are proportional to the time rate of change of flux, their amplitudes are proportional to accelerations that occur during the aircraft maneuver. This is in contrast to the permanent and induced fields where the amplitude depends only upon the relative orientation between the aircraft and the Earth's field. The maneuvers of the aircraft in the Earth's field generate time changes is the fluxes in the aircraft's conducting sheets or closed-loop structures. The resulting flow of current in turn creates a magnetic field that points in a direction perpendicular to the plane of the sheet or conducting loop and has an amplitude which varies with both the aircraft's orientation and its rate of change in orientation.

Both the Tolles and Lawson report and an earlier report by Tolles [2] give derviations of the equations resulting from these three types of noise sources. These reports show that there are three permanent terms, five induced terms, and eight eddy-current terms, or a total of sixteen terms which contribute to the maneuver signal. An attempt at the solution of the system of sixteen equations was made in 1961 by Leliak [3] for sinusoidal maneuvers. This led to the interesting discovery that the equations were singular or almost singular.

II. Magnetic Field Equations

The field that is detected by the magnetometer during an aircraft maneuver is the superposition of fields from the permanent, the induced, and the eddy-current sources [1]. If we let H_d denote the detected field, then we can write

$$H_d = H_{pd} + H_{id} + H_{ed} \tag{1}$$

where H_{pd}, H_{id}, and H_{ed} denote the field detected from the permanent sources, the induced sources, and the eddy-current sources of magnetic field, respectively.

The sources of permanent magnetic field are attached to the aircraft and do not vary with maneuvers. The permanent magnets can be expressed in terms of three components p_1, p_2, and p_3 parallel to the transverse, longitudinal, and vertical (T, L, V) axes of the aircraft. The magnetic field that is detected by the magnetometer is given by the projection of the vector (p_1, p_2, p_3) on the unit vector pointing in the direction of the Earth's magnetic field vector, $\overline{H}_e$. Hence the field which is detected due to the permanent magnetic sources can be written as

$$H_{pd} = \sum_{i=1}^{3} p_i u_i \tag{2}$$

where u_1, u_2, and u_3 are the direction cosines formed by H_e and the axes of the aircraft. The direction cosines are given by

$$u_1 = \cos X, \quad u_2 = \cos Y, \quad \text{and} \quad u_3 = \cos Z \tag{3}$$

where X, Y, and Z are the direction angles formed by the (T, L, V) axes of the aircraft and the Earth's field vector, $\overline{H}_e$.

The induced field at the detector input is proportional to $H_e \cos X$, $H_e \cos Y$, $H_e \cos Z$, and the fields are not usually aligned with the principal axes of the aircraft. Since the magnetic field which is detected is a projection of these fields on (T, L, V) axes, the detected signal from the induced field can be written as a double sum

$$H_{id} = \sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} u_i u_j. \tag{4}$$

Since $u_i u_j = u_j u_i$, we can require that

$$a_{ij} = a_{ji} \tag{5}$$

with no sacrifice in the general nature of (4).

The Earth's field must be added to these signals, which makes it difficult to isolate any constant term. The usual procedure is to remove the constant term from the equations by a high-pass filtering operation. Since

$$\sum_{i=1}^{3} u_i^2 = 1 \tag{6}$$

it is possible to eliminate u_3^2 and if we neglect the constant term we can require that

$$a_{33} = 0. \tag{7}$$

This leaves only five independent constants in (4) to be evaluated. In the notation given in [1] the constants of interest are

$$a_{11} = (TT - VV) H_e, \quad a_{22} = (LL - VV) H_e$$
$$a_{12} = \tfrac{1}{2} (TL + LT) H_e, \quad a_{13} = \tfrac{1}{2} (TV + VT) H_e$$
$$a_{23} = \tfrac{1}{2} (LV + VL) H_e \tag{8}$$

where, for example, TL denotes a longitudinal component produced at the detector by the transverse component of $\overline{H}_e$.

The eddy-current field at the detector input is proportional to the time derivative of $H_e \cos X$, $H_e \cos Y$, $H_e \cos Z$ and the field is not usually aligned with the principal axes of the aircraft. As stated previously, the magnetic field detected by the magnetometer is the projection of the incident field on the T, L, and V axes of the aircraft. Therefore, the detected signal from the eddy-current sources can be written as a double sum

$$H_{ed} = \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij} u_i \dot{u}_j \tag{9}$$

where $\dot{u}$ is the derivative of u with respect to time. By taking the derivative of (6) with respect to time and substituting into (9), $u_3 \dot{u}_3$ can be eliminated. This leaves only eight independent constants in (9), since we can require that

$$b_{33} = 0 \tag{10}$$

with no loss in the general nature of the equations. The eight constants b_{ij} are independent of aircraft maneuver, but vary with the size, shape, and electrial conductivity of the materials used in the structure of the aircraft and with the location of the magnetometer in this structure.. In the notation of [1], the eddy-current constants are

$$\begin{aligned} b_{11} &= (tt - vv) H_e, & b_{22} &= (ll - vv) H_e, \\ b_{12} &= lt H_e, & b_{13} &= vt H_e, \\ b_{21} &= tl H_e, & b_{23} &= vl H_e, \\ b_{31} &= tv H_e, & b_{32} &= lv H_e. \end{aligned} \tag{11}$$

The lower case eddy-current constants are analogous to the induced constants given by (8). Equations (1), (2), (4), and (9) describe the noise signal which is detected by the magnetometer as the aircraft manuevers in the Earth's field. The constraints given by (5), (6), (7), and (10) limit the number of independent coefficients which contribute to the maneuver signal to sixteen. This paper addresses the problem of determining these coefficients for small maneuvers. A more detailed description of the underlying physics and derivation of these equations is contained in [1-3].

III. Small Signal Equations

When the aircraft is maneuvering about a straight line path we can assume that the direction cosines are given by

$$u_i(t) = v_i(t) + U_i \tag{12}$$

where U_i are the direction cosines associated with the linear path and are considered to be constant in time. As the aircraft maneuvers, each of the direction cosines deviates from the path direction by the amount v_i, where v_i is time varying. The maneuvers are assumed to be small so that

$$|v_i| \ll l. \tag{13}$$

The signal which is generated as the aircraft maneuvers about the flight path is given by

$$s(t) = H_d[U + v(t)] - H_d(U) \tag{14}$$

and after substituting (1) into (14) and neglecting all but the first order terms, we arrive at the following small signal maneuver equation:

$$s(t) = \sum_{i=1}^{3} (p_i + \sum_{j=1}^{3} a_{ij} U_i) v_i + \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij} U_j \dot{v}_i. \tag{15}$$

Equation (15) shows that for small maneuvers the signals generated by the permanent and induced terms vary with v, while the eddy-current terms vary with $\dot{v}$. As we will show presently, this property can be used to decouple (15) into two sets of eight equations for the sixteen unknown coefficients. Before proceeding to the solution, we can simplify (15) by recalling from (6) that only two of the three direction cosine maneuver signals are independent. Thus if (12) is substituted into (6), and second order terms are neglected, we find that

$$\sum_{i=1}^{3} U_i v_i = 0 \tag{16}$$

and so it is possibe by applying (16) to eliminate v_3 from (15). As a result (15) simplifies to

$$s(t) = w_1 v_1 + w_2 v_2 + w_3 \dot{v}_1 + w_4 \dot{v}_2 \tag{17}$$

where the constants w_1 and w_2 depend only upon the permanent and induced coefficients of the magnetic field. The are given by

$$w_1 = p_1 - p_3 U_i/U_3 + 2 \sum_{j=1}^{3} (a_{ij} - a_{3j} U_i/U_3) U_j, \quad i = 1, 2. \tag{18}$$

The constants w_3 and w_4 depend only upon the eddy-current terms and can be written as

$$w_{i+2} = \sum_{j=1}^{3} (b_{ij} - b_{3j} U_i/U_3) U_j, \quad i = 1, 2. \tag{19}$$

Equation (17) shows that for small deviations from a straight line path, the maneuver signal $s(t)$ is a weighted sum of v_1 and v_2 and their time derivatives. From (18) and (19) these weights are regarded as constants, which depend upon the particular flight path and the magnetic and eddy-current properties of the aircraft. The method of solution is to first solve for these flight path constants w_i for a particular aircraft heading and aircraft maneuvers. If the aircraft heading is changed to a different beaming angle a new set of flight path constants can be found. Each additional flight path generates additional equations of the form shown by (18) and (19). From these it is then possible to solve for the unknown values of p_i, a_{ij}, and b_{ij}.

IV. Flight Path Constants

In this section we combine linear filtering techniques with correlation integrals to solve (17) for the flight path constants w_i. Let $y(t)$ be the output from a bandpass linear filter L operating on the field H_d which is detected by the magnetometer. Since constant terms are rejected by the filter,

$$LH_d(U) = 0. \tag{20}$$

Hence from (14),

$$y(t) = LH_d(u) = Ls(t) \tag{21}$$

and after substituting (17) into (21),

$$y(t) = \sum_{i=1}^{4} x_i w_i \tag{22}$$

where the path constants w_i are defined by (18) and (19). The variables x_i are obtained by filtering the direction cosines and are given by

$$x_1 = Lu_1\,, \quad x_2 = Lu_2\,, \quad x_3 = \dot{x}_1\,, \quad x_4 = \dot{x}_2\,. \tag{23}$$

To solve for the path constants w_i from (22) one can correlate each x_i with the filtered magnetometer output $y(t)$. Therefore we write

$$\Gamma_i = \langle y, x_i\rangle, \quad i = 1, 2, 3, 4 \tag{24}$$

where $\langle x, y\rangle$ is the inner product

$$\langle x, y\rangle = \int_0^T y(t)\, x(t)\, dt. \tag{25}$$

By substituting (22) into (24), we arrive at the following system of four linear equations for the four path constants:

$$\Gamma_i = \sum_{j=1}^{4} C_{ij}\, w_j, \quad i = 1, 2, 3, 4 \tag{26}$$

where the elements of the matrix C are given by

$$C_{ij} = \langle x_i, x_j\rangle. \tag{27}$$

The above system of equations can be solved by standard techniques. We require that the aircraft make small maneuvers about a straight line path in such manner that the direction cosine correlation matrix C is not singular. This means that the diagonal terms C_{ii} cannot be zero, and it is desirable that the off diagonal terms C_{ij} be made small. One way to do this is have a sequence of pitch maneuvers followed by a sequence of roll maneuvers.

V. Eddy-Current Terms

The first two flight path constants w_1 and w_2 depend upon the permanent and induced terms, while the last two w_3 and w_4 contain only eddy-current terms. Equation (19) gives the relationship between the eddy-current coefficients b_{ij} and the w_3 and w_4 for a particular flight path. Recalling that $b_{33} = 0$ there are eight unknown b_{ij} contained given by the equations for w_3 and w_4. If the flight path is changed and the maneuvers are repeated then new values for w_3 and w_4 will be obtained. From (19) there are two new equations for the unknown b_{ij} which correspond to the direction cosines (U_1, U_2, U_3) for the new path. In general, the flight paths are horizontal and only the bearing angle changes from one path to the next. Thus U_3 does not change, while the direction cosines U_1 and U_2 are varied so that the bearing angle is given by

$$\tan\theta = U_1/U_2\,. \tag{28}$$

Examination of (19) shows that each new bearing angle generates two new linearly independent equations. After data are collected from four different bearing angles, the system of equations can be solved for the eight unknown b_{ij} by standard techniques.

As discussed above all of the eddy-current terms can be found, although the flight paths are restricted to a plane. As we show in the next section, this is not true for the permanent and induced terms and it is necessary to alter U_3 to separate some of the induced terms from the permanent terms.

VI. Dip Angle Singularity

The permanent and induced terms can be found from the first two flight path constants w_1 and w_2. Examination of (18) shows, however, that only seven independent equations can be developed by changing the bearing angle alone. To solve the system of equations, it is convenient to introduce two new variables

$$A_{11} = 2\,a_{11} - p_3/U_3$$

$$A_{22} = 2\,a_{22} - p_3/U_3\,. \tag{29}$$

Rewriting (18),

$$w_i = p_i + A_{ii}\,U_i + 2 \sum_{j=1,\, j\neq 1}^{3} (a_{ij} - a_{3j}/U_3)\,U_j,$$
$$i = 1, 2. \tag{30}$$

Equation (30) contains seven unknowns p_1, p_2, A_{11}, A_{22}, a_{12}, a_{13}, and a_{23}. Each bearing change generates two new equations. After data have been collected from four bearings standard least mean squared error techniques can be used to extract the seven unknowns from the eight equations.

The singular nature of (1) was previously reported [3]. The small signal equations had not been developed at that time and the exact cause of the singularity was not located. Since changes in the bearing angle leaves U_3 unaltered, then from (29) it is impossible to separate p_3 from a_{11} and a_{22}.

To resolve the ambiguity it is necessary to change U_3. The simplest way to do this is to take data at a second dip angle. This is not always convenient as it requires flying the aircraft to a second geographic location. As an alternative, U_3 could be changed by performing small maneuvers with the aricraft in a climb, or a dive, or possibly on its side. All of this would make for a very interesting ride. This could be a serious problem for if the ambiguity is left unresolved, then the resulting compensation is valid for only one geographic location.

Flight tests conducted at various magnetic latitudes were used to determine the seriousness of the problem and the effectiveness of any potential correction techniques. As the flight tests show hysteresis effects could overwhelm this error. This greatly complicates the problem of resolving the ambiguity. Care should be taken in any future flight test program which is to study this effect to ensure that the aircraft does not land during the data collection process, as hysteresis effects could render the data useless. As an alternative, data could be collected for the case of large maneuvers at a single location. This brings us back once again to the problem of an unpleasant ride.

VII. Results from Flight Test Data

During October, November, and December of 1975 a series of aircraft maneuvers was performed on P-3C aircraft at Naval Air Development Center, Warminster, Pa. Magnetometer data were recorded from the outputs of the AN/ASQ-81 (u) helium magnetometer and the three-axis vector magnetometer outputs from the ASA-65 system. The data were later processed on the IBM 370 computer at the Dallas site of Texas Instruments Incorporated. The computer program had been written for a TI-9900 16-bit microprocessor which is planned for use in future in-flight compensation [4]. The IBM 370 computer simulation of this program was used for data analysis for ease of data access and because the TI 9900 hardware is being developed.

The computer program constructs direction cosines from ASA-65 vector magnetometer outputs. The direction cosines and the total field from the helium magnetometer were filtered by finite-duration impulse-response (FIR) digital filters [5] with a bandpass from 0.1 to 0.6 Hz. After filtering the correlation matrix is formed and the flight path constants were found by the technique given in Section IV. After pitch and roll maneuver data were collected at four bearings the system of equations was solved by the Gauss Seidel [6] technique for fifteen unknown coefficients. Most of the data was taken in the Warminster area and no attempt was made to resolve the dip angle ambiguity problem. Equation (29) was solved by arbitrarily putting

$$a_{11} = 0. \tag{31}$$

Performance was measured by the figure of merit (FOM) which is defined as the sum of the twelve peak-to-peak outputs from the magnetometer when the aircraft is performing roll, pitch, and yaw maneuvers on each of the four combined headings. The maneuver amplitudes are standardized to 10° roll, 3° pitch, and 5°yaw.

The first day that data was recorded was October 6, 1975. Table I shows the calculation of the FOM for the uncompensated aircraft on this day.

The maneuvers were accomplished by the aircraft autopilot in the FOM flight and in the learning flight. Consequently, the maneuver data was as pure as possible with the P-3C aircraft. Under these conditions, Table II shows that compensation improved the FOM a factor of 25. When compensation was applied to the learning data, little or no maneuver correlated signals could be observed above the output noise level. The residue FOM shown in Table II could be due to inaccuracies caused by noise, time variations in the magnetic properties of the aircraft and/or hysteresis, altitude variations in the magnetic field, or a combination of these effects.

Several other compensation flights were made with the maneuver programmer and, as shown in Table III, similar results were obtained.

Ten of the twelve data runs given in Table III have an FOM of 0.7 or less. Except for the two high values of FOM obtained on December 2, no significant trends are apparent in this data. Training the aircraft compensator with bearing angles of 3°, 5°, and 7° off cardinal headings produced FOMs of 0.56, 0.55, and 0.58, respectively. This is no better or worse than the results obtained using cardinal headings for the training. The poor FOM that was obtained on December 2, with 4 MK-46 torpedos was attributed to a large spike which was observed during the turn from East to South. Post flight analysis of the data indicated that the sensor had malfunctioned and the remaining data taken of December2 was with a degraded sensor. In any event, sensor problems make it difficult to draw definite conclusions about weapon storage from this data. A new sensor was installed prior to the December 3 flight, but only a limited amount of data was taken on that day.

The FOM tests summarized in Table III were conducted under nearly ideal conditions. The aircraft autopilot performed the training maneuvers and the FOM flight was made immediately after the training flight. This minimized magnetic hysteresis effects. To study the stability of the compensation and possible hysteresis effects, training data from each day was tested against FOM runs of different days during the week of October 6.

Table IV shows that a single takeoff and landing of the aircraft can have a significant effect on its magnetic properties. It has long been known that magnetism may be induced in iron by vibration. In the 16th century Gilbert showed that an iron bar hammered in the Earth's field become magnetized. Changes in magnetism alter the hysteresis curve, which in turn alters the permanent and induces terms for the maneuver signal. This data suggests that for an FOM less than 1.5 gamma or so, the compensation coefficients will have to be updated on each flight. This could be done either by a dedicated compensation trim flight, or by an adaptive algorithm which continuously updates coefficients for random flight paths.

Finally, a series of flights were made where the training maneuvers were performed manually by the pilot. With the exception of the October 10 flight, all pilot maneuvers were taken in the region of Key West. The results are given in Table V.

The pilot maneuver data differs from the automatic pilot data in two respects. First the pilot did not maintain a constant altitude during his maneuvers which introduced gradient effects. The second difference is that pilot maneuvers were conducted at a maneuver frequency of about 0.1 Hz while the automatic pilot performed the maneuvers at a frequency of 0.2 Hz. There are three

TABLE I
Uncompensated FOM (Gamma) (Oct. 5, 1975)

	Heading				
Maneuver	N	E	S	W	
Pitch	1.97	0.80	3.58	1.48	
Roll	0.84	0.68	1.20	0.44	
Yaw	0.28	0.26	0.30	0.15	
Total	3.09	1.74	5.08	2.07	11.98 gamma

TABLE II
Ccompensated FOM (Gamma) (Oct. 5, 1975)

	Heading				
Maneuver	N	E	S	W	
Pitch	0.04	0.03	0.03	0.03	
Roll	0.02	0.03	0.06	0.11	
Yaw	0.02	0.04	0.02	0.03	
Total	0.08	0.10	0.11	0.16	.45 gamma

TABLE III
FOM Using P-3C Autopilot

Date	Description of Training Data	FOM (gamma) Uncompensated	Compensated
10/6	Standard compensation #2	11.98	0.45
10/7	Standard compensation #4	12.96	0.59
10/7	Standard compensation #5	11.41	0.50
10/7	Standard compensation #6	11.57	0.70
10/8	Standard compensation	12.28	0.53
10/10	3° off cardinal heading	12.68	0.56
10/10	5° off cardinal heading	10.58	0.55
10/10	7° off cardinal heading	12.82	0.58
12/2	Standard compensation	10.46	0.96
12/2	4 mark 46 weapons	10.68	1.32
12/2	3 mark 46 weapons	8.85	0..64
12/3	2 mark 46 weapons	11.87	0.65
Mean FOM		11.51 ($\bar{\sigma}$=1.20)	0.67 ($\bar{\sigma}$=.24)

TABLE IV
Hysteresis Effect on Compensated FOM (gamma)

Date of Training	Date of FOM Flight October 6	October 7	October 8	October 10
10/6	0.45	1.04	1.16	1.38
10/7	0.73	0.50	1.21	1.23
10/8	1.32	1.24	0.53	2.03
10/10	1.26	1.23	1.95	0.56

TABLE V
FOM Under Different Pilot Maneuvers

Date of Training	Desciption of Training Data	FOM (gammas) Uncompensated	Compensated
10/10	Pilot – Warminster (cardinal)	12.11	2.94
11/21	Pilot intercardinal	15.14	3.35
11/21	Pilot cardinal	15.14	2.83
11/24	Pilot intercardinal	14.26	0.90
11/25	Pilot 22° W of N	13.37	2.63
11/25	Pilot 22° E of N	12.33	2.65
Mean FOM		13.72 ($\bar{\sigma}$=1.34)	2.55 ($\bar{\sigma}$=0.85)

TABLE VI
Improved Pilot Maneuver FOMs

	Compensation FOM (gamma)			
	No Altitude Compensation		Bandpass Filter with Altitude Compensation	
Date	Whitening Filter	Bandpass Filter	Altitude Compensation on all the Time	Altitude Compensation Off During Yaw Maneuver
10/10	1.03	2.94	1.84	1.23
11/21	1.70	3.35	1.61	1.00
11/21	1.63	2.83	1.25	1.12
11/24	1.37	0.90	0.99	1.07
11/25	1.23	2.63	2.63	2.64
11/25	1.11	2.65	1.36	0.76
Mean FOM	1.35	2.55	1.61	1.30

possible sources of frequency related error:

1) the Tolles and Lawson coefficients are frequency dependent (this is expected from materials which exhibit hysteresis);

2) lower signal-to-noise ratio for the pilot maneuver due to f^{-d} geological noise;

3) different transfer functions as a function of frequency for the various sensors (e.g., a small time delay between the scalar and vector sensors).

Pressure sensors located near the magnetometer were used to estimate altitude changes. The altitude change was combined with an estimate of the vertical gradient field (gammas per foot) in order to correct for the larger changes in altitude which occur during the pilot maneuvers. However, during the yaw maneuver the pressure sensor generates a false signal. In order to maintain a low FOM, it is common practice to turn off the altitude compensation during the yaw maneuver.

To study frequency effects, the bandpass filter was replaced by a whitening filter. In addition to reducing $1/f$ noise, this filter tended to emphasize the second harmonic of the pilot maneuver training data thus making the frequency content of the training data more like that of the FOM test data. The results of these experiments for the six data sets are shown in Table VI.

As Table VI shows, both optimal frequeny filtering and altitude compensation improve the FOM resulting from pilot maneuvers. On the average, the results were about the same, provided that the altitude compensator is turned off during yaw maneuvers. Otherwise, the whitening filter gave the best results. Although the whitening filter improved all of the data sets, one data set taken on November 25 was not improved by altitude compensation. If this set were removed from Table VI, then the mean FOM for altitude compensation with the compensator switched off during yaw maneuvers would have been 1.04 gammas. This represents a 20 percent improvement in the mean FOM.

If one does not have the luxury of switching the altitude sensor on and off, then it was found that the altitude compensation procedure served only to increase the mean FOM over that which could be achieved by prewhitening the data. Even though the performance was good, the whitening filter approach clearly suffers, since the physics of the vertical gradient is treated only statistically. Likewise, the altitude compensator suffers, since the sensor that estimates the vertical gradient gives a false signal during yaw maneuvers. In principle, this deficiency could be corrected with better altitude sensors. But even if the altitude compensation were perfect, frequency effect such as hysteresis and $1/f$ noise make it unlikely that FOMs as low as those shown in Table III could be achieved from pilot maneuver data, without resorting to an advanced filtering technique.

References

[1] W.E. Tolles and J.D. Lawson, "Magnetic compensation of MAD equipped aircraft," Airborne Instruments Lab. Inc., Mineola, N.Y., Rep. 201-1, June 1950.

[2] W.E. Tolles, "Compensation of induced magnetic fields in MAD equipped aircraft," Airborne Instruments Lab., O SRD 1386, sec. 6.1 - sr20-320, Apr. 21, 1943.

[3] P. Leliak, "Identification and evaluation of magnetic field sources of magnetic airborne detector equipped aircraft, *IRE Trans. Aerosp. Navig. Electron.*, Sept. 1961.

[4] S.H. Bickel, "An error analysis of a real time microprocessor algorithm for the magnetic compensation of aircraft," to be published.

[5] L.R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing.* New York: Prentice-Hall, 1975.

[6] F.B. Hilderbrand, *Introduction to Numerical Analysis.* New York: McGraw-Hill, 1956.

Samuel H. Bickel was born in York, Pa., on April 15, 1936. He received the B.S.E.E. degree from the Drexel Institute of Technology, Philadelphia, Pa., the M.S. degree from the University of Southern California, Los Angeles, and the Ph.D. degree from the University of California at Berkeley in 1958, 1960, and 1964, respectively.

From 1963 to 1965 he was a member of the technical staff of the MITRE Corporation, where his work included antenna noise temperature, simulation studies, electromagnetic scattering theory, and polarization studies. Following this he joined Texas Instruments Incorporated in Dallas, Tex., where he developed spectral techniques for analyzing radiometric infrared signatures. In 1974 he developed a technique for extracting ASW tactical parameters from the magnetic gradient eoherence tensor. During 1977, he analyzed SAR radar target signature data and developed a feedback procedure for real time correction of quadratic phase errors. Other work experience in the ASW group includes a LMS regression procedure for estimating target position from multiple sensor measurements and analysis of MAD detection range in the presence of geological noise. In January of 1979, he joined the Atlantic Richfield Company as a research associate where his current interests are devoted to the deconvolution of vibroseis seismic data for the purpose of geophysical exploration.

Dr. Bickel is a member of Eta Kappa Nu.