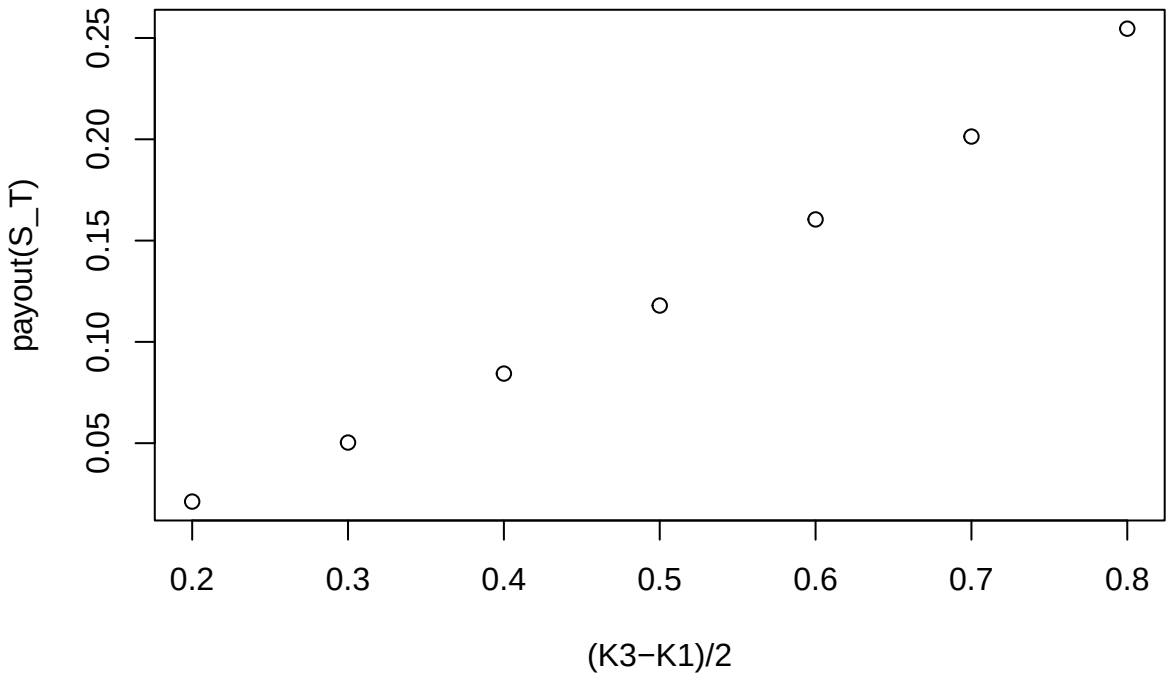
Q3 (c) (ii)

```
## Simulating a pure jump process: Discretized ###################################
set.seed(1234)

## Sample log(J) as specified in part (a)
sampleLogJump <- function(p_d, p_u, M_u, M_d){
  u = runif(1)
  ans = 0
  if (u>=p_d){
    ans = (-1/M_u)* log(abs(1- (u-p_d)/p_u))
  } else {
    ans = (1/M_d)*log(abs(u/p_d))
  }
  return(ans)
}

# Compute payout from sampled S_T (last value)
computeButterflyPay = function(ST, K1, K2, K3){
  pay = 0
  if (ST <= K1){
    pay = 0
  } else if (ST <= K2) {
    pay = ST-K1
  } else if (ST <= K3) {
    pay = K3-ST
  }
  return(pay)
}


###############
# parameters #
r = 0.05
sigma = 0.15
T=1
S0 = 1
M_u = 3.04
M_d = 3.08
p_u = 0.34
p_d = 1-p_u
lam = 0.1
N = 1000 # timesteps
n = 500 # simulations
k = p_u*M_u/(M_u-1) + (1-p_u)*M_d/(M_d+1) -1
delta_t = T/N
```

```
#### part(i) Algorithm to price butterfly option as a function of K1
priceButterfly = function(K1){
  K3 = 2*S0-K1
  K2 = (K1+K3)/2
  avPay = 0
  for (i in 1:n){
    St = S0
    for(j in 1:N){
      U <- runif(1)
      J = 1
      if(U <= lam * T/ N){
        # JUMP
        logJ = sampleLogJump(p_d, p_u, M_u, M_d)
        J = exp(logJ)
      }
      Z <- rnorm(1)
      dynamics = (1 + (r - lam*k)*delta_t + sigma*sqrt(delta_t)*Z + (J-1))
      St = St*dynamics # the last St will be S_T
    }
    pay = computeButterflyPay(St, K1, K2, K3)
    avPay = avPay + pay/n # only keep the average
  }
  return(exp(-r*T)*avPay) # return discounted
}

##### PLOTTING
x <- c()
y <- c()
for (k1 in seq(0.6, 0.9, 0.05)){
  y <- c(y, priceButterfly(k1))
  k3 = 2*S0 - k1
  x <- c(x, k3-k1)
}

plot(x,y, xlab = "(K3-K1)/2", ylab = "payout(S_T)")
```



How does the computed option value depend on (K3-K1)/2?

In a linear trend.

Why is your observation reasonable?

$(K_3 - K_1)/2 = S_0 - K_1$ This is just a function of $-K_1$ since S0 is a specified constant. The payout is only linear in the strikeprices. The payout it is negativeley linear with K_1 and positively linear with K_3. But K_3 and K_1 are negativeley correlated, so the payout is positiveley linear with $-K_1$ which is the form of the horizontal axis.