3 Dominance

3.3 Weak dominance

Definition: weakly dominates, weakly dominated, weakly dominating strategy.

For 2 strategies $s_i^{(1)}, s_i^{(2)} \in S_i$, $s_i^{(1)}$ weakly dominates $s_i^{(2)}$ if for all $s_{-i} \in S_{-i}$, $u_i(s_i^{(1)}, s_{-i}) \geq u_i(s_i^{(2)}, s_{-i})$ and this inequality is strict for at least one s_{-i}. In this case, $s_i^{(2)}$ is weakly dominated.

If s_i weakly domintes all strategies $s_i' \in S_i - \{s_i\}$, then s_i is a weakly dominating strategy.

Example.

	X	Y	Z
A	3, 3	1, 1	4, 1
B	2, 1	0, 1	3, 1

X weakly dominates Y and Z.

Y, Z do not weakly dominate each other.

Lemma 6. If for all $i \in N$, s_i^* is a weakly dominating strategy, then s^* is a Nash equilibrium.

Note. A weakly dominated strategy could still appear in a NE.

3.4 Iterated elimination of weakly dominated strategies (IEWDS)

Using example above, eliminate Y and Z.

	X
A	3, 3
B	2, 1

A weakly dominates B, eliminate B.

	X
A	3, 3

Claim: NE.

IEWDS: Repeatedly remove weakly dominated strategies until only one profile left.

Theorem 7. If IEWDS results in one strategy profile, then this is a NE.

(Proof: Exercise.)

IEWDS vs IESDS.

IESDS: NE is unique.

IEWDS: NE might not be unique.

Exercise.

Apply IEWDS is 2 different ways to get 2 NEs.

	D	E	F
A	1, 1	1, 0	2, 1
B	1, 1	0, 0	0, 0
C	0, 0	0, 0	1, 1

3.5 Application of weak dominance: Auctions

Set up of an auction. A seller puts one item up for an auction. Potential buyers put in bids to buy the item. Seller decides who wins (usually the highest bidder) and the price they pay.

Open vs closed bid auctions.

Open: Buyers repeatedly bid higher, until no one else bids.
Highest bidder pays their bid price. (Harder to analyse.)

Closed: Each buyer submits one bid secretly. (Strategic game setup.)

Types of closed bid auctions.

- First price auction: Highest bid wins, winner pays their bid.

  \$1 \$10 \$100 wins, pays \$100.

  This does not simulate open auctions, as someone willing to pay \$100 would win by bidding a bit more than \$10.

- Second price auction: Highest bid wins, winner pays the second highest bid. Example above: Bid of \$100 wins, pays \$10.

Set up for a closed bid second price auction.

n buyers $N = \{1, \dots, n\}$. 1 item for auction.

Buyer i thinks the item has value v_i "valuation"

Suppose buyer i submits bid b_i, giving strategy profile $b = (b_1, \dots, b_n)$.

The winner is the buyer who submits the highest bid, they pay the price equal to the second highest bid. If there is a tie for the highest bid, then the winner is one with the lowest index i.

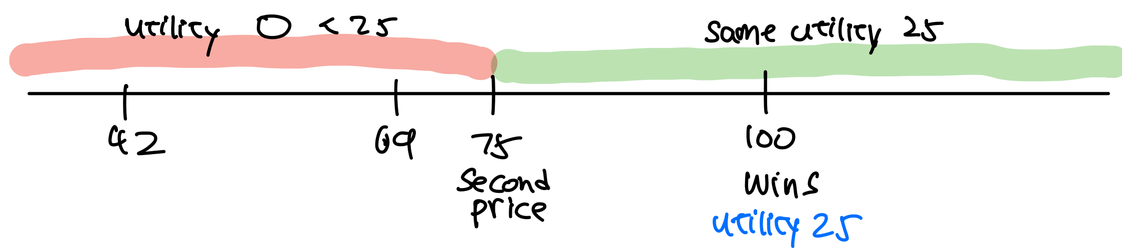
Given b, $u_i(b) = \begin{cases} v_i - \max\limits_{j \neq i} b_j & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$

Bidding one's valuation.

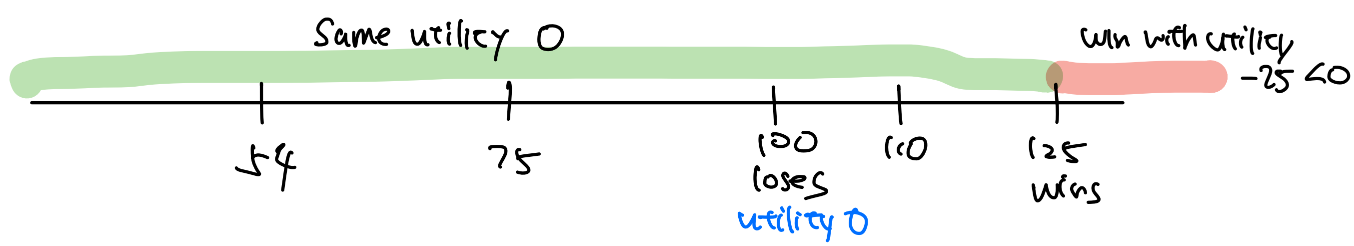
We claim that bidding one's Valuation is the best way to play.

Suppose our valuation is \$100. Would we bid something else?

① We win. 100 is the highest bid.



② We lose



In all cases, bidding our valuation is weakly dominating.