CO 456 Fall 2024: Assignment 1 problems

Due: Wednesday September 18 at 11:59pm EDT

Preamble.

- You will receive a Crowdmark link for each problem of the assignment. Submit your solutions on Crowdmark. Your submission must be clear and legible, and in the correct orientation. You need to submit your solutions to the correct problems, otherwise it will not be marked.
- Each of the problems is worth 20 marks.
- You need to justify all of your solutions, unless you are explicitly asked not to.
- You are graded on both your accuracy and your presentation (see below). A correct solution that is poorly presented may not receive full marks. The markers will not spend a lot of time figuring out something you write that is not clear.
- When discussing assignment problems on Piazza, use private posts if you think that you might spoil an answer or major thinking point of a problem.
- Read the assignment policies on the course outline for what is allowed and not allowed in working on the assignment.
- Reproduction, sharing or online posting of this document is strictly forbidden.

Guidelines to writing solutions. Your goal is to present your solutions so that your reader can easily understand what you are writing and be convinced of your arguments. So the quality of your solutions is very important. Here are some guidelines that you should keep in mind when writing your solutions.

- Write in complete sentences. Separate major steps in the solution into paragraphs.
- When writing a proof, make sure that every statement follows from earlier statements. Cite results or assumptions when you use them. Only use results from the lectures.
- Use proper notation. Mathematics is a precise language, and improper use of notation can easily cause confusion.
- Define a variable before you use it. It can be as simple as "Let $p = 3x$ for some $x \in \mathbb{Z}$" if this is the first time you are using x.
- Do not include irrelevant facts. Only state facts that are necessary in the argument.

Assignment problems.

A1-1. **Prisoner's dilemma**

(a) In prisoner's dilemma, we let H represent "sHare" and T represent "sTeal". Consider an instance of prisoner's dilemma with the following payoff table, representing the utilities u_1, u_2.

		Player II H	Player II T
Player I	H	$5, 5$	$0, 10$
Player I	T	$10, 0$	$0.1, 0.1$

Now suppose the two players actually care a little bit about each other's winnings. Define the happiness utility of both players given a strategy profile s by

$$u_1'(s) = u_1(s) + \alpha \cdot u_2(s), \quad u_2'(s) = u_2(s) + \alpha \cdot u_1(s),$$

where $\alpha \geq 0$ is a constant representing how much a player cares about the other.

i. Determine all values of α where (H, H) is a Nash equilibrium with respect to u_i'. (In other words, how much should you care in order to share? Because sharing is caring.)

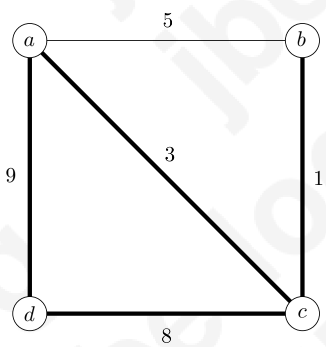
ii. Determine all values of α where the traditional (T, T) is the only Nash equilibrium with respect to u_i'?

(b) We now model climate change problem as an oversimplified strategic game, similar to prisoner's dilemma. We assume that climate change is bad. Suppose there are 10 countries. Each country can decide to either do something to solve the climate problem, or do nothing. If a country wants to help solve the climate problem, it will cost them 3. If a country does nothing, it will cost them 0; however, the climate gets bad, so every country incur an additional cost of 1. Determine all possible Nash equilibria in this game.

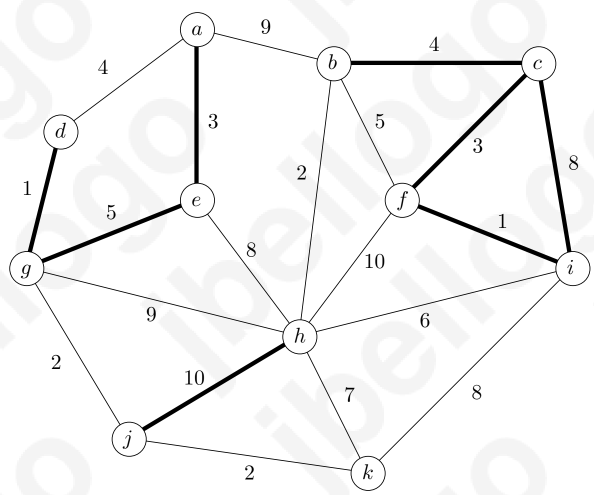
A1-2. **A strategic game on graphs**

Consider a game that is played on a connected graph $G = (V, E)$, and each edge e has an associated positive edge cost $c_e > 0$. Each vertex represents a player in the game. A strategy for a player at vertex v is a subset of all edges incident with v. Given a strategy profile s, we let G_s be the spanning subgraph of G that consists of all edges that are picked by at least one player. If G_s is not connected, then the utility of all players is $-\infty$. If G_s is connected, then the utility of a player at vertex v is the sum of $-c_e$ over all edges e in the player's strategy.

For example, suppose G is the following graph with edges labelled with their costs. If the player strategies are $s_a = \emptyset, s_b = \{bc\}, s_c = \{ac, cd\}, s_d = \{ad, cd\}$, then G_s consists of the bolded edges, which is a connected spanning subgraph. The utilities of the players are then $u_a(s) = 0, u_b(s) = -1, u_c(s) = -11, u_d(s) = -17$.



(a) Describe a quick method for finding the best response of a vertex v. Then illustrate your method with the example given below for vertex h, with the bolded edges representing ones picked by vertices other than h.



(b) For a general graph G and costs c, characterize (with proof) all possible Nash equilibria in this game.

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A1-3. **Business competition by setting prices**

Two farmers Abby and Berry are selling rice for the season. The cost of producing one kilogram of rice is \$2 for both of them. They have the ability to set their own prices for the rice that they sell, Abby at $\$p_A$ per kilogram, and Berry at $\$p_B$ per kilogram. If one farmer sets their price more than 10 cents per kilogram below the other farmer, then all customers will buy from them. They will get a demand of $10000 - 1000p$ kilograms for the price p that they set, while the other farmer gets no buyers. If the two farmers set their prices within 10 cents of each other, then they will each get a demand of $5000 - 500p$ kilograms for the price p that they set. They will only produce the amount of rice equal to the demand that they expect to have.

For example, if $p_A = 5$ and $p_B = 6$, then Abby can sell $10000 - 1000 \cdot 5 = 5000$ kilograms of rice, getting a total utility of $5000 \cdot (5-2) = 15000$, while Barry gets nothing. If $p_A = 5$ and $p_B = 5.1$, then Abby can sell $5000 - 500 \cdot 5 = 2500$ kilograms of rice with utility $2500 \cdot (5-2) = 7500$, while Berry can sell $5000 - 500 \cdot 5.1 = 2450$ kilograms of rice with utility $2450 \cdot (5.1 - 2) = 7595$.

Determine the best response function for each farmer on the prices that they set. Then determine all possible Nash equilibria (if they exist). Calculate the profit that each farmer receives at these equilibria.

Note: Assume that the prices are continuous.

A1-4. **Lectures 1-3 review questions**

(a) Consider the strategic game played by two students Alice and Bob. There is a limited amount of time for each of them to solve either a hard problem or an easy problem. Each student is only allowed to solve one problem, though they are allowed to solve the hard problem together. If they solve the hard problem together, then they are both expected to get a grade of 100. If only one of them solve the hard problem, then they cannot solve it and would get a grade of 20 for their effort. Alternatively, either student can solve the easy problem on their own to get a grade of 70. Formulate this as a strategic game, and write out the payoff table for Alice and Bob. Then determine all Nash equilibria. (No justifications required.)

(b) For the 2-player game defined by the payoff table below, identify the entries that are associated with the best response functions for either player. And write down all possible Nash equilibria in this game. (No justifications required.)

Player 2

Player 1		E	F	G	H	I
	A	6, 1	1, 4	2, 8	7, 4	5, 1
	B	3, 1	2, 8	1, 5	0, 3	3, 4
	C	5, 0	8, 6	7, 7	6, 4	8, 0
	D	1, 1	4, 2	5, 9	3, 1	5, 3

(c) Consider the Cournot's oligopoly model with 2 firms, where firm 2 has a more costly way of producing the units. Assume all the set up are the same, except the production cost for firm 1 is $C_1(q_1) = cq_1$, and the production cost for firm 2 is $C_2(q_2) = 2cq_2$, where c is still a constant with $0 < 2c < \alpha$. Find the best response function for each firm, and use it to find a Nash equilibrium. (You only need to show and explain the main steps. You do not need to show the detailed calculations.)

(d) Use IESDS on the following strategic game to find a Nash equilibrium. You only need to state the sequence of strategies that are eliminated, and then state the Nash equilibrium.

Player 2

Player 1		E	F	G	H	I
	A	4, 1	1, 6	4, 3	3, 8	7, 5
	B	5, 9	3, 5	2, 1	5, 3	6, 2
	C	7, 2	6, 7	5, 4	1, 1	3, 6
	D	1, 3	2, 1	7, 1	4, 6	5, 2