CO 456 Fall 2024 Lecture 10 (Monday October 7)

Part II: Mechanism design

Idea.

We want to design games so that players are incentivized to achieve certain global goals.

Examples.

- Election: The most popular candidate should win.
- Sports tournaments: Best team should win, teams play their best.

Problem.

Players are selfish, they will find loopholes and advantages to avoid playing as the designer intended.

Goal of mechanism design.

Design rules so that players who play for themselves will also achieve the global goals.

6 Ideal auctions

6.1 Good properties

General set up of an auction.

An auctioneer is selling some thing(s), players bid on the items.
The auctioneer needs to decide the rules for:

- who wins what item.
- who pays what amount.

Nice things to have in an auction. Recall: second-price auction is nice.

- It is easy to play: a player bidding their valuation is a dominant strategy. "Truthful bidding"
- If players give truthful bids, then their utility is never negative.

Definition: DSIC. An auction is dominant-strategy incentive-compatible (DSIC) if truthful bidding is a dominant strategy, and yields a non-negative utility.

Note: dominant means weakly dominating without the requirement of strict inequality in one case.

An auction where we give the item for free to player 1 is DSIC. (Need more conditions.)

Social welfare.

Social welfare is the sum total of values (not utilities) received by all players.

For one-item auction, only one player receives the item. If player i wins the item, then they receive value v_i. Social welfare is v_i.

One possible goal is the maximize social welfare. This is easy if players bid truthfully.

Definition: welfare-maximizing.

An auction is welfare-maximizing if truthful bidding results in maximum social welfare.

One more good property.

Auctions needs to be implemented efficiently (calculating max social welfare and the payment).

(Second price auction can be implemented efficiently.)

Definition: ideal auction.

An auction is ideal if

- It is DSIC.
- It is welfare-maximizing.
- It can be implemented efficiently.

Theorem 14. The second-price auction is ideal.

(Spoiler: This is the only ideal one-item auction.)

6.2 Example: Sponsored search auction

Search engines may show ads as the first entries of a search result. Selecting which ads to show and their order of placement is often done through auctions.

Model.

- There are $k \geq 1$ slots for sponsored links on a search result page.
- There are advertisers N bidding for these slots with related keyword.
- Each slot has a "click through rate" CTR, the probability that it is clicked. Higher slot = higher CTR.

For each slot j, let $\alpha_j \in [0,1]$ be its CTR. Assume $\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \dots$.
[Assume CTR is independent of quality of ads.]

General approach to finding ideal auctions.

Need to determine who wins what, and who pays what.

- Assuming truthful bidding, how can we assign the items so that it is welfare-maximizing and efficient?
- Given the way we assign items, how can we set prices so it is DSIC?

Social welfare for sponsored search auctions.

An advertiser i has valuation v_i, meaning each click on the ad is worth this much to them.
If they are assigned a slot with CTR x_i, then they receive value $v_i x_i$.
Social welfare is $\sum_{i \in N} v_i x_i$.

Example. 2 slots, 3 players. CTR: $\alpha_1 = 0.7, \alpha_2 = 0.5$. Valuations: $v_1 = 10, v_2 = 9, v_3 = 2$.

If we assign slot 1 to player 3, slot 2 to player 1, then social welfare is $2 \cdot 0.7 + 10 \cdot 0.5 = 6.4$.

Welfare-maximizing rule.

Assign slot i to the advertiser with the i-th highest valuation / bid.
Simple sorting algorithm makes this efficiently implemented.

Payment rule.

We want a DSIC payment rule based on welfare-maximization.
[If we ask the i-th slot winner to pay the $i+1$-st price multiplied by their CTR, it is not DSIC.] Need Myerson's Lemma to find DSIC payment rule.
Note: Efficiency is important, tons of auctions run every second.