CO 456 Fall 2024 Lecture 3 (Wednesday September 11)

2 Nash equilibrium

2.3 Cournot's oligopoly model

Recall: Given cost function $C_i(q_i) = cq_i$ and price function $P(q) = \max\{0, \alpha - \sum_j q_j\}$, the best response function for each firm is

$$B_i(q_{-i}) = \begin{cases} \{(\alpha - c - \sum_{j\neq i} q_j)/2\} & \alpha - c - \sum_{j\neq i} q_j > 0 \\ \{0\} & \text{otherwise} \end{cases}$$

NE for the two-firm case.

Suppose $q^* = (q_1^*, q_2^*)$ is a NE. They are best responses to each other:

$q_1^* \in B_1(q_2^*)$, $q_2^* \in B_2(q_1^*)$

Check: We can assume $q_1^*, q_2^* > 0$.

Then $q_1^* = (\alpha - c - q_2^*)/2$, $q_2^* = (\alpha - c - q_1^*)/2$.

Solve to get $q_1^* = q_2^* = \frac{\alpha - c}{3}$. NE is $\left(\frac{\alpha-c}{3}, \frac{\alpha-c}{3}\right)$.

Price at NE: $\alpha - q_1^* - q_2^* = \frac{\alpha + 2c}{3}$

Profit at NE: $u_i(q^*) = q_i^*(\alpha - c - q_1^* - q_2^*) = \frac{(\alpha - c)^2}{9}$.

What if firms collude?

Suppose they produce Q units in total, and split the profit.

Total profit: $Q(\alpha - c - Q)$ Maximized when $Q = \frac{\alpha - c}{2}$.

Profit at max is $\left(\frac{\alpha-c}{2}\right)\left(\alpha - c - \frac{\alpha - c}{2}\right) = \frac{(\alpha-c)^2}{4}$.

Each firm profits $\frac{(\alpha-c)^2}{8} > \frac{(\alpha-c)^2}{9}$.

What if there are large number of firms?

n firms: NE $q_i^* = (\alpha - c - \sum_{j\neq i} q_j^*)/2$. $\Rightarrow q_i^* = \frac{\alpha - c}{n+1}$.

Price: $P(q^*) = \alpha - n \cdot \frac{\alpha - c}{n+1} = \frac{1}{n+1}\alpha + \frac{n}{n+1} \cdot c \to c$ as $n \to \infty$

production cost.

Profit $\to 0$.

3 Dominance

3.1 Strict dominance

Definition: strictly dominates, strictly dominated, strictly dominating strategy.

For two strategies $s_i^{(1)}, s_i^{(2)} \in S_i$ of player i, we say that $s_i^{(1)}$ strictly dominates $s_i^{(2)}$ if $u_i(s_i^{(1)}, s_{-i}) > u_i(s_i^{(2)}, s_{-i})$ for all $s_{-i} \in S_{-i}$.

In this case, $s_i^{(2)}$ is strictly dominated.

If s_i strictly dominates all $s_i' \in S_i - \{s_i\}$, then s_i is a strictly dominating strategy.

Example.

	X	Y	Z
A	4, 2	1, 3	2, 1
B	2, 3	0, 1	3, 1

P2: X strictly dominates Z.

Z is strictly dominated.

No strictly dominating strategy.

Lemma 2.

If $s_i \in S_i$ is a strictly dominating strategy for player i and $s^* \in S$ is a NE, then $s_i^* = s_i$.

Lemma 3.

If $s^* \in S$ is a NE, then s_i^* is not strictly dominated $\forall i \in N$.

3.2 Iterated elimination of strictly dominated strategies (IESDS)

	X	Y	Z
A	4, 2	1, 3	2, 1
B	2, 3	0, 1	3, 1

P2: Z is strictly dominated (by X). Never in NE. Eliminate.

$\rightarrow$

	X	Y
A	4,2	1,3
B	2,3	0,1

P1: B is strictly dominated (by A). Eliminate.

$\rightarrow$

	X	Y
A	4,2	1,3

P2: X stricted dominated.

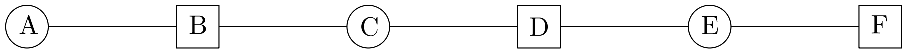
$\rightarrow$

	Y
A	1,3

NE

IESDS: Repeatedly eliminate strictly dominated strategies until only only one strategy profile left. Claim: This is the only NE.

Example. (Facility location game.) Two firms are each given a permit to open one store in one of the 6 towns along a highway. Firm 1 can open in A, C or E; firm 2 can open in B, D or F. Assume towns are equally spaced and equally populated. Customers in a town will go to the closest store. Where should the firms open the store?



Payoff table:

		Firm 2		
		B	D	F
Firm 1	A	1,5	2,4	3,3
	C	4,2	3,3	4,2
	E	3,3	2,4	5,1

1: C strictly dominates A. Eliminate A.
2: D " " F. " F.

→

	B	D
C	4,2	3,3
E	3,3	2,4

1: C strictly dominates E.
2: D " " B.

→

	D
C	$\boxed{3,3}$

NE.

Note: Extending this to any number of towns, we still get the centre 2 towns as NE.

Results on IESDS.

Theorem 4.

Let G be a strategic game. If IESDS ends with only one strategy profile s^*, then s^* is the unique NE of G.

This theorem is the immediate consequence of the following result.

Theorem 5.

Let G be a strategic game where s_i is a strictly dominated strategy for player i. Let G' be obtained from G by eliminating s_i from S_i. Then s^* is a NE of G if and only if s^* is a NE of G'.

Sketch proof of Theorem 5.