CO 456 Fall 2024 Lecture 1 (Wednesday September 4)

Part I: Strategic games

1 Introduction to strategic games

1.1 Prisoner's dilemma

Game show setting. 2 players. \$10. Each decide "share" or "steal".

- Both share ⇒ both get \$5.
- One share, one steal ⇒ steal player gets \$10, share player gets \$0.
- Both steal ⇒ both get \$0.1.

Strategic game: a player's benefit depends on their decisions and other players' decisions.

Best way to play prisoner's dilemma?

Play steal: Regardless of the other player's move, it is more beneficial to play steal than to play share.

Opponent plays steal: we get 0.1 for steal, 0 for share.

Opponent plays share: We get 10 for steal, 5 for share.

This is a "strictly dominant" strategy. (Rare.)

[Aside: This does not maximize "social welfare", total benefit of all players.]

1.2 Strategic game settings

Definition: strategic game. A strategic game is defined by specifying a set $N = \{1, \dots, n\}$ of players. For each player $i \in N$, there is a set of strategies S_i to play, and a utility function

$u_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$. (Benefit of player i given the strategies played by all players.)

Example with prisoner's dilemma.

2 players $N = \{1, 2\}$. Strategies $S_1 = S_2 = \{\text{share}, \text{steal}\}$

u_1, u_2 utility functions, e.g. $u_1(\underset{\text{P1 plays}}{\text{steal}}, \underset{\text{P2 plays}}{\text{share}}) = \underset{\text{P1 gets}}{10}$

Summarize using a table:

		P2 share	P2 steal
P1	share	5,5	0, 10
P1	steal	10, 0	0.1, 0.1

Each cell records utilities of P1, P2 in order.

Assumptions about strategic games.

① All players are rational and selfish (maximize own utility).

② All players have knowledge of all game parameters.

③ All players make one move simultaneously.

Definition: strategy profile. If player i plays strategy $s_i \in S_i$, then the strategy profile is the vector $s = (s_1, \dots, s_n) \in S_1 \times \cdots \times S_n$.

Player i gets $u_i(s)$ in utility.

1.3 Bach or Stravinsky?

Two players want to go to a concert. Player 1 likes Bach, player 2 likes Stravinsky, but they both prefer to be with each other.

P2

		Bach	Stravinsky
P1	Bach	2, 1	0, 0
	Stravinsky	0, 0	1, 2

No dominant strategy.
Look for strategy profiles that are more likely to happen.

In both cases, neither player gets higher utility by switching strategy. These are stable states called Nash equilibria.

2 Nash equilibrium

Basic concept. Looking for strategy profiles where no player is incentivized to switch their strategy. (Their utility cannot improve by switching.)

(Slightly wonky) Notations. Let $S = S_1 \times \cdots \times S_n$. set of all strategy profiles.
Let S_{-i} be the set of all strategy profiles excluding S_i. (Drop S_i from $S_1 \times \cdots \times S_n$)
If $s \in S$, then $s_{-i} \in S_{-i}$ is the profile obtained by dropping s_i.
If player i switches their strategy from s_i to s_i', then the new strategy profile is denoted $(s_i', s_{-i}) \in S$.

Definition: Nash equilibrium.

A strategy profile $s^* \in S$ is a Nash equilibrium if

$$u_i(s^*) \geq u_i(s_i', s^*_{-i}) \text{ for all } s_i' \in S_i,$$ for all players $i \in N$.

[for all players, their utility does not increase when only they switch strategies.]