MATHEMATICAL ANALYSIS 1
HOMEWORK 11

(1) Using Maclaurin polynomials, determine $\alpha \in \mathbb{R}$ so that
$$f(x) = (\arctan 2x)^2 - \alpha x \sin x$$
is infinitesimal of order 4 with respect to $\varphi(x) = x$ as $x \to 0$.

(2) Compute $f^{(6)}(0)$ where f is the function
$$f(x) = \sinh(x^2 + 2\sin^4 x).$$

(3) Determine the order and the principal part as $x \to +\infty$ with respect to $\varphi(x) = \frac{1}{x}$ of the infinitesimal function
$$f(x) = \sqrt[3]{1 + 3x^2 + x^3} - \sqrt[5]{2 + 5x^4 + x^5}.$$

(4) Determine the order and the principal part as $x \to 0$ with respect to $\varphi(x) = x$ of the infinitesimal function
$$f(x) = \sqrt[3]{1 - x^2} - \sqrt{1 - \frac{2}{3}x^2} + \sin\frac{x^4}{18}.$$

(5) For different values of $\alpha \in \mathbb{R}$, as $x \to 0$, determine the order of the following infinitesimal function with respect to $\varphi(x) = x$:
$$f(x) = \ln\cos x + \ln\cosh(\alpha x).$$

(6) Compute the following indefinite integrals using integration by parts:

(a) $\int \cos^4 x \text{ dx}$
(b) $\int \ln(\sqrt[3]{1 + x^2}) \text{ dx}$
(c) $\int \ln^2 x \text{ dx}$
(d) $\int x \arctan x \text{ dx}$

(7) Compute $\int \frac{1}{(1+x^2)^2} \text{ dx}$. *[Hint: compute* $\int \frac{1}{1+x^2} \text{ dx}$ *using integration by parts with* $f(x) = \frac{1}{1+x^2}$ *and* $g'(x) = 1$.*]*

(8) Compute the following indefinite integrals using substitution:

(a) $\int \frac{e^{2x}}{e^x+1} \text{dx}$
(b) $\int \frac{1+\cos x}{1-\cos x} \text{dx}$
(c) $\int \frac{1}{\sinh x} \text{dx}$
(d) $\int \frac{1}{e^{4x}+1} \text{dx}$

(9) (a) Recall what the *derivatives* of $\arctan x$ and of $\arctan\frac{1}{x}$ are.
(b) Use the previous part to prove that
$$\arctan\frac{1}{x} = \frac{\pi}{2} - \arctan x, \qquad \forall x > 0.$$