Università degli Studi di Roma "Tor Vergata"
Dipartimento di Matematica

Analysis 1 (Engineering Sciences) 2025-2026

Instructor: Prof. Jonathan Ben-Artzi

Final Examination — *Call 1 of 6*

27 January 2026

First Name (CAPITALS): ____________________

Last Name (CAPITALS): ____________________

Matricola: ____________________

Grading Summary

Quest.	1	2	3	4	5	6	7	8	9	10	**Total**
Points	1	1	1	1	1	1	1	1	1	1	**10**
Score											

Quest.	11	12	13	14	15	**Total**
Points	3	3	3	3	3	**15**
Score						

FINAL GRADE	**/ 25**

Examination Rules:

- **Duration**: 2 hours and 30 minutes.
- **NO** cellphones, **NO** calculators, **NO** books, **NO** notes, and **NO** headphones.
- Write full solutions clearly within the provided spaces.
- Part B will only be graded if the student achieves a score of at least 9/10 in Part A.
- Any student caught copying or engaging in academic misconduct will face disciplinary action.
- Use only blue or black ink. Additional paper will be provided upon request.

Do not turn this sheet over until instructed to do so.

Part A

Exercise 1

Find the following limit. Explain your answer. ______/ 1 p.

$$\lim_{x \to -1} \frac{x+1}{\sqrt{6x^2+3}+3x}.$$

Exercise 2

Find the following limit. Explain your answer. ______/ 1 p.

$$\lim_{x \to -\infty} \frac{3^x - 3^{-x}}{3^x + 3^{-x}}.$$

Exercise 3

———/ 1 p.

Let f be continuously differentiable on $(-1, 1)$, with $f(0) = 0$ and $|f'(x)| \leq 2$ for all $x \in (-1, 1)$. Show that the equation $f(x) = 7 + \sin x$ has no solution in $(-1, 1)$.

Exercise 4

———/ 1 p.

Describe and sketch the set $A = \{(x, y) \in \mathbb{R}^2 : xy \geq 0\}$.

Exercise 5

Find the equation of the tangent line to $y = \frac{x}{1+x^2}$ at $x_0 = 1$. ______/ 1 p.

Exercise 6

Let f be continuously differentiable on (a, b) and let $x_0 \in (a, b)$ satisfy $f(x_0) = 0$. Prove that there exists exactly one number L such that ______/ 1 p.

$$\lim_{x \to x_0} \frac{f(x) - L(x - x_0)}{x - x_0} = 0.$$

Find L.

Exercise 7

Let $\{a_n\}_{n\in\mathbb{N}}$ be a real sequence. Suppose that $\lim_{n\to\infty} |a_n|$ exists. Does $\lim_{n\to\infty} a_n$ necessarily exist? Explain your answer. ______/ 1 p.

Exercise 8

Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ converges, diverges or is indeterminate. Justify your answer. ______/ 1 p.

Exercise 9

Evaluate the indefinite integral $\int x^2 e^x$ dx using integration by parts. ______/ 1 p.

Exercise 10

Compute the area enclosed between the graphs of $y = \sqrt{x}$ and $y = x^2$. ______/ 1 p.

Part B

Exercise 11

Let $f : \mathbb{R} \to \mathbb{R}$, let $x_0 \in \mathbb{R}$ and suppose that $\lim_{x\to x_0} f(x) = \ell$ for some $\ell \in \mathbb{R}$. Prove the uniqueness of the limit. That is, prove that if $\lim_{x\to x_0} f(x) = L$ then $L = \ell$. _____/3 p.

Exercise 12

Let $f : \mathbb{R} \to \mathbb{R}$ be continuously differentiable. State and prove the Mean Value Theorem (Lagrange's Theorem) for f. In your proof you may rely on Rolle's Theorem without proving it. ______/ 3 p.

Exercise 13

Using Maclaurin polynomials, determine $\alpha \in \mathbb{R}$ so that ______/ 3 p.

$$f(x) = (\arctan 2x)^2 - \alpha x \sin x$$

is infinitesimal of order 4 with respect to $\varphi(x) = x$ as $x \to 0$.

Exercise 14

Determine the order and the principal part with respect to $\varphi(x) = \frac{1}{x}$ as $x \to +\infty$ of the function $f(x) = \sin(\sqrt{x^2 - 1} - x)$. ____/ 3 p.

Exercise 15

Show that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ without using the Taylor expansion of the sine function. You may use the inequality $x < \tan x$ which holds for all $x > 0$ sufficiently small. _____/ 3 p.