MATHEMATICAL ANALYSIS 1
HOMEWORK 8

(1) Recall the translation, rescaling and reflection functions defined on $\mathbb{R}$ (with $c > 0$):

$$t_c(x) = x + c \qquad s_c(x) = cx \qquad r(x) = -x.$$

For a function $f : \mathbb{R} \to \mathbb{R}$ that is differentiable for all $x \in \mathbb{R}$, differentiate with respect to x the following functions:

(a) $(f \circ r)(x)$
(b) $(r \circ f)(x)$
(c) $(s_c \circ f)(x)$
(d) $(f \circ s_c)(x)$
(e) $(f \circ t_c)(x)$
(f) $(t_c \circ f)(x)$
(g) $(f \circ (s_c \circ r))(x)$
(h) $(f \circ (r \circ t_c))(x)$

(2) Differentiate with respect to x the following functions (wherever they differentiable):

(a) $f(x) = 3x\sqrt[3]{1+x^2}$
(b) $f(x) = x \ln x$
(c) $f(x) = \cos(e^{x^2+1})$
(d) $f(x) = -\frac{1}{(4x-1)^3}$
(e) $f(x) = \arcsin\sqrt{x}$
(f) $f(x) = x\tan(x^3)$

(3) Write the equation of the tangent line at x_0 to the graph of the following functions:

(a) $f(x) = \ln(3x-2), \quad x_0 = 2$
(b) $f(x) = \frac{x}{1+x^2}, \quad x_0 = 1$
(c) $f(x) = e^{\sqrt{2x+1}}, \quad x_0 = 0$
(d) $f(x) = \sin\frac{1}{x}, \quad x_0 = \frac{1}{\pi}$

(4) Verify that $f(x) = 2x^5 + x^3 + 5x$ is invertible on $\mathbb{R}$ and that f^{-1} is differentiable on $\mathbb{R}$. Compute $(f^{-1})'(y_0)$ at $y_0 = 0$ and $y_0 = 8$.

(5) Find the maximum and minimum of the following functions on the given interval:

(a) $f(x) = \sin x + \cos x, \quad [0, 2\pi]$
(b) $f(x) = x^2 - |x+1| - 2, \quad [-2, 1]$

(6) Verify that $f(x) = \ln(2+x) + 2\frac{x+1}{x+2}$ vanishes only at $x_0 = -1$.

(7) Determine the number of zeroes and critical points of

$$f(x) = \frac{x\ln x - 1}{x^2}.$$