Università degli Studi di Roma "Tor Vergata"
Dipartimento di Matematica

Analysis 1 (Engineering Sciences) 2025-2026

Instructor: Prof. Jonathan Ben-Artzi

Final Examination — *Call 1 of 6*

27 January 2026

First Name (CAPITALS): ______________________

Last Name (CAPITALS): ______________________

Matricola: ______________________

Grading Summary

Quest.	1	2	3	4	5	6	7	8	9	10	**Total**
Points	1	1	1	1	1	1	1	1	1	1	**10**
Score											

Quest.	11	12	13	14	15	**Total**
Points	3	3	3	3	3	**15**
Score						

FINAL GRADE	**/ 25**

Examination Rules:

- **Duration**: 2 hours and 30 minutes.
- **NO** cellphones, **NO** calculators, **NO** books, **NO** notes, and **NO** headphones.
- Write full solutions clearly within the provided spaces.
- Part B will only be graded if the student achieves a score of at least 9/10 in Part A.
- Any student caught copying or engaging in academic misconduct will face disciplinary action.
- Use only blue or black ink. Additional paper will be provided upon request.

Do not turn this sheet over until instructed to do so.

Part A

Exercise 1

Find the following limit. Explain your answer. ______/ 1 p.

$$\lim_{x\to -1} \frac{x+1}{\sqrt{6x^2+3}+3x}.$$

Exercise 2

Find the following limit. Explain your answer. ______/ 1 p.

$$\lim_{x\to 1} \frac{\ln x}{e^x - e}.$$

Exercise 3

Describe and sketch the set $A = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 > 0\}$.

______/ 1 p.

Exercise 4

Let $z = \dfrac{1+i}{1-i}$. (i) Write z in Cartesian form. (ii) Write z in exponential form. (iii) Compute z^{2023}.

______/ 1 p.

Exercise 5

Find the equation of the tangent line to $y = x^3$ at the point $(1, 1)$. _____/ 1 p.

Exercise 6

Verify that $f(x) = 2x^5 + x^3 + 5x$ is invertible on $\mathbb{R}$ and that f^{-1} is differentiable. Compute $(f^{-1})'(0)$ and $(f^{-1})'(8)$. _____/ 1 p.

Exercise 7

Consider the sequence $a_n = \frac{n!}{n^{100}}$, $n \in \mathbb{N}_+$. Does it have a limit as $n \to \infty$? If so, what is it? ____/ 1 p.

Exercise 8

Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ converges, diverges or is indeterminate. Justify your answer. ____/ 1 p.

Exercise 9

Evaluate the indefinite integral $I = \int e^x \sin x \, \mathrm{dx}$. ______/ 1 p.

Exercise 10

Determine whether $\int_e^\infty \frac{1}{x(\ln x)^2} \, \mathrm{dx}$ converges, and compute its value if it does. ______/ 1 p.

Part B

Exercise 11

Prove that the ceiling function $f(x) = \lceil x \rceil$ is left-continuous at every $x_0 \in \mathbb{R}$. ______/ 3 p.

Exercise 12

Let $f : \mathbb{R} \to \mathbb{R}$ be defined and continuous on $\mathbb{R}$, let $x_0 \in \mathbb{R}$ and let $F(x) = \int_{x_0}^{x} f(y)\,\mathrm{d}y$. State and prove the Fundamental Theorem of Integral Calculus. In your proof you may rely on the Integral Mean Value Theorem and the Squeeze Theorem without proving them. ___/ 3 p.

Exercise 13

Using Maclaurin polynomials, determine $\alpha \in \mathbb{R}$ so that 	——/ 3 p.

$$f(x) = (\arctan 2x)^2 - \alpha x \sin x$$

is infinitesimal of order 4 with respect to $\varphi(x) = x$ as $x \to 0$.

Exercise 14

Compute the indefinite integral $I = \int \cos^4 x \; \mathrm{dx}$ using integration by parts. ______/ 3 p.

Exercise 15

Determine the order and the principal part with respect to $\varphi(x) = \frac{1}{x}$ as $x \to +\infty$ of the function $f(x) = \sin(\sqrt{x^2 - 1} - x)$.

______/ 3 p.