MATHEMATICAL ANALYSIS 1
THINGS TO REMEMBER

1. Trigonometry

1.1. Fundamental trigonometric identities.

$$
\begin{aligned}
\sin^2(\alpha) + \cos^2(\alpha) &= 1 \\
\tan^2(\alpha) + 1 &= \sec^2(\alpha) \\
1 + \cot^2(\alpha) &= \csc^2(\alpha) \\
\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \\
\cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \\
\sin(2\alpha) &= 2\sin(\alpha)\cos(\alpha) \\
\cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha) \\
\sin(-\alpha) &= -\sin(\alpha) \qquad \text{(sine is odd)} \\
\cos(-\alpha) &= \cos(\alpha) \qquad \text{(cosine is even)} \\
\sin(\alpha) - \sin(\beta) &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \\
\cos(\alpha) - \cos(\beta) &= -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)
\end{aligned}
$$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0

1.2. Inverse trigonometric functions.

The inverse trigonometric functions are defined by restricting the domains appropriately:

- $\arcsin : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$ is the inverse of sin restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- $\arccos : [-1,1] \to [0,\pi]$ is the inverse of cos restricted to $[0,\pi]$.
- $\arctan : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$ is the inverse of tan restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$.

2. Complex numbers

Property	**Formula**
Modulus	$\lvert z\rvert = \sqrt{a^2+b^2}$
Argument	$\arg(z) = \arctan(b/a)$
Conjugate	$\bar{z} = a - bi$
Real part	$\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$
Imaginary part	$\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$
Polar form	$z = r(\cos\theta + i\sin\theta)$
Exponential form	$z = re^{i\theta}$
Multiplication	$z \cdot w = rs\, e^{i(\theta+\phi)}$
Division	$\frac{z}{w} = \frac{r}{s}\, e^{i(\theta-\phi)}$
De Moivre's	$z^n = r^n e^{in\theta}$
nth roots	$\sqrt[n]{z} = \sqrt[n]{r}\, e^{i(\theta+2\pi k)/n}$

3. Derivatives

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} x^\alpha &= \alpha x^{\alpha-1} && (\forall \alpha \in \mathbb{R}) \\
\frac{\mathrm{d}}{\mathrm{d}x} \sin x &= \cos x \\
\frac{\mathrm{d}}{\mathrm{d}x} \cos x &= -\sin x \\
\frac{\mathrm{d}}{\mathrm{d}x} \tan x &= 1 + \tan^2 x = \frac{1}{\cos^2 x} \\
\frac{\mathrm{d}}{\mathrm{d}x} \arcsin x &= \frac{1}{\sqrt{1-x^2}} \\
\frac{\mathrm{d}}{\mathrm{d}x} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\
\frac{\mathrm{d}}{\mathrm{d}x} \arctan x &= \frac{1}{1+x^2} \\
\frac{\mathrm{d}}{\mathrm{d}x} a^x &= (\ln a)\, a^x \qquad (\forall a > 0) && \text{in particular, } \frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x \\
\frac{\mathrm{d}}{\mathrm{d}x} \log_a |x| &= \frac{1}{(\ln a)\, x} \qquad (\forall a > 0,\ a \neq 1) && \text{in particular, } \frac{\mathrm{d}}{\mathrm{d}x} \ln |x| = \frac{1}{x}
\end{aligned}
$$

4. Taylor

$$
e^x = \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n), \quad x \to 0.
$$

$$
\begin{aligned}
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n) \\
&= \sum_{k=1}^{n} (-1)^{k-1} \frac{x^k}{k} + o(x^n), \qquad x \to 0.
\end{aligned}
$$

$$
\begin{aligned}
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} - \cdots + (-1)^m \frac{x^{2m+1}}{(2m+1)!} + o(x^{2m+2}) \\
&= \sum_{k=1}^{m} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2m+2}), \qquad x \to 0.
\end{aligned}
$$

$$
\begin{aligned}
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} - \cdots + (-1)^m \frac{x^{2m}}{(2m)!} + o(x^{2m+1}) \\
&= \sum_{k=0}^{m} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2m+1}), \qquad x \to 0.
\end{aligned}
$$

$$
\begin{aligned}
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \cdots + \binom{\alpha}{n} x^n + o(x^n) \\
&= \sum_{k=0}^{n} \binom{\alpha}{k} x^k + o(x^n)
\end{aligned}
$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 - \cdots + (-1)^n x^n + o(x^n) = \sum_{k=0}^{n} (-1)^k x^k + o(x^n)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o(x^3)$$

5. Integrals

$$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1)$$

$$\int \frac{1}{x} \, \mathrm{d}x = \ln|x| + C \quad (\text{for } x > 0 \text{ or } x < 0)$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int \frac{1}{1+x^2} \, \mathrm{d}x = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, \mathrm{d}x = \arcsin x + C$$

$$\int \sinh x \, \mathrm{d}x = \cosh x + C$$

$$\int \cosh x \, \mathrm{d}x = \sinh x + C$$