

**MATHEMATICAL ANALYSIS 1**  
**HOMEWORK 11**

- (1) Using Maclaurin polynomials, determine  $\alpha \in \mathbb{R}$  so that

$$f(x) = (\arctan 2x)^2 - \alpha x \sin x$$

is infinitesimal of order 4 with respect to  $\varphi(x) = x$  as  $x \rightarrow 0$ .

- (2) Compute  $f^{(6)}(0)$  where  $h$  is the function

$$f(x) = \sinh(x^2 + 2 \sin^4 x).$$

- (3) Determine the order and the principal part as  $x \rightarrow +\infty$  with respect to  $\varphi(x) = \frac{1}{x}$  of the infinitesimal function

$$f(x) = \sqrt[3]{1 + 3x^2 + x^3} - \sqrt[5]{2 + 5x^4 + x^5}.$$

- (4) Determine the order and the principal part as  $x \rightarrow 0$  with respect to  $\varphi(x) = x$  of the infinitesimal function

$$f(x) = \sqrt[3]{1 - x^2} - \sqrt{1 - \frac{2}{3}x^2} + \sin \frac{x^4}{18}.$$

- (5) For different values of  $\alpha \in \mathbb{R}$ , as  $x \rightarrow 0$ , determine the order of the following infinitesimal function with respect to  $\varphi(x) = x$ :

$$f(x) = \ln \cos x + \ln \cosh(\alpha x).$$

- (6) Compute the following indefinite integrals using integration by parts:

(a)  $\int \cos^4 x \, dx$

(c)  $\int \ln^2 x \, dx$

(b)  $\int \ln(\sqrt[3]{1 + x^2}) \, dx$

(d)  $\int x \arctan x \, dx$

- (7) Compute  $\int \frac{1}{(1+x^2)^2} \, dx$ . [Hint: compute  $\int \frac{1}{1+x^2} \, dx$  using integration by parts with  $f(x) = \frac{1}{1+x^2}$  and  $g'(x) = 1$ .]

- (8) Compute the following indefinite integrals using substitution:

(a)  $\int \frac{e^{2x}}{e^x + 1} \, dx$

(c)  $\int \frac{1}{\sinh x} \, dx$

(b)  $\int \frac{1 + \cos x}{1 - \cos x} \, dx$

(d)  $\int \frac{1}{e^{4x} + 1} \, dx$

- (9) (a) Recall what the *derivatives* of  $\arctan x$  and of  $\arctan \frac{1}{x}$  are.  
(b) Use the previous part to prove that

$$\arctan \frac{1}{x} = \frac{\pi}{2} - \arctan x, \quad \forall x > 0.$$