Università degli Studi di Roma "Tor Vergata"
Dipartimento di Matematica

Analysis 1 (Engineering Sciences) 2025-2026
Instructor: Prof. Jonathan Ben-Artzi

Final Examination — *Call 1 of 6*

27 January 2026

First Name (CAPITALS): _____

Last Name (CAPITALS): _____

Matricola: _____

Grading Summary

Quest.	1	2	3	4	5	6	7	8	9	10	Total
Points	1	1	1	1	1	1	1	1	1	1	**10**
Score											

Quest.	11	12	13	14	15	Total
Points	3	3	3	3	3	**15**
Score						

FINAL GRADE	/ 25

Examination Rules:

- **Duration**: 2 hours and 30 minutes.
- **NO** cellphones, **NO** calculators, **NO** books, **NO** notes, and **NO** headphones.
- Write full solutions clearly within the provided spaces.
- Part B will only be graded if the student achieves a score of at least 9/10 in Part A.
- Any student caught copying or engaging in academic misconduct will face disciplinary action.
- Use only blue or black ink. Additional paper will be provided upon request.

Do <u>not</u> turn this sheet over until instructed to do so.

Part A

Exercise 1

As $x \to +\infty$, the function $f(x) = \ln\left(9 + \sin\frac{2}{x}\right) - 2\ln 3$ can be written in the form $f(x) = \frac{b}{x^{\alpha}} + o(x^{-\alpha})$. Find b and α.

Exercise 2

Find the following limit. Explain your answer.

$$\lim_{x \to 0} \frac{\sin(5x)}{x}.$$

Exercise 3

Describe and sketch the set $A = \{(x, y) \in \mathbb{R}^2 : xy \geq 0\}$.

1 p.

Exercise 4

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Let $z = 2 - 2i$ and $w = -1 + i\sqrt{3}$. (i) Write z and w in exponential form. (ii) Compute $z \cdot w$ 1 p. in Cartesian and exponential form. (iii) Compute z/w in exponential form. (iv) Compute $z\overline{z}$.

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Exercise 5

Find the equation of the tangent line to $y = x^3$ at the point $(1, 1)$.

Exercise 6

Compute the derivative $\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_{\sqrt{x}}^{x^2} \sin(y^3)\,\mathrm{d}y\right]$ for $x > 0$.

Exercise 7

Determine the order and the principal part with respect to $\varphi(x) = x$ as $x \to 0$ of

$$f(x) = \frac{e^x}{1+x^2} - 1.$$

Exercise 8

Show that if $\sum_{n=0}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$.

Exercise 9

Compute the average value of $f(x) = \frac{1}{3}(\sin x + 5\cos x)$ on the interval $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Exercise 10

Determine whether $\int_e^\infty \frac{1}{x(\ln x)^2}$ dx converges, and compute its value if it does.

Part B

Exercise 11

Prove that the ceiling function $f(x) = \lceil x \rceil$ is left-continuous at every $x_0 \in \mathbb{R}$.

$\underline{}/$
3 p.

Exercise 12

Let $f : \mathbb{R} \to \mathbb{R}$, let $x_0 \in \mathbb{R}$ and suppose that $\lim_{x \to x_0} f(x) = \ell$ for some $\ell > 0$. Prove that $f > 0$ on a neighborhood of x_0 (potentially excluding x_0 itself).

Exercise 13

Evaluate the integral $\int_0^1 \frac{1}{(x+1)^2}\, dx$ and deduce that

$$\lim_{n\to\infty} n\left(\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}\right) = \frac{1}{2}$$

Exercise 14

Write the Maclaurin expansion of $f(x) = e^{x^2} \sin 2x$ up to order $n = 5$.

_____/
3 p.

Exercise 15

Determine the order and the principal part with respect to $\varphi(x) = \frac{1}{x}$ as $x \to +\infty$ of the function $\quad$ 3 p.
$f(x) = \sin(\sqrt{x^2 - 1} - x)$.