Università degli Studi di Roma "Tor Vergata"
Dipartimento di Matematica

Analysis 1 (Engineering Sciences) 2025-2026
Instructor: Prof. Jonathan Ben-Artzi

Final Examination — *Call 1 of 6*

27 January 2026

First Name (CAPITALS): _____

Last Name (CAPITALS): _____

Matriculation: _____

Grading Summary

Quest.	1	2	3	4	5	6	7	8	9	10	Total
Points	1	1	1	1	1	1	1	1	1	1	**10**
Score											

Quest.	11	12	13	14	15	Total
Points	3	3	3	3	3	**15**
Score						

FINAL GRADE	/ 25

Examination Rules:

- **Duration**: 2 hours and 30 minutes.
- **NO** cellphones, **NO** calculators, **NO** books, **NO** notes, and **NO** headphones.
- Write full solutions clearly within the provided spaces.
- Part B will only be graded if the student achieves a score of at least 9/10 in Part A.
- Any student caught copying or engaging in academic misconduct will face disciplinary action.
- Use only blue or black ink. Additional paper will be provided upon request.

Do <u>not</u> turn this sheet over until instructed to do so.

Part A

Exercise 1

Let $f(x) = 5 + 2x \sin \frac{1}{x}$. Does the limit $\lim_{x \to +\infty} f(x)$ exist? Explain your answer.

Exercise 2

Determine the domain and the behvior at the end-points of the domain of the function $f(x) = \frac{x^3 - x^2 + 3}{x^2 + 3x + 2}$.

Exercise 3

For $f(x) = \frac{1}{x}$, $A = [1, +\infty)$, determine $\sup_A f$, $\inf_A f$, and whether the maximum and minimum are attained on A.

Exercise 4

Find the domain of the function $f(x) = \ln(x^2 - 1)$.

Exercise 5

Calculate the second derivative of $f(x) = e^{-x^2}$.

_____/
1 p.

Exercise 6

Find the maximum and minimum of $f(x) = x^2 - |x + 1| - 2$ on $[-2, 1]$.

_____/
1 p.

Exercise 7

Is the sequence $a_n = n \cos\left(\frac{n+1}{n} \cdot \frac{\pi}{2}\right)$ convergent, divergent or indeterminate? Explain your answer.

Exercise 8

Calculate the sum of the geometric series $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$. Explain your answer.

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Exercise 9

Evaluate $\int \frac{5x-3}{(x-1)(x-3)}dx$.

Exercise 10

Evaluate the improper integral $\int_0^\infty xe^{-2x}dx$.

Part B

Exercise 11

Let $f : \mathbb{R} \to \mathbb{R}$, let $x_0 \in \mathbb{R}$ and suppose that $\lim_{x \to x_0} f(x) = \ell$ for some $\ell < 0$. Prove that $f < 0$ $\underline{\phantom{3 \text{ p.}}}$/ 3 p. on a neighborhood of x_0 (potentially excluding x_0 itself).

Exercise 12

Let $f : X \to Y$ and $g : Y \to Z$ both be one-to-one functions. Show that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$

Exercise 13

Compute the indefinite integral $\int \frac{1}{(1+x^2)^2}\ dx$.

3 p.

Exercise 14

Write the Maclaurin expansion of $f(x) = \dfrac{x}{\sqrt[6]{1+x^2}} - \sin x$ up to order $n = 5$.

Exercise 15

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For the function $f(x) = \ln(x + \sqrt{x^2 + 1})$, determine: (i) The domain. (ii) The critical points.   3 p.
(iii) The local maximum and minimum values. (iv) The inflection points. (v) The points where
the function is not differentiable. (vi) All asymptotes, and (vii) Sketch the function.