

MATHEMATICAL ANALYSIS 1
HOMEWORK 11

- (1) Using Maclaurin polynomials, determine $\alpha \in \mathbb{R}$ so that

$$f(x) = (\arctan 2x)^2 - \alpha x \sin x$$

is infinitesimal of order 4 with respect to $\varphi(x) = x$ as $x \rightarrow 0$.

- (2) Compute $f^{(6)}(0)$ where h is the function

$$f(x) = \sinh(x^2 + 2 \sin^4 x).$$

- (3) Determine the order and the principal part as $x \rightarrow +\infty$ with respect to $\varphi(x) = \frac{1}{x}$ of the infinitesimal function

$$f(x) = \sqrt[3]{1 + 3x^2 + x^3} - \sqrt[5]{2 + 5x^4 + x^5}.$$

- (4) Determine the order and the principal part as $x \rightarrow 0$ with respect to $\varphi(x) = x$ of the infinitesimal function

$$f(x) = \sqrt[3]{1 - x^2} - \sqrt{1 - \frac{2}{3}x^2 + \sin \frac{x^4}{18}}.$$

- (5) For different values of $\alpha \in \mathbb{R}$, as $x \rightarrow 0$, determine the order of the following infinitesimal function with respect to $\varphi(x) = x$:

$$f(x) = \ln \cos x + \ln \cosh(\alpha x).$$

- (6) Compute the following indefinite integrals using integration by parts:

(a) $\int \cos^4 x \, dx$

(b) $\int \ln(\sqrt[3]{1 + x^2}) \, dx$

(c) $\int \ln^2 x \, dx$

(d) $\int x \arctan x \, dx$

- (7) Compute $\int \frac{1}{(1+x^2)^2} \, dx$. [Hint: compute $\int \frac{1}{1+x^2} \, dx$ using integration by parts with $f(x) = \frac{1}{1+x^2}$ and $g'(x) = 1$.]

- (8) Compute the following indefinite integrals using substitution:

(a) $\int \frac{e^{2x}}{e^x + 1} \, dx$

(b) $\int \frac{1+\cos x}{1-\cos x} \, dx$

(c) $\int \frac{1}{\sinh x} \, dx$

(d) $\int \frac{1}{e^{4x}+1} \, dx$

- (9) (a) Recall what the *derivatives* of $\arctan x$ and of $\arctan \frac{1}{x}$ are.

- (b) Use the previous part to prove that

$$\arctan \frac{1}{x} = \frac{\pi}{2} - \arctan x, \quad \forall x > 0.$$