

# Università degli Studi di Roma “Tor Vergata”

Dipartimento di Matematica

## Analysis 1 (Engineering Sciences) 2025-2026

Instructor: Prof. Jonathan Ben-Artzi

Final Examination — *Call 1 of 6*

27 January 2026

First Name (CAPITALS): \_\_\_\_\_

Last Name (CAPITALS): \_\_\_\_\_

Matricola: \_\_\_\_\_

### Grading Summary

Quest.	1	2	3	4	5	6	7	8	9	10	Total
Points	1	1	1	1	1	1	1	1	1	1	10
Score											

Quest.	11	12	13	14	15	Total
Points	3	3	3	3	3	15
Score						

FINAL GRADE	/ 25
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### Examination Rules:

- **Duration:** 2 hours and 30 minutes.
- **NO** cellphones, **NO** calculators, **NO** books, **NO** notes, and **NO** headphones.
- Write full solutions clearly within the provided spaces.
- Part B will only be graded if the student achieves a score of at least 9/10 in Part A.
- Any student caught copying or engaging in academic misconduct will face disciplinary action.
- Use only blue or black ink. Additional paper will be provided upon request.

**Do not turn this sheet over until instructed to do so.**

## Part A

### Exercise 1

Find the following limit. Explain your answer.

\_\_\_\_\_/1 p.

$$\lim_{x \rightarrow -1} \frac{x+1}{\sqrt{6x^2+3}+3x}.$$

### Exercise 2

Find the following limit. Explain your answer.

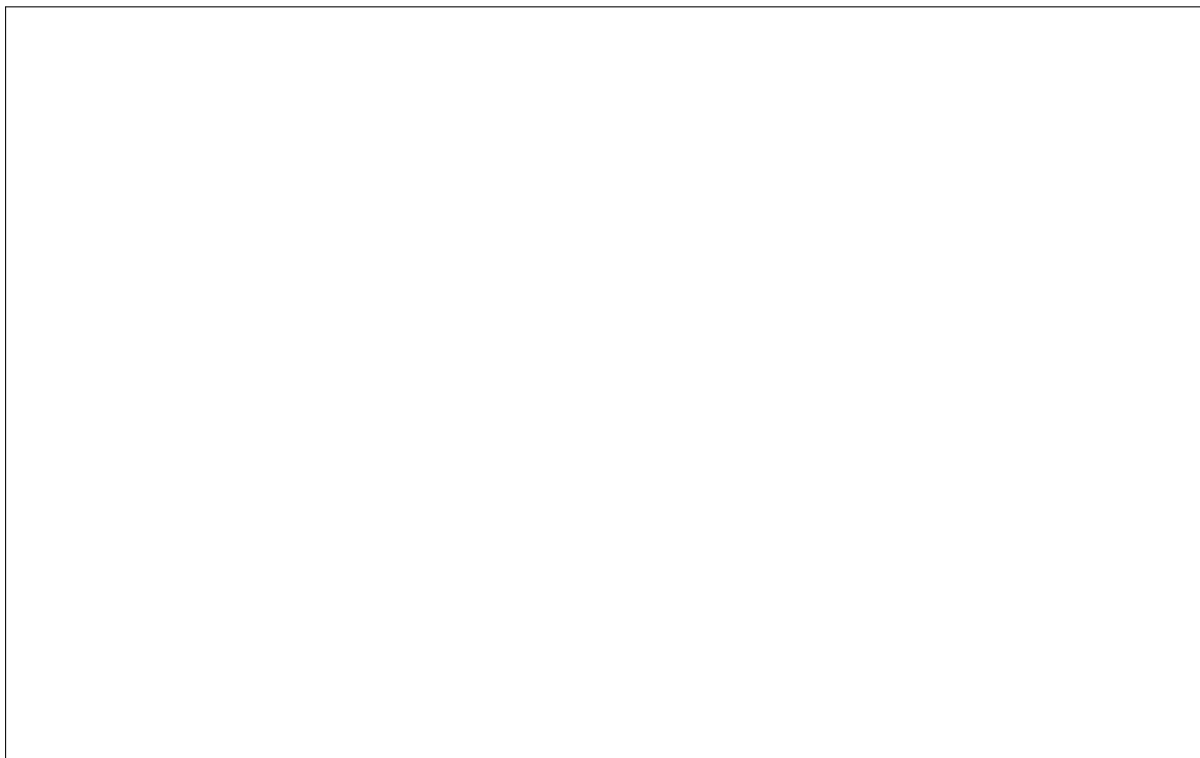
\_\_\_\_\_/1 p.

$$\lim_{x \rightarrow 1} \frac{\ln x}{e^x - e}.$$

**Exercise 3**

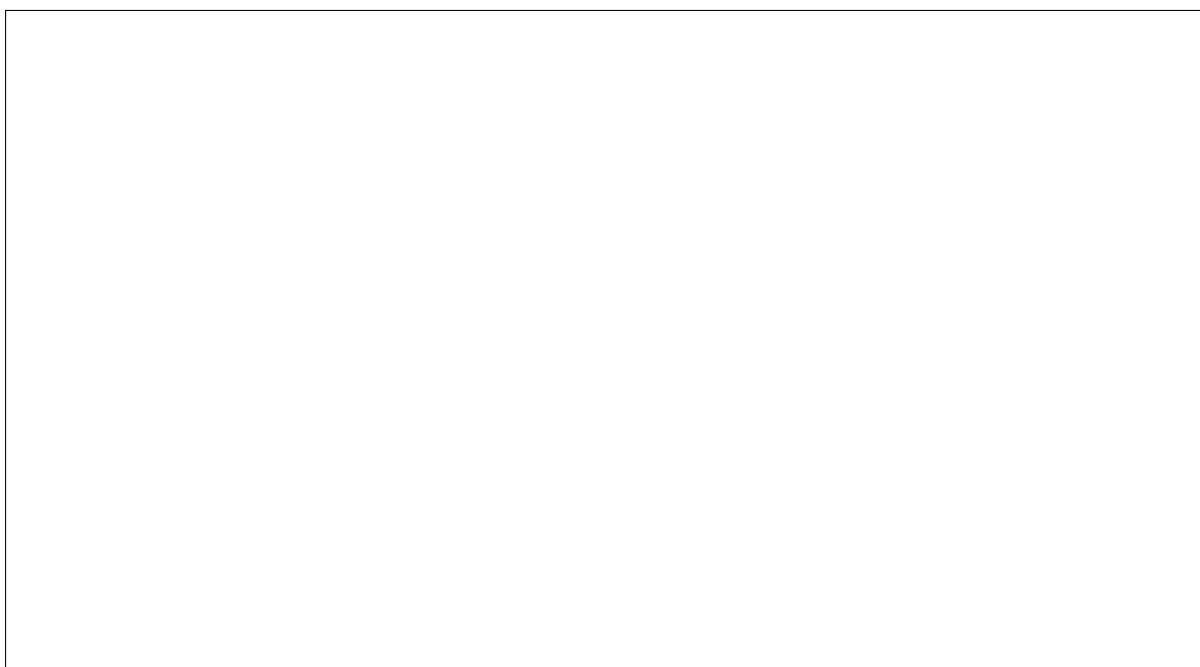
Describe and sketch the set  $A = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 > 0\}$ .

\_\_\_\_\_/   
 1 p.

**Exercise 4**

Let  $z = \frac{1+i}{1-i}$ . (i) Write  $z$  in Cartesian form. (ii) Write  $z$  in exponential form. (iii) Compute  $z^{2023}$ .

\_\_\_\_\_/   
 1 p.



**Exercise 5**

Find the equation of the tangent line to  $y = x^3$  at the point  $(1, 1)$ .

\_\_\_\_\_/   
 1 p.

**Exercise 6**

Verify that  $f(x) = 2x^5 + x^3 + 5x$  is invertible on  $\mathbb{R}$  and that  $f^{-1}$  is differentiable. Compute  $(f^{-1})'(0)$  and  $(f^{-1})'(8)$ .

\_\_\_\_\_/   
 1 p.

**Exercise 7**

Consider the sequence  $a_n = \frac{n!}{n^{100}}$ ,  $n \in \mathbb{N}_+$ . Does it have a limit as  $n \rightarrow \infty$ ? If so, what is it?

\_\_\_\_\_/1 p.

**Exercise 8**

Determine whether the series  $\sum_{n=1}^{\infty} \frac{n}{2^n}$  converges, diverges or is indeterminate. Justify your answer.

\_\_\_\_\_/1 p.

**Exercise 9**

Evaluate the indefinite integral  $I = \int e^x \sin x \, dx$ .

\_\_\_\_\_/   
 1 p.

**Exercise 10**

Determine whether  $\int_e^\infty \frac{1}{x(\ln x)^2} \, dx$  converges, and compute its value if it does.

\_\_\_\_\_/   
 1 p.

## Part B

### Exercise 11

Prove that the ceiling function  $f(x) = \lceil x \rceil$  is left-continuous at every  $x_0 \in \mathbb{R}$ .

\_\_\_\_\_  
3 p.

**Exercise 12**

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined and continuous on  $\mathbb{R}$ , let  $x_0 \in \mathbb{R}$  and let  $F(x) = \int_{x_0}^x f(y) \, dy$ . State and prove the Fundamental Theorem of Integral Calculus. In your proof you may rely on the Integral Mean Value Theorem and the Squeeze Theorem without proving them.

\_\_\_\_\_  
3 p.



**Exercise 13**

Using Maclaurin polynomials, determine  $\alpha \in \mathbb{R}$  so that

$$f(x) = (\arctan 2x)^2 - \alpha x \sin x$$

is infinitesimal of order 4 with respect to  $\varphi(x) = x$  as  $x \rightarrow 0$ .

\_\_\_\_\_  
3 p.

**Exercise 14**

Compute the indefinite integral  $I = \int \cos^4 x \, dx$  using integration by parts.

\_\_\_\_\_  
3 p.

**Exercise 15**

Determine the order and the principal part with respect to  $\varphi(x) = \frac{1}{x}$  as  $x \rightarrow +\infty$  of the function  $f(x) = \sin(\sqrt{x^2 - 1} - x)$ . \_\_\_\_\_/ 3 p.

