

MATHEMATICAL ANALYSIS 1
HOMEWORK 13

- (1) Compute the area under the graph of $f(x) = |\ln x|$ on the interval $[e^{-1}, e]$.
 (2) Determine the following function:

$$F(x) = \int_{-1}^x (|y-1| + 2) dy.$$

- (3) Compute the area between f and g :

- (a) $f(x) = |x|$ and $g(x) = \sqrt{1-x^2}$.
 (b) $f(x) = x^2 - 2x$ and $g(x) = -x^2 + x$.

- (4) **Improper integrals I.** Compute the following improper integrals:

- (a) $\int_0^{+\infty} \frac{1}{x^2+3x+2} dx$. (c) $\int_0^{+\infty} \frac{x}{(x+1)^3} dx$.
 (b) $\int_2^{+\infty} \frac{1}{x\sqrt{x-2}} dx$. (d) $\int_{-1}^1 \frac{1}{\sqrt{x(x-4)}} dx$.

- (5) **Improper integrals II.** Discuss the convergence of the following improper integrals:

- (a) $\int_0^{+\infty} \frac{\sin x}{x\sqrt{x}} dx$. (c) $\int_0^{+\infty} x e^{-x} dx$.
 (b) $\int_0^{+\infty} \frac{1}{\ln^2(2+e^x)} dx$. (d) $\int_0^\pi \frac{x-\frac{\pi}{2}}{\cos x \sqrt{\sin x}} dx$.

- (6) For which $n \in \mathbb{N}$ does the following integral converge

$$\int_2^{+\infty} \frac{x}{\sqrt{(x^2+3)^n}} dx \quad ?$$

Compute the integral for the smallest n for which is converges.

- (7) For which $\alpha \in \mathbb{R}$ does the integral

$$\int_2^3 \frac{x(\sin(x-2))^\alpha}{\sqrt{x^2-4}} dx$$

converge? What is its value when $\alpha = 0$?

- (8) **Geometric series.**

- (a) Let $r \in \mathbb{R}$ and let $\{r^k\}_{k=0}^\infty = (1, r, r^2, \dots)$ be a geometric sequence. Show that the partial sum $s_n = \sum_{k=0}^n r^k$ satisfies

$$s_n = \frac{1-r^{n+1}}{1-r} \quad \text{for } r \neq 1.$$

- (b) Using a geometric series, write the number $2.3\overline{17} = 2.3171717\dots$ as a fraction.

- (9) **Positive-term series.** Study the convergence of the following series:

- (a) $\sum_{k=1}^\infty \frac{\ln k}{k^\alpha}$ for $\alpha > 0$. (c) $\sum_{k=1}^\infty \sin \frac{1}{k}$
 (b) $\sum_{k=1}^\infty \frac{1}{2^k-1}$ (d) $\sum_{k=1}^\infty \frac{k+3}{\sqrt[3]{k^9+k^2}}$

- (10) **Alternating sign series.** Study the convergence of the following series:

- (a) $\sum_{k=1}^\infty \sin(k\pi + \frac{1}{k})$. (c) $\sum_{k=1}^\infty (-1)^k \ln(\frac{1}{k} + 1)$.
 (b) $\sum_{k=1}^\infty (-1)^{k+1} \frac{k^2}{k^3+1}$. (d) $\sum_{k=1}^\infty (-1)^k \sqrt{\frac{k^3+3}{2k^3-5}}$.

- (11) **Absolute convergence.** Do the following series converge absolutely?

- (a) $\sum_{k=1}^\infty \frac{(-1)^{k+1} 5^{k-1}}{(k+1)^{24k+2}}$. (b) $\sum_{k=1}^\infty \frac{\cos 3k}{k^3}$.