

Università degli Studi di Roma “Tor Vergata”

Dipartimento di Matematica

Analysis 1 (Engineering Sciences) 2025-2026

Instructor: Prof. Jonathan Ben-Artzi

Final Examination — *Call 1 of 6*

27 January 2026

First Name (CAPITALS): _____

Last Name (CAPITALS): _____

Matricola: _____

Grading Summary

Quest.	1	2	3	4	5	6	7	8	9	10	Total
Points	1	1	1	1	1	1	1	1	1	1	10
Score											

Quest.	11	12	13	14	15	Total
Points	3	3	3	3	3	15
Score						

FINAL GRADE / **25**

Examination Rules:

- **Duration:** 2 hours and 30 minutes.
- **NO** cellphones, **NO** calculators, **NO** books, **NO** notes, and **NO** headphones.
- Write full solutions clearly within the provided spaces.
- Part B will only be graded if the student achieves a score of at least 9/10 in Part A.
- Any student caught copying or engaging in academic misconduct will face disciplinary action.
- Use only blue or black ink. Additional paper will be provided upon request.

Do not turn this sheet over until instructed to do so.

Part A

Exercise 1

Find the following limit. Explain your answer.

1 p.

$$\lim_{x \rightarrow 1} \frac{\ln x}{e^x - e}.$$

Exercise 2

Find the following limit. Explain your answer.

1 p.

$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{x}.$$

Exercise 3

Sketch $f(x) = \log_{10}(x^2)$ and specify what are $\text{dom}(f)$ and $\text{im}(f)$.

1 p.

Exercise 4

For $f(x) = 1/x$ and $A = [-1, 0)$, determine $\sup_A f$ and $\inf_A f$, and state whether they are attained.

1 p.

Exercise 5

Compute the derivative of $f(x) = \arcsin(\sqrt{x})$ for $x \in (0, 1)$.

_____ /
1 p.

Exercise 6

Find the equation of the tangent line to $y = \frac{x}{1+x^2}$ at $x_0 = 1$.

_____ /
1 p.

Exercise 7

Let $g(x) = \frac{x^2}{x^2+1}$. Compute the sum $\sum_{n=0}^{\infty} g(x)^n$ for any $x \in \mathbb{R}$ (with the convention $0^0 = 1$). _____ / 1 p.

Exercise 8

Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ converges, diverges or is indeterminate. Justify your answer. _____ / 1 p.

Exercise 9

Compute the average value of $f(x) = \frac{1}{3}(\sin x + 5 \cos x)$ on the interval $[\frac{\pi}{2}, \frac{5\pi}{2}]$.

1 p.

Exercise 10

Evaluate the definite integral $\int_0^1 \arctan x \, dx$.

1 p.

Part B

Exercise 11

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous on the closed interval $[a, b]$. State and prove the Intermediate Value Theorem for f . (In your proof you may rely on the corollary to Bolzano's Theorem which states that if f, g are continuous on $[a, b]$ and $f(a) < g(a)$, $f(b) > g(b)$, then there exists an intermediate point x_0 where $f(x_0) = g(x_0)$)

3 p.

Exercise 12

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuously differentiable. Assume that $f'(x) > 0$ for all $x \in \mathbb{R}$. Prove that 3 p. /
 f is injective.

Exercise 13

Determine the order and the principal part as $x \rightarrow +\infty$ with respect to $\varphi(x) = \frac{1}{x}$ of the 3 p. infinitesimal function

$$f(x) = \sqrt[3]{1 + 3x^2 + x^3} - \sqrt[5]{2 + 5x^4 + x^5}.$$

Exercise 14

Compute the indefinite integral $I = \int \cos^4 x \, dx$ using integration by parts.

_____ /
3 p.

Exercise 15

Using the expression for the derivatives of $\arctan x$, show that

$$\arctan \frac{1}{x} = \frac{\pi}{2} - \arctan x, \quad \forall x > 0.$$

_____ /
3 p.

