MATHEMATICAL ANALYSIS 1
HOMEWORK 7

Since we are at the mid-point of the semester, this assignment is minimal. Just one exercise, to deepen your understanding of the theorems that we've seen this week:

(1) In this problem we prove the **corollary to the Existence of Zeroes theorem** regarding a zero of a continuous function on an open interval that may be infinite (you are guided in the steps below):
   (a) State the corollary (it is Corollary 7.2 in the lecture notes).
   (b) Suppose that $\ell_\alpha < 0 < \ell_\beta$ (state that the case $\ell_\alpha > 0 > \ell_\beta$ follows a similar proof, and is therefore omitted).
   (c) By Theorem 5.2 of the lecture notes there exists a neighborhood of α on which $f < 0$ (here you need to consider the two cases of α being finite or $-\infty$).
   (d) A similar argument is applied for a neighborhood of β.
   (e) In these neighborhoods pick points a and b.
   (f) Argue that Theorem 7.1 (existence of zeroes) is now applicable for f on $[a, b]$.
   (g) Conclude the proof.

Please use this opportunity to review the main concepts/theorems that we have seen so far (this is not an exhaustive list, please see your notes and lecture notes online):

- Elements of logic
  - Proof by induction
  - Proof by contradiction
- Functions
  - Injectivity, surjectivity, invertibility
  - Monotonicity (also of sequences)
  - The elementary functions
- Sequences
  - Subsequences, The **Bolzano-Weierstrass Theorem**
- Complex numbers
- Limits
  - The number e
  - Asymptotes
  - The $\varepsilon - \delta$ formalism
  - Continuity and types of discontinuities
  - Left and right limits
  - Algebra of limits
  - Indeterminate ("meaningless") limits
  - Comparison theorems
- Asymptotic behavior of functions
  - Landau symbols
  - Order of a function, principal part
- Continuous functions on intervals
  - **Bolzano's Theorem** (existence of zeroes)
  - **Intermediate Value Theorem**
  - **Weierstrass' Theorem**
  - Uniform continuity and Lipschitz
  - **Heine-Cantor Theorem**
  - Invertibility