# MATHEMATICAL ANALYSIS 1
# HOMEWORK 10

(1) Prove the following proposition: (it is Proposition 9.4 in the lecture notes)

**Proposition.** *Any Maclaurin polynomial of an even function contains only even powers. Any Maclaurin polynomial of an odd function contains only odd powers.*

(2) Let $p_n(x) = a_0 + a_1 x + \cdots a_n x^n$ and $q_m(x) = b_0 + b_1 x + \cdots b_m x^m$ be two polynomials of orders $n$ and $m$ respectively. Write the formula for $p_n(x) \cdot q_m(x)$ *(pay attention: $n$ and $m$ might be different).* Explain your answer.

(3) Suppose that two functions $f, g$ can be written as

$$f(x) = p_n(x) + o(x^n) \qquad g(x) = q_m(x) + o(x^m)$$

as $x \to 0$, where $p_n, q_m$ are as in the previous question. Express the product $f(x) \cdot g(x)$. What is the order of the error? Explain your answer.

(4) Write $(Tf)_{n,x_0}(x)$ for the following $f, n, x_0$:

(a) $f(x) = e^x$, $n = 4$, $x_0 = 2$
(b) $f(x) = \ln x$, $n = 3$, $x_0 = 3$
(c) $f(x) = 7 + x - 3x^2 + 5x^3$, $n = 2$, $x_0 = 1$
(d) $f(x) = \sin x$, $n = 6$, $x_0 = \frac{\pi}{2}$
(e) $f(x) = \sqrt{2x+1}$, $n = 3$, $x_0 = 4$

(5) Write the Maclaurin expansions up to the indicated order with Peano's remainder:

(a) $f(x) = x \cos 3x - 3 \sin x$, $n = 2$
(b) $f(x) = e^{x^2} \sin 2x$, $n = 5$
(c) $f(x) = \ln \frac{1+x}{1+3x}$, $n = 4$
(d) $f(x) = \frac{x}{\sqrt[6]{1+x^2}} - \sin x$, $n = 5$

(6) Prove that there exists a neighborhood of 0 on which the following inequality holds:

$$2\cos(x + x^2) \leq 2 - x^2 - 2x^3.$$

(7) (a) Write the Maclaurin polynomial of $e^x$ of order $n$ with Lagrange's remainder.
(b) What is the minimal $n$ we should take if we want to approximate the number $e$ to within $\frac{1}{1000000}$ (one millionth)? Justify your answer.