Homework 10

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Exercise (3.39). *Compute the order of each element $T \in D_4$.*

Solution. $\mathcal{O}(R_0) = 1$
$\mathcal{O}(R_{90}) = 4$
$\mathcal{O}(R_{180}) = 2$
$\mathcal{O}(R_{270}) = 4$
$\mathcal{O}(F_v) = 2$
$\mathcal{O}(F_h) = 2$
$\mathcal{O}(F_{D_1}) = 2$
$\mathcal{O}(F_{D_2}) = 2$

Theorem (3.42). *If G is a cyclic group, then G is abelian.*

Proof. Let G be generated by a and let g and h be elements of G. $g = a^m$ and $h = a^n$, $m, n \in \mathbb{Z}$. $g * h = h * g$
So, $a^n * a^m = a^m * a^n$
$a^{n+m} = a^{m+n}$ Since addition is cumulative, this cyclic group is abelian. □

Exercise (3.44).
1. *Give an example of an infinite group that is abelian but not cyclic.*
2. *Give an example of a finite group that is abelian but not cyclic.*

Solution.
1. $\{\mathbb{R} \setminus \{0\}, \cdot\}$
2. The group of $\mathbb{Z}_4 \cdot \mathbb{Z}_2$

Exercise (3.46). *Give examples of Groups G in which*

1. $Z(G) = \{e\}$
2. $Z(G) = G$
3. $\{e\} \subsetneq Z(G) \subsetneq G$

Solution.
1. $Z(D_3) = \{R_)\}$
2. $\{2\mathbb{Z}, +\}$

3. $Z(D_4) = \{R_0, R_{180}\}$

Exercise (3.47)**.** *1. Consider* $H = \{R_0, [flipacrossahorizontalline]\}$ *a subgroup of* D_4*. Write out the left cosets of* H*. Also write out the right cosets of* H*.*

2. Consider $K = \{[0]_{12}, [3]_{12}, [6]_{12}, [9]_{12}\}$ *a subgroup of* $\mathbb{Z}_{12}$*. Write out the left and right cosets.*

Solution. 1. The left cosets of: $R_0H = \{R_0, F_H\}$; $R_{90}H = \{R_{90}, D_2\}$; $R_{180}H = \{R_{180}, F_V\}$; $R_{270}H = \{R_{270}, D_1\}$; $F_HH = \{F_H, R_0\}$; $F_VH = \{F_V, R_{180}\}$; $F_{D_1}H = \{F_{D_1}, R_{270}\}$; $F_{D_2}H = \{F_{D_2}, R_{90}\}$

Lemma 1 (3.48)**.** *Let* H *be a subgroup of* G *and let* g *and* $g\prime$ *be elements of* G*. Then the cosets* gH *and* $g\prime H$ *are either identical or disjoint.*

Proof. Let $g, g\prime \in G$ and $h \in H$ $g * h = g\prime h$ Since $h = h$, for gh to equal $g\prime h$, g must equal $g\prime$. If $gh \neq g\prime h$ $g \neq g\prime$ □

Theorem (3.49)**.** *Let* G *be a finite group with subgroup* H*. Then* $|H|$ *divides* $|G|$*.*

Proof. Write your proof here. □

Scholium 1 (3.51)**.** *Let* G *be a finite group with a subgroup* H*. Then* $[G : H] = |G|/|H|$*.*

Proof. Write your proof here. □

Corollary (3.52)**.** *Let* G *be a finite group with an element* g*. Then* $\mathcal{O}(g)$ *divides* $|G|$*.*

Proof. Write your proof here. □