Reflection 2

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Part 1: Useful Mistakes

A lot of the mistakes I have made were mistakes I didn't originally notice, only until the grading process did I discover the mistakes. These mistakes help me realize the rigor required for proofs. Some specific mistakes I have made, are not properly doing if and only if proofs, I assume one side and prove the other, but I don't properly prove it by additionally assuming the other side and proving the first side. This is an easy mistake to correct in the future, I just need to finish the proof. Another mistake I make are in induction proofs, where I select the wrong base case. This too is an easy mistake to fix, since arguably in an induction proof the base case is the trivial step the real work comes from proving the claim works for the next item. I just need to double check the base case is indeed the correct base case. Another mistake I have made a few times is assuming one thing, and then proving my assumption. This goes against the spirit of an implication, as I should assume something, and then prove something different. To remedy this, I need to be more mindful while solving proofs.

Part 2: Types of Argument

The first type of argument is proof by contradiction. The type of argument was used in proving Theorem 2.28. This argument works well when dealing with a claim that assumes something is true, and as a result something else must also be true. Specifically, this works well when the assumption deals with a property of a Graph. An outline of this proof would be:

Proof. Assume towards contradiction the negation of some part (or all) of the claim. By unpacking definitions, find a contradiction. Since there is a contradiction, our assumption must be false. So we have the negation of assumption, which was the negation of the claim. Therefore, we end with the claim, by using double negation. $\square$

The second type of argument is proof by induction. This type of argument was used in proving Theorem 2.51. This argument works well when dealing with claims that involve manipulating a previous number/graph/etc. with the results still following some property. Summations are a specific example of this. An outline of this proof would be:

Proof. Prove the claim holds true for the base case. Now assume the claim works for the previous iteration and show the claim works for the next iteration (so assume it works for k and prove it works for $k+1$). This assumption (the induction hypothesis) will mostly likely be used for a substitution during the process of proving it works for the next iteration. $\square$

The third type of argument is proof by contrapositive. This type of argument was used in proving Theorem 2.31. This argument works well with conditionals. This argument style is an alternative to proof by implication. An outline for this proof would be:

Proof. If the claim is: If P, then Q, you would start by supposing the negation of Q. By using some proof techniques, you would need to show that the negation of P is true (so P is false). $\square$

Part 3: Three Polished Proofs

Corollary (2.58)**.** *If G is a tree with n vertices, then G has $n-1$ edges.*

Proof. A tree is a connected, planar graph, by Theorem 2.46, so by Theorem 2.55, $|V| - |E| + |F| = 2$. Since G has n vertices, let $n = V$, so $n - |E| + 1 = 2$. Since G is a tree and therefore is planar, it only has 1 face, the unbounded face. $n - 1 = |E|$. Therefore the number of edges is $n - 1$. $\square$

My proof for Corollary 2.58 has a interesting result since this theorem shows that there are many ways to prove a claim. We have proved this claim previously, and the fact that this claim can be proven using different arguments if important for proof writing, as there are many ways to write a proof.

Corollary (2.61)**.** *The graph K_5 is not planar.*

Proof. Assume towards contradiction K_5 to be a planar graph. Since K_5 is a complete graph, it contains no loops and has unique edges for each pair of distinct vertices. So by Theorem 2.59, If $|V| \geq 3$, then $|E| \leq 3|V| - 6$. K_5 has 5 vertices and 10 edges. $10 \nleq 3 * 5 - 6$ contradiction, so K_5 is not planar. $\square$

Corollary 2.61 has an important argument, that K_5 is planar. K_5 is a graph that we have been looking at since the start of the semester, and it important that we can now definitely proved that the 5-station problem is not possible. The ability to determine if a graph is planar in also important.

Theorem (2.29)**.** *Let G be a graph. Let C be a subgraph of G that consists of the vertices and edges that belong to a circuit in G. Then $deg_C(v)$ is even for every vertex v of C.*

Proof. Let C be a subgraph of G that consists of the vertices and edges that belong to a circuit in G. Consider taking a walk through C. Since C is a circuit, by definition a walk must end at the same vertex it began at, and edges can not be repeated. So, for every time you walk into a vertex, you must walk away from the vertex on another edge, since you cannot repeat edges and must return at the starting vertex, so the walk cannot end at that vertex, unless it is the first vertex in the walk. So for every passage through a vertex, there must be a multiple of 2 edges with endpoints at that vertex (one edge could be used for entry, and the other for exiting the vertex, and this process could be repeated m times if the degree of the vertex is $2m$. Therefore, for n amount of passages through a vertex, there are $2n$ edges with endpoints at that vertex, so the degree of that vertex is $2n$, which is even since 2 times a integer is even. □

My proof for Theorem 2.29 has 3 characteristics of good style in the exposition. First, I clearly define the notation and variables used in the proof, such as C and G. Second, I provide definitions of terms, such as circuit. Lastly, my proof is concise and does not contains information that is not used. These are all examples of good style.