Fundamental Principles of Counting

Contents

1.1 The Rules of Sum and Product

The Rule of Sum: If a first task can be performed in m ways, while a second task can be performed in n ways, and the two tasks cannot be performed simultaneously, then performing either task can be accomplished in any one of $m + n$ ways.

Example 1.1

A college library has 40 textbooks on sociology and 50 textbooks dealing with anthropology. By the rule of sum, a student at this college can select among $40 + 50 = 90$ textbooks in order to learn more about one or the other of these two subjects.

Example 1.2

The rule can be extended beyond two tasks as long as no pair of the tasks can occur simulatneously. For instnace, a computer science instructor who has, say, five introductory books each on C++, FORTRAN, Java, and Pascal can recommend any one of these 20 books to a student who is interested in learning a first programming language.

Example 1.3

The computer science instructor of Example 1.2 has two colleagues. One of these colleagues has three textbooks on the analysis of algorithms, and the other has five such textbooks. If n denotes the maximum number of different books on this topic that this instructor can borrow from them, then $5 \leq n \leq 8$, for here both colleagues *may* own copies of the same textbook(s).

The Rule of Product: If a procedure can be broken down into first and second stages, and if there are m possible outcomes for the first stage and if, for each of these outcomes, there are n possible outcomes for the second stage, then the total procedure can be carried out, in the designated order, in mn ways.

Example 1.4

In trying to reach a decision on plant expansion, an administrator assigns 12 of her employees to two committees. Committee A consists of five members and is to investigate possible favorable results from such an expansion. The other seven employees, committee B, will scrutinize possible unfavorable repercussions. Should the administrator decide to speak to just one committee member before making her decision, then by the rule of sum there are 12 employees she can call upon for input. However, to be a bit more unbiased, she decides to speak with a member of committee A on Monday, and then with a member of committee B on Tuesday, before reaching a decision. Using the following principle, we find that she can select two such employees to speak with in $5 \times 7 = 35$ ways.

Example 1.5

The drama club of Central University is holding tryouts for a spring play. With six men and eight women auditioning for the leading male and female roles, by the rule of product the director can cast his leading couple in $6 \times 8 = 48$ ways.

Example 1.6

Here various extensions of the rule are illustrated by considering the manufacture of license plates consisting of two letters followed by four digits.

a. If no letter or digit can be repeated, there are $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3,276,000$ different possible plates.
b. With repetitions of letters and digits allowed, $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6,760,000$ different license plates are possible.
c. If repetitions are allowed, as in part (b), how many of the plates have only vowels (A, E, I, O, U) and even digits? (0 is an even integer.)

Example 1.7

In order to store data, a computer's main memory contains a large collection of circuits, each of which is capable of storing a *bit*—that is, one of the *bi*nary digi*t*s 0 or 1. These storage circuits are arranged in units called (memory) cells. To identify the cells in a computer's main memory, each is assigned a unique name called its *address*. For some computers, such as embedded microcontrollers (as found in the ignition system of an automobile), an address is represented by an ordered list of eight bits, collectively referred to as a *byte*. Using the rule of product, there are $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 256$ such bytes. So we have 256 addresses that may be used for cells where certain information may be stored.
A kitchen appliance, such as a microwave oven, incorporates a small computer whose main memory may contain hundreds of cells. Some older computers (such as the PDP 11 family) contained thousands of memory cells and used two-byte addresses to identify these cells in their main memory. Such addresses are made up of two consecutive bytes, or 16 consecutive bits. Thus there were $256 \times 256 = 2^8 \times 2^8 = 2^16 = 65,536$ available addresses that could be used to identify cells in th emain memory. Other computers using addressing systems of four bytes. This 32-bit architecture is used in the Pentium and SPARC processors, where there are many as $2^8 \times 2^8 \times 2^8 \times 2^8 = 2^{32} = 4,294,967,296$ addresses for use in identifying the cells in the main memory. When a programmer deals with the Alpha processor, he or she considers memory cells with eight-byte addresses. Each of these addresses comprises $8 \times 8 = 64$ bits, and there are $2^{64} = 18,446,744,073,709,551,616$ possible addresses for this architecture. (Of course, not all of these possibilities are actually used.)

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