Introduction

Practice Questions

John C. Hull

Contents

1. Bearish Option strategies

Explain carefully the difference between selling a call option and buying a put option.

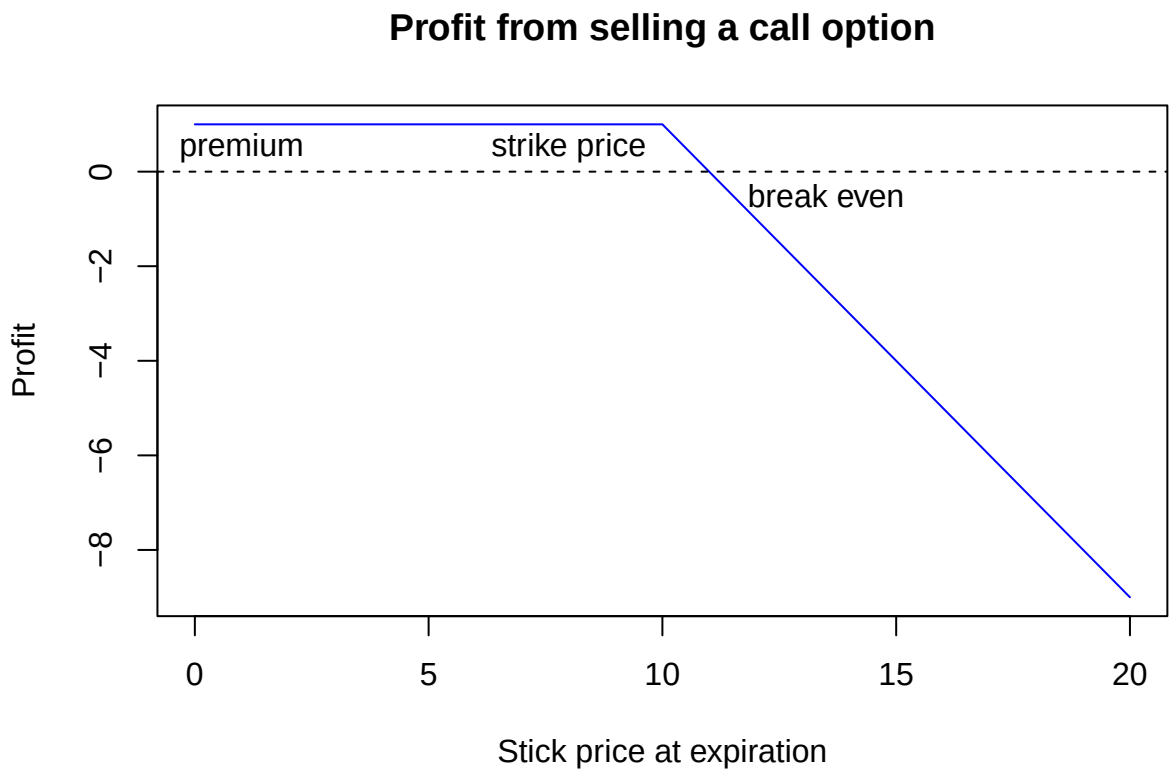
Selling a call option (short call)

Description You sell someone the right to buy a stock from you at a specific price (strike price) before a certain date (expiration). You collect a premium up front.

Your outlook You're bearish or neutral—you think the stock won't rise above the strike price.

Profit and loss Your maximum profit is the premium you receive (if the stock stays at or below the strike price). Your maximum loss is unlimited (if the stock goes way up, you may have to sell it at the strike price, which could be much lower than market value.

$$-\max(S_T - K, 0) = \min(K - S_T, 0)$$



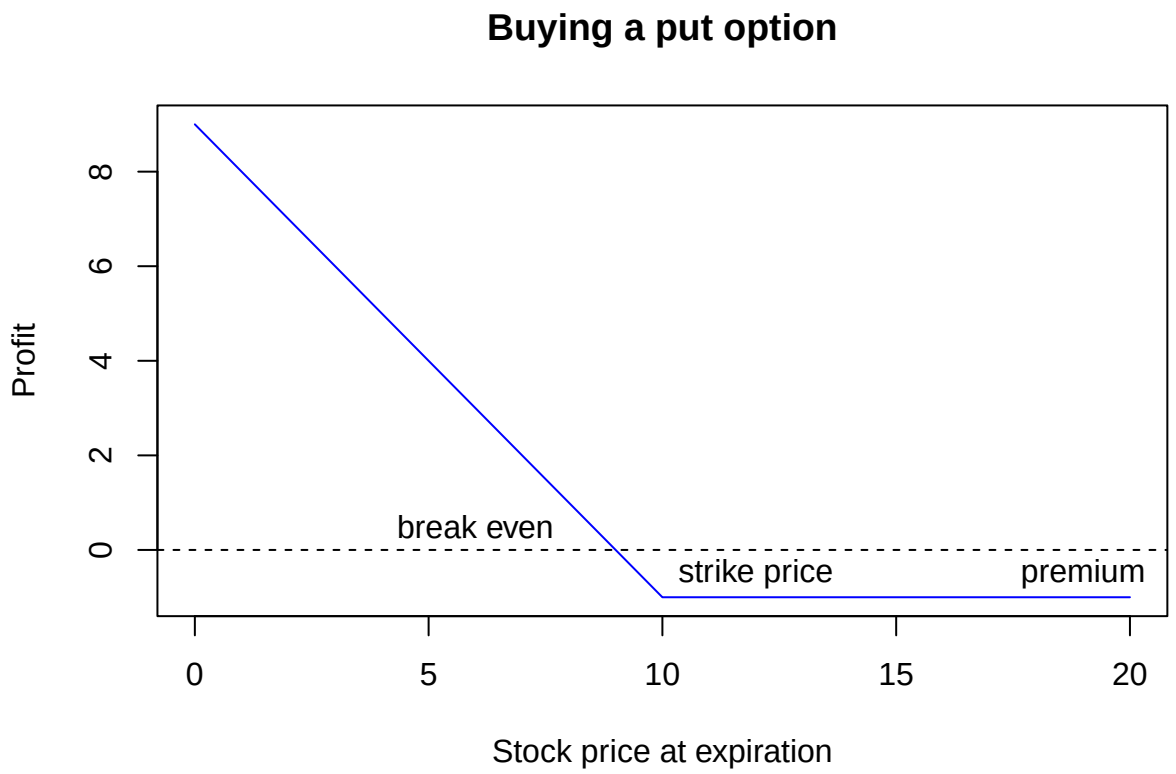
Buying a put option (long put)

Description You buy the right to sell a stock at a specific price (strike price) before a certain date. You pay a premium for this right.

Your outlook You're bearish—you think the stock will fall below the stike price.

Profit and loss Your maximum profit is the strike price - premium (if the stock goes to $0). Your maximum loss is the premium youpaid (if the stock stays above the strike price).

$$\max(K - S_T, 0)$$



Conclusion

In both cases, the potential payoff is $K - S_T$. When you write a call option, the payoff is negative or zero. (This is because the counterparty chooses whether to exercise.) When you buy a put option, the payoff is zero or positive. (This is because you choose whether to exercise.)

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2. Short forward contract for foreign exchanges

An investor enters into a short forward contract to sell 100,000 British pounds for U.S. dollars at an exchange rate of 1.3000 USD per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.2900 and (b) 1.3200?

To determine the gain or loss from a short forward contract, we look at the difference between the forward price (the agreed price) and the spot price (the actual price at maturity), multiplied by the contract size.

A short forward contract will be selling GBP. The investor benefits if the spot rate is lower than the forward rate.

Given:

- Contract type: Short forward (selling GBP)
- Contract size: 100,000 GBP
- Forward rate: 1.3000 USD/GBP

- Spot rate at maturity:
    a. 1.2900 USD/GBP
    b. 1.32000 USD/GBP

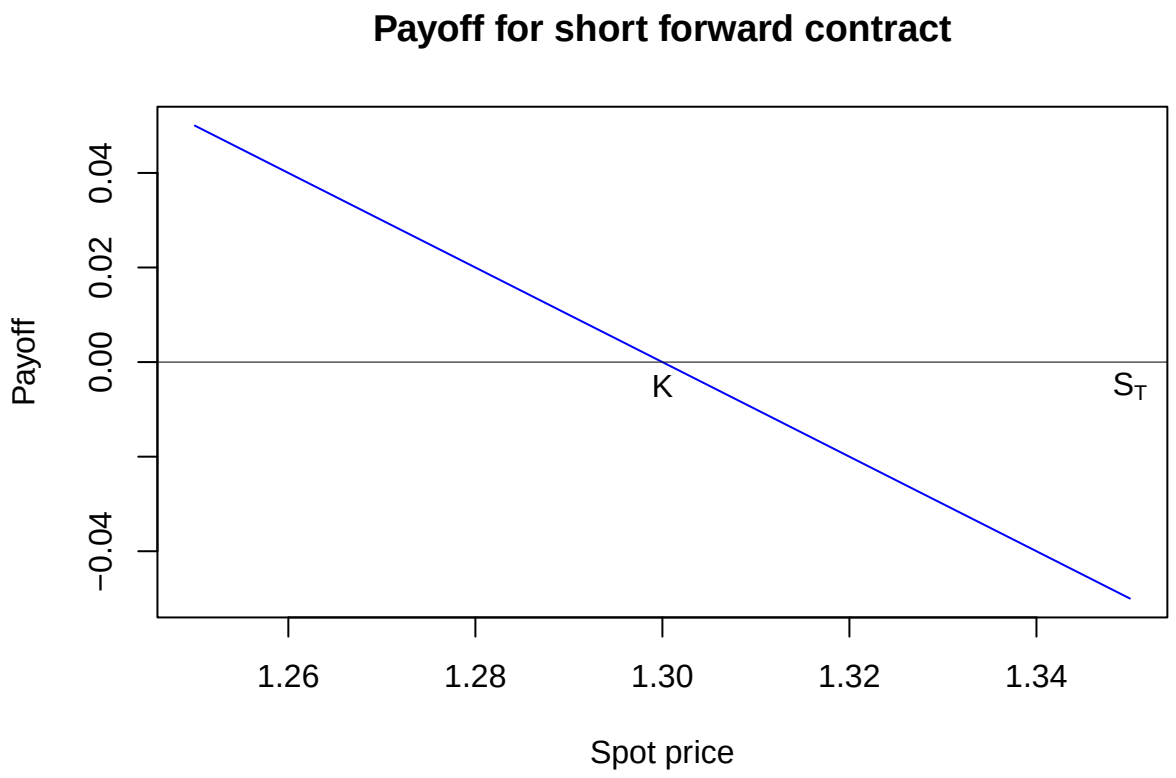
Formula for a short forward

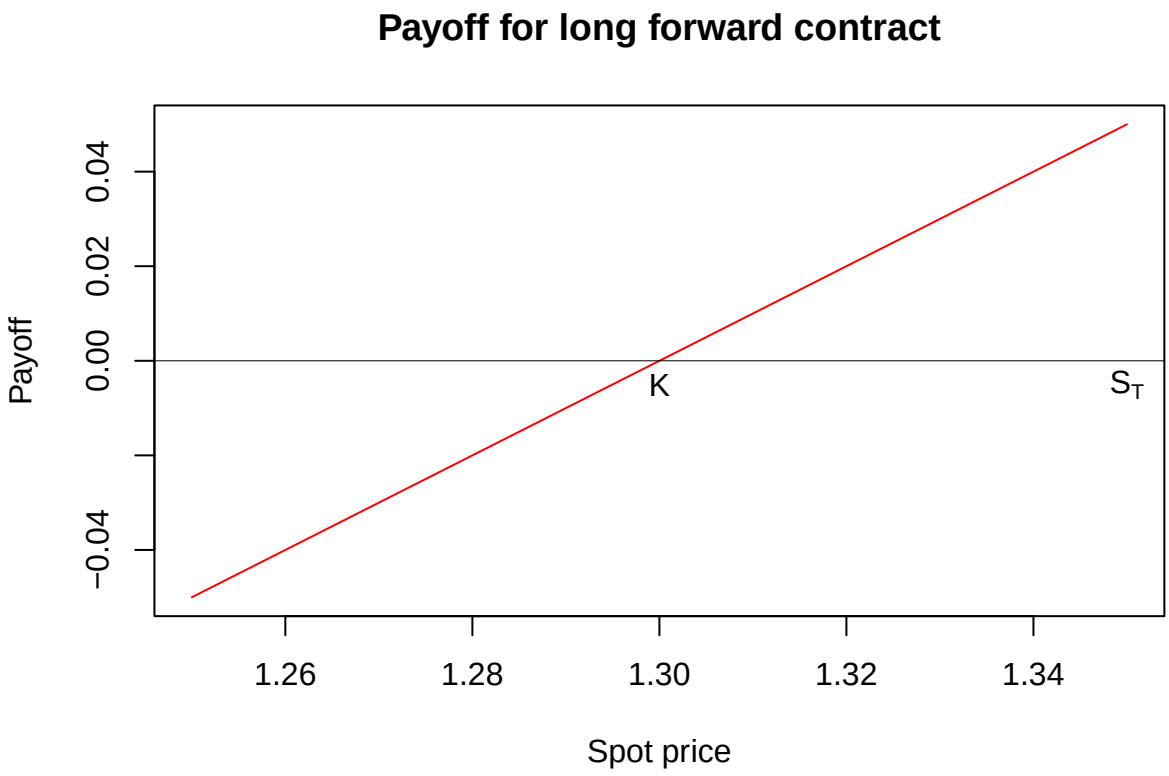
$$\begin{aligned}\text{Gain/Loss} &= (\text{Forward rate} - \text{Spot rate}) \times \text{Contract size} \\ &= (K - S_T) \times C\end{aligned}$$

Where:

- S_T is the spot price.
- K is the delivery price.
- C is the contract size.

$$\begin{aligned}&\text{for spot price 1.2900 USD/GBP:} \\ &\qquad\text{Gain/Loss} = (1.3000 - 1.2900) \times 100,000 \\ &\qquad\qquad = \$1,000 \\ &\text{for spot price 1.3200 USD/GBP:} \\ &\qquad\text{Gain/Loss} = (1.3000 - 1.3200) \times 100,000 \\ &\qquad\qquad = (\$2,000)\end{aligned}$$





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3. Short futures contracts for commodities

A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound and (b) 51.30 cents per pound?

A trader that enters into a short futures contract gains when the price of the underlying asset drops and loses when the pice rises.

Given:

- Short (selling) position in cotton futures
- Initial futures price: 50.00 cents/pound
- Contract size: 50,000 pounds
- Spot price at maturity:
  a. 48.20 cents per pound
  b. 51.30 cents per pound

Short Futures Formula:

$$\begin{aligned}\text{Gain/Loss} &= (\text{Initial futures price} - \text{Spot price at maturity}) \times \text{Contract size} \\ &= (K - S_T) \times C\end{aligned}$$

Where:

- K Futures price
- S_T Spot price at maturity
- C Contract size

$$\begin{aligned}&\text{for spot price 48.20:} \\ &\qquad \text{Gain/Loss} = (50 - 48.20) \times 50,000 \\ &\qquad\qquad = \$900 \\ &\text{for spot price 51.30:} \\ &\qquad \text{Gain/Loss} = (50 - 51.30) \times 50,000 \\ &\qquad\qquad = (\$650)\end{aligned}$$

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4. Short put option analysis

Suppose that you write a put contract with a strike price of $40 and an expiration date in 3 months. The current stock price is $41 and the contract is on 100 shares. What have you committed yourself to? How much could you gain or lose?

When you write (i.e., sell) a put option, you are giving the buyer the right, but not the obligation, to sell you the stock at the strike price.

Given:

- Position: you wrote a put (short put)
- Strike price: $40
- Current stock price: $41
- Shares per contract: 100
- Time to expiration: 3 months

What writing a put means

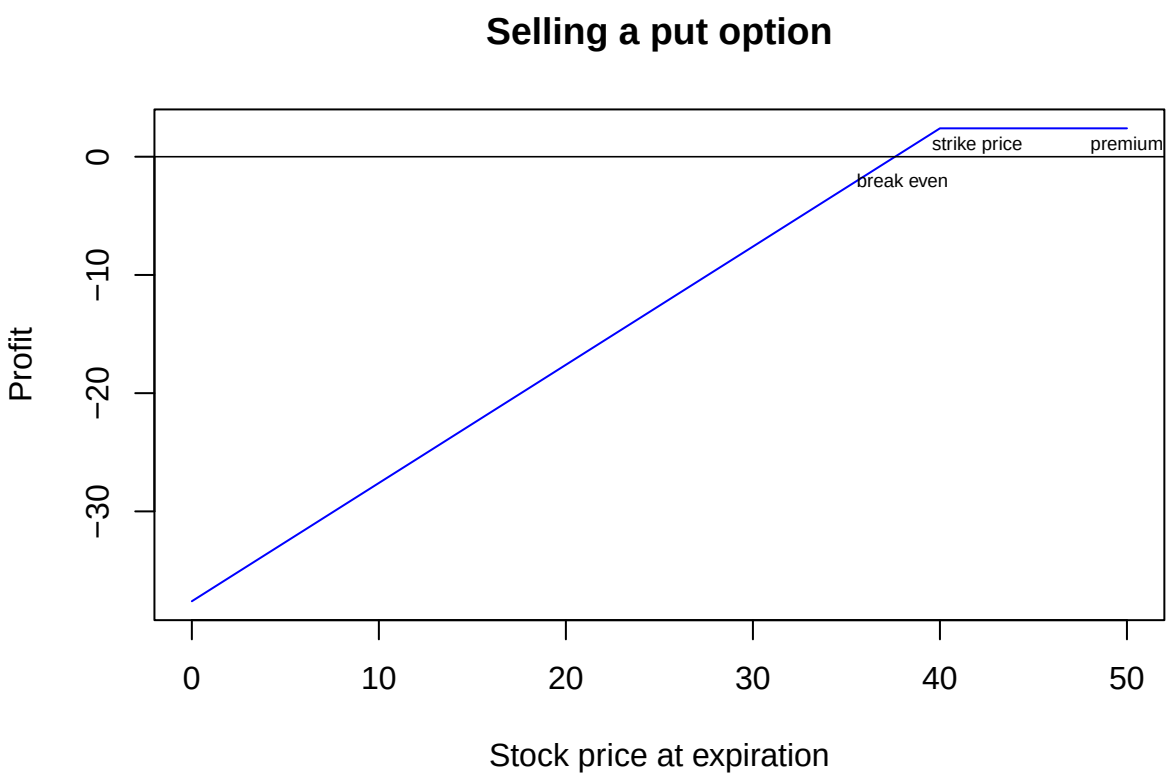
You've committed to buying 100 shares at $40 each if the option buyer chooses to exercise the put. This happens only if the stock price drops below $40, because the buyer can then sell shares to you for more than market value.

What is the maximum gain/loss

The maximum gain is the premium you received for selling the put. This occurs if the option expires and the stock price is above the strike price of $40.

The maximum loss happens if the stock price goes to $0. You still have to buy the stock at $40 and the loss is

$$\begin{aligned}\text{Max loss} &= (\text{Strike price} - 0) \times 100 \\ &= 40 \times 100 \\ &= (\$4,000)\end{aligned}$$



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5. Buying stocks versus long options

You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with a strike price of $30 costs $2.90. You have $5,800 to invest. Identify two alternative investment strategies, one in the stock and the other in an option on the stock. What are the potential gains and losses from each?

You are comparing direct stock investment vs. using options for leverage.

Given:

- Current stock price: $29
- Call option (3-month, strike $30): Costs $2.90
- Your capital: $5,800
- Each option covers 100 shares

Strategy 1: Buy the stock

$$\begin{aligned}\text{Shares bought} &= \frac{\$5,800}{\$29} \\ &= 200 \text{ shares}\end{aligned}$$

In this scenario, you gain or lose $1 per share for every $1 move in the stock price.

Strategy 2: Buy a call option for the stock

$$\begin{aligned}\text{Contracts} &= \frac{\$5,800}{\$290} \\ &\approx 20 \text{ call contracts (each for 100 shares)}\end{aligned}$$

Break-even stock price Strike + Premium = \$30 + \$2.90 = \$32.90.

You lose all you investment if the stock is at or below \$30 at expiration. However, your gains are potentially unlimited on the up side.

NOTE You have to sell the options before expiration.

Conclusion

Buying stock gives steady exposure—less risky but lower potential upside. Buying calls gives explosive potential gains but a real chance of losing everything.

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6. Long put options for hedging stock positions

Suppose that you own 5,000 shares worth \$25 each. How can put options be used to provide you with insurance against a decline in the value of your holding over the next 4 months?

This is a protective put strategy—a method of hedging a stock position against downside risk.

Given:

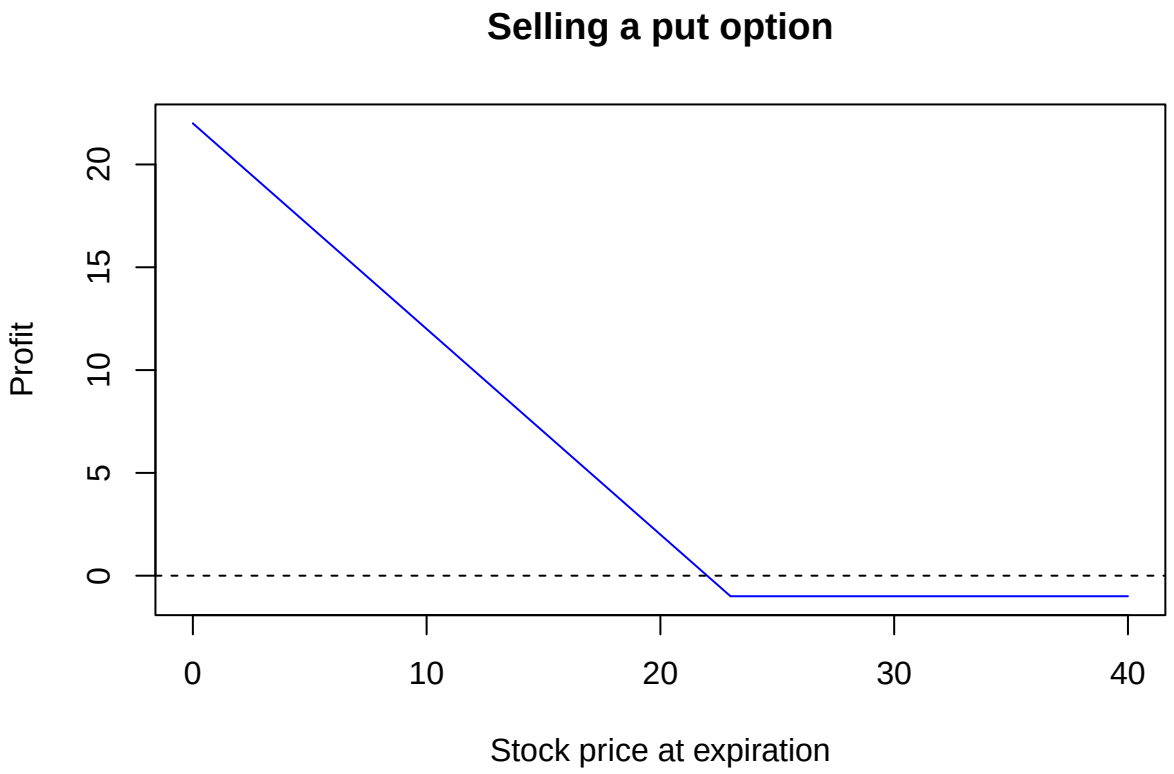
- You own 5,000 shares
- Current stock price: \$25

- Total value: $125,000
- You're worried the price might drop in the next 4 months

Using put options as insurance (hedge)

To protect yourself, you can buy put options with a strike price around $25 and an expiration 4 months out. Each put option covers 100 shares.

A put gives you the right to sell the stock at the strike price—no matter how low the market price falls.



How many puts to buy?

If you own 5,000 shares, each put contract you buy is for 100 shares. Therefore, you need 50 put contracts to fully protect your position.

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7. Stock options in relation to the firm

When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss.

When a company issues stock (primary market) The company receives funds directly from the sale. This money can be used for operations, expansion, paying debt, etc.

When a stock option is created (a call or put on an exchange) Stock options are usually created and traded on options exchanges like the CBOE. These are contracts between investors—not between investors and the company. So if you buy a call option, the company receives nothing.

Exceptions: Employee stock options There's is one exception: Employee stock options (ESOs) are granted by companies to employees. When exercised, employees pay the exercise price to the company → company receives funds and issues new shares. But this happens only when/if the option is exercised.

Conclusion

So in most cases, stock options do not raise funds for a company—only stock issuance does, and employee options may if exercised.

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8. Speculating and hedging uses for futures contracts

Explain why a futures contract can be used for either speculation or hedging.

Futures contracts are powerful tools because they serve two very different purposes depending on who's using them: speculation or hedging.

What is a futures contract?

A futures contract is an agreement to buy or sell an asset at a fixed price on a future date. This could be for commodities, currencies, stock indices, interest ratest, etc.

Speculation: Making a bet on price movement

A speculator uses futures to try to profit from price changes, without owning the underlying asset. For example, if you believe oil prices will rise, so you go long (buy) an oil futures contract at \$75/barrel. If oil rises to \$80/barrel your profit is \$5. If oil falls to \$70/barrel your loss is \$5.

Hedging: Reducing risk of price fluctuations

A hedger uses futures to protect against adverse price movements in something they already own or plan to buy/sell. For example, if your a wheat farmer and you're worried wheat prices might fall before harvest. You could sell wheat futures now to lock in the price. If the price falls, you will lose money on the crop, but you gain on the future. If the price rises, you earn money on the crop, but lose on the future.

Final thought

Futures are zero-sum—one party's gain is another's loss. Speculators add liquidity, while hedgers seek stability.

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9. Call options analysis

Suppose that a March call option to buy a share for \$50 costs \$2.50 and is held until March. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

The holder of the option will exercise and make a profit, after expiration date, if the price increases above \$52.50.

Given:

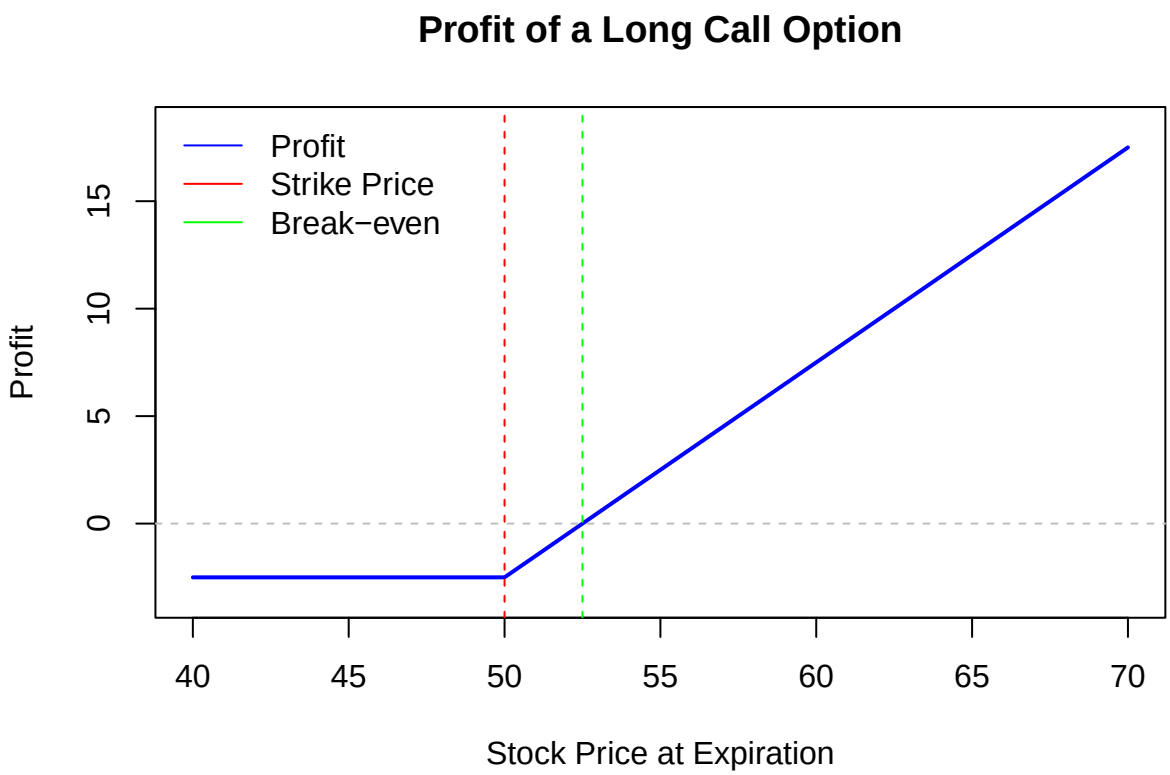
- Option type: Call
- Strike price: \$50
- Premium: \$2.50
- Expires: March

Profit formula for a long call at expiration

$$\text{Profit} = \begin{cases} S - \text{strike} - \text{premium}, & \text{if} \quad S > 50 \\ -\text{premium}, & \text{if} \quad S \leq 50 \end{cases}$$

Plugging in the numbers we get:

$$\text{Profit} = \begin{cases} S - 52.50, & \text{if} \quad S > 50 \\ -2.50, & \text{if} \quad S \leq 50 \end{cases}$$



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10. Put options analysis

Suppose that a June put option to sell a share for \$60 costs \$4 and is held until June. Under what circumstances will the seller of the option (i.e., the party with the short position) make a profit? Under what

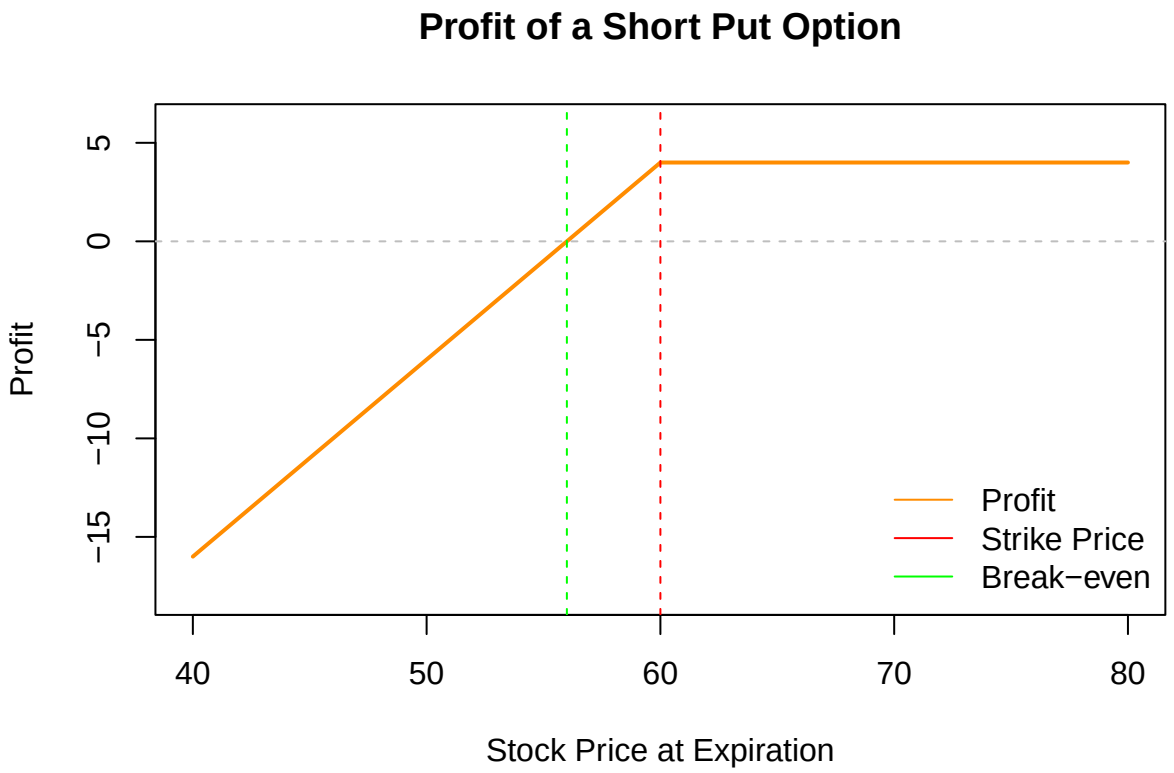
circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.

Given:

- Option type: Put
- Strike price: $60
- Premium: $4
- Expiration: June

This is the short side of a put option.

The buyer of the put will exercise the option if the stock price at expiration date is less than $60. You will collect $4 no matter what, but if the stock price drops below $56 $(60 - 4)$, then you will lose on the sale.



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11. Call option cash flows

It is May and a trader writes a September call option with a strike price of $20. The stock price is $18 and the option price is $2. Describe the trader's cash flows if the option is held until September and the stock price is $25 at that time.

Given:

- Current month: May
- Option type: call option (short position—the trader wrote the option)
- Strike price: \$20
- Option price (premium): \$2 (received by the trader)
- Stock price at expiration (September): \$25

What happens at expiration?

Since the stock price at expiration (\$25) is above the strike price (\$20), the call option will be exercised. That means the option buyer will buy the stock for \$20, and the trader must deliver the stock at that price.

Trader's cash flows:

Initial cash inflow (in May): The trader receives \$2 per option from selling (writing) the call.

At expiration (in September): The trader is obligated to sell a share at \$20, while the market price is \$25. This results in a \$5 loss per share (they miss out on the extra \$5).

Net cash flow (profit/loss):

$$\text{Total} = \$2 - \$5 = -\$3$$

Final answer

The trader receives \$2 when the option is sold in May, but loses \$5 when the option is exercised in September. Therefore the net loss is \$3 per share.

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12. Put option analysis

A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?

Given:

- Option type: Put option (short position—the trader wrote the option)
- Strike price: \$30
- Option price (premium): \$4 (received by the trader)
- Expiration: December

When does the trader make a gain?

The trader profits when the put option is not exercised (because the trader is the one who sold the option). The put option gives the buyer the right to sell the stock at \$30. If the stock price at expiration is above \$30, the option will not be exercised because the buyer can sell the stock at a higher market price. So, the trader will make a gain if the stock price at expiration is $>$ \$30 (because the option will expire worthless, and the trader keeps the \$4 premium).

Break-even point

The break-even is the point where the trader's profit from the option premium is exactly offset by any losses from having to buy the stock at the strike price.

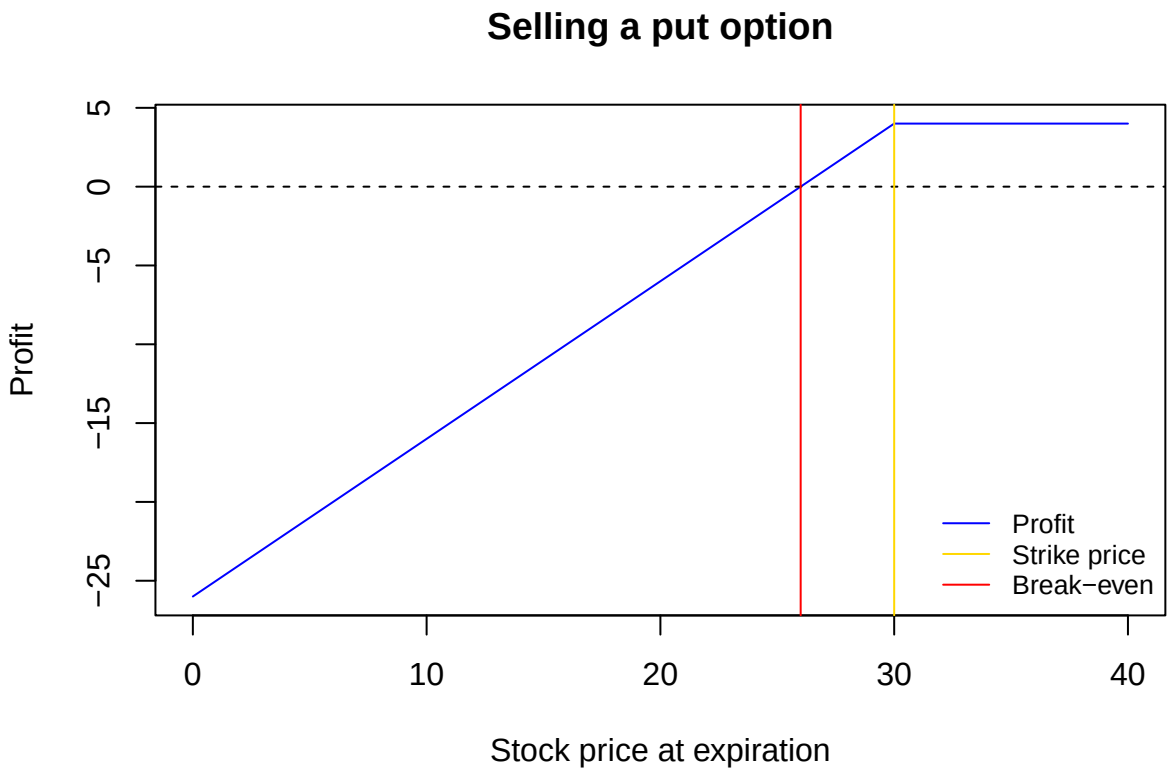
$$\text{Break-even} = \text{Strike price} - \text{Premium received} = 30 - 4 = \$26$$

When does the trader lose?

If the stock price falls below \$30, the buyer of the option may exercise the option, and the trader will have to buy the stock at \$30—even though it's worth less in the market. The trader will lose money if the stock price at expiration is below \$26.

Conclusion

The trader makes a gain when the stock price at expiration is above \$30 because the put option will not be exercised, and the trader keeps the full premium (\$4).



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13. Foreign currency option hedging strategies

A company knows that it is due to receive a certain amount of a foreign currency in 4 months. What type of option contract is appropriate for hedging?

When a company is due to receive a foreign currency in the future (in this case, in 4 months), it faces the risk of currency fluctuations that could affect the value of the payment it will receive. To hedge against the risk of the foreign currency depreciating (i.e., the value of the currency dropping relative to the company's domestic currency), the company can use a foreign currency option.

The appropriate option contract for this scenario is a long put option on the foreign currency (for hedging receivables). The company is expecting to receive a foreign currency payment in the future. If the foreign currency depreciates against the domestic currency by the time the company receives it, the value of the payment will be lower in domestic currency terms. A put option gives the company the right (but not the obligation) to sell the foreign currency at a predetermined strike price. This ensures that the company can lock in a minimum exchange rate, protecting itself from a fall in the value of the foreign currency. In simple terms, a put option on the foreign currency allows the company to sell the foreign currency at the strike price (if the spot rate is less favorable) when it receives the payment in 4 months. This provides a hedge if the foreign currency depreciates.

Key details of the hedging strategy

- **Strike price:** The agreed-upon exchange rate at which the company can sell the foreign currency in 4 months.
- **Premium:** The cost of purchasing the put option. This cost must be weighted against the potential savings from hedging.
- **Expiry:** The option expires when the foreign currency payment is due (in 4 months), giving the company the right to sell the foreign currency at the agreed strike price.

Alternative hedging options

Forward contracts Instead of using options, the company could enter into a forward contract to lock in the exchange rate. However, forwards are obligatory and less flexible than options (they don't allow for profit if the market moves in your favor).

Conclusion

A put option on the foreign currency is the appropriate tool to hedge receivables in a foreign currency, as it gives the company the flexibility to protect itself from depreciation while still allowing it to benefit if the foreign currency appreciates.

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14. Foreign currency forward contract and option hedging strategies

A U.S. company expects to have to pay 1 million Canadian dollars in 6 months. Explain how the exchange rate risk can be hedged using (a) a forward contract and (b) an option.

When a U.S.company expects to pay in a foreign currency in the future, it faces the risk that the CAD could appreciate relative to the U.S. dollar (USD), which would increase the cost of the payment. To mitigate this risk, the company can use two common hedging strategies: forward contracts and options.

Given:

- Amount to be paid: 1,000,000 CAD
- Time horizon: 6 months
- The company is concerned that the CAD might appreciate, making the payment more expensive in USD terms.

Hedging using a forward contract

How it works: A forward contract allows the U.S. company to lock in the exchange rate at which it will convert USD to CAD in the future. The company enters into a forward contract with a bank or financial institution to buy 1 million CAD at a specific exchange rate, to be settled in 6 months. The forward contract guarantees that the company will pay a fixed rate for the CAD, regardless of how the spot exchange rate fluctuates.

Steps:

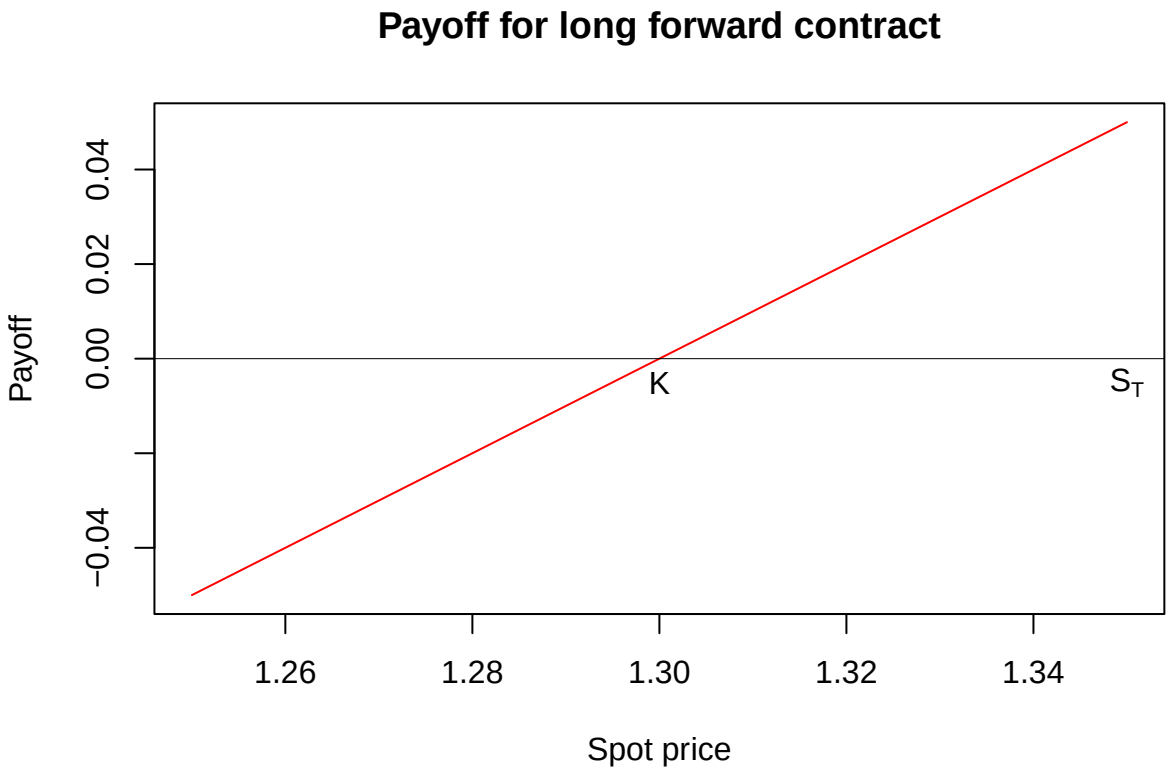
1. **Agreement:** The company agrees to buy (long) 1 million CAD at a predetermined forward rate (e.g., 1 USD = 1.30 CAD) for delivery in 6 months.
2. **At maturity:** In 6 months, the company will pay USD to get 1 million CAD at the agreed-upon exchange rate of 1.30 CAD/USD (assuming the rate is 1.30).
3. **Risk mitigation:** This hedging strategy eliminates exchange rate risk, as the company knows exactly how much USD it will need to pay for the CAD, even if the spot rate has changed.

Pros:

- **Certainty:** The company knows exactly how much USD it will need to pay in 6 months.
- **No cost:** Typically there are no upfront premium for a forward contract, but there may be fees or spread differences.

Cons:

- **No flexibility:** If the exchange rate moves favorably, in the company's benefit, it is still bound by the forward rate.



Hedging using an option

How it works: A call option gives the company the right, but not the obligation, to buy 1 million CAD at a specific strike price in 6 months. The company buys a call option on CAD, which gives the company the right to buy CAD at the strike price. The company can decide whether to exercise the option depending on the spot exchange rate at the time of expiration.

Steps:

1. **Agreement:** The company buys a call option to purchase 1 million CAD at a specified strike price (e.g., 1 USD = 1.30 CAD).
2. **At maturity:** If the spot rate is below the strike price, the company will not exercise the option and will simply convert its USD at the lower spot rate. If the spot rate is above the strike price, the company will exercise the option and purchase CAD at the strike price.
3. **Risk mitigation:** The option gives the company the flexibility to benefit from favorable exchange rate movements while still protecting against unfavorable movements.

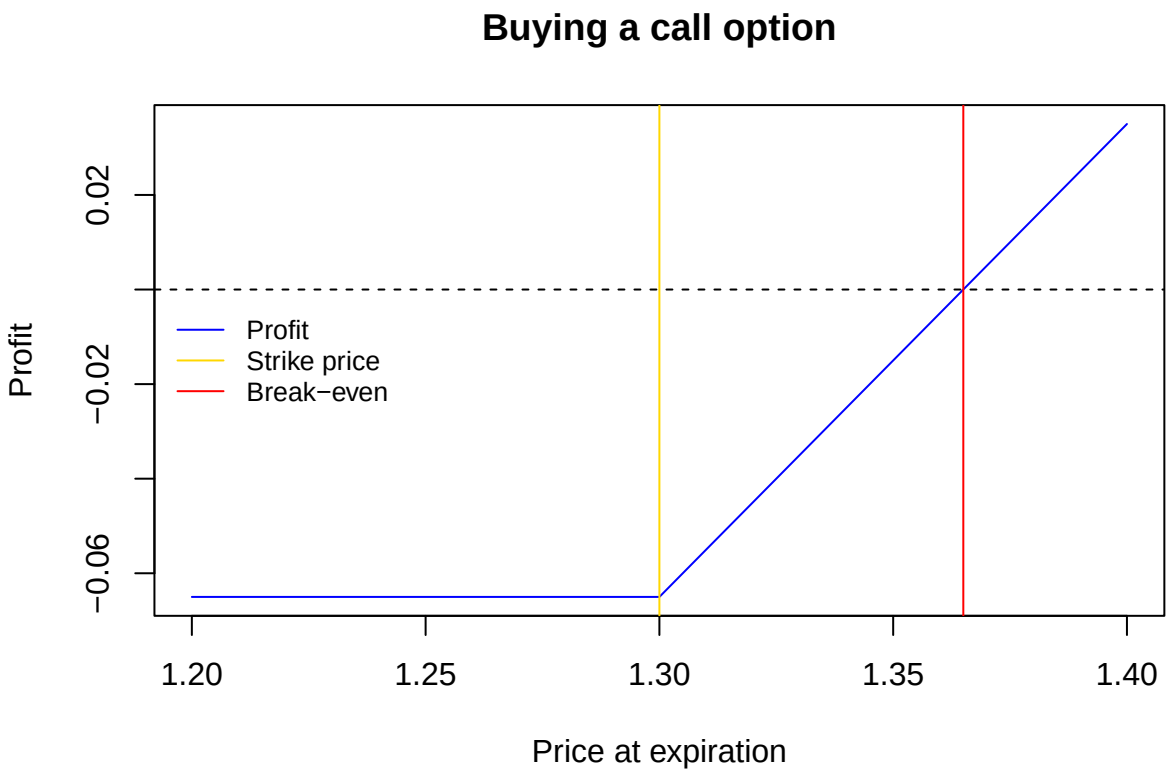
Pros:

- **Flexibility:** The company can choose not to exercise the option if the exchange rate moves in its favor.

- **Limited downside:** The maximum loss is the premium paid for the option (e.g., $30,000), whereas the potential gain is theoretically unlimited if the currency appreciates significantly.

Cons:

- **Premium cost:** The company must pay an upfront premium to purchase the option, which is a cost even if the option is not exercised.
- **Uncertainty in cost:** The exact amount of USD to be spent will depend on the final exchange rate and the premium paid.



Conclusion

- **Forward contract:** Ideal if the company wants certainty and wants to completely lock in the exchange rate for the future transaction. It eliminates the risk but does not allow the company to benefit from favorable exchange rate movements.
- **Option:** Ideal if the company wants flexibility and is willing to pay an upfront premium for the right to exercise only if the exchange rate moves unfavorably. The option provides protection while also allowing the company to benefit from favorable currency movements.

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15. Foreign currency short forward contracts

A trader enters into a short forward contract on 100 million yen. The forward exchange rate is $0.0090 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) $0.0084 per yen and (b) $0.0101 per yen?

In a short forward contract, the trader is agreeing to sell a foreign currency (in this case, 100 million yen) at a predetermined forward exchange rate. If the spot exchange rate at the end of the contract is lower than the forward exchange rate, the trader will gain, as they can buy the yen at a lower price in the open market and still sell it at the agreed higher price. Conversely, if the spot exchange rate is higher than the forward exchange rate, the trader will lose.

Given:

- Contract amount: 100 million yen
- Forward exchange rate: \$0.0090 per yen
- Scenarios:
    a. Spot exchange rate: \$0.0084 per yen
    b. Spot exchange rate: \$0.0101 per yen

Scenario A: Spot exchange rate @ \$0.0084 Yen

Here the trader selling (shorting) a forward contract will make a profit.

$$\begin{aligned}\text{Profit per yen} &= \text{Forward rate} - \text{Spot rate} \\ &= 0.0090 - 0.0084 \\ &= 0.0006 \\ \text{Profit} &= \text{Profit per yen} \times \text{Contract amount} \\ &= 0.0006 \times 100,000,000 \\ &= \$60,000\end{aligned}$$

Scenario B: Spot exchange rate @ \$0.0101 Yen

In this scenario the trader will take a loss.

$$\begin{aligned}\text{Loss per yen} &= \text{Forward rate} - \text{Spot rate} \\ &= 0.0090 - 0.0101 \\ &= (0.0011) \\ \text{Loss} &= \text{Loss per yen} \times \text{Contract amount} \\ &= -0.0011 \times 100,000,000 \\ &= (\$110,000)\end{aligned}$$

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16. Derivative trader types

The CME Group offers a futures contract on long-term Treasury bonds. Characterize the traders likely to use this contract?

The CME Group's long-term Treasury bond futures contract is a financial instrument designed to allow traders to hedge, speculate, or manage interest rate risk. This contract is typically based on U.S. Treasury bonds with a notional value of \$100,000 and a maturity of 15 years or more. The contract is primarily used by various types of traders, each with different objectives.

Hedgers

Institutional investors (e.g., insurance companies, pension funds, mutual funds): These traders use the futures contracts to hedge against interest rate fluctuations that could affect the value of their long-term bond holdings.

For example, if a pension fund holds long-term Treasury bonds, it may use futures to protect the portfolio from potential interest rate hikes, which would lower bond prices. By selling futures, the fund can offset losses in the bond portfolio with gains from the short futures position if rates rise.

Banks and financial institutions Banks may use these futures contracts to manage the risk related to their bond portfolios, especially if they hold significant amounts of long-term debt securities. They might hedge the exposure to interest rate changes that can affect the value of their portfolios.

Corporations with long-term debt Companies that have issued long-term bonds or are exposed to future interest rate movements might use these futures contracts to lock in favorable rates, thereby reducing the risk associated with fluctuating borrowing costs.

Speculators

Hedge funds and proprietary trading firms Speculators who believe that interest rates are going to move in a particular direction may use these futures contracts to profit from changes in the value of long-term Treasury bonds. If a speculator expects rates to fall (and bond prices to rise), they would buy futures contracts. Conversely, if they expect rates to rise (and bond prices to fall), they would sell futures contracts.

Individual traders Some individual traders might speculate on interest rate changes using these contracts as a lower-cost alternative to trading physical Treasury bonds. This allows them to gain exposure to U.S. government debt without having to trade the bonds directly, which can involve higher capital requirements and liquidity concerns.

Arbitrageurs

Arbitrage traders These traders attempt to take advantage of pricing inefficiencies between the futures and the underlying Treasury bonds or related instruments. They might trade Treasury bond futures alongside cash Treasury bonds or other related futures contracts to profit from small price discrepancies between the futures market and the cash market.

Central banks and sovereign wealth funds

Central banks, espeically those holding large amounts of Treasury securities, may use these futures contracts for portfolio management or to hedge their foreign exchange reserves. Sovereign wealth funds, with large investments in U.S. Treasuries, might also use these futures for risk management and to control the interest rate exposure of their portfolios.

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17. Nature of options and futures

"Options and futures are zero-sum games." What do you think is meant by this?

The phrase "Options and futures are zero-sum games" means that, in these markets, the total profits and losses for all participants balance out to zero. This is a key characteristic of derivative markets like options and futures.

18. Option strategy: long forward / long European put

Describe the profit from the following portfolio: a long forward contract on an asset and a long European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.

This is a classic combination of a long forward contract and a long European put option on the same underlying asset with the same maturity and a strike price of the put equal to the forward price at initiation.

Key definitions:

- F_0: forward price of the asset at initiation
- S_T: spot price of the asset at maturity
- $K = F_0$: strike price of the put option (same as forward price)
- The put option is European, so it can only be exercised at maturity
- We ignore transaction costs and assume no arbitrage

Payoffs at Maturity:

$$\begin{aligned} &\text{Long Forward Contract:} \\ &\qquad \text{Profit} = S_T - F_0 \\ &\text{Long Put Option with strike } K = F_0\text{:} \\ &\qquad \text{Profit} = \max(F_0 - S_T, 0) \end{aligned}$$

The total profit from the portfolio:

$$\text{Total profit} = (S_T - F_0) + \max(F_0 - S_T, 0)$$

Case 1: $S_T \geq F_0$

- The forward contract gains: $S_T - F_0$
- The put expires worthless: $\max(F_0 - S_T, 0) = 0$
- Total profit: $S_T - F_0$

Case 2: $S_T < F_0$

- The forward loses: $S_T - F_0$ (negative)
- The put gains: $F_0 - S_T$
- Total profit: $(S_T - F_0) + (F_0 - S_T) = 0$

Conclusion

- If $S_T \geq F_0$, profit $= S_T - F0$
- If $S_T < F_0$, profit $= 0$

Interpretation

This combination of a long forward and a long put is synthetically equivalent to a long call option with strike price equal to the forward price. This is a useful arbitrage-free result known as put-call parity in derivative pricing.

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19. Index Currency Option Note (ICON)

In the 1980s, Bankers Trust developed *index currency option notes* (ICONs). These were bonds in which the amount received by the holder at maturity varied with a foreign exchange rate. One example was its trade with the Long Term Credit Bank of Japan. The ICON specified that if the yen/USD exchange rate, S_T, is greater than 169 yen per dollar at maturity (in 1995), the holder of the bond receives \$1,000. If it is less than 169 yen per dollar, the amount received by the holder of the bond is:

$$1,000 - \max\left[0, 1,000\left(\frac{169}{S_T} - 1\right)\right]$$

When the exchange rate is below 84.5, nothing is received by the holder at maturity. Show that this ICON is a combination of a regular bond and two options.

We will show how the Index Currency Option Note (ICON) can be replicated using a combination of a straight bond and two options.

ICON Payoff Structure

At maturity:

- If $S_T > 169$ yen/USD, the holder receives \$1,000
- If $S_T \leq 169$ Holder receives:

$$1,000 - \max\left[0, 1,000\left(\frac{169}{S_T} - 1\right)\right]$$

This simplifies to:

$$\text{Payoff} = \min\left[1,000, 1,000\left(\frac{S_T}{169}\right)\right]$$

Because:

$$\begin{aligned} 1,000 - 1,000\left(\frac{169}{S_T} - 1\right) &= 1,000\left(2 - \frac{169}{S_T}\right) \\ &= \text{Only valid when} \frac{169}{S_T} > 1 \Rightarrow S_T < 169 \end{aligned}$$

Or more directly, the ICON payoff is:

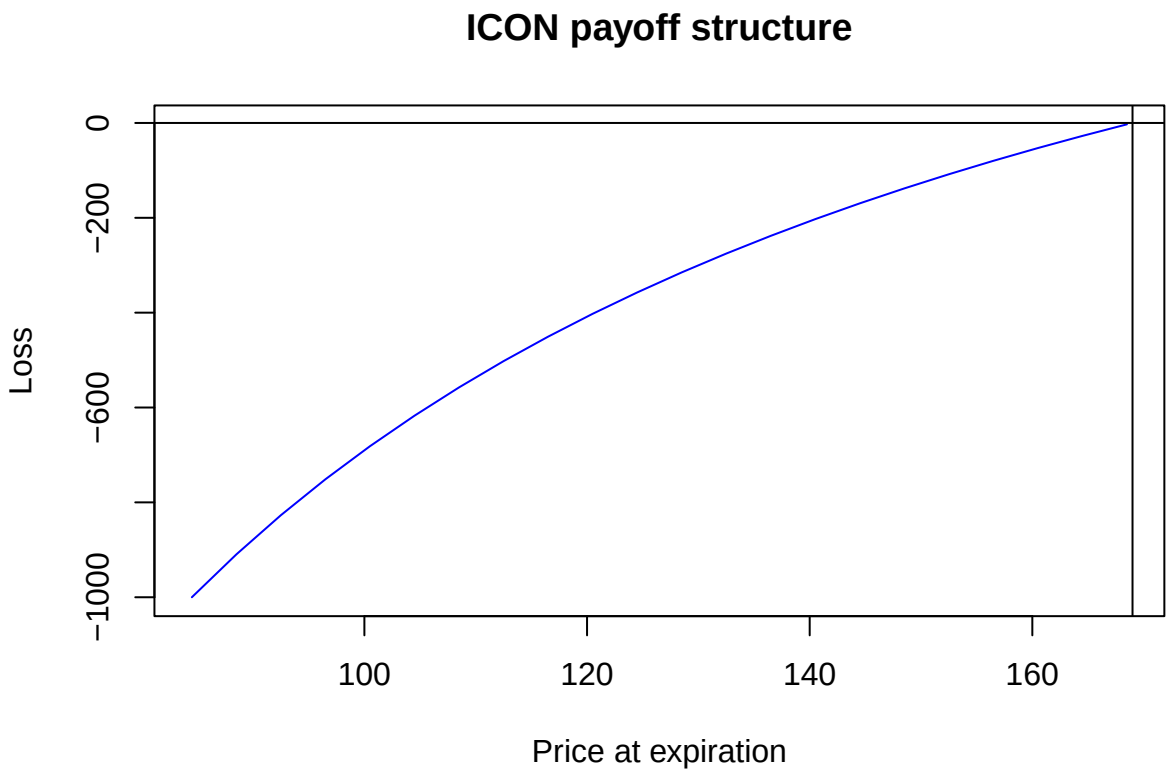
$$\text{Payoff} = \begin{cases} 1,000, & \text{if } S_T \geq 169 \\ 1,000 \cdot \frac{S_T}{169}, & \text{if } S_T < 169 \end{cases}$$

Step 1: Interpret the payoff

This looks like a bond that is worth \$1,000 minus an amount that increases as the exchange rate falls below 169.

So the loss due to unfavorable exchange rate movements is:

$$-\max\left[0, 1,000\left(\frac{169}{S_T} - 1\right)\right]$$



Let's isolate this component and interpret it.

Step 2: Define the loss component

Let:

$$L(S_T) = 1,000 \left(\frac{169}{S_T} - 1 \right) = 1,000 \cdot \left(\frac{169 - S_T}{S_T} \right)$$

This is positive when $S_T < 169$, and zero otherwise.

This form suggests exposure to an inverse relationship with the exchange rate—similar to a put option payoff structure.

But it's a bit more complex because of the inverse $\frac{1}{S_T}$ term.

Step 3: Show it as a Bond + Options

Let's construct the ICON payoff using:

- A straight zero-coupon bond paying $1,000
- A long position in a digital option or binary option (to truncate the loss)
- A short position in a put option on the exchange rate

But a cleaner replication is:

$$\text{ICON} = \text{Straight bond} - \text{Put option on } \frac{1}{S_T}$$

Let's use financial engineering to express it more precisely.

Step 4: Final decomposition

The ICON is equivalent to:

1. A \$1,000 zero-coupon bond (pays full amount if exchange rate is favorable)
2. Short a put option on the foreign exchange rate with:
   - Strike price: 169 yen/USD
   - Underlying: the exchange rate S_T
   - Notional amount: scaled such that the option payout when $S_T < 169$ is:

$$1,000\left(\frac{169}{S_T}-1\right)$$

3. Long a digital put option (binary put) with strike at 84.5 yen/USD:
   - Ensures that if the exchange rate falls below 84.5, the ICON pays nothing
   - This truncates losses and acts as a floor

Conclusion: The ICON is a combination of:

- A straight \$1,000 bond
- Short a put option on the YEN/USD exchange rate with strike 169
- Long a digital put option (or structured floor) to ensure minimum payoff is \$0 when exchange rate falls too far.

This structure allows Bankers Trust to reduce coupon payments based on exchange rate movements, effectively embedding currency risk into a bond payoff using options.

---

20. Forward spread trade

On July 1, 2021, a company enters into a forward contract to buy 10 million Japanese yen on January 1, 2022. On September 1, 2021, it enters into a forward contract to sell 10 million Japanese yen on January 1, 2022. Describe the payoff from this strategy.

This strategy involves entering into two offsetting forward contracts:

- July 1, 2021: Enter long forward to buy 10 million Yen on January 1, 2022.
- September 1, 2021: Enter short forward to sell 10 million Yen on January 1, 2022.

Both contracts settle on January 1, 2022.

Key observations:

- The company has no net exposure to yen on January 1, 2022, because it will buy and sell the same amount (¥10 million) on the same day.
- However, the forward rates locked in on July 1 and September 1 are likely different.
- This creates a spread in the forward prices, which gives rise to a certain (riskless) profit or loss, depending on the direction of the spread.

Let's define

- F_1: Forward rate to buy yen on July 1 (i.e., cost in USD per ¥1)
- F_2: Forward rate to sell yen on September 1 (i.e., revenue in USD per ¥1)

Then the payoff per yen on January 1, 2022 is:

$$\text{Payoff per yen} = F_2 - F_1$$

So, the total payoff is:

$$\text{Payoff} = 10,000,000 \times (F_2 - F_1)$$

Interpretation:

- If $F_2 > F_1$: You make a profit—sell the yen at a higher price than you agreed to buy.
- If $F_2 < F_1$: You incur a loss—you buy yen at a higher price than you can sell them for.

Economic Meaning

This is effectively a speculative position on the change in forward rates between July 1 and September 1. It's a forward spread trade.

- You are locking in a forward rate arbitrage, betting on how forward rates move over time.
- You are not exposed to future spot rate changes—only the change in forward pricing between the two dates.

---

21. Foregin exchange arbitrage opportunities

Suppose the USD/sterling spot and forward exchange rates are as follows:

Spot	1.2580
90-day forward	1.2556
180-day forward	1.2518

What opportunities are open to an arbitrageur in the following situations?

a. A 180-day European call option to buy 1 pound for $1.22 costs 2 cents.
b. A 90-day European put option to sell 1 pound for $1.29 costs 2 cents.

180-day European call option to buy 1 GBP for $1.22, costing $0.02

- Strike price: $1.22
- Cost of option: $0.02
- Forward rate (180-day): $1.2518

This call option gives the right to buy GBP at $1.22, but in 180 days GBP is expected to cost $1.2518 (forward rate). That means:

- If you buy the call for $0.02 now and exercise it in 180 days, you could immediately sell the pound for $1.2518 in the forward market.
- Profit = $1.2518 - $1.22 - $0.02 = $0.0318 per pound

This option is underpriced relative to the forward rate.

90-day European put option to sell 1 GBP for $1.29, costing $0.02

- Strike price: $1.29
- Cost of option: $0.02
- Forward Rate (90-day): $1.2556

This put gives the right to sell GBP for $1.29, while the expected market value is only $1.2556 in 90 days.

- Buy GBP forward for $1.2556
- Use the put to sell it at $1.29
- Profit = $1.29 - $1.2556 - $0.02 = $0.0144 per pound

This option is underpriced compared ot the forward rate.

---

22. Exercising options

A trader buys a call option with a strike price of $30 for $3. Does the trader ever exercise the option and lose money on the trade? Explain your answer.

A call option is only exercised when the stock price is above the strike price. However, a trader can exercise the option and lose money if they wanted to if the stock is less than the strike price minus the premium. Here that would be a stock price that is below $33.

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23. Put option analysis

A trader sells a put option with a strike price of $40 for $5. What is the trader's maximum gain and maximum loss? How does your answer change if it is a call option? Graph each using R.

Put option (sold by trader)

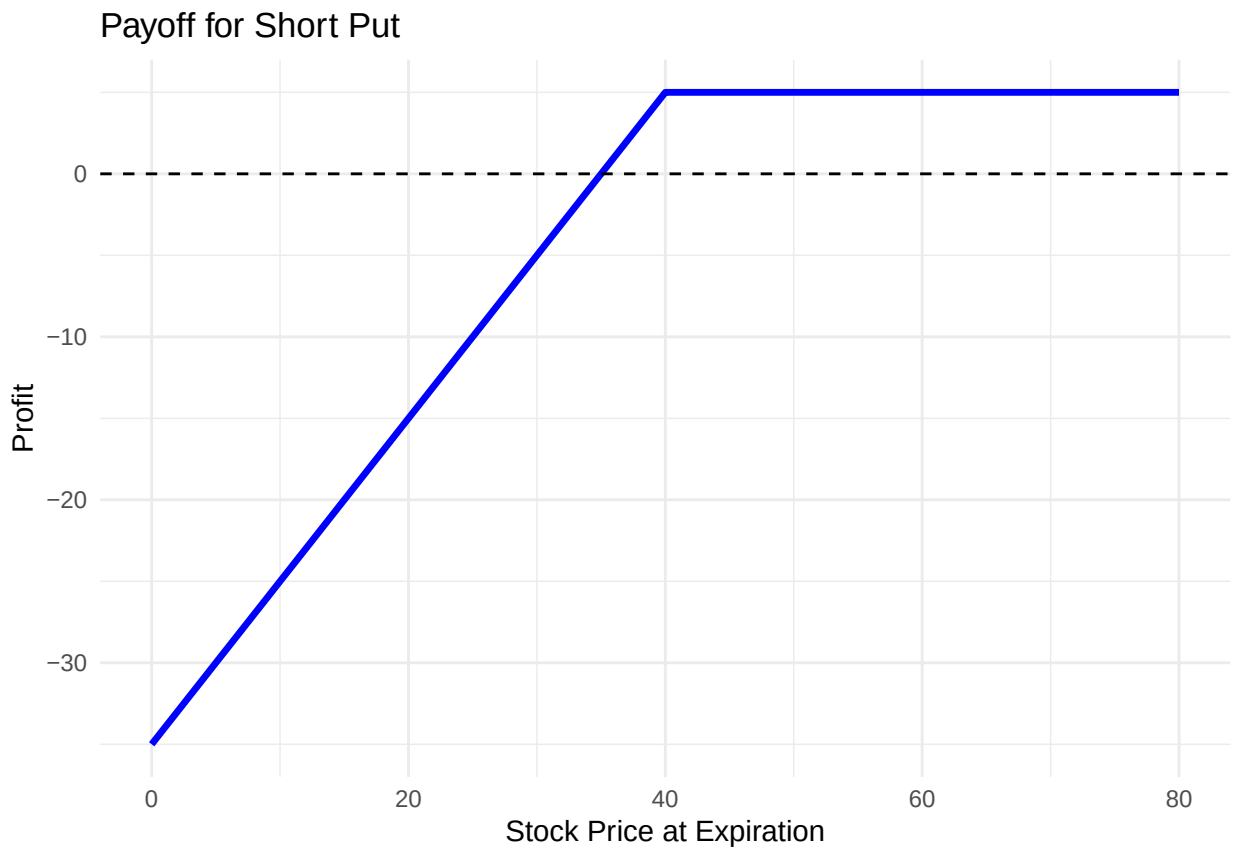
- Strike price: $40
- Premium received: $5
- Short put: Trader must buy the asset at $40 if the buyer exercises

Maximum gain: The buyer will only exercise if the stock price is below $40. But if the stock price stays above or equal to $40, the option expires worthless, and the seller keeps the $5 premium.

Maximum loss: Occurs if the stock price falls to zero. The trader buys at $40, asset is worth $0, the loss is $40, but received $5 premium so net loss is $35.

For a put option the profit to strike price graph is as follows:

```
## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use `linewidth` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
```

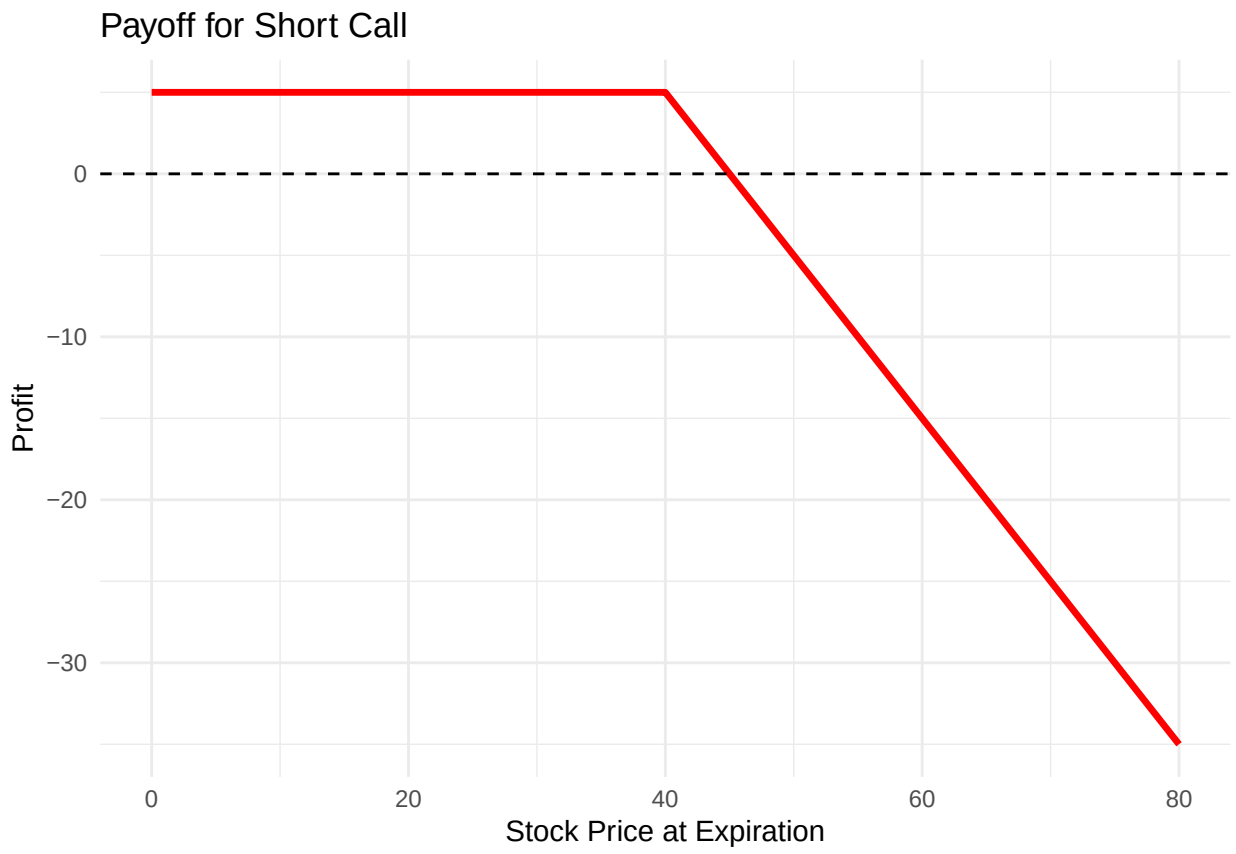


Call option (sold by trader)

- Strike price: $40
- Premium received: $5
- Short call: Trader must sell the asset at $40 if the buyer exercises

Maximum gain: If the stock price $\leq$ $40 the option expires worthless. The maximum gain is $5.

Max loss: There's unlimited downside since the stock can rise infinitely. If the stock hits $100, the trader sells at $40, takes a $60 loss minus the $5 premium. net loss is $55.



24. Nature of options

"Buying a put option on a stock when the stock is owned is a form of insurance." Explain this statement.

When a trader buys a put option on a stock, they are buying the right to sell a stock at the strike price. This means that, if the stock falls below the strike price, the trader's loss on the stock is capped by the difference between what he bought the stock for and the strike price plus the premium he paid for the option.

25. Buying stock versus buying call options

On May 21, 2020, the spot ask price of Apple stock is \$316.50 and the ask price of a call option with a strike price of \$320 and a maturity date of September is \$21.70. A trader is considering two alternatives: buy 100 shares of the stock and buy 100 September call options. For each alternative, what is (a) the upfront cost, (b) the total gain if the stock price in September is \$400, and (c) the total loss if the stock price in September is \$300. Assume that the option is not exercised before September and positions are unwound at option maturity.

Given:

- Spot price of Apple (AAPL): $316.50
- Call option strike price: $320
- Call option premium (ask): $21.70
- Maturity: September
- Trader considers buying either:

1. 100 shares of stock
2. 100 call options (each option = 1 share)

- Scenarios for AAPL in September:

i. Price = $400
ii. Price = $300

Strategy 1: Buy 100 Shares of Stock

- Upfront cost: $100 \times 316.50 = \$31,650$
- Gain if stock price = $400: $(400 - 316.40) \times 100 = \$8,350$
- Loss if stock price = $300: $(300 - 316.50) \times 100 = -\$1,650$

Strategy 2: Buy 100 Call options (strike = $320, premium = $21.70)

Remember that call options give you the right to buy at $320. If the stock ends below $320, option is worthless.

- Upfront cost: $100 \times 21.70 = \$2,170$
- Gain if stock price = $400:
  - Intrinsic value of option = $400 - $320 = $80
  - Total value of 100 options = $8,000
  - Profit = $8,000 - $2,170 = $5830
- Loss if stock price = $300:
  - Option expires worthless
  - Loss = Premium paid = -$2,170

---

26. Nature of arbitrage

What is arbitrage? Explain the arbitrage opportunity when the price of a dually listed mining company stock is $50 (USD) on the New York Stock Exchange and $60 (CAD) on the Toronto Stock Exchange. Assume that the exchange rate is such that 1 U.S. dollar equals 1.21 Canadian dollars. Explain what is likely to happen to prices as traders take advantage of this opportunity.

Arbitrage is the practice of simultaneously buying and selling the same (or equivalent) asset in different markets to profit from a price discrepancy—without taking any risk. It's a key concept in finance because it helps ensure market efficiency.

Scenario summary

- Stock price: $50 USD on NYSE - $60 CAD on the TSX
- Exchange rate: 1 USD = 1.21 CAD

Converting the NYSE price to CAD: $\$50 USD \times 1.21 = \$60.50 CAD$

This means that the TSX listing is slightly cheaper.

Arbitrage opportunity

1. Buy the stock on the TSX at $60 CAD
2. Sell it simultaneously on the NYSE at $50 USD
3. Convert the proceeds: $50 USD × 1.21 = $60.50 CAD

What happens next (Market reaction)

As arbitrageurs take advantage of this price difference, you will get a convergence of the prices.

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27. Forward contracts versus options

Trader A enters into a forward contract to buy an asset for $1,000 in one year. Trader B buys a call option to buy the asset for $1,000 in one year. The cost of the option is $100. What is the difference between the positions of the traders? Show the profit as a function of the price of the asset in one year for the two traders.

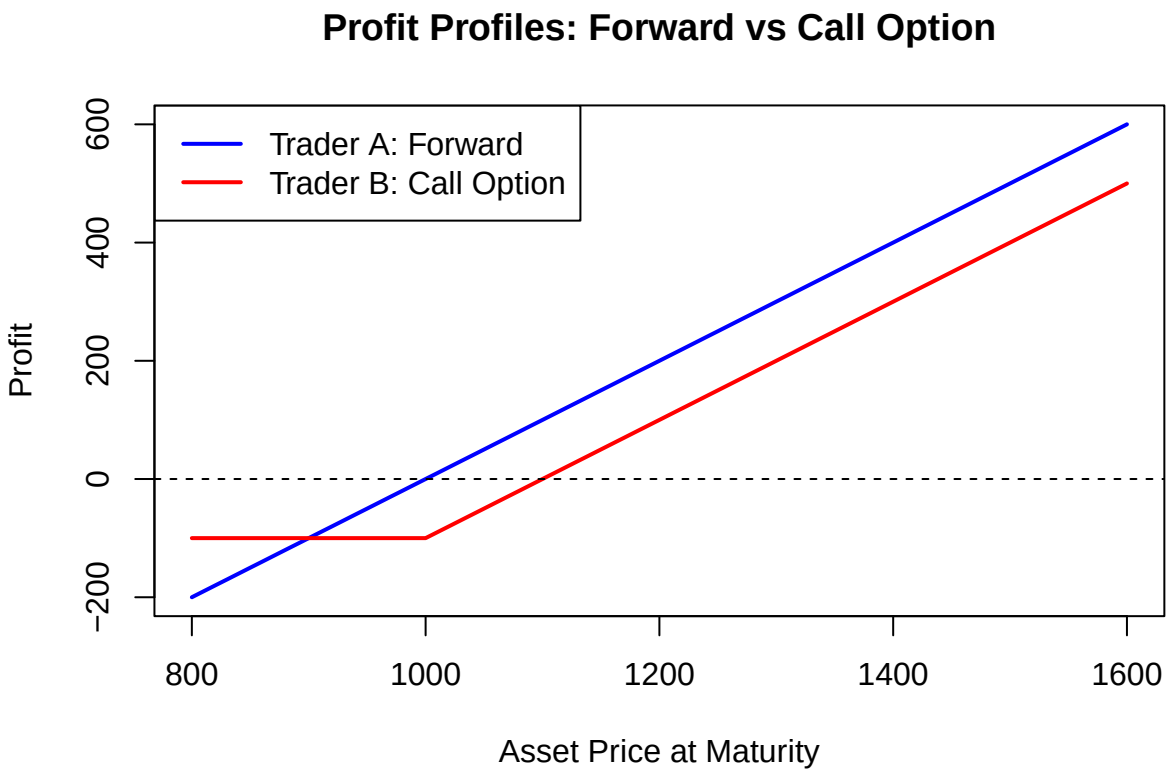
Trader A: Forward contract

- Trader A agrees to buy the asset in one year for $1,000.
- No upfront cost
- Profit at maturity is: $\text{Profit}_A = S_T - 100$.

Trader B: Call option

- Trader B buys a call option to buy the asset at $1,000, paying $100 upfront.
- The profit at maturity is: $\text{Profit}_B = \max(S_T - 1,000, 0) - 100$

A forward contract is a contract to buy the asset in a future date. An option gives the buyer an option, but not the obligation, to buy the asset at a future date. The option has a premium ($100) because of this benefit.



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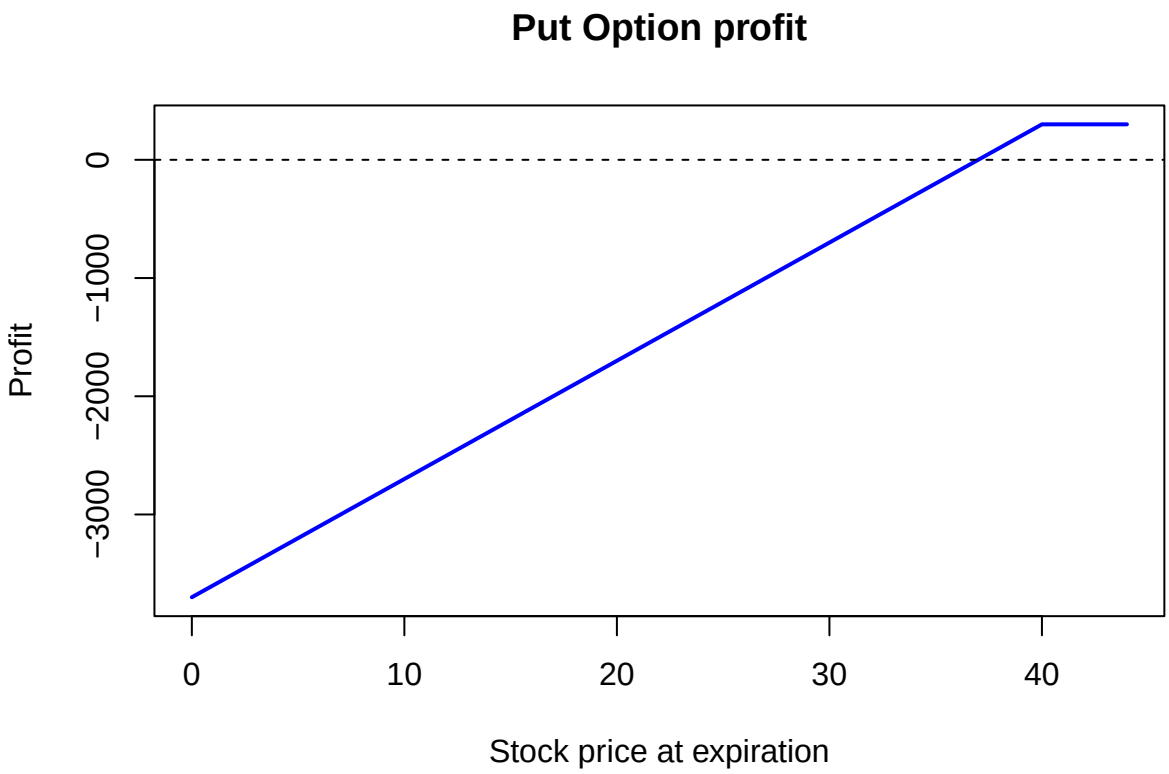
28. Short put option analysis

In March, a U.S. investor instructs a broker to sell one July put option contract on a stock. The stock is $42 and the strike price is $40. The option price is $3. Explain what the investor has agreed to. Under what circumstances will the trade prove to be profitable? What are the risks?

The investor is shorting (selling) a put option for $3. The investor is agreeing to buy if the stock price drops to $40 or below. The maximum profit the investor can realize is the $3 premium he is asking for. The risk is, if the price does fall below $40, the investor will begin to have losses. The maximum loss is ($4,000).



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29. Forward contracts versus options

A U.S. company knows it will have to pay 3 million euros in three months. The current exchange rate is 1.1500 dollars per euro. Discuss how forward and options contracts can be used by the company to hedge its exposure.

If a U.S. company knows it will need to pay 3 million euros in three months, it faces foreign exchange risk: if the euor strengthens against the dollar, it will cost more USD to meet the obligation. The company can use forward contracts or options to hedge this risk.

Forward contract hedge (Fixed hedge)

A forward contract locks in an exchange rate today for a transaction that will happen in the future. If the company enters into a forward contrat for 1.1520 USD/EUR, the company agrees to pay:

$$3,000,000 \times 1.520 = \$3,456,000$$

- If the euro appreciates to 1.20 then the company avoids a loss.
- If the euro depreciates to 1.10 then the company misses out on cheaper conversion.

The advantage of forward contracts is that they eliminate uncertainty and lock in prices. The disadvantage is that you cannot benefit on favorable exchange rate changes.

Options contract hedge (Flexible hedge)

A call option on euros gives the right (but not obligation) to buy euros at a fixed rate (strike price) in 3 months. To purchase a call option the company will have to pay a premium. Suppose the premium is \$0.02 per euro.

$$3,000,000 \times 0.02 = \$60,000$$

- If EUR/USD > 1.15 then the option is exercised and losses are limited.
- IF EUR/USD < 1.15 then you let the option expire, and buy euros at the cheaper spot rate.

The advantage of options over forward contracts is that you get to take advantage of exchange rate changes, but you must pay a premium for that benefit.

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30. Bull call spread option strategy

A stock price is \$29. A trader buys one call option contract on the stock with a strike price of \$30 and sells a call option contract on the stock with a strike price of \$32.50. The market prices of the options are \$2.75 and \$1.50, respectively. The options have the same maturity date. Describe the trader's position.

This strategy is called a bull call spread.

The trader buys 1 call option with a strike price of \$30 for \$2.75 and sells 1 call option with a strike price of \$32.50 for \$1.50. Both options are on the same stock and have the same expiration date.

$$\text{Net premium paid} = 2.75 - 1.50 = \$1.25 \text{ per share}$$

Each option contract typically controls 100 shares, so the total cost is:

$$\$1.25 \times 100 = \$125$$

Payoff profile at expiration

Let S_T be the stock price at expiration.

1. If $S_T \leq 30$:
   - Both options expire worthless.
   - Net loss = \$1.25 per share
2. If $30 < S_T < 32.50$:
   - The \$30 call is in the money
   - The \$32.50 call is still out of the money
   - Profit increases linearly from -\$1.25 to a max:

$$\text{Profit} = S_T - 30 - 1.25$$

3. If $S_T \geq 32.50$:
   - Both calls are exercised
   - Profit is capped because the trader loses on the short call beyond \$32.50

$$\text{Max profit} = (32.50 - 30) - 1.25 = \$1.25 \text{ per share}$$

Summary

Stock price at expiry	Net profit per share
$\leq$ \$30	-\$1.25
\$31	-\$0.25
\$32	+\$0.75
$\geq$ \$32.50	+\$1.25 (max profit)

Why use this strategy?

The trader is moderately bullish—they expect the stock to rise, but not drastically. The bull call spread lowers the cost compared to just buying a single call but limits the upside.

---

31. Forward contract arbitrage

The price of gold is currently \$1,200 per ounce. The forward price for delivery in 1 year is \$1,300 per ounce. An arbitrageur can borrow money at 3% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

Given:

- Spot price of gold: \$1,200/oz
- Forward price (1-year) = \$1,300/oz
- Risk-free borrowing rate = 3% per annum
- Storage cost = \$0
- Gold provides no income

Theoretical forward price formula:

$$F = S \times e^{rT}$$

Where:

- F = forward price
- S = spot price (\$1,200)
- r = risk-free rate (0.03)
- T = time = 1 year

$$\begin{aligned} F_{\text{theoretical}} &= 1200 \times e^{0.03} \\ &\approx 1200 \times 1.03045 \\ &\approx \$1,236.54 \end{aligned}$$

The forward price is \$1,300 > \$1,236.54. The forward is overpriced, which means you should sell the forward and buy the spot to lock in arbitrage.

Arbitrage strategy

1. Borrow \$1,200 at 3% interest
2. Buy 1 oz of gold at \$1,200
3. Enter a forward contract to sell 1 oz of gold in 1 year at \$1,300
4. Wait 1 year:
    - Deliver the gold under the forward contract and receive \$1,300
    - Repay loan: $1200 \times e^{0.03} \approx \$1,236.54$

Profit

$$1300 - 1236.54 = \$63.46$$

Conclusion

An arbitrageur can lock in a risk-free profit of \$63.46 per ounce by:

- Buying gold now
- Locking in a future sale via forward
- Financing the purchase at 3%

---

32. Buying a put option versus placing a stop-loss order

On May 21, 2020, an investor owns 100 Apple shares. The share price is about \$316 and a December put option with a strike price of \$290 costs \$21.30. The investor is comparing two alternatives to limit downside risk. The first involves buying one December put option contract with a strike price of \$290. The second involves instructing a broker to sell the 100 shares as soon as Apple's price reaches \$290. Discuss the advantages and disadvantages of the two strategies.

This involves two popular risk managment strategies: buying a put option (a type of portfolio insurance) vs. placing a stop-loss order.

Scenario Recap

- Stock owned: 100 Apple shares
- Current price: \$316
- Goal: Limit downside risk
- Option: Buy a December put option (strike = \$290, premium = \$21.30)
- Alternative: Place a stop-loss sell order at \$290

Strategy 1: Buy a December Put Option (strike \$290)

How it works

- You pay \$21.30 per share (\$2,130 total) for the right (not obligation) to sell at \$290 until expiration in December.
- If Apple's price falls below \$290, you can still sell at \$290.
- If the price rises, you keep your shares and let the put expire.

Advantages

- **Guaranteed minimum price:** You can always sell at $290 no matter how low the market goes.
- **Upside potential retained:** If Apple rises above $316, you keep the gains.
- **Works in volatile or gapping markets:** Protects even if the price gaps down past $290 overnight.

Disadvantages

- **Upfront cost:** You lose the $2,130 premium if Apple stays above $290.
- **Time decay:** Option loses value as it approaches expiry.
- **No trigger for automatic sale:** You must exercise or sell the option actively.

Strategy 2: Stop-Loss Order at $290

How it works

- You tell your borker to sell your 100 shares if Apple drops to $290 or below.

Advantages

- **No cost to set up:** Unlike a put, you don't pay a premium.
- **Simple and automatic:** If price hist $290, your shares are sold without further action.

Disadvantages

- **Not guaranteed execution price:** If the price gaps below $290, you may sell at a much lower price (e.g., $280).
- **No protection below stop price:** It's a trigger to market sell, not a fixed sale price.
- **No flexibility:** If the price briefly drops to $290 and then rebounds, you're out of the market.

Final Thoughts

- If you want full protection and are willing to pay for it, go with the put option.
- If you want free protection and are okay with the risk of selling below $290, go with the stop-loss.

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33. Structured bond / Contingent payment bond

A bond issued by Standard Oil some time ago worked as follows. The holder received no interest. At the bond's maturity the company promised to pay $1,000 plus an additional amount based on the price of oil at that time. The additional amount was equal to the product of 170 and the excess (if any) of the price of a barrel of oil at maturity over $25. The maximum additional amount paid was $2,550 (which corresponds to a price of $40 per barrel). Show that the bond is a combination of a regular bond, a long position in call options on oil with a strike price of $25, and a short position in call options on oil with a strike price of $40.

This type of bond issued by Standard Oil is an example of a structured bond or contingent payment bond—essentially a zero-coupon bond with an embedded option feature tied to the price of oil.

Bond terms recap

- No interest payments (i.e., zero-coupon bond)
- At maturity, the holder receives:
  - $1,000 base amount (guaranteed)
  - Plus: an additional amount $= 170 \times \max(0, P_{\text{oil}} - 25)$
    - Capped when oit hits $40
    - So, the maximum extra payout $= 170 \times (40 - 25) = 170 \times 15 = 2,550$

Step-by-step decomposition

We want to show that the bond is made up of:

1. A regular bond paying $1,000 at maturity
2. A long call option on oil with:
   - Strike price: $25
   - Quantity: 170 barrels
3. A short call option on oil with:
   - Strike price: $40
   - Quantity: 170 barrels

Regular bond component

This is simple—the investor receives $1,000 at maturity, regardless of the oil price. This is equivalent to holding a zero-coupon bond with a face value of $1,000.

Long call option @ $25

The formula

$$170 \times \max(0, P_{\text{oil}} - 25)$$

is exactly the payoff of 170 call options on oil with a strike price of $25.

- If the oil price is above $25, the call options are in the money.
- Each call gives: $\max(0, P_{\text{oil}} - 25)$
- Multiply that by 170 and that's the total extra payment.

So, the investor is effectively long 170 call options at $25.

Short call option @ $40

The total payment is capped at $2,550—which happens only if oil hits or exceeds $40.

- That's equivalent to saying: the bondholder doesn't get any more upside beyond $40.
- To cap the payoff, the issuer is essentially shorting a call with a strike at $40.
- That offests any gains above $40.

Mathematically, this is the same as:

$$170 \times [\max(0, P_{\text{oil}} - 25) - max(0, P_{\text{oil}} - 40)]$$

Which simplifies to:

$$170 \times \text{Payoff of a bull call spread from 25 to 40}$$

---

34. Structured protection option strategy

Suppose that in the situation of the following table, a corporate treasurer said: "I will have 1 million pounds to sell in 6 months. If the exchange rate is less than 1.19, I want you to give me 1.19. If it is greater than 1.25, I will accept 1.25. If the exchange reate is between 1.19 and 1.25, I will sell the sterling for the exchange rate." How could you use options to satisfy the treasurer?

Table 3: Spot and forward quotes for the exchange ratebetween USD and GBP on May 21, 2020

Spot	Bid	Ask
Spot	1.2217	1.2220
1-month forward	1.2218	1.2222
3-month forward	1.2220	1.2225
6-month forward	1.2224	1.2230

To meet the corporate treasurer's request—a structured protection where:

- They get at least 1.19 USD/GBP if the exchange reate drops.
- They get no more than 1.25 USD/GBP if the exchange rate rises.
- But they can benefit from any rate between 1.19 and 1.25

You can use a collar strategy in the foreign exchange options market.

Goal recap

If USD/GBP in 5 months is...	Treasurer wants:
< 1.19	To sell at 1.19 (protection)
Between 1.19 and 1.25	To sell at market rate
> 1.25	To sell at 1.35 (cap gains)

Solution: Use a collar (options strategy)

1. Buy a 6-month put option on GBP with a strike price of 1.19. This guarantees a minimum sale price of 1.19 (downside protection).
2. Sell a 6-month call option on GBP with a strike price of 1.25. This gives up upside beyond 1.25, but the premium from selling the call can offset the cost of the put.

How the collar works

Spot rate in 6 months	Net results
< 1.19	Use the put to sell at 1.19
1.19 to 1.25	Let the options expire and sell at spot rate
> 1.25	Called away at 1.25 (via short call)

So, the treasurer effectively locks in a range of outcomes: min = 1.19, max = 1.25, and benefits from spot rates in between.

Implementation notes

- Contract size should be 1 million GBP.
- Use 6-month GBP/USD options
- Verify that the put premium $\approx$ call premium to make the strategy zero-cost, or low-cost.
- The available spot and forward rates in the table don't affect this options structure directly, but you would use forward rates as teh base case if not hedgin with options.
