T-304-CACS

Name: Jóhann Berentsson

Kennitala: 050188-2489

Explain your solution concisely but clearly. Include all derivation steps. Make sure a fellow student would be able to understand what you mean.

1. Find the largest possible volume V of a cylindrical metal can having a given surface area A.

Solution: Volume of a cylinder: $V = h\pi r^2$		Surface area of a cylinder: $A = (2\pi r^2) + (2\pi rh)$	
1. Isolate h for surface. $A = (2\pi r^2) + (2\pi rh)$	2. Insert the volume formula. $h = \frac{A - 2\pi r^2}{2pir}$	3. Find derivative. $V(r) = \frac{Ar}{2} - \pi r^3$	4. Critical point. $V'(r) = \frac{A}{2} - 3\pi r^2 = 0$
$2\pi r^2 = A - 2\pi r^2$	$V = h\pi r^2$	$V'(r) = \frac{Ar}{2} - 3\pi r^2$	$3\pi r^2 = \frac{A}{2}$
$h = \frac{A - 2\pi r^2}{2\pi r}$	$V = \left(\frac{A - 2\pi r^2}{2\pi r}\right)\pi r^2$	$V'(r) = \frac{A}{2} - 3\pi r^2$	$r^2 = \frac{A}{(3 \times 2)\pi}$
	$V = \frac{(A\pi r^2) - (2\pi^2 r^4)}{2\pi r}$		$r = \sqrt{\frac{A}{(3 \times 2)\pi}}$
	$V = \frac{A\pi r^2}{2\pi r} - \frac{2\pi^2 r^4}{2\pi r}$		$r = \sqrt{\frac{A}{6\pi}}$
	$V = \frac{Ar}{2} - \pi r^3$		

5. Find h for volume
$$r = \sqrt{\frac{A}{6\pi}} \qquad h = \frac{\frac{2A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad V = h \times \pi \times r^{2}$$

$$h = \frac{A-2\pi r^{2}}{2\pi r} \qquad h = \frac{\frac{2A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad V = \frac{\sqrt{6\pi A}}{3\pi} \times \pi$$

$$h = \frac{A-2\pi(\sqrt{\frac{A}{6\pi}})^{2}}{2\pi(\sqrt{\frac{A}{6\pi}})} \qquad h = \frac{2A}{3} \times \frac{1}{\sqrt{\frac{2\pi A}{6\pi}}} \qquad V = \frac{\sqrt{6\pi A} \times \pi \times A}{3\pi \times 1 \times 6\pi}$$

$$h = \frac{A-2\pi(\frac{A}{6\pi})}{2\pi(\sqrt{\frac{A}{6\pi}})} \qquad h = \frac{2A}{3} \times \frac{1}{2\pi\sqrt{\frac{A}{6\pi}}} \qquad V = \frac{\sqrt{6\pi A} \times \pi \times A}{3\pi \times 1 \times 6\pi}$$

$$h = \frac{A-2\pi(\frac{A}{6\pi})}{2\pi(\sqrt{\frac{A}{6\pi}})} \qquad h = \frac{2A}{3} \times \frac{1}{2\pi\sqrt{\frac{A}{6\pi}}} \qquad V = \frac{\sqrt{6\pi A} \times A}{3\pi \times 1 \times 6\pi}$$

$$h = \frac{A-\frac{2\pi A}{6\pi}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad h = \frac{2A}{3} \times \frac{1}{2\pi\sqrt{A}}$$

$$h = \frac{A-\frac{A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad h = \frac{2A}{3} \times \frac{\sqrt{6\pi}}{2\pi\sqrt{A}}$$

$$h = \frac{2A}{3} \times \frac{\sqrt{6\pi}}{2\pi\sqrt{A}}$$

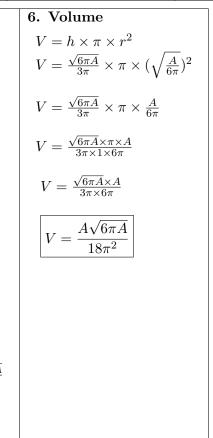
$$h = \frac{2A}{3} \times \frac{\sqrt{6\pi}}{2\pi\sqrt{A}}$$

$$h = \frac{2A\sqrt{6\pi A}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad h = \frac{(2A)(\sqrt{6\pi})}{(3)(2\pi\sqrt{A})}$$

$$h = \frac{2A\sqrt{6\pi A}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad h = \frac{2A\sqrt{6\pi A}}{6\pi\sqrt{A}}$$

$$h = \frac{2A\sqrt{6\pi A}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad h = \frac{\sqrt{12\pi A}}{6\pi\sqrt{A}}$$

$$h = \frac{3A-A}{2\pi\sqrt{\frac{2\pi A}{6\pi}}} \qquad h = \frac{\sqrt{6\pi A}}{3\pi}$$



Deadline: Week 7, Tuesday 23:59 T-304-CACS

2. Consider the function

$$f(x) = \sqrt{\frac{1}{3 + e^x}}$$

- 1. Find a Taylor approximation of this function around 0 up to order 2.
- 2. Use the result to approximate f(0.3). How many decimal digits are correct in the approximation?

Solution:	

3. Consider the function

$$f(x) = \arctan x + ax$$

- 1. For what values of a does this function have a local minimum?
- 2. Find the (x, y) coordinates of the local minimum in terms of a.

Solution:	