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Explain your solution concisely but clearly. Include all derivation steps. Make sure a fellow student would be able to understand what you mean.

1. Find the largest possible volume V of a cylindrical metal can having a given surface area A.

Solution: Volume of a cylinder: $V = h\pi r^2$ | Surface area of a cylinder: $A = (2\pi r^2) + (2\pi rh)$

1. Isolate h for surface.

$A = (2\pi r^2) + (2\pi rh)$

$2\pi r^2 = A - 2\pi r^2$

$h = \frac{A-2\pi r^2}{2\pi r}$

2. Insert the volume formula.

$h = \frac{A-2\pi r^2}{2pir}$

$V = h\pi r^2$

$V = (\frac{A-2\pi r^2}{2\pi r})\pi r^2$

$V = \frac{(A\pi r^2)-(2\pi^2 r^4)}{2\pi r}$

$V = \frac{A\pi r^2}{2\pi r} - \frac{2\pi^2 r^4}{2\pi r}$

$V = \frac{Ar}{2} - \pi r^3$

3. Find derivative.

$V(r) = \frac{Ar}{2} - \pi r^3$

$V'(r) = \frac{Ar}{2} - 3\pi r^2$

$V'(r) = \frac{A}{2} - 3\pi r^2$

4. Critical point.

$V'(r) = \frac{A}{2} - 3\pi r^2 = 0$

$3\pi r^2 = \frac{A}{2}$

$r^2 = \frac{A}{(3\times 2)\pi}$

$r = \sqrt{\frac{A}{(3\times 2)\pi}}$

$r = \sqrt{\frac{A}{6\pi}}$

5. Find h for volume

$r = \sqrt{\frac{A}{6\pi}}$

$h = \frac{A-2\pi r^2}{2\pi r}$

$h = \frac{A-2\pi(\sqrt{\frac{A}{6\pi}})^2}{2\pi(\sqrt{\frac{A}{6\pi}})}$

$h = \frac{A-2\pi(\frac{A}{6\pi})}{2\pi(\sqrt{\frac{A}{6\pi}})}$

$h = \frac{A-\frac{2\pi A}{6\pi}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{A-\frac{A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{\frac{3A}{3}-\frac{A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{\frac{3A-A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{\frac{2A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{\frac{2A}{3}}{2\pi\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{2A}{3} \times \frac{1}{\sqrt{\frac{2\pi A}{6\pi}}}$

$h = \frac{2A}{3} \times \frac{1}{2\pi\sqrt{\frac{A}{6\pi}}}$

$h = \frac{2A}{3} \times \frac{1}{\frac{2\pi\sqrt{A}}{\sqrt{6\pi}}}$

$h = \frac{2A}{3} \times \frac{\sqrt{6\pi}}{2\pi\sqrt{A}}$

$h = \frac{(2A)(\sqrt{6\pi})}{(3)(2\pi\sqrt{A})}$

$h = \frac{2A\sqrt{6\pi}}{6\pi\sqrt{A}} = \frac{\sqrt{12\pi A}}{\sqrt{6\pi A}}$

$h = \frac{\sqrt{6\pi A}}{3\pi}$

6. Volume

$V = h \times \pi \times r^2$

$V = \frac{\sqrt{6\pi A}}{3\pi} \times \pi \times (\sqrt{\frac{A}{6\pi}})^2$

$V = \frac{\sqrt{6\pi A}}{3\pi} \times \pi \times \frac{A}{6\pi}$

$V = \frac{\sqrt{6\pi A}\times\pi\times A}{3\pi\times 1\times 6\pi}$

$V = \frac{\sqrt{6\pi A}\times A}{3\pi\times 6\pi}$

$$\boxed{V = \frac{A\sqrt{6\pi A}}{18\pi^2}}$$

2. Consider the function

$$f(x) = \sqrt{\frac{1}{3 + e^x}}$$

   1. Find a Taylor approximation of this function around 0 up to order 2.
   2. Use the result to approximate $f(0.3)$. How many decimal digits are correct in the approximation?

Solution:

3. Consider the function

$$f(x) = \arctan x + ax$$

   1. For what values of a does this function have a local minimum?
   2. Find the (x, y) coordinates of the local minimum in terms of a.

Solution: