Derivatives

$f(x) = sin(x)$ $f'(x) = cos(x)$ $f''(x) = -sin(x)$ $f'''(x) = -cos(x)$
$f(x) = 2sin(x) + 3cos(x)$ $f(x) = ag(x) + bh(x)$ $f'(x) = aq'(x) + bh'(x)$
$f(x) = e^x$ $f'(x) = e^x$
$f(x) = e^(2sin(x) + 4cos(x))$ $f(x) = e^x$ $f'(x) = e^x$
$g(x) = 2sin(x) + 4cos(x)$ $g'(x) = 2cos(x) - 4sin(x)$

Derivative

$f(x) = sqrt(x)cos^2(x + 1)$

Midterm

TODO: add midterm to calendar 2. or 3. of October.

- Optimisation (Local / Gloal)
- Derivatives
- L'Hopital
- Word Problem

L'Hopital

$lim_{x\to 0}\frac{x-xcos(x)}{x-sin(x)}$
$f(0)=\frac{0-0\times 1}{0-0}=\frac{0}{0}=0$
$=1-(cos(x)-xsin(x))=1-cos(x)+xsin(x)$
Use product rule on the counter.
$h(x)=x-xcos(x)$
$h'(x)=1-[xcos(x)]\frac{d}{dx}=1-((x)(-sin(x))+(1)(cos(x)))=1-(-xsin(x)+cos(x))$
$g'(x)=[x-sin(x)]\frac{d}{dx}=1-cos(x)$
$f'(x)=\frac{h'(x)}{g'(x)}=\frac{[\frac{x-xcos(x)}{x-sin(x)}]\frac{d}{dx}}{[x-sin(x)\frac{d}{dx}]}=$
$f'(0)=\frac{1-1+0\times 0}{1-1}=0$
$f''(x)=\frac{h''(x)}{g''(x)}=\frac{xcos(x)+sin(x)}{sin(x)}$
$h''(x)=[1-cos(x)+xsin(x)]\frac{d}{dx}=0-(-sin(x))+[xsin(x)]\frac{d}{dx}$
$=(x)(cos(x))+(1)(sin(x))=xcos(x)+sin(x)$
$=sin(x)+(xcos(x))+sin(x)$
$g''(x)=[1-cos(x)]\frac{d}{dx}=0-(-sin((x))=sin(x))$
$f''(0)=\frac{0+0+0}{0}=\frac{0}{0}$
$f'''(x)=\frac{h'''(x)}{g'''(x)}$
$h'''(x)=\frac{[sin(x)+xcos(x)+sin(x)]\frac{d}{dx}}{[sin(x)]\frac{d}{dx}}=\frac{cos(x)+cos(x)+sin(x)+cos(x)}{cos(x)}$
$h'''(0)=\frac{1+1+1}{1}=\frac{3}{1}=3$

Word Problem

Given a parameter P, what rectangle has the largest area.

maximize a function = find the derivative

$P = 2x + 2y$

$A(x, y) = x \times y$

$2y = P - 2x$

$y = \frac{P}{2} - \frac{2x}{2} = \frac{P}{2} - x$

$A(x, y) = x \times (\frac{P}{2} - x) = \frac{P}{2}x - x^2$

$A(x) = \frac{P}{2}x - x^2$

$A'(x) = \frac{P}{2} - 2x$

$\frac{P}{2} - 2x = 0 => x = \frac{P}{4}$

$2y = P - 2x$

$y = \frac{P}{2} - \frac{2x}{2} = (\frac{P}{2} - x)$

$y = (\frac{P}{2} - \frac{P}{4}) = \frac{P}{4}$

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␣␣␣␣P/4
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|␣␣␣␣␣␣␣␣␣␣|
|␣␣␣␣␣␣␣␣␣␣|␣p
|␣␣␣␣␣␣␣␣␣␣|␣-
|␣␣␣␣␣␣␣␣␣␣|␣4
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$\frac{P}{4} \times \frac{P}{4} = \frac{P2}{16}$

Optimisation

take derivative, set to 0.
If bounded, check ends. $f(x) = xe^-2$ on range $[-\frac{1}{2}, 2]$
product rule and chain rule
$f'(x) = [xe^(- x^2)]\frac{d}{dx}$
$h(x) = x$
$h'(x) = 1$
$g(x) = e^-2$
$g'(x) = [f(g(x))]\frac{d}{dx} = f'(g(x)) \times g'(x)$
$f(x) = e^x$
$g(x) = -x^2$
$f'(x) = e^x$
$g'(x) = -2x$
$f'(x) = e^{-x^2} \times (-2x) + e^{-x^2}$
$= -2x^2e^{-x^2} + e^{-x^2} = 0$
We need ln to get rid of the e.
$ln(-2x^2e^{-x^2} + e^{-x^2}) = ln(0)$
$2ln(2xe^{-x^2} + e^{-x^2}) = ln(0)$
$2 \times (-x^2) \times (-x^2) \rightarrow 2 \times a \times a = 2a^2 \rightarrow 2(-x^2)^2 = -2x^4$
this is simpler
$e^{-x^2}(2x^2 + 1)$
$-2x^2 + 1 = 0$

Taylor series

Not on midterm

Intergrals

U-substitution	Chain Rule
integration by-parts	Product Rule

$$\begin{array}{ll} u = x & dv = e^x dx \\ \frac{du}{dx} = 1 & v = e^x \\ du = dx & \end{array}$$

Example 1

$\int v dv = uv - \int v dv$
$\boxed{\int x e^x dx} = x e^x - \int e^x dx$
$= x e^x - e^x + C$
$= e^x (x-1) + C$

Example 2

$\int_0^1 x^2 \sqrt{(}1-x) dx$ Integration by parts.
$\int x\sqrt{x} = \int x^{\frac{3}{2}}$
$\int_0^1 x^2 \sqrt{(}1-x) dx = [-\frac{2}{3} x^2 (1-x)^{\frac{3}{2}}]_0^1 + \boxed{\frac{2}{3} \int_0^1 ex(1-x)^{\frac{3}{2}} dx}$
Plug in the numbers.
$[-\frac{2}{3} \times \frac{4}{5} \times x(1-x)^{\frac{5}{2}}]_0^1 + \frac{2}{3} \times \frac{4}{5}$
$[-\frac{2}{3} \times \frac{4}{5} \times \frac{2}{7} \times (1-x)^{\frac{7}{2}}]_0^1 = \frac{16}{105}$

Example 3

U-substitution
$\int_0^1 x^2 \times \sqrt{1-x} = -\int_1^0 (1-u)^2 \times \sqrt{u} du$
$\boxed{u = 1-x}$
$\boxed{dx = -du}$
$= \int_0^1 (1-2u-u^2)\sqrt{u} du = \int_0^1 (u^{\frac{1}{2}} - 2u^{\frac{3}{2}} + u^{\frac{5}{2}}) du$
$\int a + b - c = \int a + \int b - \int c$
$[\frac{2}{3} u^{\frac{3}{2}}]_0^1 - [-\frac{4}{5} u^{\frac{4}{5}}]_0^1 + [\frac{2}{7} u^{\frac{7}{2}}]_0^1 = \frac{70-84+30}{105} = \boxed{\frac{16}{105}}$

How do I know which to use? Do 100 intergrals!

Examaple 3

$\int_0^1 x^2\sqrt{1-x}dx$ Integrate by-parts.

$$\boxed{\int udu = uv - \int vdu}$$

$u = x^2$ $\frac{du}{dx} = 2x$ $du = 2xdx$

$dv = \sqrt{1-x}$ $v = \frac{2(1-x)^{\frac{3}{2}}}{3}$

$= (1-x)^{\frac{1}{2}} = \frac{(1-x)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2(1-x)^{\frac{3}{2}}}{3}$

$= [\frac{2}{3}x^2(1-x)^{\frac{3}{2}}]_0^1 - \frac{2}{3}\int(1-x)^{\frac{3}{2}}\sqrt(1-x)$

$[-\frac{2}{3}x^2(1-x)^{\frac{3}{2}}]_0^1 + \frac{2}{3}\int(1-x)^{\frac{3}{2}}2xdx$

There might be some errors in this example.

Linear Algebra

1.

$\int_a^b = F(b) - F(a)$

$sin(x)$ and $sin(x+\frac{\pi}{3})$

$sin(x) = sin(x+\frac{\pi}{3})$

$sin(x) = sin(x)cos(\frac{\pi}{3}) + cos(x)sin(\frac{pi}{3})$

$sin(x) = \frac{1}{2}sin(x) + \frac{\sqrt{3}}{2}cos(x)$

$sin(x) - \frac{1}{2}sin(x) = \frac{\sqrt{3}}{2}cos(x)$

$\frac{1}{2}sin(x) = \frac{\sqrt{3}}{2}cos(x)$

$\frac{1}{2}sin(x) = \frac{\sqrt{3}}{2}cos(x)$

$sin(x) = \sqrt{3}cos(x)$

$tan(x) = \sqrt{3}cos(x)$

$tan(x) = \sqrt{3}$

$k \in \mathbb{Z}$

$$\boxed{x = \frac{\pi}{3} + k\pi}$$

$\int_{\frac{-2\pi}{3}}^{\frac{\pi}{3}} sin(x + \frac{\pi}{3}) - sin(x)dx$
Use angle sum to get:
$\int_{\frac{-2\pi}{3}}^{\frac{\pi}{3}} cos(x + \frac{\pi}{6})dx$
Plug the number in:
$sin(\frac{\pi}{3} + \frac{\pi}{6}) - sin(\frac{-2\pi}{3} + \frac{\pi}{6}) = 2$

2.

$$\boxed{\begin{array}{l} A = (1, -5, 5) \\ B = (11, 5, 0) \\ C = (3, 9, 10) \end{array}}$$

This is a triangle.

(a) Side lengths

$||\overline{AC}|| = \sqrt{(3-1)^2 + (9+5)^2 + (10-5)^2} = \sqrt{2^2 + 14^2 + 5^2} = 15$
$||\overline{AB}|| = \sqrt{180}$
$||\overline{BC}|| = 15$
The dot product tells you in how much in A directis is B going.
The cross product gives us a 3D vector.
Right hand rule tells us if it is going towards us or away from us.

(b) Angle from A

$$\boxed{\begin{array}{l} A = (a, b, c) \\ B = (d, e, f) \end{array}}$$

$AB = |A| \times |B| \times cos(p)$
Dot product $AB = ad + be + cf$
Vectors:
$\overline{AC} = ((3-1), (9+5), (10-5)) = (2, 14, 5)$
$\overline{AB} = ((11-1), (5+5), (0-5)) = (10, 10, -5)$
$\overline{BC} = ((3-11), (9-5), (10-0)) = (-8, 4, 10)$
$\overline{AC} \times \overline{AB} = 20 + 140 - 25 = 135$
Magnitude of the vector:
$||\overline{AB}|| = 6\sqrt{5}$
$||\overline{AC}|| = 15$
$135 = 6\sqrt{5} \times 15 \times cos(\varphi)$
$135 = 90\sqrt{5} \times cos(\varphi)$

$cos(\varphi) = \frac{135}{90\sqrt{5}}$

$arccos(\frac{135}{90\sqrt{5}})$

(c) Area of triangle

Cross product $|A \times B| = |A| \times |B| sin(\varphi)$

$|\overline{AB} \times \overline{AC}| = |\overline{AB}| \times |\overline{AC}| \times sin(\varphi) = 6\sqrt{5} \times 15 \times sin(0.8) = 149.86$

(d) Projection of $\overline{AC}$ to $\overline{AB}$

$proj_{\overline{AB}}\ \overline{AC} = \frac{\overline{AB} \times \overline{AC}}{|\overline{AB}|} = \frac{135}{6\sqrt{5}} = \frac{9\sqrt{5}}{2}$
Dot product: $\overline{AB} \times \overline{AC} = 135$
$proj_{\overline{AB}}\ \overline{AC} = \text{vector} \times \text{scaler} = (2, 14, 5) \times \frac{9\sqrt{5}}{2} = (9\sqrt{5}, 63\sqrt{5}, \frac{45\sqrt{5}}{2})$

(e) $Proj_{\overline{AB}}\overline{AC}$