

$A_1 m(t) \cos(2\pi f_0 t)$ \leftarrow Señal DSB-CS

$$\mathcal{F}\{A_1 m(t) \cos(2\pi f_0 t)\}$$

Sabemos que:

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Luego

$$\cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

Con lo anterior

$$A_1 m(t) \left[\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \right] = A_1 m(t) \cos(2\pi f_0 t)$$

$$= \frac{A_1 m(t) e^{j2\pi f_0 t}}{2} + \frac{A_1 m(t) e^{-j2\pi f_0 t}}{2}$$

Así:

$$\mathcal{F}\{A_1 m(t) \cos(2\pi f_0 t)\} = \mathcal{F}\left\{\frac{A_1 m(t) e^{j2\pi f_0 t}}{2}\right\}$$

$$+ \mathcal{F}\left\{\frac{A_1 m(t) e^{-j2\pi f_0 t}}{2}\right\} =$$

$$\frac{A_1}{2} \left(\mathcal{F}\{m(t) e^{j2\pi f_0 t}\} + \mathcal{F}\{m(t) e^{-j2\pi f_0 t}\} \right)$$

Ahora, si

$$F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = X(\omega)$$

Luego

$$\begin{aligned} F\{x(t)e^{j\omega_0 t}\} &= \int_{-\infty}^{\infty} x(t) e^{j\omega_0 t} e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} x(t) e^{j\omega_0 t - j\omega t} dt = \\ &\int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt = X(\omega - \omega_0) \end{aligned}$$

Por lo tanto

$$\begin{aligned} A_1 \left(F\{m(t)e^{j2\pi f_0 t}\} + F\{m(t)e^{-j2\pi f_0 t}\} \right) \\ = \frac{A_1}{2} \left[M(\omega - 2\pi f_0) + M(\omega + 2\pi f_0) \right] \end{aligned}$$

Luego del Mixer 1, se ve que

$$A_1 m(t) \cos^2(2\pi f_0 t) = \frac{A_1 m(t)}{2} + \frac{A_1 m(t) \cos(4\pi f_0 t)}{2}$$

Con lo que se espera de Fourier es:

$$\mathcal{F}\left\{\frac{A_1 m(t)}{2} + \frac{A_1 m(t)}{2} \cos(4\pi f_0 t)\right\}$$

$$= \frac{A_1}{2} \left(\mathcal{F}\{m(t)\} + \mathcal{F}\{m(t) \cos(4\pi f_0 t)\} \right)$$

Muestramente

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Por lo cui:

$$\cos(4\pi f_0 t) = \frac{e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}}{2}$$

A₃ guchs:

$$\frac{A_1}{2} \left(\mathcal{F}\{m(t)\} + \mathcal{F}\left\{\frac{m(t)e^{j4\pi f_0 t}}{2}\right\} + \mathcal{F}\left\{\frac{m(t)e^{-j4\pi f_0 t}}{2}\right\} \right)$$

$$= \frac{A_1}{2} \left(M(w) + \frac{M(w - 4\pi f_0)}{2} + \frac{M(w + 4\pi f_0)}{2} \right)$$

Luego del Lowpass filter la señal guchs

$$\frac{A_1}{2} m(t)$$

Con lo cui es evidente que su espectro de Fourier

es

$$\mathcal{F} \left\{ \frac{A_1}{2} m(t) \right\} = \frac{A_1}{2} M(w)$$

y por ultimo despues del sumo amplitud
queda

$m(t)$

$$\mathcal{F} \{ m(t) \} = M(w)$$