

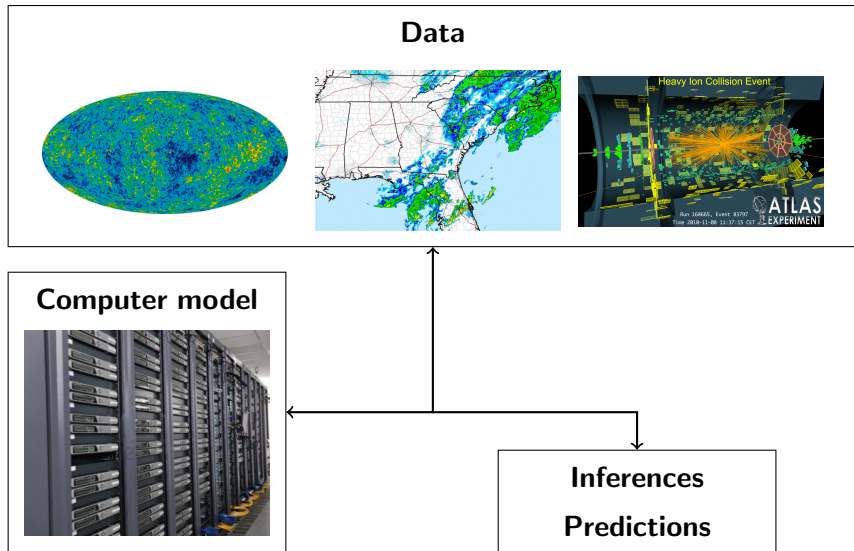
QGP parameter extraction via a global  
analysis of event-by-event flow  
coefficient distributions

Jonah Bernhard

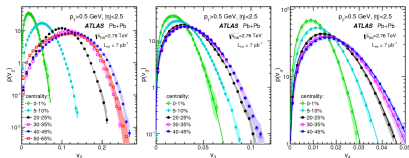
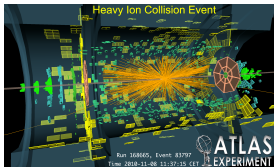
JET group meeting

March 24, 2014

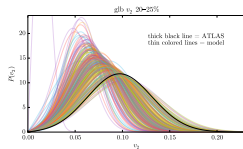
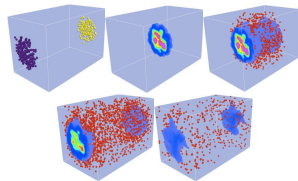
# Model-to-data comparison



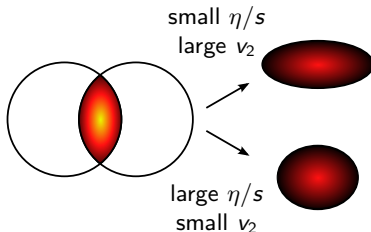
# Model-to-data comparison: heavy-ion collisions



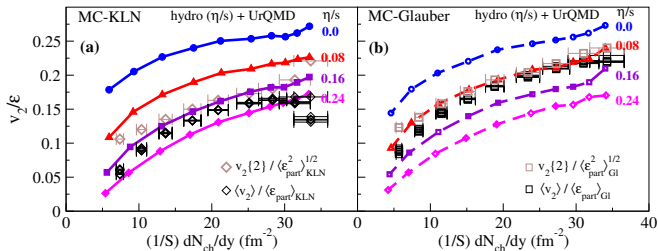
**Model**  
Initial conditions,  
 $\tau_0, \eta/s, \dots$



# Measuring QGP $\eta/s$



- Observe experimental  $v_n$ .
- Run model with variable  $\eta/s$ .
- Constrain  $\eta/s$  by matching  $v_n$ .



H. Song, S. A. Bass, U. Heinz, T. Hirano and C. Shen, PRL **106**, 192301 (2011).

# Extracting QGP properties

- |  |            |  |
|--|------------|--|
| <ul style="list-style-type: none"><li>■ Average flow <math>\langle v_n \rangle</math>.</li><li>■ Vary only <math>\eta/s</math>, other parameters fixed.</li><li>■ Only several discrete values.</li><li>■ Qualitative constraints lacking uncertainty.</li></ul> | $\implies$ | <ul style="list-style-type: none"><li>■ Event-by-event flow <math>P(v_n)</math>.</li><li>■ Vary all salient parameters: <math>\eta/s</math>, <math>\tau_0</math>, IC parameters, ...</li><li>■ Continuous parameter space.</li><li>■ Quantitative constraints including uncertainty.</li></ul> |
|--|------------|--|

## Challenges

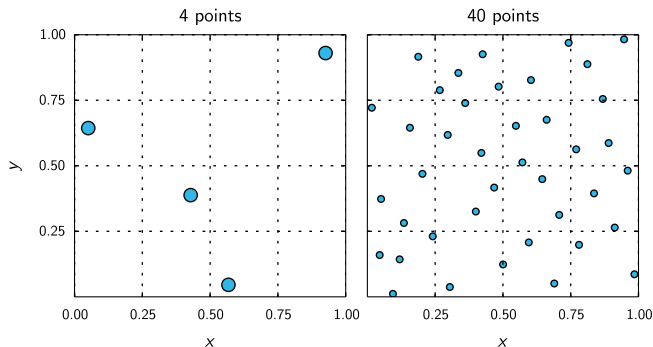
- Event-by-event models very computationally expensive,  $\sim 1$  hour per event.
- Need  $\mathcal{O}(10^3)$  events per parameter-point to study fluctuations.
- Must vary all parameters simultaneously.

## Strategies

- Evaluate model at efficient pre-determined parameter points.
  - Latin-hypercube sampling.
- Interpolate between explicitly calculated points.
  - Gaussian process emulator.

# Latin-hypercube sampling

- Random set of parameter points.
- Maximizes CPU time efficiency.
- Skeleton of parameter space.



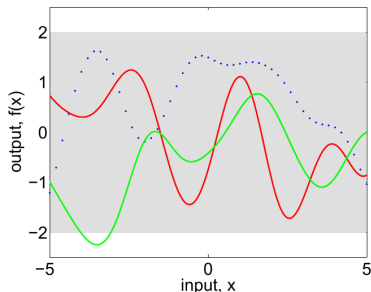
# Gaussian processes

- A Gaussian process is a collection of random variables, any finite number of which have a joint Gaussian distribution.
- Instead of drawing variables from a distribution, functions are drawn from a process.

Require a covariance function, e.g.

$$\text{cov}(x_1, x_2) \propto \exp\left[-\frac{(x_1 - x_2)^2}{2\ell^2}\right]$$

Nearby points correlated, distant points independent.

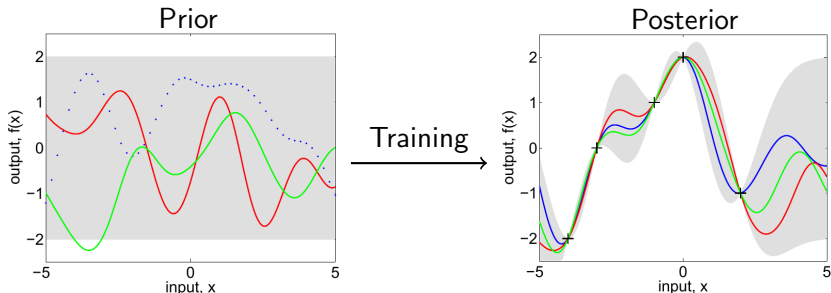


*Gaussian Processes for Machine Learning,*  
Rasmussen and Williams, 2006.



# Gaussian process emulators

- Prior: the model is a Gaussian process.
- Posterior: Gaussian process conditioned on model outputs.



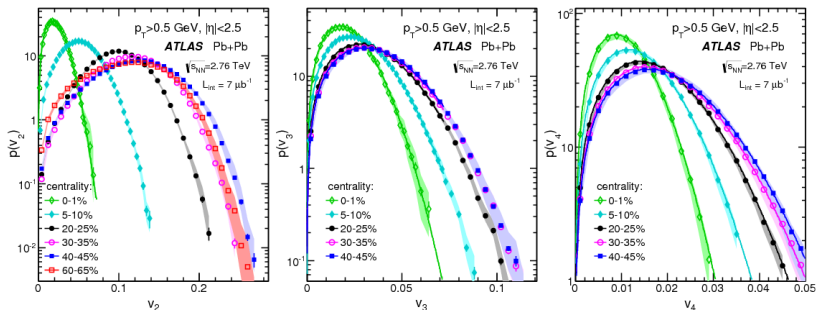
- Emulator is a fast surrogate to the actual model.
  - More certain near calculated points.
  - Less certain in gaps.

# Experimental data

- ATLAS event-by-event flow distributions  $v_2$ ,  $v_3$ ,  $v_4$ .
- Fit to Rice / Bessel-Gaussian distribution

$$P(v_n) = \frac{v_n}{\delta_{v_n}^2} e^{-\frac{(v_n)^2 + (v_n^{\text{RP}})^2}{2\delta_{v_n}^2}} I_0\left(\frac{v_n^{\text{RP}} v_n}{\delta_{v_n}^2}\right)$$

- Reduce to parameters  $v_n^{\text{RP}}$ ,  $\delta_{v_n}$ .



ATLAS Collaboration, JHEP **1311**, 183 (2013).

Modern version of Duke+OSU model VISHNU  
(Viscous Hydro and UrQMD):

- MC-Glauber & MC-KLN initial conditions

H.-J. Drescher and Y. Nara, Phys. Rev. C **74**, 044905 (2006).

- Viscous hydro

H. Song and U. Heinz, Phys. Rev. C **77**, 064901 (2008).

- Cooper-Frye sampler

Z. Qiu and C. Shen, arXiv:1308.2182 [nucl-th].

- UrQMD (Ultrarelativistic Quantum Molecular Dynamics)

S. Bass *et. al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998).

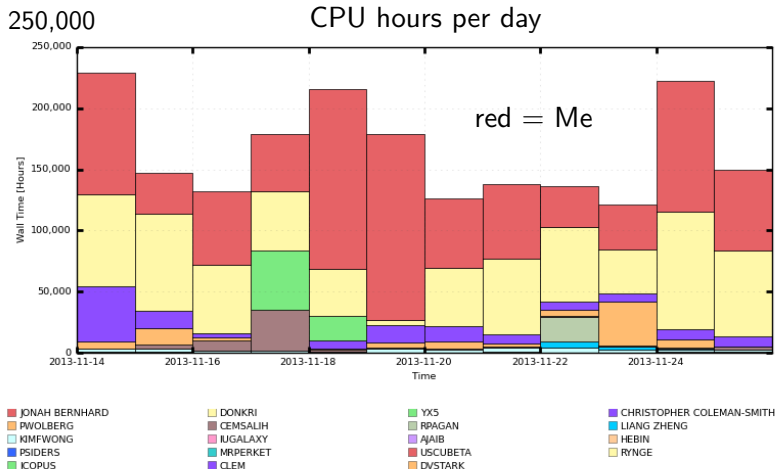
M. Bleicher *et. al.*, J. Phys. G **25**, 1859 (1999).

→ Tailored for running many events on Open Science Grid.

# Computer experiment design

- Six centrality bins 0–5%, 10–15%, ... 50–55%.
- 256 Latin-hypercube points, five input parameters:
  - Normalization
  - IC-specific parameter
  - Thermalization time  $\tau_0$
  - Viscosity  $\eta/s$
  - Shear relaxation time  $\tau_\Pi$
- Massive parallelization on Open Science Grid.
- Completed 1000–2000 events per centrality bin and input-parameter point.
  - 3.5 million total
  - $0.5 \mu\text{b}^{-1}$  (ATLAS:  $7 \mu\text{b}^{-1}$ )

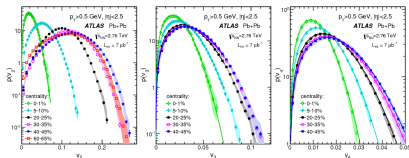
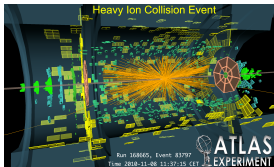
# Open Science Grid usage



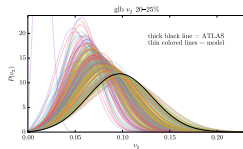
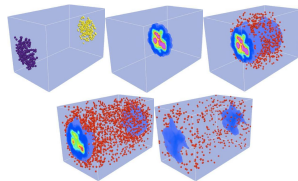
Maximum: 228,948 Hours, Minimum: 121,492 Hours, Average: 164,621 Hours, Current: 149,737 Hours

Completed KLN design (1.5 million events) in two weeks.

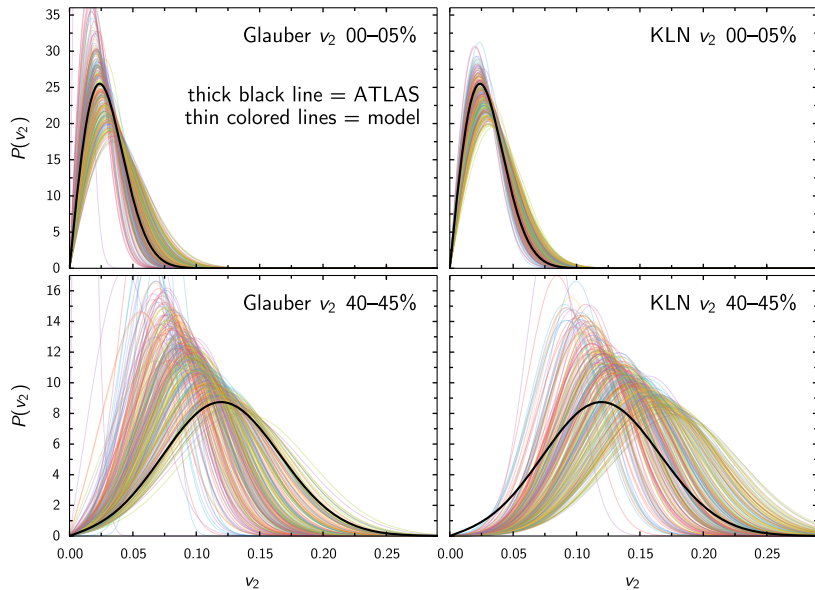
# Model-to-data comparison: heavy-ion collisions



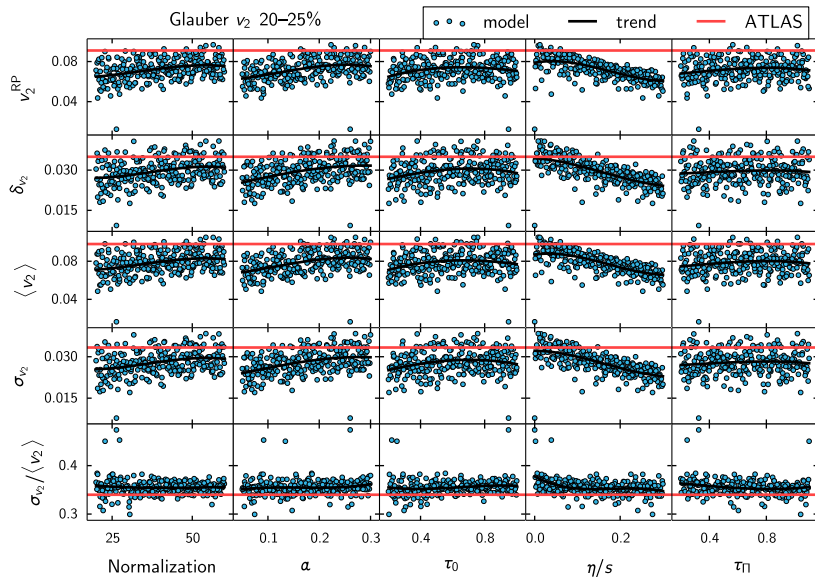
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# Model flow distributions

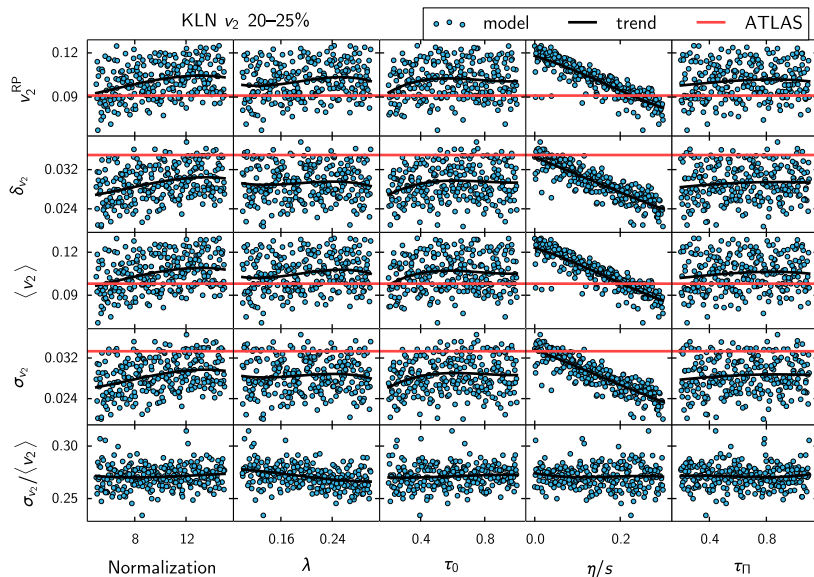


# Input-output summary

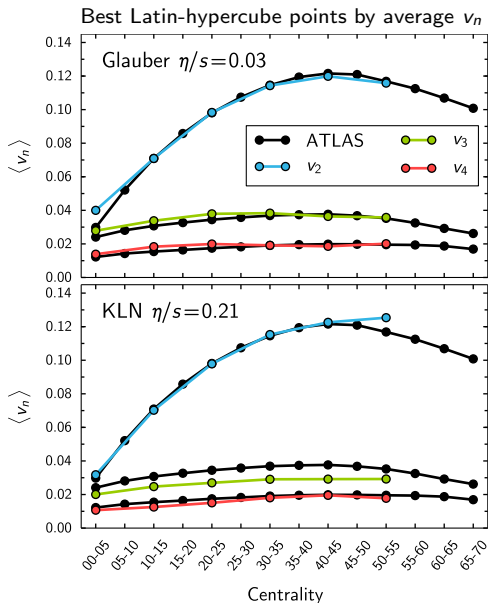




# Input-output summary

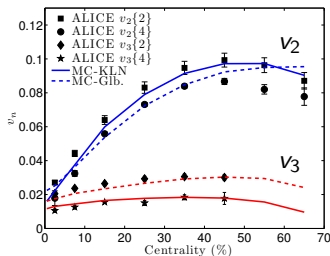


# Best parameter points



OSU results, same model

dashed: Glauber  $\eta/s = 0.08$   
solid: KLN  $\eta/s = 0.20$



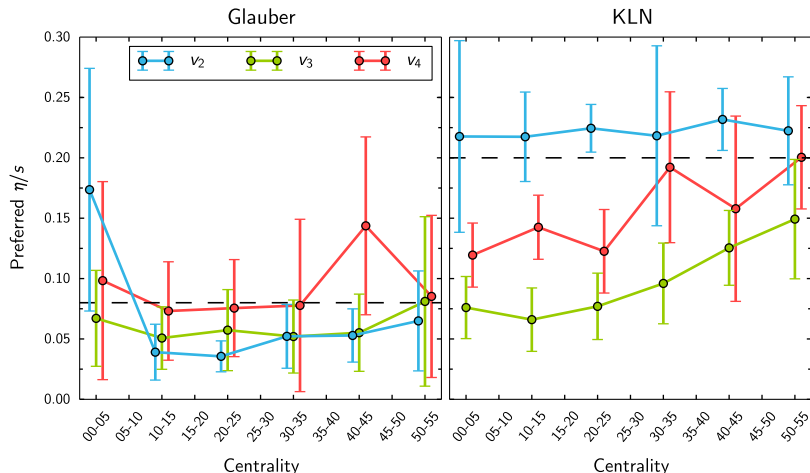
Z. Qiu, C. Shen, and U. Heinz,  
Phys. Lett. B **707**, 151 (2012).

# Constraining $\eta/s$

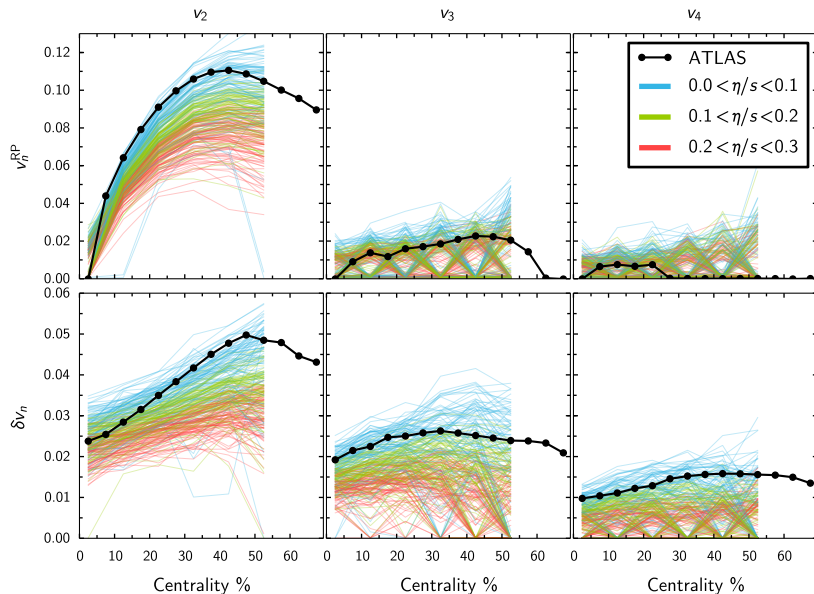
Points: average  $\eta/s$  of best 10 Latin-hypercube points by average  $v_n$

Error bars: standard deviation of best 10

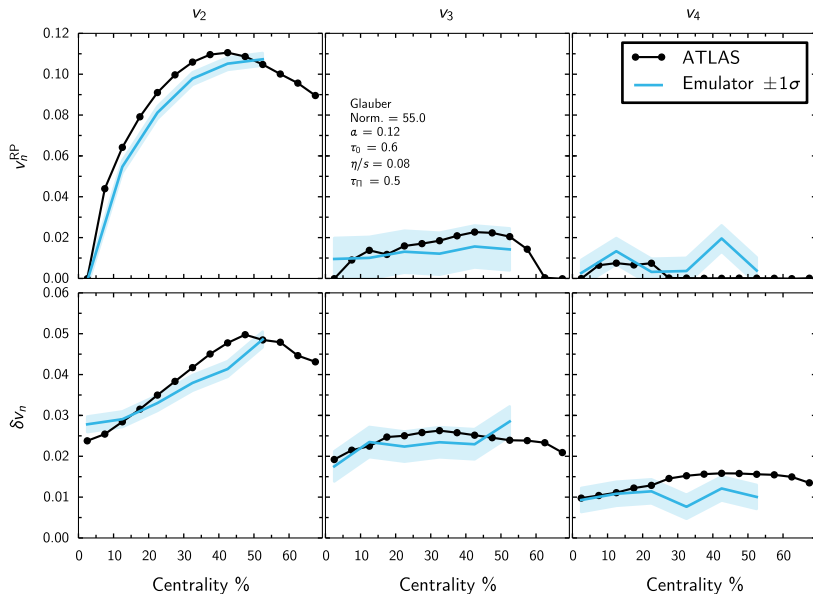
Dashed lines: canonical  $\eta/s$  (Glauber 0.08, KLN 0.20)



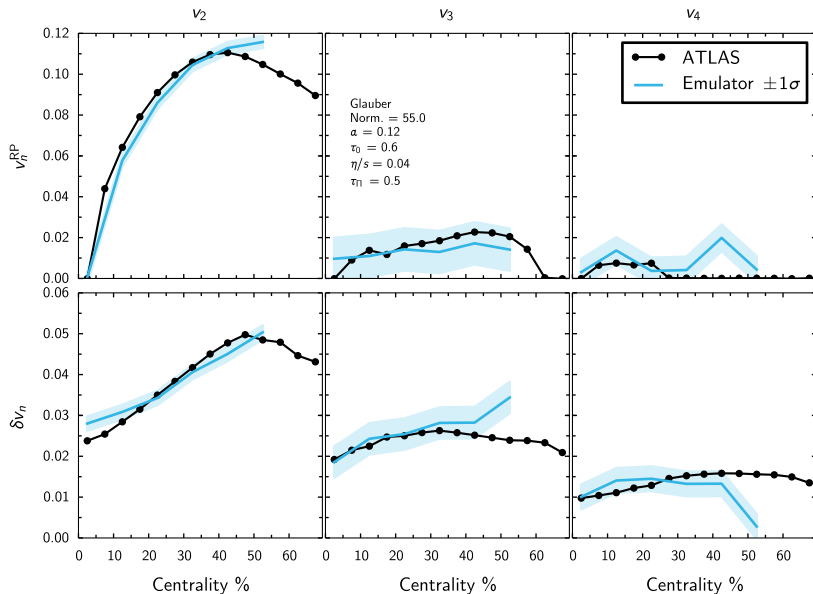
# Bessel-Gaussian parameters (Glauber)



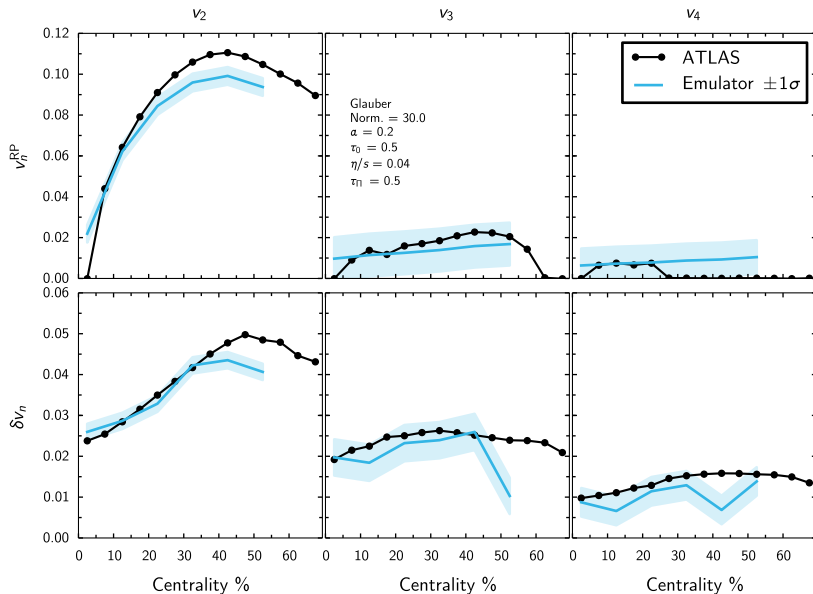
# Emulated Bessel-Gaussian parameters



# Emulated Bessel-Gaussian parameters



# Emulated Bessel-Gaussian parameters



# Summary & outlook

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- Framework for massive event-by-event model-to-data comparison: new level of knowledge-extraction capability.
- Preliminary results consistent with previous work.
- Improve flow distribution parameter estimates.
- Validate emulator.
- Calculate posterior distributions  
→ extract optimal values of parameters with uncertainty.
- Calibrate simultaneously on other observables, e.g. multiplicity.
- Repeat with more advanced models, especially initial conditions.



backup slides

# Generating Gaussian processes

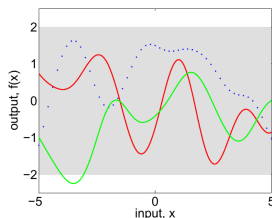
- Choose a set of input points  $X_*$ .
- Choose a covariance function, e.g.

$$k(x_i, x_j) = \exp[-(x_i - x_j)^2/2]$$

and create covariance matrix  $K(X_*, X_*)$ .

- Generate MVN samples (GPs)

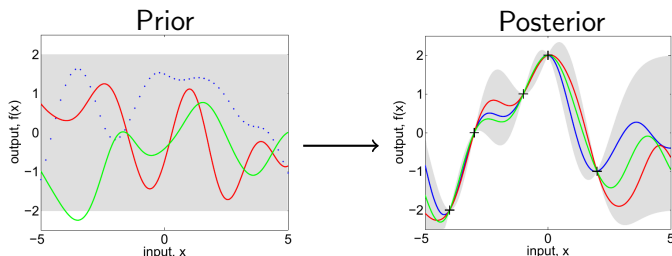
$$\vec{f}_* \sim \mathcal{N}[\vec{0}, K(X_*, X_*)].$$



# Training the emulator

- Make observations  $\vec{f}$  at training points  $X$ .
- Generate conditioned GPs

$$\vec{f}_* | X_*, X, \vec{f} \sim \mathcal{N}[K(X_*, X)K(X, X)^{-1}\vec{f}, \\ K(X_*, X_*) - K(X_*, X)K(X, X)^{-1}K(X, X_*)].$$

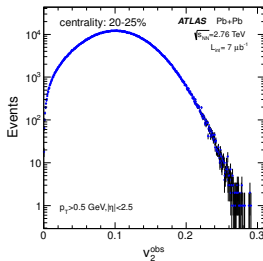
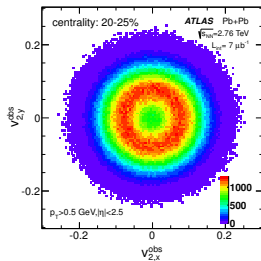


- Flow vectors follow bivariate Gaussian

$$P(\vec{v}_n) = \frac{1}{2\pi\delta_{v_n}^2} e^{-\frac{(\vec{v}_n - \vec{v}_n^{\text{RP}})^2}{2\delta_{v_n}^2}}.$$

- Integrate out angle

$$P(v_n) = \frac{v_n}{\delta_{v_n}^2} e^{-\frac{(v_n)^2 + (v_n^{\text{RP}})^2}{2\delta_{v_n}^2}} I_0\left(\frac{v_n^{\text{RP}} v_n}{\delta_{v_n}^2}\right).$$



# Finite multiplicity and unfolding

- Observed flow smeared by finite multiplicity and nonflow

$$P(v_n^{\text{obs}}) = \int P(v_n^{\text{obs}}|v_n)P(v_n) dv_n$$

where  $P(v_n^{\text{obs}}|v_n)$  is the response function.

- Pure statistical smearing  $\rightarrow$  Gaussian response

$$P(v_n^{\text{obs}}|v_n) = \frac{v_n^{\text{obs}}}{\delta_{v_n}^2} e^{-\frac{(v_n^{\text{obs}})^2 + (v_n)^2}{2\delta_{v_n}^2}} I_0\left(\frac{v_n v_n^{\text{obs}}}{\delta_{v_n}^2}\right).$$

- $v_n^{\text{RP}}$  unaffected; width increased as

$$\delta_{v_n}^2 \rightarrow \delta_{v_n}^2 + 1/2M.$$

- Results still qualitative.
- Some Bessel-Gaussian fit parameters are incorrect (MLE two-solution problem / low SNR).
- Uncertainties not quantified.
- Parameter space not systematically explored.