QGP parameter extraction via a global analysis of event-by-event flow coefficient distributions

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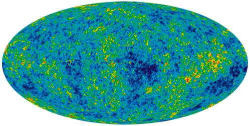
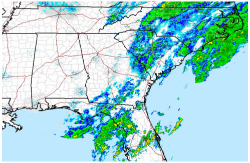
Preliminary exam

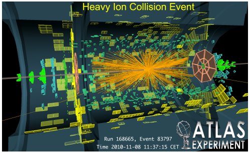
January 6, 2014

Model-to-data comparison

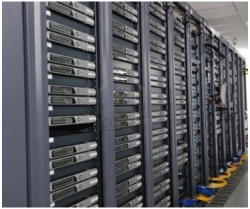
Data







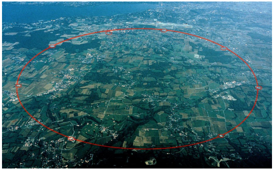
Computer model

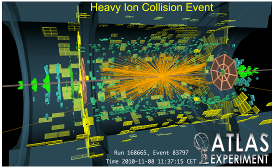


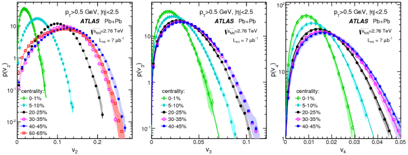
Inferences

Predictions

Model-to-data comparison: heavy-ion collisions



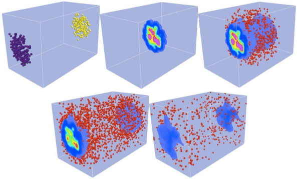


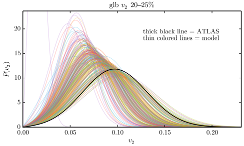


Model

Initial conditions,

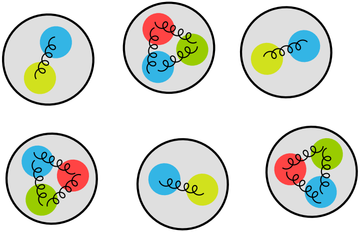
τ_0, η/s, ...



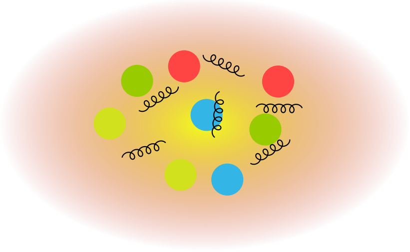


Hot QCD matter

Normal matter



Quark-gluon plasma



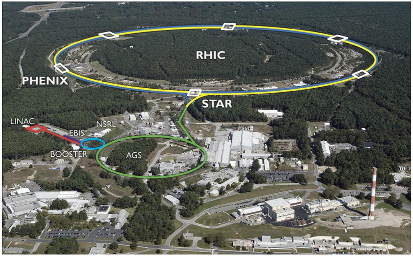
- Quarks and gluons confined to hadrons.
- Bound by strong nuclear force.
- Described by Quantum Chromodynamics (QCD).

- QCD crossover transition $T \sim 165$ MeV $\sim 10^{12}$ K.
- Deconfined quarks and gluons.
- Hot and dense, short mean free path (fluid-like).

Relativistic heavy-ion collisions

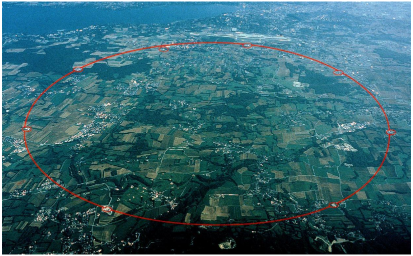
- Postulated that the universe was one large QGP in the first microseconds after the Big Bang.
- Small amounts created in relativistic heavy-ion collisions.

RHIC / BNL



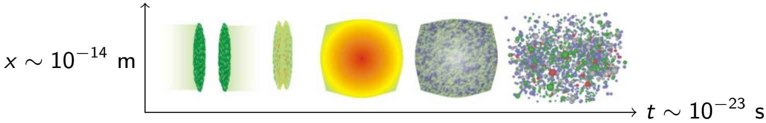
Au+Au, Cu+Cu, U+U
$\sqrt{s} \leq 200$ GeV

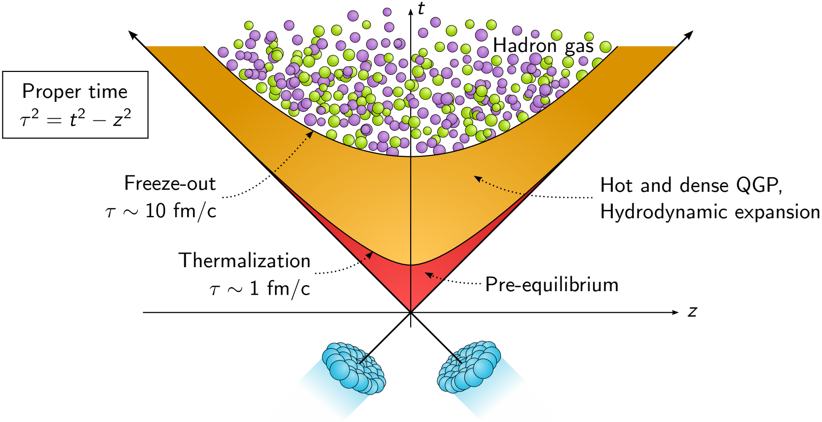
LHC / CERN



Pb+Pb
$\sqrt{s} = 2.76$ TeV

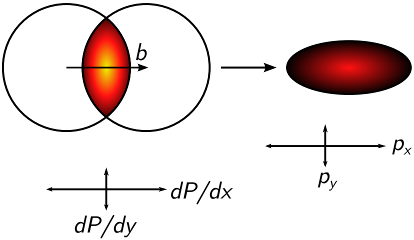
Spacetime evolution





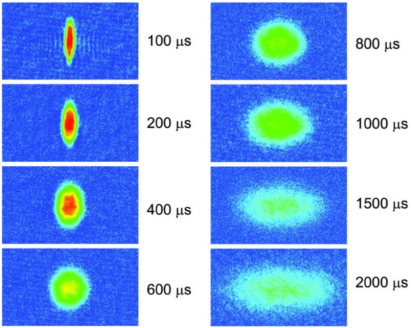
Collective behavior

Strongly-interacting fluids exhibit collective behavior



Pressure gradient $\rightarrow$ fluid flow:

$$(\epsilon + P)\frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} P$$



K. O'Hara, S. Hemmer, M. Gehm, S. Granade, J. Thomas, Science **298**, 2179 (2002).

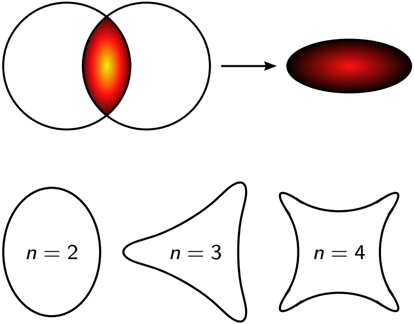
Initial-state spatial anisotropy $\Longrightarrow$ Final-state momentum anisotropy

Flow

Momentum anisotropy parameterized by Fourier coefficients v_n

$$\frac{dN}{d\phi} \propto 1 + \sum_n v_n \cos[n(\phi - \psi_n)]$$

ϕ: Angle of transverse momentum
ψ_n: Reaction-plane angle (phase)

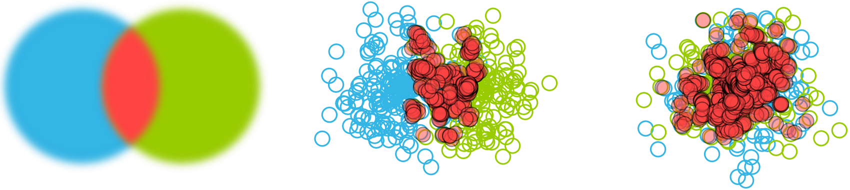


Flow provides essential evidence for the existence of a strongly-interacting QCD phase.

Event-by-event fluctuations

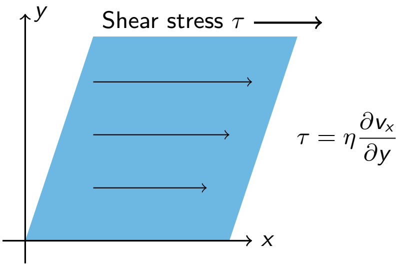
- Average: symmetric nuclei, almond-shape overlap.
  - Large v_2, small $v_4, v_6, \ldots$, vanishing $v_3, v_5, \ldots$
- Event-by-event: randomly distributed nucleons, irregular overlap.
  - All v_n nonzero.
  - Flow probability distributions $P(v_n)$.

Average peripheral | Fluctuating peripheral | Fluctuating central



Viscosity

- Shear viscosity $\eta =$ fluid's resistance to shear flow.
- Strongly-interacting fluid
  $\rightarrow$ short mean free path
  $\rightarrow$ small η
- Viscosity damps collective behavior (flow).



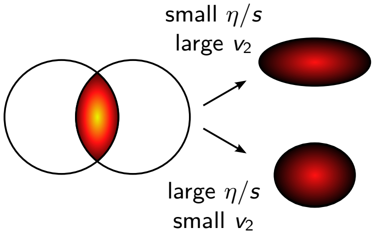
$$\eta \sim n m v_{\mathrm{avg}} \ell_{\mathrm{mf}} \sim \epsilon \ell_{\mathrm{mf}} / v_{\mathrm{avg}} \sim \epsilon t_{\mathrm{mf}}$$

QGP specific shear viscosity

Specific shear viscosity = dimensionless ratio to entropy density, η/s.

$$\eta \sim \epsilon t_{\text{mf}}, \; s \sim n \quad \Longrightarrow \quad \eta/s \sim (\epsilon/n) t_{\text{mf}} \gtrsim 1$$

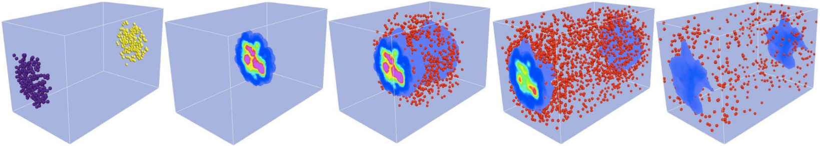
Water $\eta/s \sim 300$ at STP, Helium $\eta/s \sim 2$ at 3 K,
QGP $\eta/s \sim \mathcal{O}(10^{-1})$.



Measuring QGP η/s:

- Observe experimental v_n.
- Run model with variable η/s.
- Constrain η/s by matching v_n.

Simulations



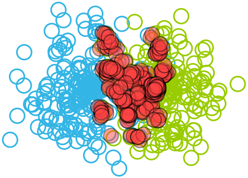
Modern event-by-event model:

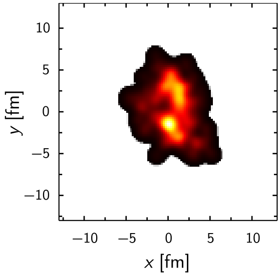
- Monte Carlo initial conditions
- (Pre-equilibrium)
- Viscous relativistic hydrodynamics
- Monte Carlo freeze-out
- Boltzmann transport

Initial conditions

- MC-Glauber model
  - Randomly samples nucleon positions.
  - Calculates energy density based on nucleon overlap.
- MC-KLN model
  - Randomly samples nucleon positions.
  - Uses effective field theory to calculate gluon densities $\rightarrow$ proportional to energy density.
- Many others.

Pb+Pb, $b = 8$ fm



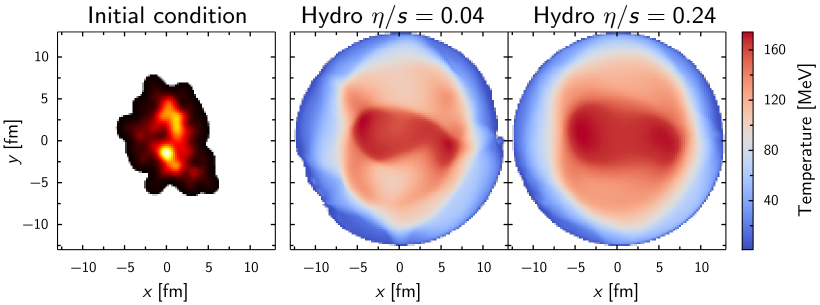


Viscous relativistic hydrodynamics

- Ignore pre-equilibrium, expand medium without interactions.
- Start hydro evolution at time τ_0 (must set explicitly).
- Conservation equations:

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu}.$$

- $\pi^{\mu\nu}$ contains dissipative effects (viscosity).
- Equation of state $P = P(\epsilon)$.

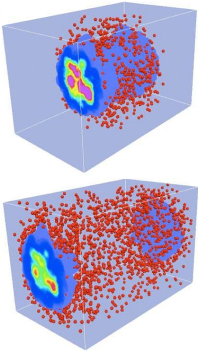


Hadronic freeze-out

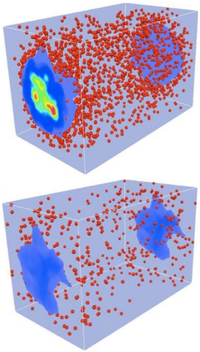
- Hydro stops at QCD transition, $T \sim 165$ MeV.
- Freezes into hadrons on hypersurface σ according to Cooper-Frye formula

$$E \frac{dN_i}{d^3p} = \int_\sigma f_i(x,p)\, p^\mu\, d^3\sigma_\mu$$

- Randomly sample to produce an ensemble of particles.



Transport

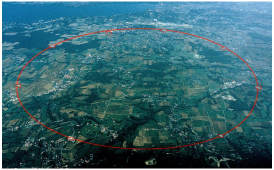


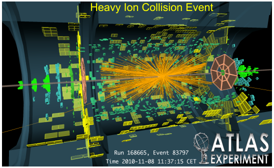
- Non-equilibrium Boltzmann transport

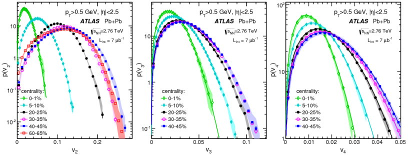
$$\frac{df_i(x,p)}{dt} = \mathcal{C}_i(x,p)$$

- Calculates final collisions and decays.
- Particles stream into "detector".

Model-to-data comparison: heavy-ion collisions



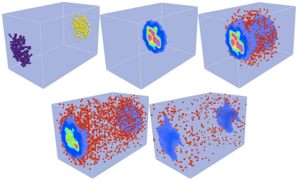


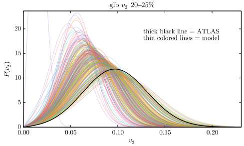


Model

Initial conditions,

τ_0, η/s, . . .





Computer experiments with slow models

Challenges

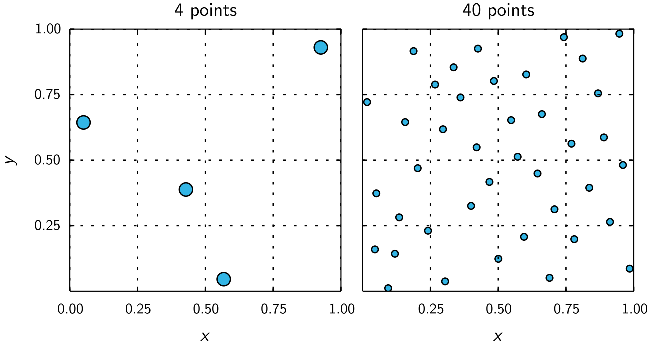
- Event-by-event models very computationally expensive, $\sim$1 hour per event.
- Need $\mathcal{O}(10^3)$ events per parameter-point to study fluctuations.
- Must vary all parameters simultaneously.

Strategies

- Evaluate model at efficient pre-determined parameter points.
  - Latin-hypercube sampling.
- Interpolate between explicitly calculated points.
  - Gaussian process emulator.

Latin-hypercube sampling

- Random set of parameter points.
- Optimally fills parameter space.
- Avoids clusters.



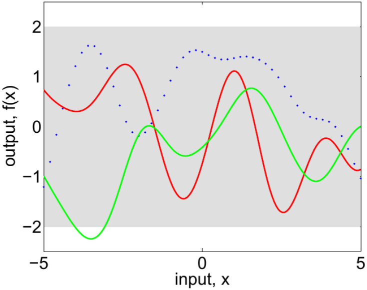
Gaussian processes

- *A Gaussian process is a collection of random variables, any finite number of which have a joint Gaussian distribution.*
- Instead of drawing variables from a distribution, functions are drawn from a process.

Require a covariance function, e.g.

$$\mathrm{cov}(x_1, x_2) \propto \exp\left[-\frac{(x_1 - x_2)^2}{2\ell^2}\right]$$

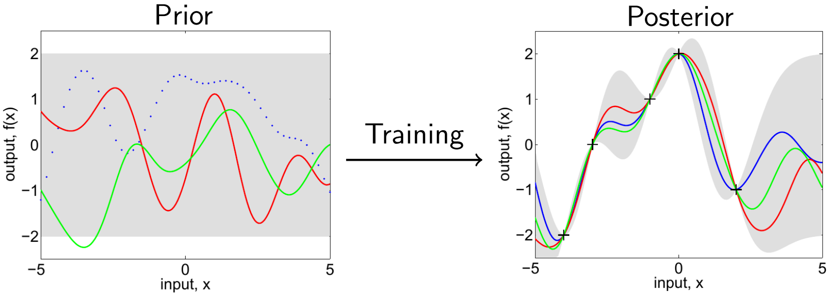
Nearby points correlated, distant points independent.



Gaussian Processes for Machine Learning, Rasmussen and Williams, 2006.

Gaussian process emulators

- Prior: the model is a Gaussian process.
- Posterior: Gaussian process conditioned on model outputs.



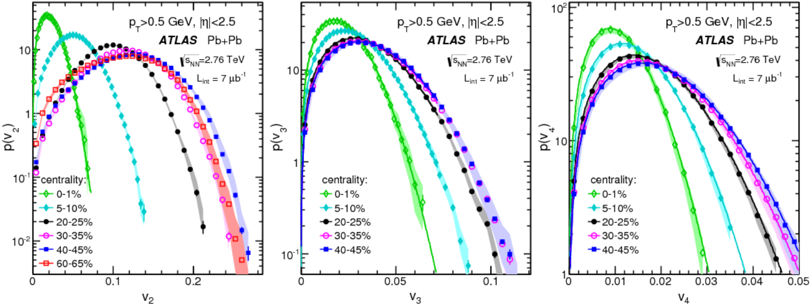
- Emulator is a fast surrogate to the actual model.
  - More certain near calculated points.
  - Less certain in gaps.

Experimental data

- ATLAS event-by-event flow distributions v_2, v_3, v_4.
- Fit to Rice / Bessel-Gaussian distribution

$$P(v_n) = \frac{v_n}{\delta_{v_n}^2} e^{-\frac{(v_n)^2 + (v_n^{RP})^2}{2\delta_{v_n}^2}} I_0\left(\frac{v_n^{RP} v_n}{\delta_{v_n}^2}\right)$$

- Reduce to parameters v_n^{RP}, δ_{v_n}.



ATLAS Collaboration, JHEP **1311**, 183 (2013).

Event-by-event model

Modern version of Duke+OSU model VISHNU
(Viscous Hydro and UrQMD):

- MC-Glauber & MC-KLN initial conditions

  H.-J. Drescher and Y. Nara, Phys. Rev. C **74**, 044905 (2006).

- Viscous hydro

  H. Song and U. Heinz, Phys. Rev. C **77**, 064901 (2008).

- Cooper-Frye sampler

  Z. Qiu and C. Shen, arXiv:1308.2182 [nucl-th].

- UrQMD (Ultrarelativistic Quantum Molecular Dynamics)

  S. Bass *et. al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998).
  M. Bleicher *et. al.*, J. Phys. G **25**, 1859 (1999).

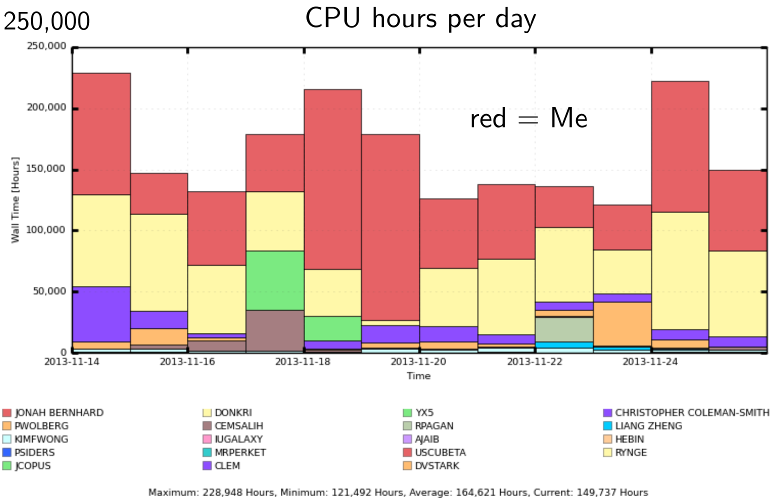
$\rightarrow$ Tailored for running many events on Open Science Grid.

Computer experiment design

- Six centrality bins 0–5%, 10–15%, . . . 50–55%.
- 256 Latin-hypercube points, five input parameters:
    - Normalization
    - IC-specific parameter
    - Thermalization time τ_0
    - Viscosity η/s
    - Shear relaxation time τ_Π

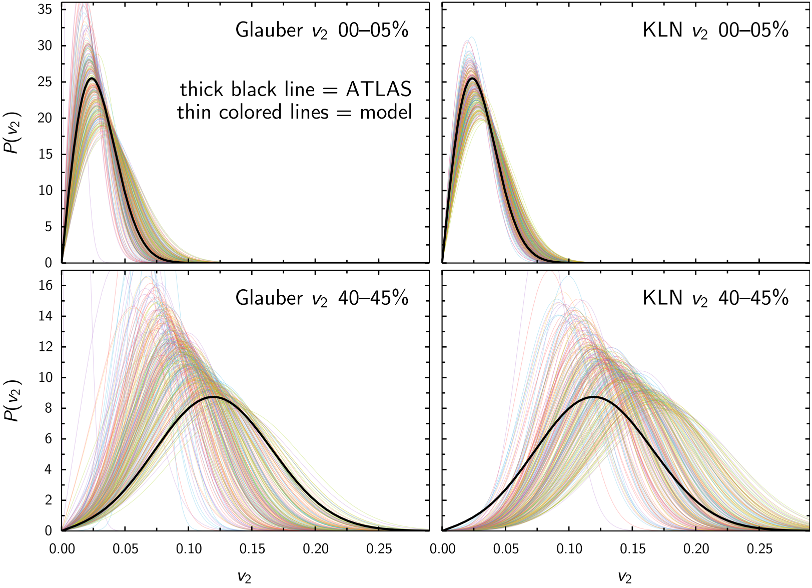
- Massive parallelization on Open Science Grid.
- Completed 1000–2000 events per centrality bin and input-parameter point.
    - 3.5 million total
    - 0.5 μb^{-1} (ATLAS: 7 μb^{-1})

Open Science Grid usage

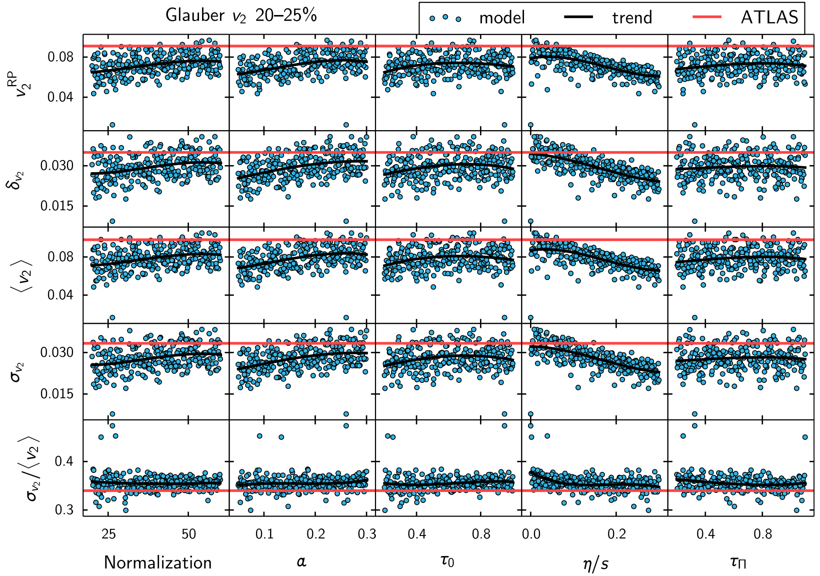


Completed KLN design (1.5 million events) in two weeks.

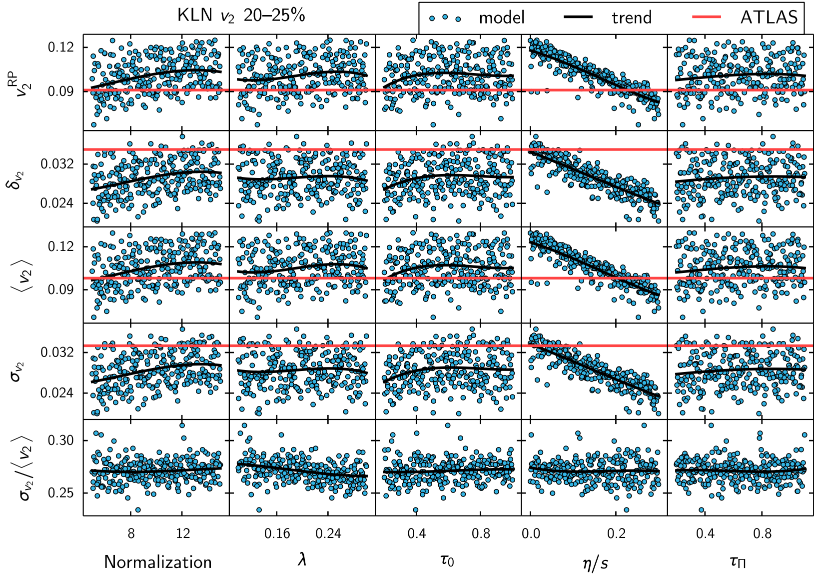
Model flow distributions



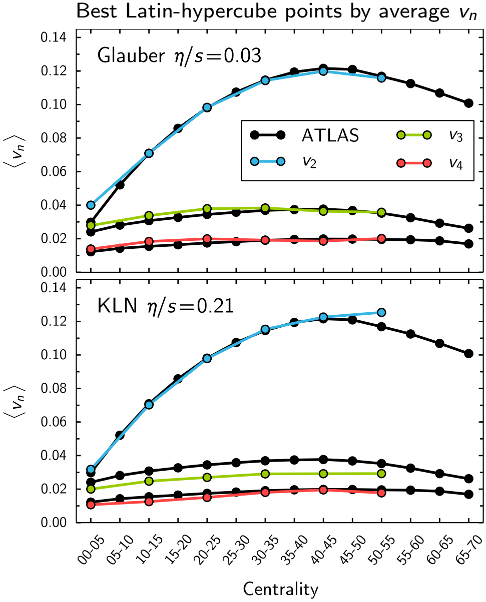
Input-output summary



Input-output summary

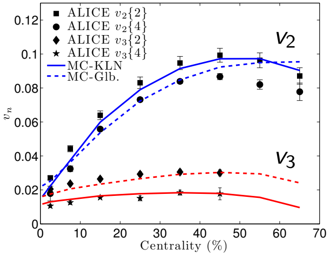


Best parameter points



OSU results, same model

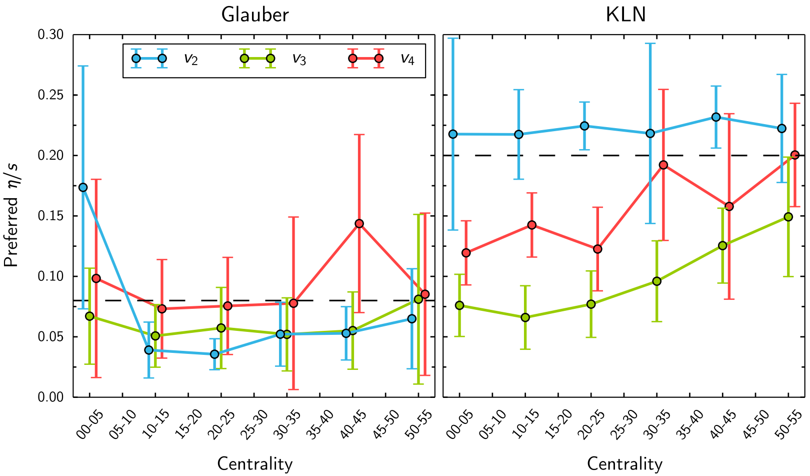
dashed: Glauber $\eta/s = 0.08$
solid: KLN $\eta/s = 0.20$



Z. Qiu, C. Shen, and U. Heinz, Phys. Lett. B **707**, 151 (2012).

Constraining η/s

Points: average η/s of best 10 Latin-hypercube points by average v_n
Error bars: standard deviation of best 10
Dashed lines: canonical η/s (Glauber 0.08, KLN 0.20)



Summary & outlook

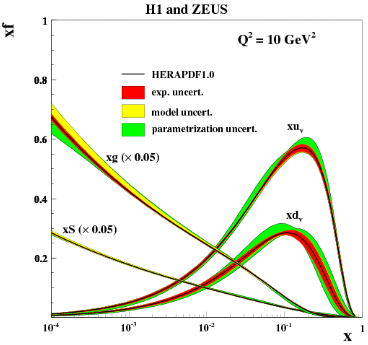
- Framework for massive event-by-event model-to-data comparison: new level of knowledge-extraction capability.
- Preliminary results consistent with previous work.

- Improve goodness of fit: beyond average flow.
- Emulator: vary single parameters independently, determine best-fit parameter values.
- Calibrate simultaneously on other observables, e.g. multiplicity.
- Repeat with more advanced models, especially initial conditions.

backup slides

Color glass condensate

- High energy / small x: parton distribution functions dominated by gluons.
- Gluons overlap coherently $\rightarrow$ condensate state.



$$x = p_T e^{\pm y} / \sqrt{s}$$

Viscous hydro

Conservation of energy and momentum:

$$\partial_\mu T^{\mu\nu} = 0$$

Stress-energy tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \pi^{\mu\nu}$$

Ideal part:

$$T^{\mu\nu}_{\text{ideal}} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

Shear viscosity correction:

$$\pi^{\mu\nu} = \eta \nabla^{\langle\mu} u^{\nu\rangle}$$

Symmetric and traceless:

$$\nabla^{\langle\mu} u^{\nu\rangle} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$$

Projection orthogonal to four velocity:

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \qquad \nabla_\mu = \Delta_\mu^\alpha \partial_\alpha$$

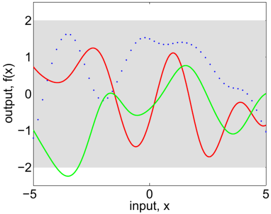
Generating Gaussian processes

- Choose a set of input points X_*.
- Choose a covariance function, e.g.

$$k(x_i, x_j) = \exp[-(x_i - x_j)^2/2]$$

  and create covariance matrix $K(X_*, X_*)$.
- Generate MVN samples (GPs)

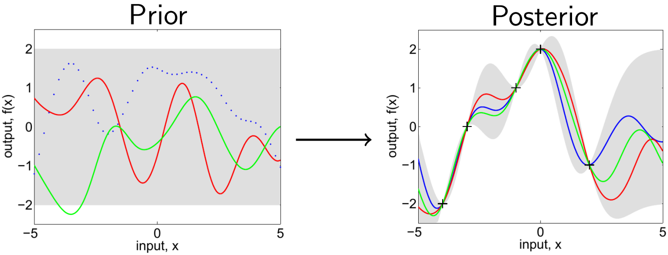
$$\vec{f}_* \sim \mathcal{N}[\vec{0}, K(X_*, X_*)].$$



Training the emulator

- Make observations $\vec{f}$ at training points X.
- Generate conditioned GPs

$$\vec{f}_*|X_*, X, \vec{f} \sim \mathcal{N}[K(X_*, X)K(X, X)^{-1}\vec{f},\\ K(X_*, X_*) - K(X_*, X)K(X, X)^{-1}K(X, X_*)].$$



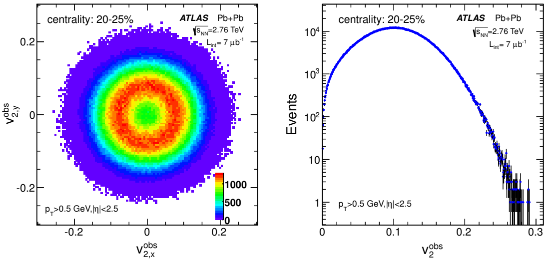
Rice / Bessel-Gaussian distribution

- Flow vectors follow bivariate Gaussian

$$P(\vec{v}_n) = \frac{1}{2\pi\delta_{v_n}^2} e^{-\frac{(\vec{v}_n - \vec{v}_n^{RP})^2}{2\delta_{v_n}^2}}.$$

- Integrate out angle

$$P(v_n) = \frac{v_n}{\delta_{v_n}^2} e^{-\frac{(v_n)^2+(v_n^{RP})^2}{2\delta_{v_n}^2}} I_0\left(\frac{v_n^{RP} v_n}{\delta_{v_n}^2}\right).$$



Finite multiplicity and unfolding

- Observed flow smeared by finite multiplicity and nonflow

$$P(v_n^{\mathrm{obs}}) = \int P(v_n^{\mathrm{obs}}|v_n)P(v_n)\,dv_n$$

where $P(v_n^{\mathrm{obs}}|v_n)$ is the response function.

- Pure statistical smearing $\rightarrow$ Gaussian response

$$P(v_n^{\mathrm{obs}}|v_n) = \frac{v_n^{\mathrm{obs}}}{\delta_{v_n}^2} e^{-\frac{(v_n^{\mathrm{obs}})^2+(v_n)^2}{2\delta_{v_n}^2}} I_0\left(\frac{v_n v_n^{\mathrm{obs}}}{\delta_{v_n}^2}\right).$$

- v_n^{RP} unaffected; width increased as

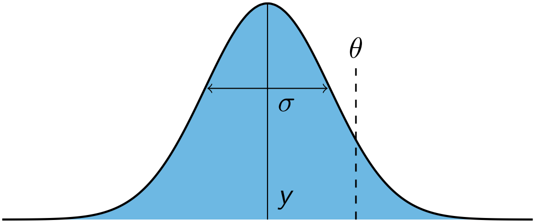
$$\delta_{v_n}^2 \rightarrow \delta_{v_n}^2 + 1/2M.$$

Likelihood

Given experimental observations y_i with errors σ_i and model predictions θ_i, what is the likelihood that the model describes reality?

$$\mathcal{L} \sim \exp\left[-\sum_i \frac{(y_i - \theta_i)^2}{2\sigma_i^2}\right]$$

Or as a null hypothesis: can the model be rejected based on comparison to the data? (e.g. If a coin is flipped N times and yields heads each time, what is the probability that it is fair?)



Linear fit example