AMERICAN OPTION PRICING IN A JUMP-DIFFUSION MODEL

By

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To my fiancée

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TABLE OF CONTENTS

page

ACKNOWLEDGMENTS 4

LIST OF FIGURES 7

ABSTRACT 8

CHAPTER

1 INTRODUCTION 9

1.1 Literature 9
1.2 Organization of the Thesis 10

2 KOU'S DOUBLE-EXPONENTIAL JUMP-DIFFUSION MODEL 12

2.1 Model Formulation 12
2.2 Properties of the Model 14
2.3 Comparison with Other Models 14
2.3.1 The Constant Elasticity of Variance Model 15
2.3.2 The Normal Jump-Diffusion Model 15
2.3.3 Models Based on t-Distribution 16
2.3.4 Stochastic Volatility Models 16
2.3.5 Affine Jump-Diffusion Models 17
2.3.6 Models Based on Lévy Processes 18
2.4 Leptokurtic Feature 18
2.5 Equilibrium for General Jump-Diffusion Models 19
2.6 The Volatility Smile 23

3 AMERICAN OPTION PRICING IN KOU'S MODEL 27

3.1 Hh Function 27
3.2 European Call and Put Options 28

4 ANALYTIC RESULT FOR AMERICAN OPTION PRICING 30

4.1 Main Result 30
4.2 Proof of the Result 33

5 CONCLUSION 36

5.1 Implication of the Main Result 36
5.2 Limitations of the Model 36
5.3 Scope of Further Research 37

APPENDIX

A DERIVATION OF THE RATIONAL EXPECTATIONS 38

B DERIVATION OF THE Υ FUNCTION 41

REFERENCES 49

BIOGRAPHICAL SKETCH 51

LIST OF FIGURES

Figure page

2-1 Comparison between density g (solid line) of Leptokurtic feature and normal distribution density (dotted line) with same mean and variance 25

2-2 Midmarket and Model-Implied Volatilities for Japanese LIBOR Caplets in May 1998 26

3-1 The Hh function for $n = 1, 3, 5$ with the Steepest Curve for $n = 5$ and the Flattest Curve for $n = 1$ 29

Abstract of Thesis Presented to the Graduate School
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Many alternative models have been developed lately to generalize the Black-Scholes option pricing model in order to incorporate more empirical features. Brownian motion and normal distribution have been used in this Black-Scholes option-pricing framework to model the return of assets. However, two main points emerge from empirical investigations: (i) the leptokurtic feature that describes the return distribution of assets as having a higher peak and two asymmetric heavier tails than those of the normal distribution, and (ii) an empirical phenomenon called "volatility smile" in option markets. Among the recent models that addressed the aforementioned issues is that of Kou (2002), which allows the price of the underlying asset to move according to both Brownian increments and double-exponential jumps. The aim of this thesis is to develop an analytic pricing expression for American options in this model that enables us to efficiently determine both the price and related hedging parameters.

CHAPTER 1
INTRODUCTION

1.1 Literature

More than forty years ago, working within the framework of Samuelson's pricing model, McKean (1965) proved the equivalence between the optimal stopping problem defining the value of an American option and a free-boundary problem. Using this equivalence, McKean determined the partial differential equation governing the value of the American option and solved it to obtain an analytical formula. This solution represents the value of the American option in terms of the exercise boundary, which has to be determined as a part of the solution. However, the use of the formula was limited due to the appearance of the slope of the critical boundary. The arbitrage framework for option valuation was developed by Black and Scholes (1973) and Merton (1973). Jump-diffusion processes were incorporated by Merton (1976) into the theory of option valuation in order to introduce a discontinuous sample path of the underlying stock's return dynamics, in contrast to the classical lognormal diffusion model of Black and Scholes. Jump-diffusion models allow us to account for large price changes due to sudden exogenous events or information. They are particularly adapted in the context of foreign exchange rates and may explain some systematic empirical biases with respect to the Black-Scholes model.

Many modifications of the Black-Scholes model based on Brownian motion and normal distribution have been conducted. Two main empirical features have received much attention: (1) the asymmetric leptokurtic feature, where the return distribution is skewed to the left and has a higher peak and heavier tails than those of the normal distribution; (2) the volatility smile. More precisely, if the Black-Scholes model is correct, then the implied volatility should be constant,but it's widely recognized that the implied volatility curve resembles a "smile", meaning that is a convex curve of the strike price. To incorporate the asymmetric leptokurtic features in asset pricing, a variety of models have been proposed, including among others, (a) a chaos theory, fractal Brownian motion, and stable processes; (b) generalized hyperbolic models, including log-t model and log-hyperbolic model; and (c) time-changed Brownian motion, including log- variance gamma model. An immediate problem with these models is that it may be difficult to obtain analytical solutions for option prices. More precisely, they might give some analytical formulae for standard European call and put options, but any analytical solutions for interest rate derivatives and path-dependent options, such as American options, barrier, and lookback options; are unlikely. In parallel development, different models are also proposed to incorporate the "volatility smile" in option pricing. Popular ones are (a) stochastic volatility and GARCH models; (b) constant elasticity of variance (CEV) model; (c) the normal jump-diffusion model by Merton (1976); (d) affine stochastic volatility and affine jump-diffusion models (Duffie); (e) models based on Lévy processes; and (f) a numerical procedure called "implied binomial trees". For the Background of these alternative models, see, for example, Hull (2000) and Carr et al. (2003). Unlike the original Black-Scholes model, although many alternative models can lead to analytic solution for European call and put options, it is difficult to do so for path-dependent options, such as American options, lookback options, and barrier options. Even numerical methods for these derivatives are not easy. For example, the convergence rates of binomial trees and Monte Carlo simulation for path-dependent options are typically much slower than those for call and put options.

1.2 Organization of the Thesis

The thesis is organized as follows. In chapter 2, Kou's model is reviewed, highlighting its internal self-consistency, ability to facilitate price computation, and is compared to most common and related models. We also review the leptokurtic feature and a rational expectations equilibrium justification of the model. Its ability to capture "volatility smiles" is illustrated as well. Chapter 3 reviews the European option pricing implication of Kou's model. Chapter 4 contains the main result of the present thesis, namely an analytic expression for the price of an American option in the context of Kou's model. Chapter 5 concludes with implications of our main result.

CHAPTER 2
KOU'S DOUBLE-EXPONENTIAL JUMP-DIFFUSION MODEL

2.1 Model Formulation

To begin, we review briefly the basic aspect of an elementary jump-diffusion. It is a stochastic process following a diffusion most of the time, except at random dates, governed by a Poisson process, when the state jumps to a random location. Jump-diffusion models were introduced in option pricing by Robert C. Merton (1976). In his model, the diffusion is a Brownian motion and the jumps are normally distributed. More recently, Kou (2002) proposed to modify Merton's by considering jumps with double exponential (or Laplace) distribution.

And advantage of the latter is its ability to allow for asymmetric jumps. This is particularly relevant when these jumps express reaction to major positive or negative news, where the impact can be dramatically different.

Let S(t) be the price of the underlying asset at time t. The simplest jump-diffusion model for the process $\{S(t)\}_t$ assumes that it is subject to jumps occurring to a Poisson process $\{N(t)\}$ with rate λ. Then, S(t) solves the Stochastic Differential Equation.

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)}(V_i - 1)\right) \tag{2–1}$$

where μ is the return of the asset, W(t) is a standard Brownian motion, N(t) a Poisson process with rate λ, and $\{V_i\}_i$ is a sequence of independent, identically distributed (i.i.d.)

In the case of Merton Y=log(V) is assumed to be normally distributed, whereas in Kou's, Y follows an asymmetric double exponential distribution with density

$$f_y = p.\eta_1 e^{-\eta_1 y} 1_{(y\geq 0)} + q.\eta_2 e^{\eta_2 y} 1_{(y<0)}$$

$$\eta_1 > 1, \eta_2 > 0$$

where $p, q > 0, p + q = 1$, represents the probabilities of upward and downward jumps.

In other words,

$$log(V) = Y =^d \left\{ \begin{array}{ll} \xi^+ & \text{with probability p} \\ \xi^- & \text{with probability q} \end{array} \right\}.$$

where ξ^+ and ξ^- are exponential random variables with mean $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$, respectively. The notation $=^d$ means equal in distribution.In the model, all sources of randomness, N(t), W(t) and Y s, are assumed to be independent.

Solving the stochastic differential equation (2-1) gives the dynamics of the asset price:

$$S(t) = S(0)exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right\}\Pi_{i=1}^{N(t)} V_i \tag{2–2}$$

Note that $E(Y) = \frac{p}{\eta_1} - \frac{q}{\eta_2}$, $Var(Y) = pq(\frac{p}{\eta_1} + \frac{q}{\eta_2})^2 + (\frac{p}{\eta_1^2} + \frac{q}{\eta_2^2})$ and

$$E(V) = E(e^Y) = q\frac{\eta_2}{\eta_2 + 1} + p\frac{\eta_1}{\eta_1 - 1} \tag{2–3}$$

$\eta_1 > 1$, $\eta_2 > 0$

$\eta_1 > 1$ is to ensure that $E(V) < \infty$ and $E(S(t)) < \infty$.

Several properties of the double exponential distribution are important for the model. First, it captures the leptokurtic feature we will develop later.The leptokurtic feature of the jump size distribution is inherited by the return distribution. Second, a unique feature of the double exponential distribution is the memoryless property. The memoryless property is a property of certain probability distributions: in our case the exponential distributions, but also the geometric distributions, wherein any derived probability from a set of random samples is distinct and has no information (i.e. "memory") of earlier samples. This property explains why the closed form solutions for various option pricing problems, including barrier, lookback, and perpetual American options, are feasible, under the double exponential jump diffusion model while it seems impossible for many other models.

2.2 Properties of the Model

As with any model, Kou's is just an approximation of reality. But a few successful models, such as Black and Scholes, provide several insights through their simple expressions. Kou's offers yet more, as it is arbitrage-free, is embedded in an equilibrium setting, and leads to closed-form expressions for several path-dependent options of great interest, such as barrier and lookback, beside European options. This self-consistency feature is further enhanced as the model captures empirical phenomena that cannot be ignored from a risk management perspective. Specifically, return distribution tend to be affected significantly by unpredictable events with effects that are different between "good" and "bad" news. This feature is expressed through a compound Poisson process with jumps that are asymmetric and double exponentially distributed. Furthermore, the ability of Kou's model to reproduce volatility smile in option pricing has been documented for interest-rate derivatives.

2.3 Comparison with Other Models

There are many alternatives models that can satisfy some of the above criteria. The main advantage of the double exponential jump-diffusion model is its simplicity. Many alternative models can only compute prices for standard call and put options. Unfortunately, analytical solutions for other equity derivatives, like the ones we got for the original Black-Scholes model, such as path-dependent options are unlikely. Even numerical methods for interest rate derivatives and path-dependent options are not easy. Therefore, it is hard to persuade practitioners to switch from the Black-Scholes model to alternative models. However, the double exponential jump-diffusion model attempts to improve the empirical implications of the Black-Scholes model while still retaining its analytical tractability. I detail more specifically comparison between the chosen model and some well-known alternative models.

2.3.1 The Constant Elasticity of Variance Model

This CEV model captures in a simple way the notion that the volatility increases as the asset price decreases. There are good reasons for considering this model: (1) the model nicely captures the leverage effect associated with asset prices; (2) there is research suggesting that the pricing of warrants in particular is improved by the use of this model; (3) it is analytically tractable; (4) it predicts skews and, with some additional adjustments for spreads, smiles; (5) one member of the family is equivalent to a non-normal and non-log-normal interest rate model (CIR), allowing us to explore aspects of the interest-rate mathematics simultaneously with the equity mathematics.

Under the CEV model, like in the double exponential jump-diffusion model, analytical solutions for path dependent options and interest rate derivatives are available. However, the CEV model does not have the leptokurtic feature. More precisely, the return distribution in the CEV has a thinner right tail than of the normal distribution. Therefore, this undesirable feature also has a consequence in terms of the implied volatility in option pricing, meaning that under the CEV model the implied volatility can only be a monotone function of the strike price. Consequently, if the implied volatility is a convex function, as frequently observed in option markets, the CEV model is unable to reproduce this implied volatility curve.

2.3.2 The Normal Jump-Diffusion Model

Merton (1976) was the first to consider a jump-diffusion model similar to the one we got. In Merton's paper, Ys are normally distributed. The normal jump-diffusion model and the double exponential jump-diffusion model can both lead to the leptokurtic feature we did not have with the constant elasticity model above, implied volatility smile, and analytical solutions for call and put options, and interests rate derivatives. However,the main difference between the two models is the analytical tractability for path-dependent options. To price American options for general jump-diffusion processes, we imperatively have to study the first passage time of a jump-diffusion process to a flat boundary. Analytical solutions for the American options can be derived from the double exponential jump-diffusion model. However, it seems impossible to get similar results for other jump-diffusion processes, including the normal jump-diffusion model.

2.3.3 Models Based on t-Distribution

The t-distribution is widely used in empirical studies of asset pricing. One problem with t-distribution, as a return distribution is that it cannot be used in models with continuous compounding. More precisely, suppose that at time 0 the daily return distribution X has a power-type tail. Then in models with continuous compounding, the asset price tomorrow $A(\Delta t)$ is given by $A(\Delta t) = A(0)e^x$. Since X has a power-type right tail, we got $E(e^x) = \infty$. Consequently,

$$E\left(A(\Delta t)\right) = E\left(A(0)e^x\right) = A(0)E(e^x) = \infty$$

In other words, the asset price tomorrow has an infinite expectation. This paradox holds for t-distribution with any degrees of freedom, as long as one considers models with continuous compounding. Furthermore, if the risk neutral return also has a power-type right tail, then the call option price is also infinite:

$$E^*\left((A(\Delta t) - K)^+\right) \geq E^*\left(A(\Delta t) - K\right) = \infty$$

The only relevant models with t-distributed returns are models with discretely compounded returns. However, in models with discrete compounding, closed-form solution are in general impossible.

2.3.4 Stochastic Volatility Models

The name of stochastic volatility derives from the models treatment of the underlying security's volatility as a random process, governed by state variables such as the price level of the underlying, the tendency of volatility to revert to some long-run mean value, and the variance of the volatility process itself, among others.

These models assume that the underlying volatility is constant over the life of the derivative, and unaffected by the changes in the price level of the underlying. However, these models cannot explain long-observed features of the implied volatility surface such as volatility smile and skew, which indicate that implied volatility does tend to vary with respect to strike price and expiration. By assuming that the volatility of the underlying price is a stochastic process rather than a constant, it becomes possible to model derivatives more accurately.

This model and the double exponential jump-diffusion model are complement each other: The stochastic volatility model can incorporate dependent structure better, while the double exponential jump-diffusion model has better analytical tractability, especially for path-dependent options and complex interest rate derivatives. One empirical phenomenon worth-mentioning is that the daily return distribution tends to have more kurtosis than the distribution of monthly returns. This is consistent with the models with jumps, but inconsistent with stochastic volatility models. More precisely, in stochastic volatility models, or essentially any models in a pure diffusion setting, the kurtosis decreases as the sampling frequency increases, while in jump models the instantaneous jumps are independent of the sampling frequency.

2.3.5 Affine Jump-Diffusion Models

Duffie at al. (2000) propose a very general class of affine jump-diffusion models which can incorporate jumps, stochastic volatility, and jumps in volatility. Both normal and double exponential jump-diffusion models can be viewed as special cases of their model. However, because of the special feature of the exponential distribution, the double exponential jump-diffusion model leads to analytical solutions for path-dependent options, which are difficult for other affine jump-diffusion models, even numerical methods are not easy. Furthermore, the double exponential model is simpler than general affine jump-diffusion models: It has fewer parameters that makes calibration easier. The double exponential jump-diffusion model attempts to strike a balance between reality and tractability.

2.3.6 Models Based on Lévy Processes

This process displays independent and stationary increments. Although the double exponential jump-diffusion model is a special case of Lévy processes, because of the special features of the exponential distribution it has analytical tractability for path-dependent options and interest rate derivatives, which are difficult for other Lévy processes.

2.4 Leptokurtic Feature

Using Kou's incremental expression of the dynamics of the asset price, the return over a small time interval Δt is given by:

$$\frac{\Delta S(t)}{S(t)} \approx \mu\Delta t + \sigma Z\sqrt{\Delta t} + B.Y \tag{2–4}$$

where Z and B are standard normal and Bernoulli random variables, respectively, with

$$P(B = 1) = \lambda\Delta t$$

and

$$P(B = 0) = 1 - \lambda\Delta t$$

and Y is given by (2-2)

The density g of the right-hand side of (2-5), being an approximation for the return $\frac{\Delta S(t)}{S(t)}$ is the following

$$g(x) = \frac{1 - \lambda\Delta t}{\sigma\sqrt{\Delta t}}\varphi\left(\frac{x - \mu\Delta t}{\sigma\sqrt{\Delta t}}\right)$$
$$+\lambda\Delta t\left\{p\eta_1 e^{\frac{(\sigma^2\eta_1^2\Delta t)}{2}} e^{-(x-\mu\Delta t)\eta_1}\Phi\left(\frac{x - \mu\Delta t - \sigma^2\eta_1\Delta t}{\sigma\sqrt{\Delta t}}\right)\right.$$

$$+q\eta_2 e^{\frac{(\sigma^2\eta_2^2\Delta t)}{2}} e^{(x-\mu\Delta t)\eta_2}\Phi\left(-\frac{x-\mu\Delta t+\sigma^2\eta_2\Delta t}{\sigma\sqrt{\Delta t}}\right)\Bigg\}$$

with

$$E_g(G) = \mu\Delta t + \lambda\left(\frac{p}{\eta_1} - \frac{q}{\eta_2}\right)\Delta t,$$

$$Var_g(G) = \sigma^2\Delta t + \left\{pq\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)^2 + \left(\frac{p}{\eta_1^2} + \frac{q}{\eta_2^2}\right)\right\}\lambda\Delta t$$

$$+\left(\frac{p}{\eta_1} - \frac{q}{\eta_2}\right)^2 \lambda\Delta t(1-\lambda\Delta t),$$

where $\varphi(.)$ is the standard normal density function.You have a representation of density function g in Fig. 2-2

2.5 Equilibrium for General Jump-Diffusion Models

Kou considers a typical rational expectations economy (Lucas 1978) in which a representative investor tries to solve a utility maximization problem $max_c E[\int_0^\infty U(c(t)), t)dt]$, where $U(c(t), t)$ is the utility function of the consumption process c(t). There is an exogenous endowment process, denoted by $\delta(t)$, available to the investor. Also given to the investor is an opportunity to invest in a security (with a finite liquidation date T_0, although T_0 can be very large) which pays dividends. If $\delta(t)$ is Markovian, it can be shown that, under mild conditions, the rational expectations equilibrium price (also called the "shadow" price) of the security, $p(t)$, must satisfy the Euler equation

$$p(t) = \frac{E\left(U_c(\delta(T), T)p(T)|\mathfrak{F}_t\right)}{U_c\left(\delta(t), t\right)}, \forall T \in [t, T_0], \qquad (2–5)$$

where U_c is the partial derivative of U with respect to c. At this price p(t), the investor will never change his or her current holdings to invest in the security, either long or short, even though he or she is given the opportunity to do so. Instead, in equilibrium the investor finds it optimal to just consume the exogenous endowment; i.e., $c(t) = \delta(t)$ for all $t \geq 0$.

In fact Kou derives explicitly the implication of the Euler equation when the endowment process $\delta(t)$ follows a general jump-diffusion process under the physical measure P:

$$\frac{d\delta t}{\delta(t-)} = \mu_1 dt + \sigma_1 dW_1(t) + d\left[\sum_{i=1}^{N(t)}(\widetilde{V}_i - 1)\right] \tag{2–6}$$

where the $\widetilde{V}_i \geq 0$ are any independent identically distributed, nonnegative random variables. To simplicity matters, all three sources of randomness, the Poisson process N(t), the standard Brownian motion $W_1(t)$, and the jump size $\widetilde{V}$, are assumed to be independent.

Although it is intuitively clear that, generally speaking, the asset price p(t) should follow a similar jump-diffusion process as that of the dividend process $\delta(t)$, a careful study of the connection between the two is needed. This is because p(t) and $\delta(t)$ may not have similar jump dynamics. Furthermore, deriving explicitly the change of parameters from$\delta(t)$ to p(t) also provides some valuable information about the risk premiums embedded in jump-diffusion models. The work here builds upon and extends the previous work by Naik and Lee (1990) in which the special case that $\widetilde{V}_i$ has a lognormal distribution is investigated. Another difference is that Naik and Lee (1990) require that the asset pays continuous dividends and there is no outside endowment process, while here the asset pays no dividends and there is an outside endowment process. Consequently, the pricing formulae are different even in the case of lognormal jumps.

For simplicity, as in Naik and Lee (1990), Kou only considers the utility function of the special forms

$$U(c,t) = e^{-\theta t}\frac{c^{\alpha}}{\alpha}$$

if $0 < \alpha < 1$

and

$$U(c,t) = e^{-\theta t} log(c)$$

if $\alpha = 0$, where $\theta > 0$

Under these types of utility functions, the rational expectations equilibrium price of (2-6) becomes

$$p(t) = \frac{E\left(e^{-\theta t}(\delta(T)^{\alpha-1}p(T))|\mathfrak{F}_t\right)}{e^{-\theta t}\left(\delta(t)^{\alpha-1}\right)} \tag{2–7}$$

Assumption. The discount rate θ should be large enough so that

$$\theta > -(1-\alpha)\mu_1 + \frac{1}{2}\sigma_1^2(1-\alpha)(2-\alpha) - \lambda\zeta_1^{(\alpha-1)},$$

where the notation $\zeta_1^{(a)}$ means $\zeta_1^{(a)} := E\left[(\widetilde{V})^{(a)} - 1\right]$.

As will been in Proposition 1 (Kou, 2002), this assumption guarantees that in equilibrium the term structure of interest rate is positive.

Proposition 1. Suppose $\zeta_1^{(a-1)} < \infty$. (1) Letting B(t,T) be the price of a zero coupon bound with maturity T, the yield $r := -\left(\frac{1}{(T-t)}) log(B(t,T)\right)$ is a constant independent of T,

$$r = \theta + (1-\alpha)\mu_1 - \frac{1}{2}\sigma_1^2(1-\alpha)(2-\alpha) - \lambda\zeta_1^{(\alpha-1)} > 0 \tag{2–8}$$

(2) Let $Z(t) := e^{rt}U_c(\delta(t),t) = e^{r-\theta}\left(\delta(t)\right)^{\alpha-1}$. Then Z(t) is a martingale under P,

$$\frac{dZ(t)}{Z(t-)} = -\lambda\zeta_1^{(\alpha-1)}dt + \sigma_1(\alpha-1)dW_1(t) + d\left[\sum_{i=1}^{N(t)}(\widetilde{V}_i^{\alpha-1} - 1)\right] \tag{2–9}$$

Using Z(t), one can define a new probability measure P^*, the Euler Equation (2-8) holds if and only if the asset price satisfies

$$S(t) = e^{-r(T-t)}E^*\left(S(t)|\mathfrak{F}_t\right), \forall T \in [t, T_0], \tag{2–10}$$

Furthermore, the rational expectations equilibrium price of a possible path dependent European option, with the payoff $\psi_S(t)$ at the maturity T, is given by

$$\psi_S(t) = e^{-r(T-t)} E^* \left(\psi_S(t) | \mathfrak{F}_t\right), \forall T \in [t, T_0], \tag{2–11}$$

Given the endowment process $\delta(t)$, it must be decided what stochastic processes are suitable for the asset price S(t) to satisfy the equilibrium requirement (2-8) or (2-11). Kou postulates a special jump-diffusion form for S(t), namely:

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma\rho dW_1(t) + \sqrt{1-\rho^2} dW_2(t) + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right), V_i = \widetilde{V}^{\beta}, \tag{2–12}$$

where $W_2(t)$ is a Brownian motion independent of $W_1(t)$. In other words, the same Poisson process affects both the endowment$\delta(t)$ and the asset price S(t), and the jump size are related through a power function, where the power $\beta \in (-\infty, \infty)$ is an arbitrary constant. The diffusion coefficients and the Brownian motion part of $\delta(t)$ and S(t), though, are totally different. It remains to determine what constraints should be imposed on this model so that the jump-diffusion model can be embedded in the rational expectations equilibrium requirement (2-8) or (2-11).

Theorem 1. (Kou, 2002) Suppose $\zeta_1^{\alpha+\beta-1} < \infty$ and $\zeta_1^{\alpha-1} < \infty$. The model (2-13) satisfies the equilibrium requirement (2-11) if and only if

$$\mu = r + \sigma_1\sigma\rho(1-\alpha) - \lambda\left(\zeta_1^{(\alpha+\beta-1)} - \zeta_1^{(\alpha-1)}\right)$$

$$= \theta + (1-\alpha)\left\{\mu_1 - \frac{1}{2}\sigma_1^2(2-\alpha) + \sigma_1\sigma\rho\right\} - \lambda\zeta_1^{(\alpha+\beta-1)}. \tag{2–13}$$

If (14) is satisfied, then under P^*,

$$\frac{dS(t)}{S(t-)} = rt + \lambda^* E^*(\widetilde{V}^\beta - 1)dt + \sigma dW^*(t) + d\left(\sum_{i=1}^{N(t)}(\widetilde{V}_i^\beta - 1)\right). \tag{2–14}$$

Here, under $P^*, W^*(t)$ is a new Brownian motion, N(t) is a new Poisson process with jump rate $\lambda^* = \lambda E(\widetilde{V}_i^{\alpha-1}) = \lambda\left(\zeta_1^{(\alpha-1)} + 1\right)$, and $\widetilde{V}_i$ are independent identically distributed random variables with a new density under P^*:

$$f^*_{\widetilde{V}}(x) = \frac{1}{\zeta_1^{(\alpha-1)} + 1} x^{\alpha-1} f_{\widetilde{V}}(x) \tag{2–15}$$

The following corollary gives a condition under which all three dynamics, $\delta(t)$ and S(t) under P and S(t) under P^*, have the same jump-diffusion form, which is very convenient for analytical calculation.

Corollary 1. (Kou, 2002) Suppose the family $\mathfrak{V}$ of distributions of the jump size $\widetilde{V}$ for the endowment process $\delta(t)$ satisfies that, for any real numbers $a \in [0, 1)$ and $b \in (-\infty,)$,

$$\widetilde{V^b} \in \mathfrak{V} \tag{2–16}$$

and

$$const.x^{a-1} f_{\widetilde{V}(x) \in \mathfrak{V}}$$

where the normalizing constant, const, is $\{\zeta_1^{(a-1)} + 1\}^{-1}$ (provided that $\zeta_1^{(a-1)} < \infty$). Then the jump sizes for the asset price S(t) under P and the jump size for S(t) under the rational expectations risk-neutral measure P^* all belong to the same family $\mathfrak{V}$.

2.6 The Volatility Smile

The volatility smile is a long-observed pattern in which at-the-money options tend to have lower implied volatilities than in- or out-of-the-money options. The pattern displays different characteristics for different markets and results from the probability of extreme moves. Typically, a quantitative analyst will calculate the implied volatility from liquid vanilla options and use models of the smile to calculate the price of more exotic options.

In the Black-Scholes model, the theoretical value of a vanilla option is a monotonic increasing function of the Black-Scholes volatility. Furthermore, except in the case of American options with dividends which early exercise could be optimal, the price is a strictly increasing function of volatility. This means it is usually possible to compute a unique implied volatility from a given market price for an option. This implied volatility is best regarded as a rescaling of option prices which makes comparisons between different strikes, expirations, and underlyings easier and more intuitive. When implied volatility is plotted against strike price, the resulting graph is typically downward sloping, or downward sloping with an upward bend at either end.

To illustrate that his model can produce "implied volatility smile" Kou considers a real data set used first in Andersen and Andreasen (2000) for two -year and nine-year caplets in the Japanese LIBOR market as of late May 1998. Figure 2-2 shows both observed implied volatility curves and calibrated implied volatility curves derived by using the futures option formula in Corollary 2, with the discount parameter D being the corresponding bond prices and the underlying asset being the LIBOR rate.

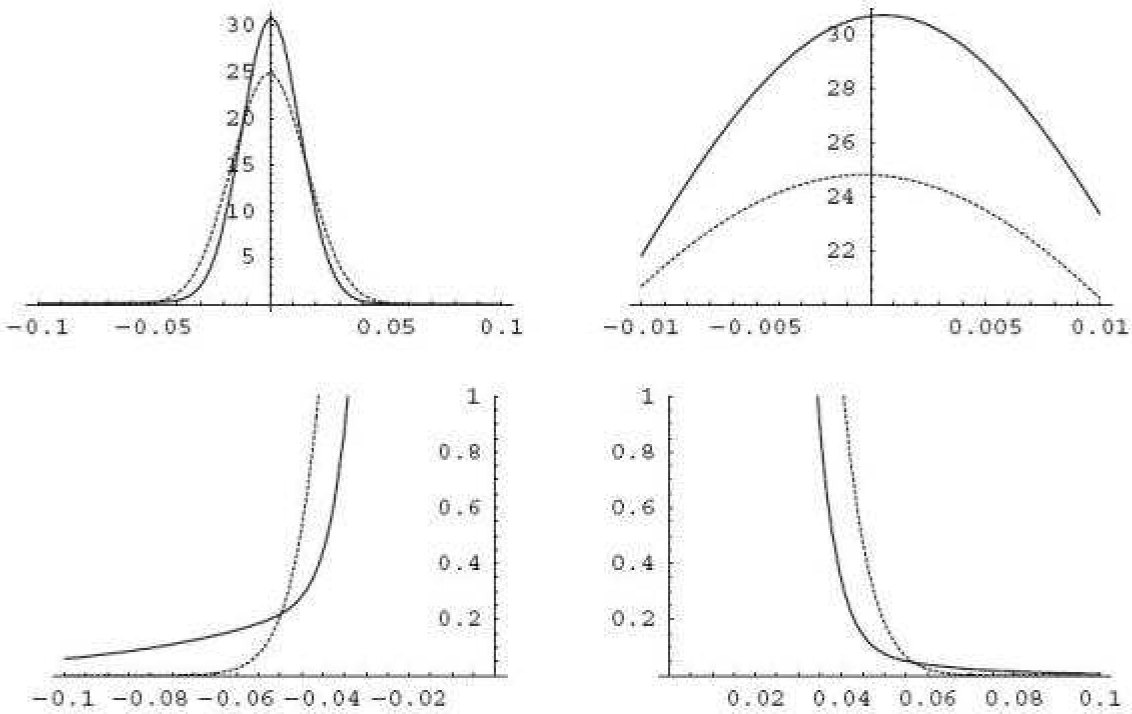

Figure 2-1. Comparison between density g (solid line) of Leptokurtic feature and normal distribution density (dotted line) with same mean and variance

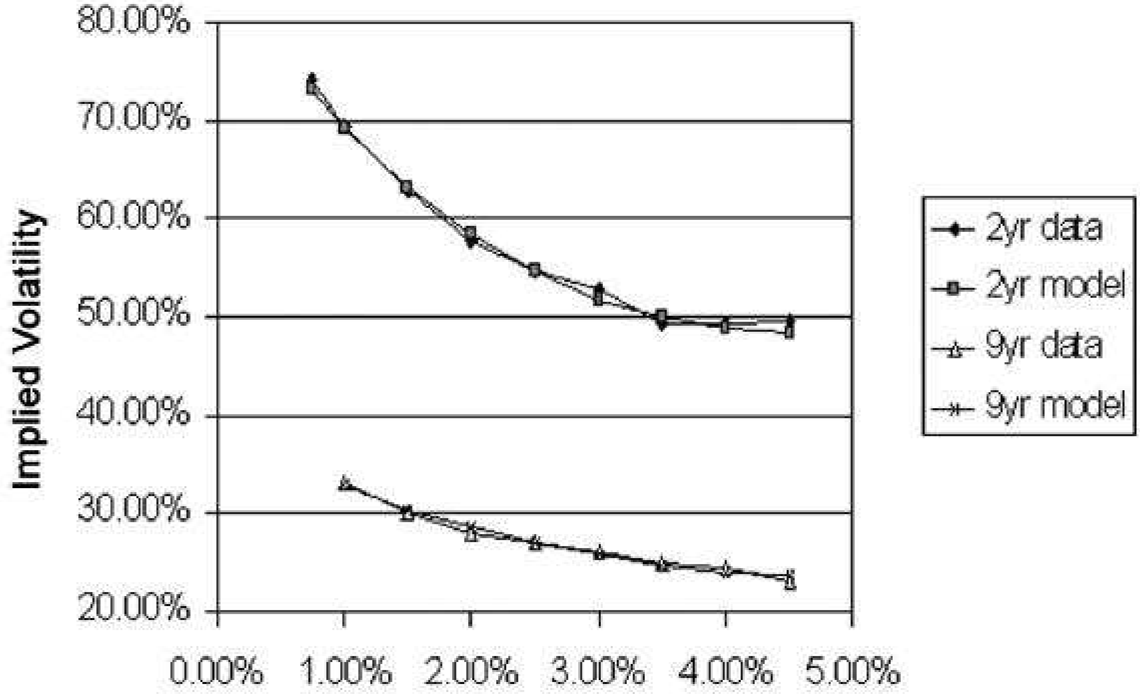

Figure 2-2. Midmarket and Model-Implied Volatilities for Japanese LIBOR Caplets in May 1998

CHAPTER 3
AMERICAN OPTION PRICING IN KOU'S MODEL

In this section I will review the main results and functions in Kou's Model that we will use for further development in the next chapter. We begin by common functions such as Hh function, a special function of mathematical physics, and alternative ones that will be used to express the European option price. For notational simplicity, I follow Kou in dropping the $*$ in the risk-neutral notation,i.e. write η_1 instead of η_1^*, etc. To compute these pricing formulas, one has to study the distribution of the sum of the double exponential random variables and normal random variables. Fortunately, this distribution can be obtained in closed form in terms of the Hh function.

3.1 Hh Function

For every $n \geq 0$, the Hh function is a non-increasing function and can be viewed as a generalization of the cumulative normal distribution function defined by

$$Hh_n(x) = \int_x^\infty Hh_{n-1}(y)dy = \frac{1}{n!}\int_x^\infty (t-x)^n e^{-t^2/2}dt \geq 0 \tag{3–1}$$

$$n = 0, 1, 2, ...$$

$$Hh_{-1}(x) = e^{-x^2/2} = \sqrt{2\pi}\varphi(x)$$

In addition,

$$Hh_n(x) = 2^{-n/2}\sqrt{\pi}e^{-x^2/2}$$

$$\times \left\{ \frac{{}_1F_1(\frac{1}{2}n + \frac{1}{2}, \frac{1}{2}, \frac{1}{2}x^2)}{\sqrt{2}\Gamma(1 + \frac{1}{2}n)} - x\frac{{}_1F_1(\frac{1}{2}n + 1, \frac{3}{2}, \frac{1}{2}x^2)}{\Gamma(\frac{1}{2} + \frac{1}{2}n)} \right\}$$

where ${}_1F_1$ is the confluent hypergeometric function. A tree-term recursion is also available for the Hh function:

$$nHh_n(x) = Hh_{n-2}(x) - xHh_{n-1}(x), n \geq 1 \tag{3–2}$$

Therefore, one can compute all $Hh_n(x), n \geq 1$, by using the normal density function and normal distribution function. The Hh function is illustrated in Figure 3-1.

3.2 European Call and Put Options

Now, let's introduce the following notation. For any given probability P, define

$$\Upsilon(\mu, \sigma, \lambda,', \eta_1, \eta_2; a, T) = P\{Z(T) \geq a\}$$

where Z(t)=$\mu t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i$ Y has a double exponential distribution with density $f_Y(y) \sim p\eta_1 \times e^{-\eta_1 y}\mathbf{1}_{y \geq 0} + q\eta_2 e^{\eta_2 y}\mathbf{1}_{y<0}$, and N(t) is a Poisson process with rate λ. The pricing formula of the call option will be expressed in terms of Υ, which in turn can be derived as a sum of Hh functions. An explicit formula for Υ is given in Theorem B.1 in Appendix B of Kou (2002) and is reproduced in an appendix here for convenience.

Theorem 2. From (2-12), the price of a European call option is given by

$$\psi_c(0) = S(0)\Upsilon\left(r + \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \widetilde{\lambda}, \widetilde{p}, \widetilde{\eta_1}, \widetilde{\eta_2} log\left(K/S(0)\right), T\right)$$
$$-Ke^{-rT} \times \Upsilon\left(r - \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \lambda, p, \eta_1, \eta_2 log\left(K/S(0)\right), T\right)$$

where $\widetilde{p} = \frac{p}{1-\zeta}.\frac{\eta_1}{\eta_1 - 1}, \widetilde{\eta_1} = \eta_1 - 1,$, and $\widetilde{\eta_2} = \eta_2 + 1, \widetilde{\lambda} = \lambda(\zeta + 1), \zeta = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1$

The price of the corresponding put option, $\psi_p(0)$, can be obtained by the put-call parity:

$$\psi_p(0) - \psi_c(0) = e^{-rT}E^*\left((K - S(T))^+ - \left(S(T) - K\right)^+\right)$$

$$= e^{-rT}E^*\left(K - S(T)\right) = Ke^{-rT} - S(0)$$

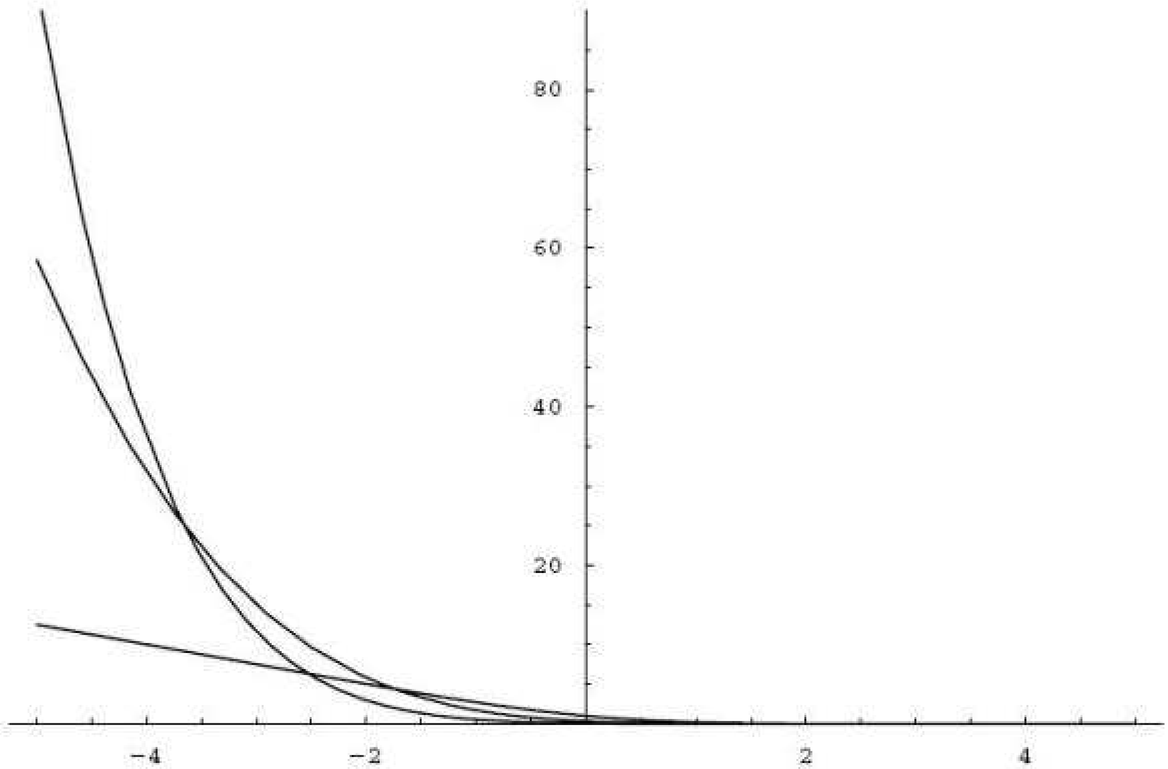

Figure 3-1. The Hh function for $n = 1, 3, 5$ with the Steepest Curve for $n = 5$ and the Flattest Curve for $n = 1$

CHAPTER 4
ANALYTIC RESULT FOR AMERICAN OPTION PRICING

4.1 Main Result

There exists no closed-form formula for the value of an American option, even when the underlying asset price process follows a time-homogeneous Geometric Brownian Motion process such as in the Black-Scholes model. In that case, a computation efficient technique to price an American option is to use the decomposition into the sum of the corresponding European option price and an early exercise premium. The EEP involves the early exercise boundary that is the main source of difficulty in pricing American options. In the classical context of Black and Scholes, the price decomposition leads to an integral equation for the early exercise boundary. Simple parametrization of this boundary have been shown to approximately solve this equation and thus resulting in efficient pricing and hedging of American options (cf Ju (1998), AitSahlia and Lai (2001).) The objective of this thesis is to present an analytical expression for the EEP that will facilitate the numerical evaluation of the early exercise strategy, the resulting American option price, as well as the corresponding hedging parameters, when the underlying asset price follows Kou's model.

Let S be the current underlying asset price and z=ln S. We shall then denote by F(t,S) the price of the American option when there are t units of time left till expiration. For $0 \leq s \leq t$, define the flow process

$$Z_s(z) = z + (r - \lambda\zeta - \frac{\sigma^2}{2})s + \sigma W_s + \sum_{i=1}^{N(s)} Y_i$$

where

$$\zeta = E(V) - 1 = p\frac{\eta_1}{\eta_1 - 1} + (1 + p)\frac{\eta_2}{\eta_2 + 1} - 1$$

is the expected relative jump (or expected jump on the log-scale). As shown in Kou (2002), the measure determining the probability space on which $\{Z_s(z)\}_s$ and all the stochastic processes involved are defined is determined by the representative agent. The main result of this thesis is stated below.

Theorem: Let $\{b(\tau) : 0 \le \tau \le t\}$ be he optimal exercise boundary when maturity is t and let $\mu = r - \lambda\zeta - \frac{\sigma^2}{2}$. Then for a put,

$$F(t,z) = F_E(t,z) + e_1(t,z) - e_2(t,z)$$

where $F_E(t,z)$ is the European put option price.

$$e_1(t,z) = rK\int_0^t e^{-rs}P[Z_s(z) \le b(t-s)]ds - \lambda\zeta e^z \int_0^t \int_{-\infty}^{b(t-s)-z-\mu s} e^w f(w,s)dwds$$

$$e_2 = \int_0^t e^{-rs}\int_{-\infty}^{b(t-s)-z-\mu s} f(w;s)\int_{b(t-s)-z-\mu s-w}^{\infty} [V(t-s, z+\mu s+w+y) - (K - e^{z+\mu s+w})]$$
$$\times p\eta_1 e^{-\eta_1 y}dydwds$$

with

$$f(w,s) = \pi_0\varphi\left(\frac{w}{\sigma\sqrt{s}}\right)$$
$$+\sum_{n=1}^{\infty}\pi_n\sum_{k=1}^{n}P_{n,k}(\widetilde{\sigma}\eta_1)^k\frac{e^{(\widetilde{\sigma}\eta_1)^2/2}}{\widetilde{\sigma}\sqrt{2\Pi}}e^{w\eta_1}Hh_{k-1}\left(-\frac{w}{\widetilde{\sigma}}+\widetilde{\sigma}\eta_1\right)$$
$$+\sum_{n=1}^{\infty}\pi_n\sum_{k=1}^{n}Q_{n,k}(\widetilde{\sigma}\eta_2)^k\frac{e^{(\widetilde{\sigma}\eta_2)^2/2}}{\widetilde{\sigma}\sqrt{2\Pi}}e^{w\eta_2}Hh_{k-1}\left(\frac{w}{\widetilde{\sigma}}+\widetilde{\sigma}\eta_2\right)$$

Here ϕ is the density functions of the standard normal distribution, and

$$\pi_n = e^{-\lambda s}\frac{(\lambda s)^n}{n!}$$

$$\widetilde{\sigma} = \sigma\sqrt{s}$$

$$P_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{n-i} p^i q^{n-i}$$

$$Q_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{n-i} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{i-k} p^{n-i} q^i$$

For $1 \le k \le n-1$

With $P_{n,n} = p^n$, $Q_{n,n} = q^n$ and $p + q = 1$

Remark: The probability $P[Z_s(z) \le b(t-s)]$ in $e_1(t,z)$ can be expressed explicitly using Theorem B.1 of Kou (2002).

Indeed,

$$P[Z_s(z) \le b(t-s)] = 1 - Q(t,s,z)$$

where letting $a = b(t-s) - z$,

$$Q(t,s,z) = \frac{e^{(\sigma\eta_1)^2 s/2}}{\sigma\sqrt{2\pi s}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k} \left(\sigma\sqrt{s}\eta_1\right)^k \times I_{k-1}\left(a - \mu s; -\eta_1, -\frac{1}{\sigma\sqrt{s}}, -\sigma\eta_1\sqrt{s}\right)$$

$$+\frac{e^{(\sigma\eta_2)^2 s/2}}{\sigma\sqrt{2\pi s}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k} \left(\sigma\sqrt{s}\eta_2\right)^k$$

$$\times I_{k-1}\left(a - \mu s; \eta_2, \frac{1}{\sigma\sqrt{s}}, -\sigma\eta_2\sqrt{s}\right)$$

$$+\pi_0 \Phi\left(-\frac{a-\mu s}{\sigma\sqrt{s}}\right)$$

with Φbeing the standard normal cumulative distribution function and

$$I_n(c;\alpha;\beta;\delta) = -\frac{e^{\alpha c}}{\alpha} \sum_{i=0}^{n} \left(\frac{\beta}{\alpha}\right)^{n-i} Hh_i(\beta c - \delta)$$

$$+\left(\frac{\beta}{\alpha}\right)\frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\Phi\left(-\beta c+\delta+\frac{\alpha}{\beta}\right)$$

for $\beta > 0$, $\alpha \neq 0$, and $n \geq -1$; and

$$I_n(c;\alpha;\beta;\delta) = -\frac{e^{\alpha c}}{\alpha}\sum_{i=0}^{n}\left(\frac{\beta}{\alpha}\right)^{n-i}Hh_i(\beta c-\delta)$$

$$-\left(\frac{\beta}{\alpha}\right)\frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\Phi\left(\beta c-\delta-\frac{\alpha}{\beta}\right)$$

for $\beta > 0$, $\alpha < 0$, and $n \geq -1$

4.2 Proof of the Result

For a jump-diffusion process driven by a Brownian motion and a compound Poisson process with arbitrary jumps, Pham (1997) shows that the value function F is convex in the state variable. We can therefore apply the generalized It$\widehat{o}$ lemma for convex functions (cf. Meyer, 1976), which helps us establish:

$$e^{-rt}F\left(t, Z_f(z)\right) - F(0,z) =$$

$$\int_0^t e^{-rs}\mathfrak{L}F\left(t-s, Z_s(z)\right)ds + \int_0^t e^{*rs}F\left(t-s, Z_s(z)\right)\sigma dW_s$$

$$+\int_0^t\int_R e^{-rs}\left[F\left(t-s, Z_s(z)+y\right) - F\left(t-s, Z_s(z)\right)\right]\widetilde{\nu}(ds,dy) \tag{4–1}$$

where $\widetilde{\nu}$ is the associated compensated Poisson measure.

Taking expectations on both sides of (4.1) with respect to the risk-neutral measure associated with the representative agent and noting that the two stochastic integrals in (4.1) are martingales, we thus have:

$$F(t,z) = T_e(t,z) - E\left[\int_0^t e^{-rs}\mathfrak{L}F(t-s, Z_s(z))ds\right]$$

where $F_E(t,z)$ is the European option price, which is known explicitly (cf Theorem 2 of Kou (2002)).

With $b(\tau)$ defined to be the optimal exercise boundary when there are τ units of time left in the life of the put option, we note that

$$\mathfrak{L}F\left(t-s, Z_s(z)\right) = 0$$

if

$$Z_s(z) > b(t-s).$$

On the other hand, when $Z_s(z) \leq b(t-s)$, we have

$$F\left(t-s, Z_s(z)\right) = K - e^{Z-s(z)}.$$

And thus

$$\mathfrak{L}F\left(t-s, Z_s(z)\right) = -rK + \lambda\zeta e^{Z_s(z)},$$

from which we infer

$$F(t,z) = F_E(t,z) + e_1(t,z) - e_2(t,z),$$

where

$$e_1(t,z) = rK\int_0^t e^{-rs}P[Z_s(z) \leq b(t-s)]ds - \lambda\zeta E\left\{\int_0^t e^{-rs}e^{Z_s(z)}1_{\{Z_s(z)\leq b(t-s)\}}ds\right\} \quad (4\text{–}2)$$

and

$$e_2(t,z) = \lambda E\left[\int_0^t \int_{A_{t,s,z}} e^{-rs}1_{\{Z_s(z)\leq b(t-s)\}} \times \{V(t-s, Z_s(z)+y) - (K - e^{Z_s(z)})\}f_Y(y)dyds\right]$$

where $1_A = 1$ if event A occurs, 0 otherwise, and

$$A_{t,s,z} = \{y \in R : Z_s(z) + y > b(t-s)\}$$

The term $P\{Z_s(z) \le b(t-s)\}$ in (4.2) above can now be trivially obtained from Theorem B.1 of Kou (2002).

Making use of Theorem B.1 in Kou (2002) again, we can also express the second integral term in (4.2) as

$$E\left\{\int_0^t e^{-rs} e^{Z_s(z)} 1_{\{Z_s(z) \le b(t-s)\}} ds\right\}$$

$$= e^z \int_0^t \int_{-\infty}^{b(t-s)-z-\mu s} e^w f(w,s) dw ds$$

where

$$f(w,s) = \pi_0 \varphi\left(\frac{w}{\sigma\sqrt{s}}\right)$$

$$+ \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k} (\widetilde{\sigma}\eta_1)^k \frac{e^{(\widetilde{\sigma}\eta_1)^2/2}}{\widetilde{\sigma}\sqrt{2\Pi}} e^{w\eta_1} Hh_{k-1}\left(-\frac{w}{\widetilde{\sigma}} + \widetilde{\sigma}\eta_1\right)$$

$$+ \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k} (\widetilde{\sigma}\eta_2)^k \frac{e^{(\widetilde{\sigma}\eta_2)^2/2}}{\widetilde{\sigma}\sqrt{2\Pi}} e^{w\eta_2} Hh_{k-1}\left(\frac{w}{\widetilde{\sigma}} + \widetilde{\sigma}\eta_2\right)$$

where $\widetilde{\sigma} = \sigma\sqrt{s}$ and $P_{n,k}$, $Q_{n,k}$ as stated in the main body of theorem.

Finally, observe that the value difference under the integral of e_2 captures the fact that the underlying asset price can jump back from the exercise region (stopping region) to the continuation (or no exercise) region.

CHAPTER 5
CONCLUSION

This thesis presents an explicit analytical expression for the price of an American option in the context of Kou's double-exponential jump-diffusion model. The latter has become very popular among practitioners as it capture fundamental empirical properties of the underlying asset while leading to closed-form formulas to price a variety of path-dependent options beside the European type.

5.1 Implication of the Main Result

The two main advantages of the analytical result presented here are first that it sets the stage for efficient numerical implementation. Specifically, the presence of a non-local term in the second element of the early exercise premium requires that the prices of the option at subsequent dates and states be known in order to be able to price at the current date and state. Fortunately, the anticipated use of Gaussian quadrature for numerical integration will fix all subsequent dates and states for which such computations will be necessary. Secondly, the analytical expression derived in this thesis will also be useful to compute directly all the hedging parameters as we can apply differentiations throughout the integral terms.

5.2 Limitations of the Model

Their are several limitations of the model. First disadvantage, the pricing formulae, although analytical, appear quite complicated. This may not seem to be a major problem since the Hh functions can be computed easily, and what appears to be lengthy might make little difference in terms of computer programming, as long as it is a closed-form solution. Secondly, their is a more serious criticism which is the difficulties with hedging. Due to the jump part, the market is incomplete, and the conventional riskless hedging arguments are not applicable in here. However, it should be pointed out that the riskless hedging is really a special property of the continuous-time Brownian motion, and it does not hold for most of the alternative models. Even within the Brownian motion framework, the riskless hedging is impossible if we want to do it in discrete time. Finally, just like all models based on Lévy processes, one empirical observation of the double exponential jump-diffusion model is that it cannot incorporate the possible dependence structure among asset returns, because the model assume independent increments.

5.3 Scope of Further Research

Here, a possible way to incorporate the dependence is by using some other point process, $\widetilde{N}(t)$, with dependent increments, to replace our Poisson process $N(t)$, while still retaining the independence between the Brownian motion, the jump sizes, and $\widetilde{N}(t)$. The modified model then will no longer have independent increments, yet will be simple enough to produce closed-form solutions. However, it seems difficult to get analytical solutions for path-dependent options by using this new process $\widetilde{N}(t)$ instead of $N(t)$.

APPENDIX A
DERIVATION OF THE RATIONAL EXPECTATIONS

Proof of Proposition 1.

(1) Since B(T,T)=1, Equation (2-8) yields

$$B(t,T) = e^{\theta(T-t)} \frac{E((\delta(T))^{\alpha-1}|\mathfrak{F}_t)}{(\delta(t))^{\alpha-1}} \tag{A–1}$$

Using the fact that

$$\left(\frac{\delta(T)}{\delta(t)}\right)^{\alpha-1} = exp(\alpha-1)\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)(T-t) + \sigma_1(\alpha-1)(W_1(T)-W_1(t)) \prod_{i=N(t)+1}^{N(T)} \widetilde{V}_i^{\alpha-1}$$

$$\begin{aligned} E\left(\prod_{i=N(t)+1}^{N(t)} \widetilde{V}_i^{\alpha-1}\right) &= \sum_{j=0}^{\infty} e^{-\lambda(T-t)} \frac{[\lambda(T-t)]^j}{j!} \zeta_1^{(\alpha-1)} + 1^j \\ &= exp\left(\lambda\zeta_1^{(\alpha-1)}(T-t)\right) \end{aligned}$$

First equation yields

$$B(t,T) = exp\left[-(T-t)\theta - (\alpha-1)\left(\mu_1 - \frac{1}{2}\sigma_1^2\right) - \frac{1}{2}\sigma_1^2(\alpha-1)^2 - \lambda\zeta_1^{(\alpha-1)}\right]$$

Note that it implies

$$e^{-r(T-t)} = E(U_c(\delta(T),T)/U_c(\delta(t),t)|\mathfrak{F}_t) \tag{A–2}$$

which shows that Z(t) is a martingale under P. Furthermore, (2-7) and (2-9), lead to

$$\begin{aligned} Z(t) &= (\delta(0))^{\alpha-1} e^{(r-\theta)t} exp\left\{(\alpha-1)\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1(\alpha-1)(W_1(t))\right\} \prod_{i=1}^{N(t)} \widetilde{V}_i^{\alpha-1} \\ &= (\delta(0))^{\alpha-1} exp\left\{-\frac{1}{2}\sigma_1^2(\alpha-1)^2 - \lambda\zeta_1^{(\alpha-1)}\right\} t + \sigma_1(\alpha-1)(W_1(t)) \prod_{i=1}^{N(t)} \widetilde{V}_i^{\alpha-1} \end{aligned}$$

Now

$$\psi_s(t) = \frac{E(U_c(\delta(T),T)\psi_s(T)|\mathfrak{F}_t)}{(U_c(\delta(t),t))} = e^{-rT}E\left\{\frac{Z(T)}{Z(t)}\psi_s(T)|\mathfrak{F}_t\right\}$$
$$= e^{-rT}E^*(\psi_s(T)|\mathfrak{F}_t)$$

Proof of Theorem 1. The Girsanov theorem for jump-diffusion processes tells us that under P^*, $W_1'(t) = W_1(t) - \sigma_1(\alpha-1)t$ is a new Brownian motion and under P^* the jump rate of N(t) is $\lambda^* = \lambda E\left(\widetilde{V}_i^{\alpha-1}\right) = \lambda\left(\zeta_1^{(\alpha-1)}+1\right)$ and $\widetilde{V}_i$ has a new density $f^*_{\widetilde{V}}(x) = (1/(\zeta_1^{(\alpha-1)}+1))x^{\alpha-1}f_{\widetilde{V}}(x)$. Therefore, dynamics of S(t) is given by

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma\left\{\rho dW_1(t) + \sqrt{1-\rho^2}dW_2(t)\right\} + \Delta\left(\sum_{i=1}^{N(t)}(V_i^\beta - 1)\right)$$

$$= \mu + \sigma_1\sigma\rho(\alpha-1)dt + \sigma\left\{\rho dW_1'(t) + \sqrt{1-\rho^2}dW_2(t)\right\} + \Delta\left(\sum_{i=1}^{N(t)}(V_i^\beta - 1)\right)$$

Because

$$E^*(\widetilde{V}_i^\beta) = \int_0^\infty x^\beta \frac{1}{\zeta_1^{(\alpha-1)}+1}x^{(\alpha-1)}f_{\widetilde{V}}(x)dx$$
$$= \frac{1}{\zeta_1^{(\alpha-1)}+1}E(\widetilde{V}^{\alpha+\beta-1}) = \frac{\zeta_1^{\alpha+\beta-1}+1}{\zeta_1^{\alpha-1}+1}$$

we have

$$\lambda^*\left\{E^*(\widetilde{V}^\beta) - 1\right\} = \lambda\left(\zeta_1^{\alpha+\beta-1} - \zeta_1^{\alpha-1}\right)$$

.

Therefore

$$\frac{dS(t)}{S(t-)} = \{\mu + \sigma_1\sigma\rho(\alpha-1) + \lambda(\zeta_{\alpha+\beta-1} - \zeta_{\alpha-1})\}dt$$

$$-\lambda^*\left\{E^*(\widetilde{V}^\beta) - 1\right\}dt + \sigma\left\{\rho dW_1'(t) + \sqrt{1-\rho^2}dW_2(t)\right\} + \Delta\left(\sum_{i=1}^{N(t)}(V_i^\beta - 1)\right)$$

Hence to satisfy the rational equilibrium requirement $S(t) = e^{-r(T-t)}E^*(S(T)|\mathfrak{F})$ we must have $\mu + \sigma_1\sigma\rho(\alpha - 1) + \lambda(\zeta_{\alpha+\beta-1} - \zeta_{\alpha-1}) = r$

So, under the measure P^*, the dynamics of S(t) is given by

$$\frac{dS(t)}{S(t-)} = rdt - \lambda^* \left\{E^*(\widetilde{V}^\beta) - 1\right\} dt + \sigma \left\{\rho dW_1'(t) + \sqrt{1-\rho^2}dW_2(t)\right\} + \Delta \left(\sum_{i=1}^{N(t)}(V_i^\beta - 1)\right)$$

from which (2-15) follows.

APPENDIX B
DERIVATION OF THE Υ FUNCTION

The explicitness of Kou's formula for the European option price rests on deriving a formula for Υ, the tail distribution of the random variable $Z(T)$. His approach is to decompose the latter into the sum of exponentials by exploiting the memoryless property. We reproduce here the main steps of his derivations which ultimately resides in his Theorem B.1.

The memoryless property of exponential random variables yields $(\xi^+ - \xi^- | \xi^+ > \xi^-) =^d \xi^+$ and $(\xi^+ - \xi^- | \xi^+ < \xi^-) =^d -\xi^-$, thus leading to the conclusion that

$$\xi^+ - \xi^- = \left\{ \begin{array}{rl} \xi^+ & \text{with probability } \eta_2/(\eta_1+\eta_2) \\ -\xi^- & \text{with probability } \eta_1/(\eta_1+\eta_2) \end{array} \right\}. \tag{B–1}$$

because the probabilities of the events $\xi^+ > \xi^-$ and $\xi^+ < \xi^-$ are $\eta_2/(\eta_1+\eta_2)$ and $\eta_1/(\eta_1+\eta_2)$, respectively. The following proposition extends (B.1.)

Proposition B.1. For every $n \geq 1$, we have the following decomposition

$$\sum_{i=1}^{n} Y_i =^d \left\{ \begin{array}{rl} \sum_{i=1}^{k} \xi_i^+ & \text{with probability } P_{n,k}, k = 1, 2, ..., n \\ -\sum_{i=1}^{k} \xi_i^- & \text{with probability } Q_{n,k}, k = 1, 2, ..., n \end{array} \right\}. \tag{B–2}$$

where $P_{n,k}$ and $Q_{n,k}$ are given by

$$P_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{n-i} p^i q^{n-i}$$

$$1 \leq k \leq n-1$$

$$Q_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{n-i} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{i-k} p^{n-i} q^i$$

$$1 \leq k \leq n-1, P_{n,n} = p^n, Q_{n,n} = q^n$$

and $\binom{0}{0}$ is defined to be one. Hence ξ_i^+ and ξ_i^- are i.i.d. exponential random variables with rates η_1 and η_2, respectively.

As a key step in deriving closed-form solutions for call and put options, this proposition indicates that the sum of the i.i.d. double exponential random variable can be written, in distribution, as a randomly mixed gamma random variable. To prove Proposition B.1, the following lemma is needed.

Lemma B.1.

$$\sum_{i=1}^{n} \xi_i^+ - \sum_{i=1}^{n} \xi_i^-$$

$$=^d \left\{ \begin{array}{ll} \sum_{i=1}^{k} \xi_i & \text{with probability } \binom{n-k+m-1}{m-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n-k}(\frac{\eta_2}{\eta_1+\eta_2})^m, k = 1, ..., n \\ -\sum_{i=1}^{l} \xi_i & \text{with probability } \binom{n-l+m-1}{n-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n}(\frac{\eta_2}{\eta_1+\eta_2})^{m-l}, l = 1, ..., m \end{array} \right\}. \quad \text{(B–3)}$$

Kou proves this result by introducing the random variables $A(n, m) = \sum_{i=1}^{n} \xi_i - \sum_{j=1}^{m} \tilde{\xi}_j$ Then

$$A(n, m) =^d \left\{ \begin{array}{ll} A(n-1, m-1) + \xi^+ & \text{with probability } \eta_2/(\eta_1 + \eta_2) \\ A(n-1, m-1) - \xi^- & \text{with probability } \eta_1/(\eta_1 + \eta_2) \end{array} \right\}.$$

$$=^d \left\{ \begin{array}{ll} A(n, m-1) & \text{with probability } \eta_2/(\eta_1 + \eta_2) \\ A(n-1, m) & \text{with probability } \eta_1/(\eta_1 + \eta_2) \end{array} \right\}.$$

via B.1.. Now suppose the horizontal axis represents the number of $\{\zeta_i^+\}$ and vertical axis represents the number of $\{\zeta_i^-\}$. Suppose we have a random walk on the integer lattice points. Starting from any point $(n, m), n, m \geq 1$, the random walk goes either one step to the left with probability $\eta_1/(\eta_1 + \eta_2)$ or one step down with probability $\eta_2/(\eta_1 + \eta_2)$, and the random walks stops once it reaches the horizontal or vertical axis. For any path from (n,m) to (k,0) , $1 \geq k \geq n$, it must reach (k,1) first before it makes a final move to (k,0). Furthermore, all the paths going from (n,m) to (k,1) must have exactly n-k lefts and m-1 downs, whence the total number of such paths is $\binom{n-k+m-1}{m-1}$. Similarly the total number of paths from (n,m) to (0,l) , $1 \geq l \geq m$, is $\binom{n-l+m-1}{n-1}$. Thus

$$A(n,m) =^d \left\{ \begin{array}{cc} \sum_{i=1}^{k} \xi_i & \text{with probability } \binom{n-k+m-1}{m-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n-k}(\frac{\eta_2}{\eta_1+\eta_2})^{m}, k = 1, ..., n \\ -\sum_{i=1}^{l} \xi_i & \text{with probability } \binom{n-l+m-1}{n-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n}(\frac{\eta_2}{\eta_1+\eta_2})^{m-l}, l = 1, ..., m \end{array} \right\}.$$

and the lemma is proven.

To prove proposition B.1, Kou follows by analogy the proof of Lemma B.1 to compute $P_{n,m}, 1 \geq k \geq n$, the probability weight assigned to $\sum_{i=1}^{k} \xi_i^+$ when we decompose $\sum_{i=1}^{k} Y_i$. It is equivalent to consider the probability of the random walk ever reach (k,0) starting from the point (i,n-i) being $\binom{n}{i} p^i q^{n-i}$. Note that the point (k,0) can only be reached from point (i,n-i) such that $k \geq i \geq n-1$, because the random walk can only go left or down, and stops once it reaches the horizontal axis. Therefore, for $1 \geq k \geq n-1$, (B-3) leads to

$$P_{n,k} = \sum_{i=k}^{n-1} P(goingfrom(i,n-i)to(k,0)).P(startingfrom(i,n-i))$$

$$= \sum_{i=k}^{n-1} \binom{i+(n-i)-k-1}{(n-i)-1} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{n-i} p^i q^{n-i}$$

$$= \sum_{i=k}^{n-1} \binom{n-k-1}{n-i-1} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{n-i} p^i q^{n-i}$$

$$= \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{n-i} p^i q^{n-i}$$

Of course $P_{n,n} = p^n$. Similarly, one can compute $Q_{n,k}$:

$$Q_{n,k} = \sum_{i=k}^{n-1} P(goingfrom(n-i,i)to(0,k)).P(startingfrom(n-i,i))$$

$$= \sum_{i=k}^{n-1} \binom{i+(n-i)-k-1}{(n-i)-1} \binom{n}{n-i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{n-i} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{i-k} p^{n-i} q^i$$

$$= \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{n-i} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{i-k} p^{n-i} q^i$$

with $Q_{n,n} = q^n$. Incidentally, we have also got $\sum k = 1n(P_{n,k} + Q_{n,k}) = 1$

B.2. Here we review results on Hh functions. First, note that $Hh_n(x) \to 0$, as $x \to \infty$, for $n \geq -1$; and $Hh_n(x) \to \infty$, as $x \to -\infty$, for $n \geq -1$; and $Hh_0(x) = \sqrt{2\pi}\phi(-x) \to \sqrt{2\pi}$, as $x \to -\infty$. Also, for every $n \geq -1$, as $x \to \infty$,

$$lim Hh_n(x) / \left\{ \frac{1}{x^{n+1}} e^{-\frac{x^2}{2}} \right\} = 1 \tag{B–4}$$

and as $x \to \infty$

$$Hh_n(x) = O(|x|^n) \tag{B–5}$$

Here (B-4) is clearly true for $n = -1$, while for $n \geq 0$ note that as $x \to_\infty$,

$$Hh_n(x) = \frac{1}{n!} \int_x^\infty (t-x)^n e^{-\frac{t^2}{2}} dt$$

$$\leq \frac{2^n}{n!} \int_{-\infty}^\infty |t|^n e^{-t^2} 2dt + \frac{2^n}{n!} \int_{-\infty}^\infty |x|^n e^{-t^2} 2dt = O(|x|^n)$$

For option pricing it is important to evaluate the integral $I_n(c;\alpha;\beta;\delta)$,

$$I_n(c;\alpha;\beta;\delta) = \int_c^\infty e^{\alpha x} Hh_n(\beta x - \delta) dx, n \geq 0 \tag{B–6}$$

for arbitrary constants α, c and β.

Proposition B.2. (Kou, 2002)

(1) If $\beta > 0$ and $\alpha \neq 0$, then for all $n \geq -1$,

$$I_n(c;\alpha;\beta;\delta) = -\frac{e^{\alpha c}}{\alpha} \sum_{i=0}^{n} \left(\frac{\beta}{\alpha}\right)^{n-i} Hh_i(\beta c - \delta) + \left(\frac{\beta}{\alpha}\right)^{n+1}$$

$$\frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi\left(-\beta c+\delta+\frac{\alpha}{\beta}\right) \tag{B–7}$$

(2) If $\beta < 0$ and $\alpha < 0$, then for all $x \geq -1$

$$I_n(c;\alpha;\beta;\delta) = -\frac{e^{\alpha c}}{\alpha}\sum_{i=0}^{n}\left(\frac{\beta}{\alpha}\right)^{n-i}Hh_i(\beta c-\delta) - \left(\frac{\beta}{\alpha}\right)^{n+1}\frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi\left(\beta c-\delta-\frac{\alpha}{\beta}\right) \tag{B–8}$$

To prove this result, Kou makes us of integration by parts to obtain: Case 1. $\beta > 0$ and $\alpha \neq 0$. Since, for any constant α and $n \geq 0$, $e^{\alpha x}Hh_n(\beta x-\delta) \to 0$ as $x \to \infty$ thanks to (B-4), integration by parts leads to

$$I_n = -\frac{1}{\alpha}Hh(\beta c-\delta)e^{\alpha c} + \frac{\beta}{\alpha}\int_c^\infty e^{\alpha x}Hh_{n-1}(\beta c-\delta)dx$$

In other words, we have a recursion, for $n \geq 0$, $I_n = -(e^{\alpha c}\alpha)Hh_n(\beta c-\delta) + (\frac{\beta}{\alpha})I_{n-1}$ with

$$I_{-1} = \sqrt{2\pi}\int_c^\infty e^{\alpha x}\varphi\left(-\beta x+\delta\right)dx$$

$$= \frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi\left(-\beta c+\delta+\frac{\alpha}{\beta}\right)$$

Solving it yields, for $n \geq -1$,

$$I_n = -\frac{e^{\alpha c}}{\alpha}\sum_{i=0}^{n}\left(\frac{\beta}{\alpha}\right)^{i}Hh_{n-i}(\beta c+\delta) + \left(\frac{\beta}{\alpha}\right)^{n+1}I_{-1}$$

$$= -\frac{e^{\alpha c}}{\alpha}\sum_{i=0}^{n}\left(\frac{\beta}{\alpha}\right)^{n-i}Hh_i(\beta c+\delta)$$
$$+\left(\frac{\beta}{\alpha}\right)^{n+1}\frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi\left(-\beta c+\delta+\frac{\alpha}{\beta}\right)$$

where the sum over an empty set is defined to be zero.

Case2. $\beta < 0$ and $\alpha < 0$. In this case, we must also have, for $n \geq 0$ and any constant $\alpha < 0, e^{\alpha x} Hh_n(\beta x - \delta) \to 0$ as $x \to \infty$, thanks to (B-5). Using integration by parts, we again have the same recursion, for $n \geq 0, I_n = -(e^{\alpha c}/\alpha)Hh_n(\beta c - \delta) + (\beta/\alpha)I_{n-1}$, but with a different initial condition

$$I_{-1} = \sqrt{2\pi} \int_c^\infty e^{\alpha x} \varphi(-\beta x + \delta) dx$$

$$= -\frac{\sqrt{2\pi}}{\beta} exp\left\{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}\right\} \phi\left(\beta c - \delta - \frac{\alpha}{\beta}\right)$$

Solving it yields (B-8), for $n \geq -1$.

Finally, we sum the double exponential and the normal random variables

Proposition B.3. (Kou, 2002)

Suppose $\{\xi_1, \xi_2, ...\}$ is a sequence of i.i.d. exponential random variables with rate $\eta > 0$, and Z is a normal variable with distribution $N(0, \sigma^2)$. Then for every $n \geq 1$, we have: (1) The density functions are given by:

$$f_{Z+\sum_{i=1}^n \xi_i}(t) = (\sigma\eta)^n \frac{e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} e^{-t\eta} Hh_{n-1}\left(-\frac{t}{\sigma} + \sigma\eta\right) \tag{B–9}$$

$$f_{Z-\sum_{i=1}^n \xi_i}(t) = (\sigma\eta)^n \frac{e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} e^{-t\eta} Hh_{n-1}\left(\frac{t}{\sigma} + \sigma\eta\right) \tag{B–10}$$

(2) The tail probabilities are given by

$$P(Z + \sum_{i=1}^n \xi_i \geq x) = (\sigma\eta)^n \frac{e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} e^{-t\eta} I_{n-1}\left(x; -\eta, -\frac{1}{\sigma}, -\sigma\eta\right) \tag{B–11}$$

$$P(Z - \sum_{i=1}^n \xi_i \geq x) = (\sigma\eta)^n \frac{e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} e^{-t\eta} I_{n-1}\left(x; \eta, \frac{1}{\sigma}, -\sigma\eta\right) \tag{B–12}$$

Proof. Case 1. The densities of $Z + \sum_{i=1}^n \xi_i$, and $Z - \sum_{i=1}^n \xi_i$. We have

$$f_{Z+\sum_{i=1}^n \xi_i}(t) = \int_{-\infty}^{\infty} f_{\sum_{i=1}^n \xi_i}(t-x) f_Z(x) dx$$

$$= e^{-t\eta}(\eta^n) \int_{-\infty}^{t} \frac{e^{x\eta}(t-x)^{n-1}}{(n-1)!} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)} dx$$

$$= e^{-t\eta}(\eta^n) e^{(\sigma\eta)^2/(2)} \int_{-\infty}^{t} \frac{(t-x)^{n-1}}{(n-1)!} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\sigma^2\eta)^2/(2\sigma^2)} dx$$

Letting $y = (x - \sigma^2\eta)/\sigma$ yields

$$f_{Z+\sum_{i=1}^n \xi_i}(t) = e^{-t\eta}(\eta^n) e^{(\sigma\eta)^2/(2)} \sigma^{n-1}$$

$$\times \int_{-\infty}^{t/\sigma - \sigma\eta} \frac{(t/\sigma - y - \sigma\eta)^{n-1}}{(n-1)!} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

$$= \frac{e^{(\sigma\eta)^2/2}}{\sqrt{2\pi}} (\sigma^{n-1}\eta^n) e^{-t\eta} Hh_{n-1}(-t/\sigma + \sigma\eta)$$

because $(1/(n-1)!) \int_{-\infty}^{a} (a-y)^{n-1} e^{-y^2/2} dy = Hh_{n-1}(a)$. The derivation of $f_{Z+\sum_{i=1}^n \xi_i}(t)$ is similar.

Case 2. $P\left(Z + \sum_{i=1}^n \xi_i \geq x\right)$ and $P\left(Z - \sum_{i=1}^n \xi_i \geq x\right)$. From (B-9), it is clear that

$$P\left(Z + \sum_{i=1}^{n} \xi_i \geq x\right) = \frac{(\sigma\eta)^n e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \int_{x}^{\infty} e^{(-i\eta)} Hh_{n-1}\left(-\frac{t}{\sigma} + \sigma\eta\right) dt$$

$$= \frac{(\sigma\eta)^n e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} I_{n-1}\left(x; -\eta, -\frac{1}{\sigma}, -\sigma\eta\right) dt$$

by (B-6). We can compute $P\left(Z - \sum_{i=1}^n \xi_i \geq x\right)$ similarly. The ultimate result in Kou's ability to derive closed-form pricing expressions is the following:

Theorem B.1. (Kou, 2002) With $\pi_n := P(N(t) = n) = e^{-\lambda T}(\lambda T)^n/n!$, I_n as in Proposition B. , and

$$Z(T) = \left(r - \lambda\zeta - \frac{\sigma^2}{2}\right) T + \sigma W(T) + \sum_{i=1}^{N(T)} Y_i,$$

we have

$$P(Z(T) \geq a) = \frac{e^{(\sigma\eta_1)^2 T/2}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k} \left(\sigma\sqrt{T}\eta_1\right)^k \times I_{k-1}\left(a - \mu T; -\eta_1, -\frac{1}{\sigma\sqrt{T}}, -\sigma\eta_1\sqrt{T}\right)$$

$$+\frac{e^{(\sigma\eta_2)^2 T/2}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k} \left(\sigma\sqrt{T}\eta_2\right)^k$$

$$\times I_{k-1}\left(a - \mu T; \eta_2, \frac{1}{\sigma\sqrt{T}}, -\sigma\eta_2\sqrt{T}\right)$$

$$+\pi_0\phi\left(-\frac{a - \mu T}{\sigma\sqrt{T}}\right)$$

To proof is an immediate consequence of decomposition (B-2)

$$P(Z(T) \geq a) = \sum_{n=0}^{\infty} \pi_n P\left(\mu T + \sigma\sqrt{T} Z + \sum_{j=1}^{n} Y_j \geq a\right)$$

$$= \pi_0 P\left(\mu T + \sigma\sqrt{T} Z \geq a\right)$$

$$+\sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k} P\left(\mu T + \sigma\sqrt{T} Z + \sum_{j=1}^{n} \xi_j^+ \geq a\right)$$

$$+\sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k} P\left(\mu T + \sigma\sqrt{T} Z - \sum_{j=1}^{n} \xi_j^- \geq a\right)$$

The result now follows via (B-11) and (B-12) for $\eta_1 > 1$ and $\eta_2 > 0$.

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BIOGRAPHICAL SKETCH

Jérémy Berros was born in Compiègne, France. He earned his Bachelor's in Mathematics, Physic and Engineering Science's degree from "Pierre et Marie Curie College". In 2008 he began his Master's degree in the Department of Industrial and Systems Engineering at the University of Florida. He will finish his Master with concentration in quantitative finance in August 2009