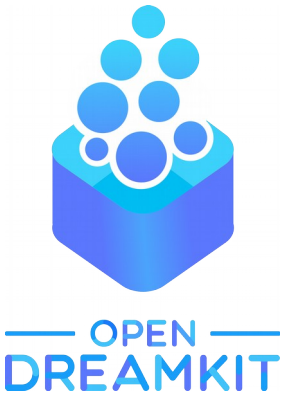
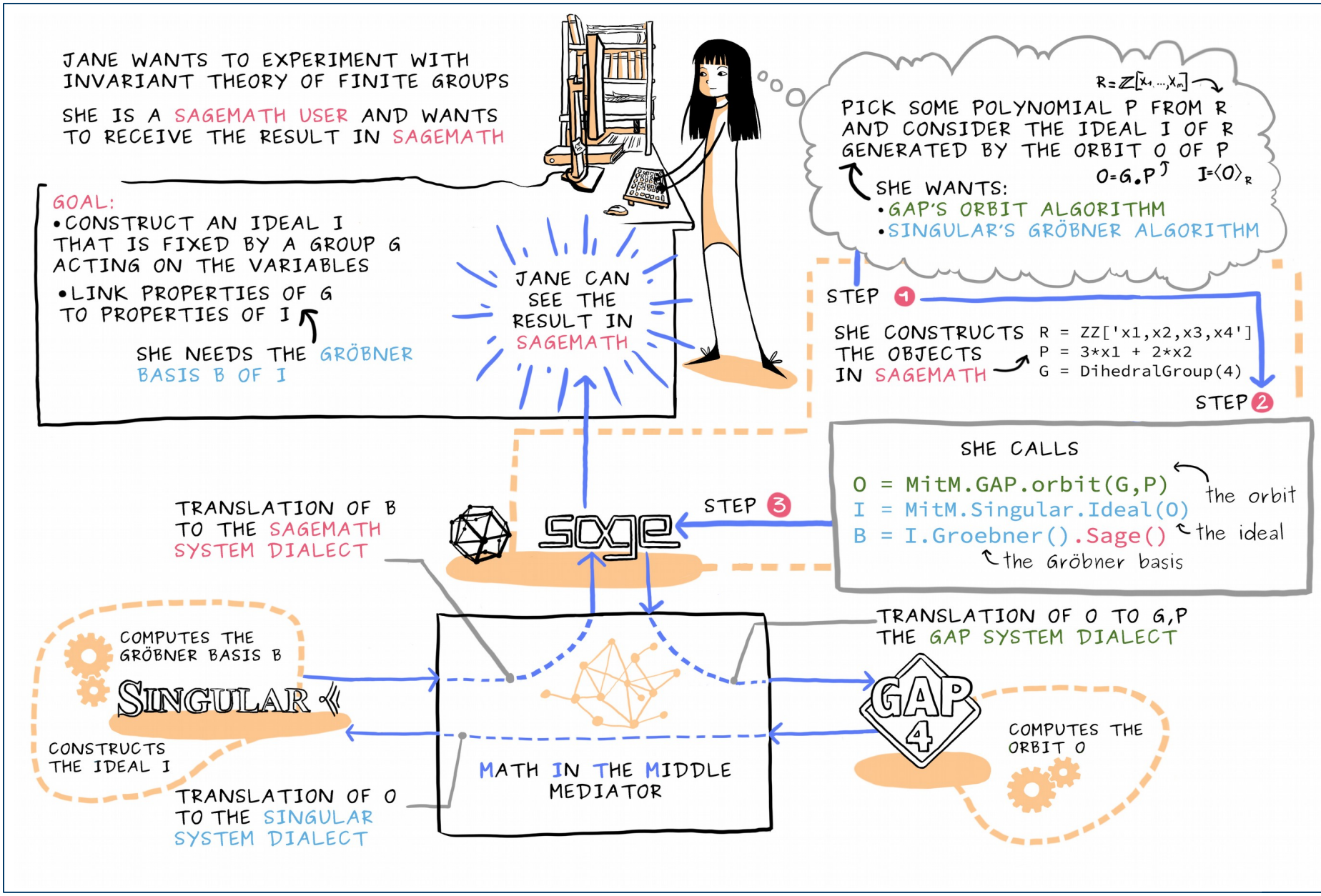
MitM: Math in the Middle

Jonas Betzendahl, 2019-11-22

Based on Kohlhase et al.'s 2017 paper:

"Knowledge-Based Interoperability for Mathematical Software Systems"




JANE WANTS TO EXPERIMENT WITH INVARIANT THEORY OF FINITE GROUPS
SHE IS A SAGEMATH USER AND WANTS TO RECEIVE THE RESULT IN SAGEMATH
GOAL:
• CONSTRUCT AN IDEAL I THAT IS FIXED BY A GROUP G ACTING ON THE VARIABLES
• LINK PROPERTIES OF G TO PROPERTIES OF I
SHE NEEDS THE GRÖBNER BASIS B OF I
JANE CAN SEE THE RESULT IN SAGEMATH
PICK SOME POLYNOMIAL P FROM R AND CONSIDER THE IDEAL I OF R GENERATED BY THE ORBIT O OF P
R = ℤ[x1,...,xm]
O = G.P
I = ⟨O⟩R
SHE WANTS:
• GAP'S ORBIT ALGORITHM
• SINGULAR'S GRÖBNER ALGORITHM
STEP 1
SHE CONSTRUCTS THE OBJECTS IN SAGEMATH
R = ZZ['x1,x2,x3,x4']
P = 3*x1 + 2*x2
G = DihedralGroup(4)
STEP 2
SHE CALLS
O = MitM.GAP.orbit(G,P)
the orbit
I = MitM.Singular.Ideal(O)
the ideal
B = I.Groebner().Sage()
the Gröbner basis
STEP 3
TRANSLATION OF B TO THE SAGEMATH SYSTEM DIALECT
sage
TRANSLATION OF O TO G,P THE GAP SYSTEM DIALECT
COMPUTES THE GRÖBNER BASIS B
SINGULAR
CONSTRUCTS THE IDEAL I
TRANSLATION OF O TO THE SINGULAR SYSTEM DIALECT
MATH IN THE MIDDLE MEDIATOR
GAP 4
COMPUTES THE ORBIT O