RCW Parcel Purchasing Problem

Jake Feldman and Deniz Altinbuken

May 3rd, 2011

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Goal: Constrained by a budget, the Conservation must determine which of these parcels to purchase to maximize the number of occupied territories at the end of a time horizon

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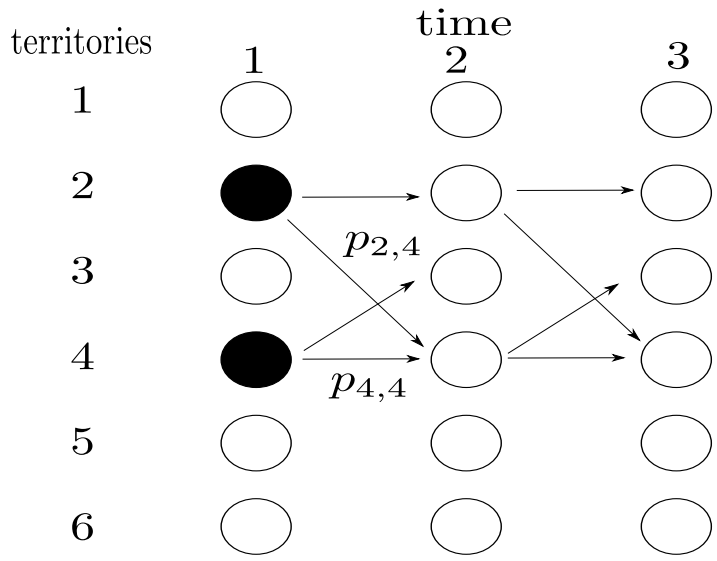

Figure: Cascade Example

Sample Average Approximation

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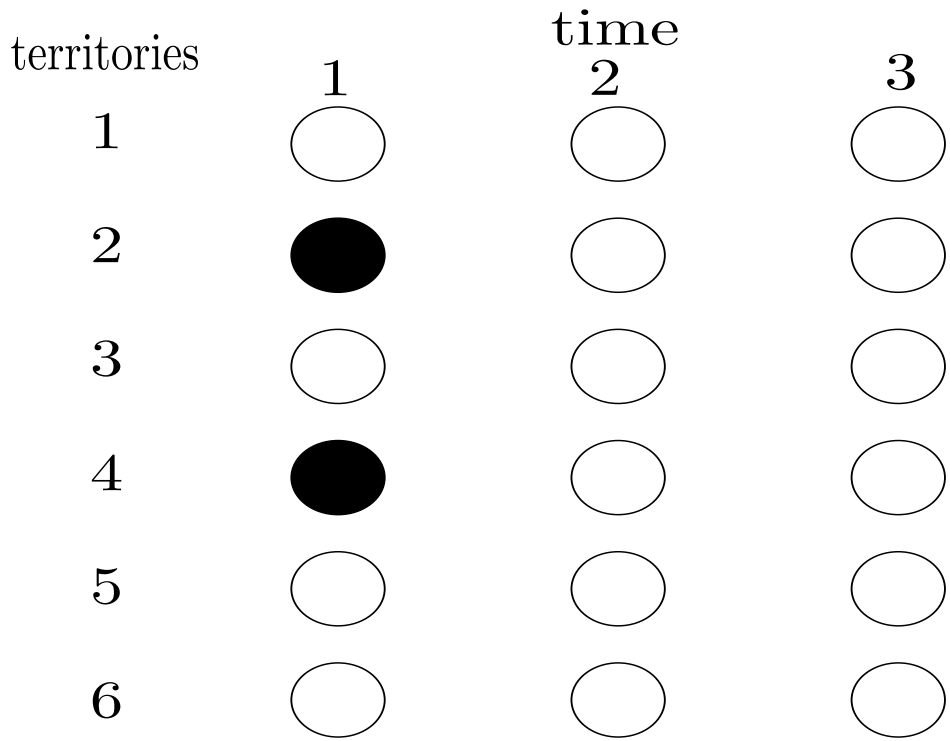
- Can formulate problem as MILP

Example of Buying with a Deterministic Cascade



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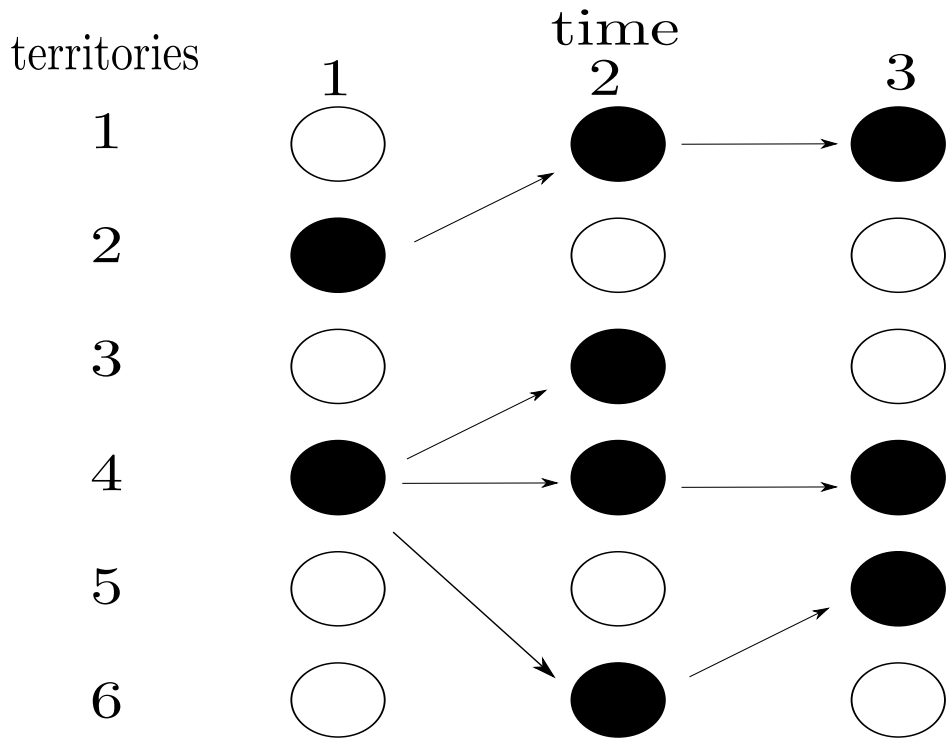

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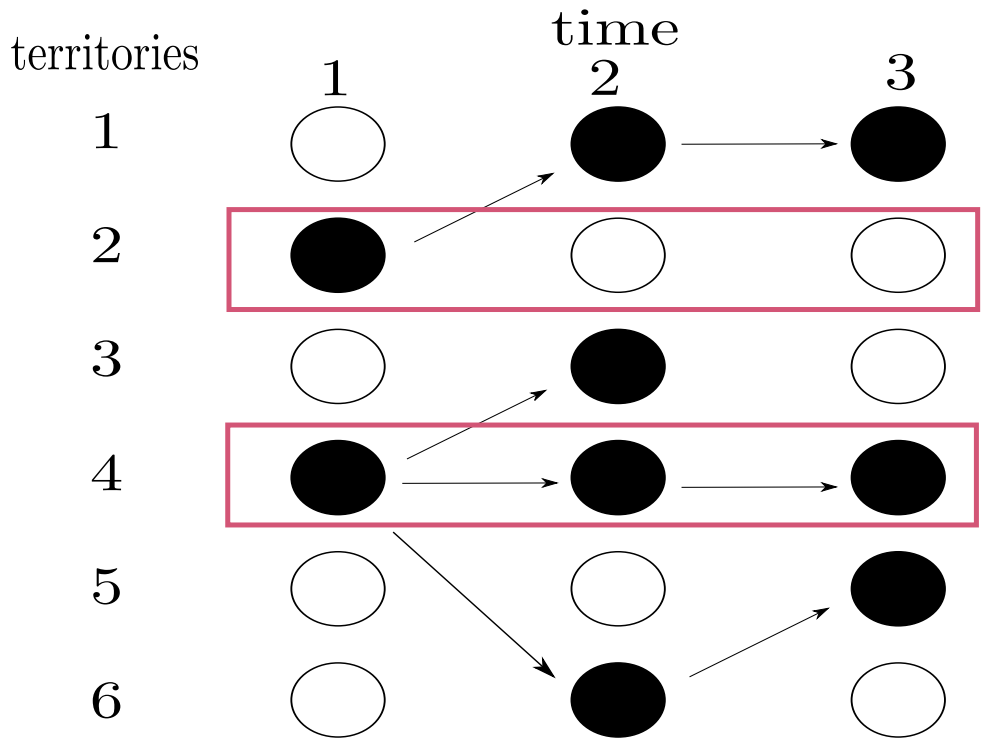

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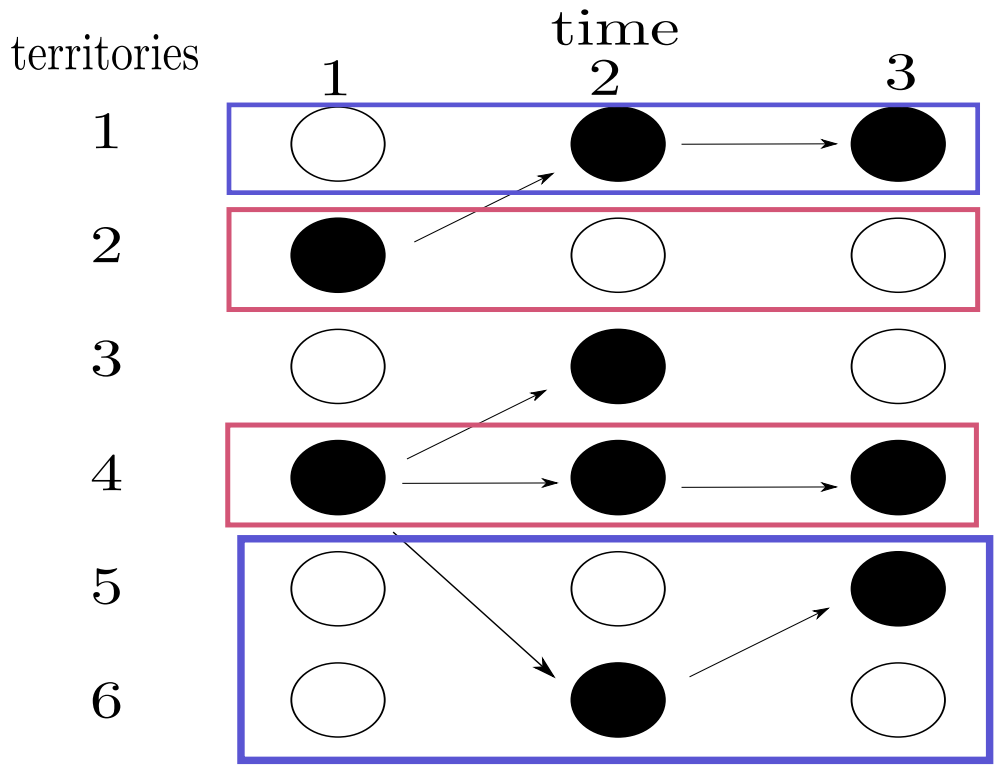

Figure: Other buying decisions

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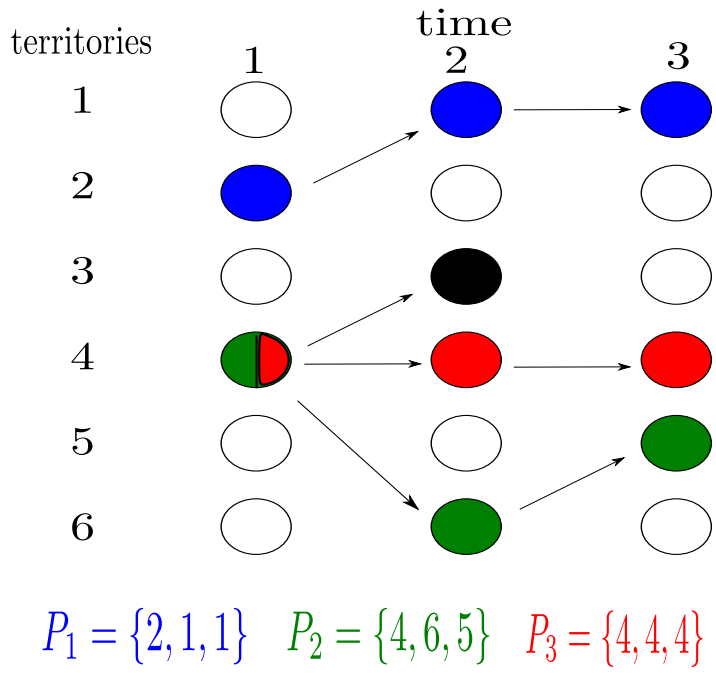

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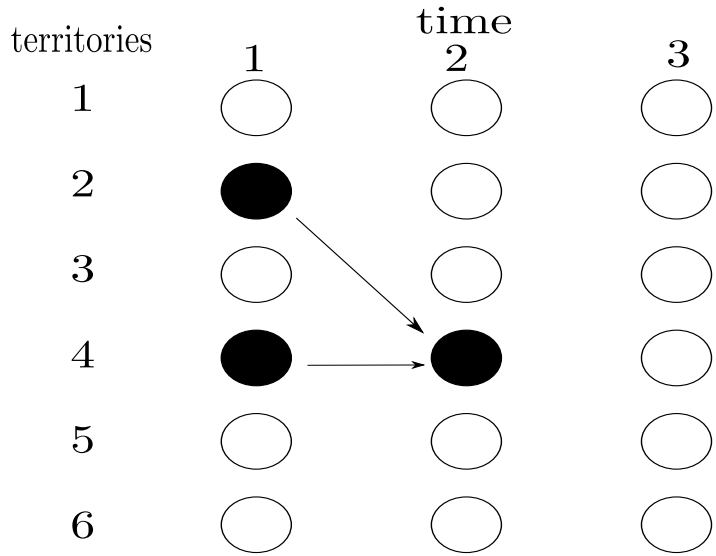

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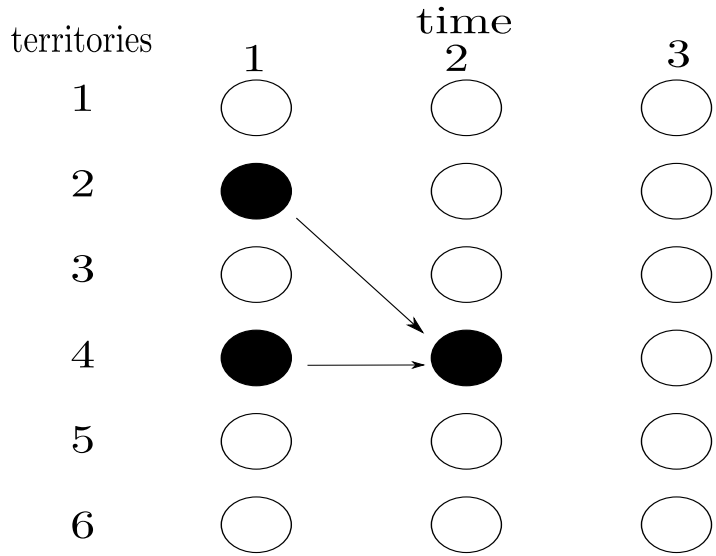

Figure: Two colonization events on single parcel

- Realistic – territories are large enough to support multiple colonizations
- Unrealistic – activations probabilities remain constant

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- Formulate dynamic buying model
  - Should produce best results

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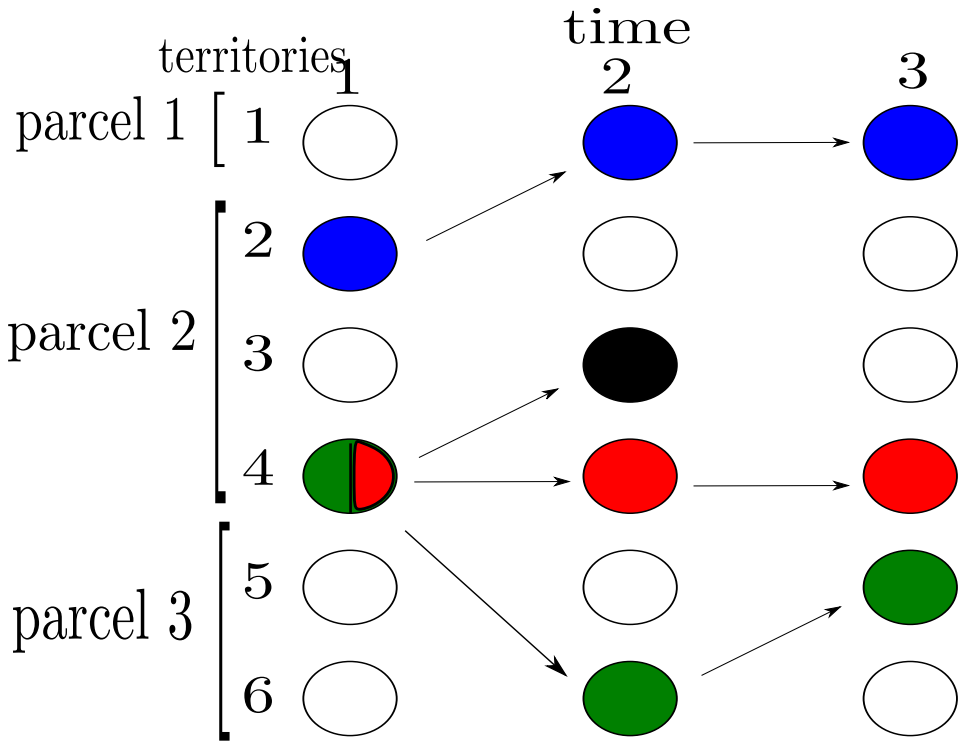
P_j : the set of paths of length T for deterministic cascade $j = 1, ..., N$

d_i : For each $p_{l,j} \in P_j$, let d_i be the number of time periods we are in parcel i.



$$P_1 = \{2, 1, 1\} \implies d_1 = 2, d_2 = 1, d_3 = 0$$

Figure: Encoding d's

c_i : Cost of parcel of i

$\alpha_{l,j}$: For each $p_{l,j}$ we associate an $\alpha_{l,j}$ that represents whether we have bought all the territories in path l

B : Our Budget

M : Large number

Reformulating Original Model Contd.

$$Z_t^* = \max_x \sum_l \sum_j \alpha_{l,j} \tag{1}$$

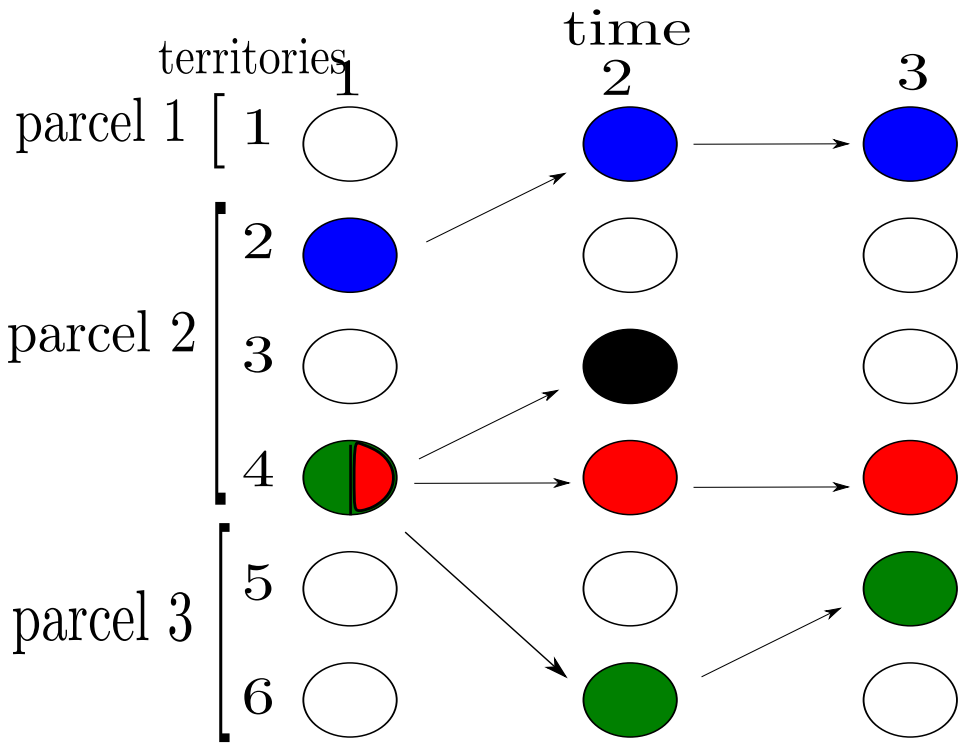
$$s.t.$$

$$\sum_i d_i x_i + M(1 - \alpha_{l,j}) \geq T \ \ \forall \ \ p_{l,j} \tag{2}$$

$$\sum_i c_i x_i \leq B \tag{3}$$

$$x_i, \alpha_{l,j} \in \{0, 1\} \tag{4}$$

Formulating Constraint



$$2x_1 + x_2 + M(1 - \alpha_1) \geq 3$$

Figure: Cascade Network

The Robust Single Stage Problem Formulation

$$\max_{x} z \tag{5}$$

$$s.t.$$

$$\sum_{i} d_i x_i + M(1 - \alpha_{l,j}) \geq T \ \ \forall \ \ p_{l,j} \tag{6}$$

$$\sum_{i} c_i x_i \leq B \tag{7}$$

$$z \leq \sum_{l} \sum_{j} \alpha_{l,j} \ \ \forall \ \ j \tag{8}$$

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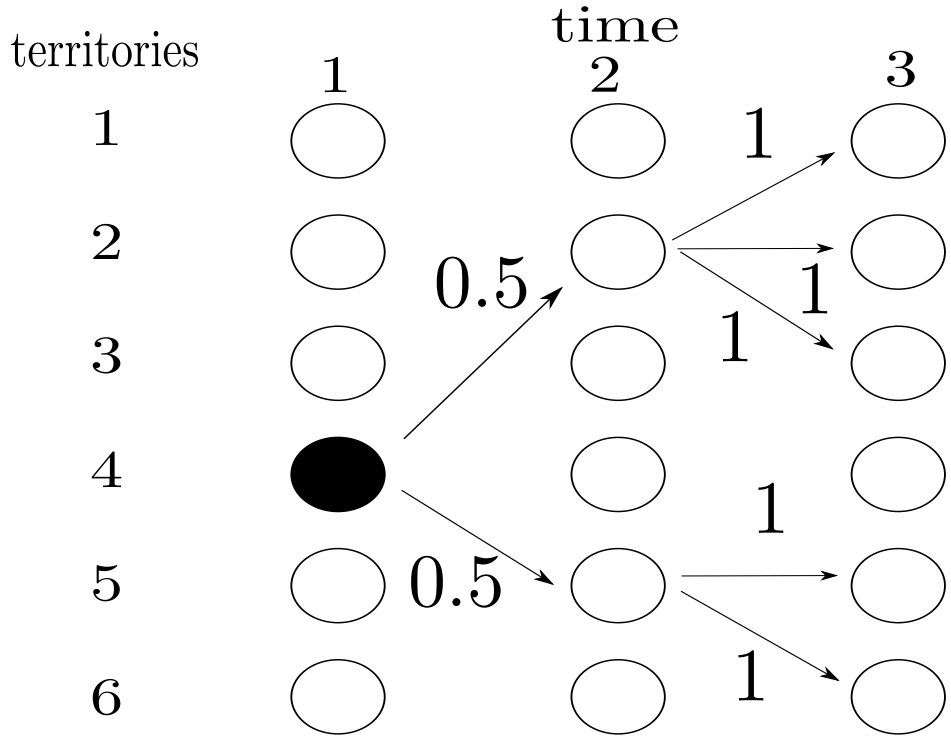

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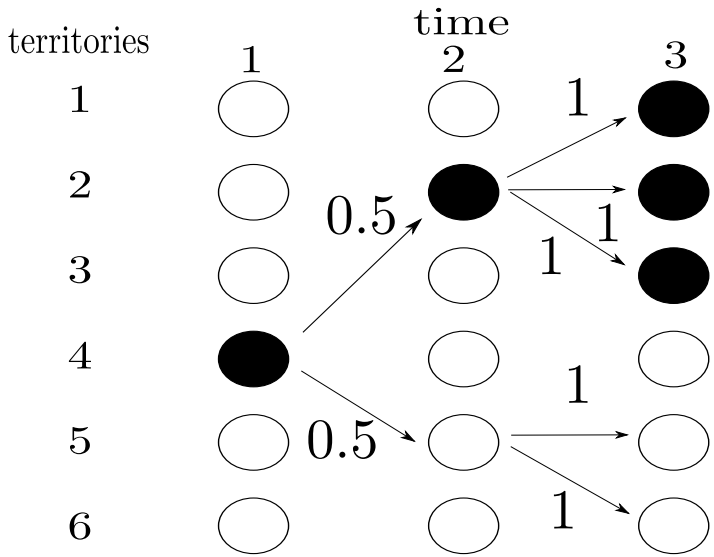

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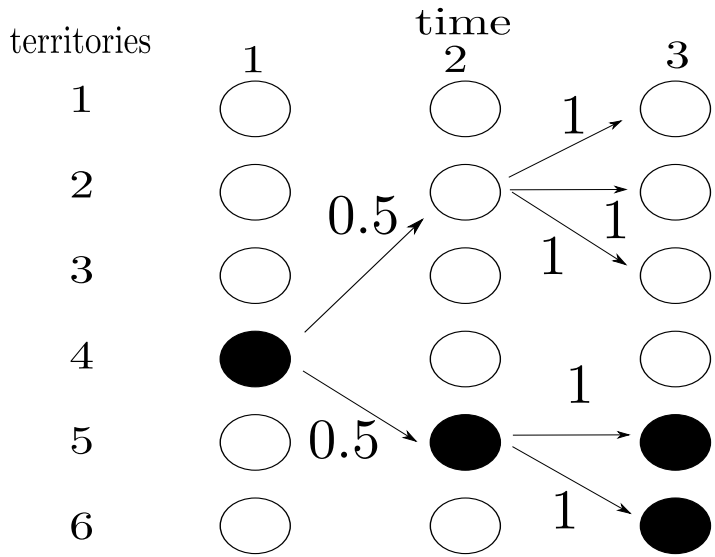

Figure: Deterministic Cascade 2

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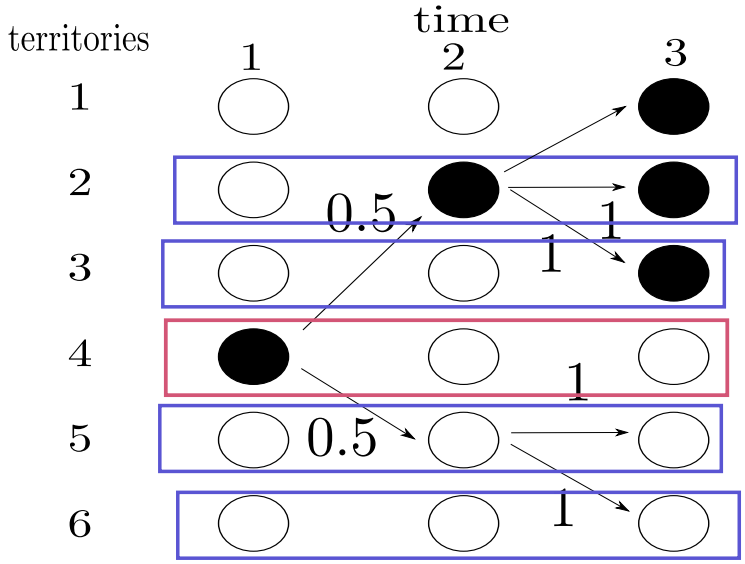
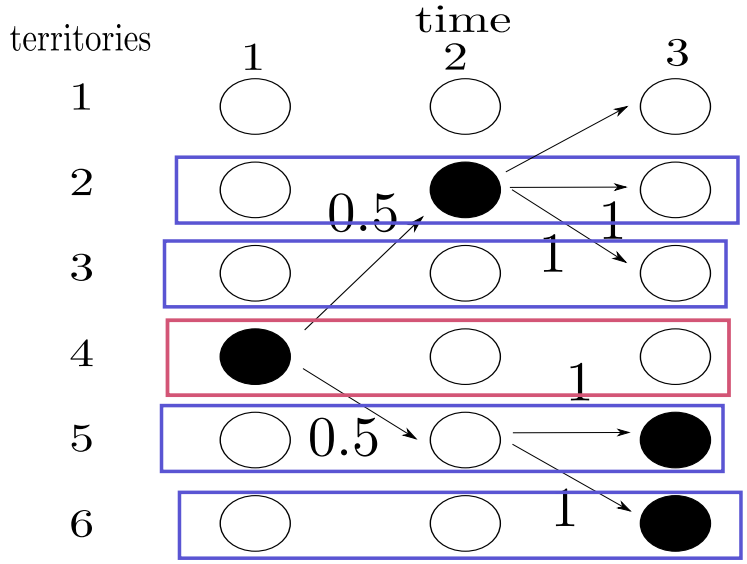


Figure: Buy only at beginning, objective function=2



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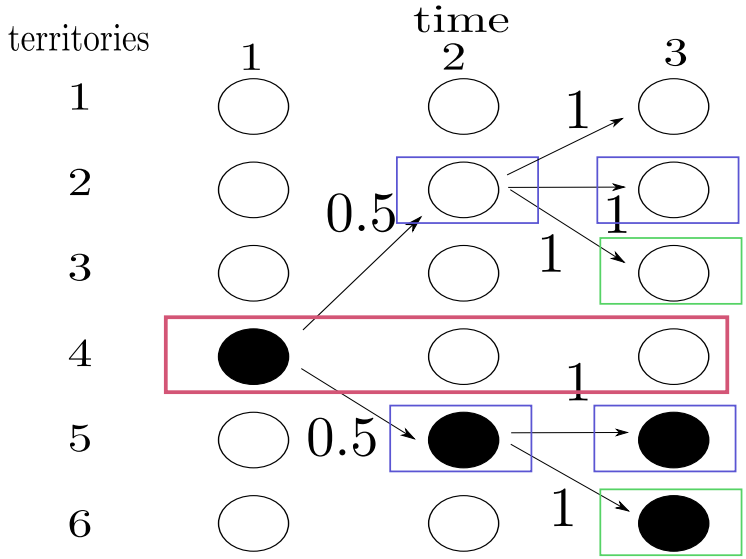

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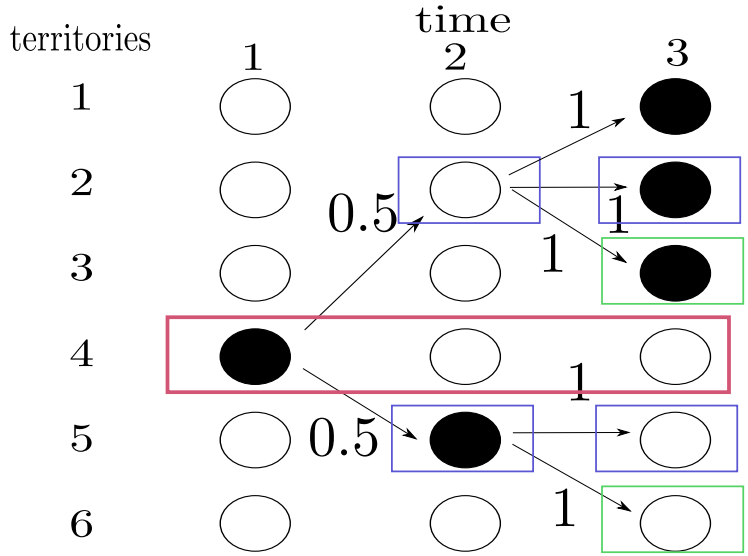

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5. Repeat steps 1-4

Two Stage Buying Model

$$Z^*_{t+1} = \max_x \sum_l \sum_j T\alpha_{l,j} + \sum_i x_{i,t+1} - \sum_i x_{i,t} \tag{10}$$

$$s.t.$$

$$\sum_i d_i(x_{i,t} + x_{i,t+1}) + M(1 - \alpha_{l,j}) \geq T \ \ \forall \ \ p_{l,j} \tag{11}$$

$$x_{i,t} + x_{i,t+1} \leq 1 \ \forall \ i \tag{12}$$

$$\sum_i c_i x_i \leq B \tag{13}$$

$$x_i, \alpha_{l,j} \in \{0, 1\} \tag{14}$$

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