Warm Up Question

- City 1
- City 2 (40 M Demand)
- City 3

Decision variables - How much do we want to ship from each rescvoir to each city?

x_{ij} : Gallons of water shipped from resevoir i to city j

x_{12} : Gallons shipped from rescvoir 1 to city 2.

Objective: Minimize cost

$$Cost = (7x_{11} + 8x_{12} + 10x_{13}) + (9x_{21} + 7x_{22} + 8x_{23}) + \text{Penalty}$$

cost to ship from Res 1 — Cost to ship from Res. 2

Constraints:

1. Can't ship more than 60M gallons from each resevoir

$$x_{11} + x_{12} + x_{13} \leq 60$$

$$x_{21} + x_{22} + x_{23} \leq 60$$

2. Demand for each city must be met

$$X_{11} + X_{21} = 40$$

$$X_{12} + X_{22} = 40$$

$$X_{13} + X_{23} = 40$$

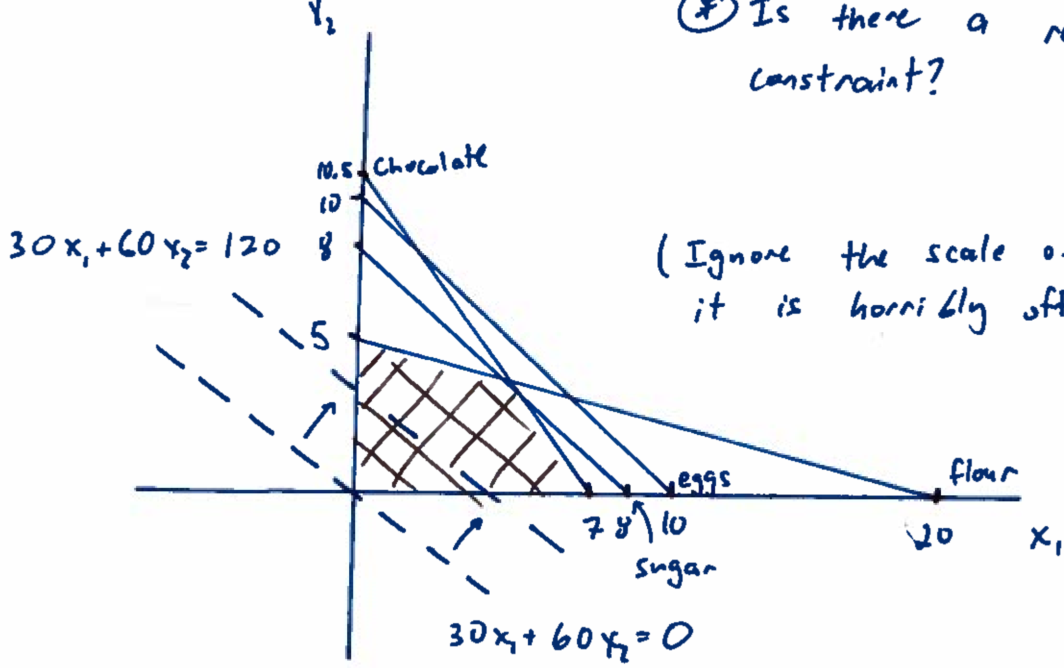
3. Can't ship negative amounts of water

$$X_{ij} \geq 0$$

Recall our bake sale LP

$$\begin{aligned} \max \quad & 30x_1 + 60x_2 \\ \text{s.t.} \quad & x_1 + 4x_2 \le 20 \\ & 9x_1 + 6x_2 \le 63 \\ & x_1 + x_2 \le 8 \\ & x_1 + x_2 \le 10 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Solve Graphically



(#) Is there a redundant constraint?

(Ignore the scale on the graph, it is horribly off)

How many solutions can an LP have? what about optimal solutions?

- exactly one optimal sol'n
- Zillions of optimal sol'n ($45x_1 + 30x_2$)
- no optimal sol'n
  - infeasible → $\boxed{\begin{aligned} \max \quad & x \\ & x \ge 5 \\ & x \le 3 \end{aligned}}$
  - unbounded → $\boxed{\begin{aligned} \max \quad & x \\ & x \ge 0 \end{aligned}}$

Any L.P. with feasible sol'ns and a bounded feasible region must possess at least one optimal solution.
Moreover, at least one optimal solution lies at a corner

Intuition: In an LP, the feasible region is a flat polygonal ramp and we walk uphill till we hit the highest point.

Simplex Method

1. Start at a feasible corner (i.e. not a corner outside feas. region)
2. Check if current point is optimal by examining rate of improvement along adjacent edges
3. Pick a direction to move along a feasible edge s.t. the obj. fcn. will improve
4. Move
5. Stop moving when you hit a boundary
6. Repeat 1-5