Parket Sisters

Linear Programming Formulation:

$$\text{MAX: } 3.0\,X_1 + 3.0\,X_2 + 5.0\,X_3$$

Subject to:

$$\text{Plastic: } 1.2\,X_1 + 1.7\,X_2 + 1.2\,X_3 \le 1{,}000$$

$$\text{Chrome: } 0.8\,X_1 + 0\,X_2 + 2.3\,X_3 \le 1{,}200$$

$$\text{Stainless Steel: } 2.0\,X_1 + 3.0\,X_2 + 4.5\,X_3 \le 2{,}000$$

$$X_1, X_2, X_3 \ge 0$$

X_1 = number of ballpoint pens

X_2 = number of mechanical pencils

X_3 = number of fountain pens

The Excel spreadsheet setup:

	A	B	C	D	E	F	G	H	I
1			Parket Sisters						
2									
3	Decision Variables:		Ballpoint pens (X1)	Mechanical pencils (X2)	Fountain pens (X3)				
4			0	0	0				
5						Profit			
6	Objective Function		3.0	3.0	5.0	0			
7									
8	Constraints:					LHS	Type	RHS	units
9	Plastic		1.2	1.7	1.2	0	<=	1000	ounce
10	Chrome		0.8	0	2.3	0	<=	1200	ounce
11	Stainless Steel		2.0	3.0	4.5	0	<=	2000	ounce

	A	B	C	D	E	F	G	H	I
1			Parket Sis						
2									
3	Decision Variables:		Ballpoint pens (X1)	Mechanical pencils (X2)	Fountain pens (X3)				
4			0	0	0				
5						Profit			
6	Objective Function		3	3	5	=SUMPRODUCT(C6:E6,C4:E4)			
7									
8	Constraints:					LHS	Type	RHS	units
9	Plastic		1.2	1.7	1.2	=SUMPRODUCT(C9:E9,$C$4:$E$4)	<=	1000	ounce
10	Chrome		0.8	0	2.3	=SUMPRODUCT(C10:E10,$C$4:$E$4)	<=	1200	ounce
11	Stainless Steel		2	3	4.5	=SUMPRODUCT(C11:E11,$C$4:$E$4)	<=	2000	ounce

The Answer report:
Microsoft Excel 9.0 Answer Report
Worksheet: [Parketsister.xls]Sheet1

Target Cell (Max)

Cell	Name	Original Value	Final Value
$F$6	Objective Function Profit	0	2766.666667

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$4	Ballpoint pens (X1)	0	700
$D$4	Mechanical pencils (X2)	0	0
$E$4	Fountain pens (X3)	0	133.3333333

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$9	Plastic LHS	1000	$F$9<=$H$9	Binding	0
$F$10	Chrome LHS	866.6666667	$F$10<=$H$10	Not Binding	333.3333333
$F$11	Stainless Steel LHS	2000	$F$11<=$H$11	Binding	0

The sensitivity report:
Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Parketsister.xls]Sheet1

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$4	Ballpoint pens (X1)	700	0	3	2	0.777777778
$D$4	Mechanical pencils (X2)	0	-1.383333333	3	1.383333333	1E+30
$E$4	Fountain pens (X3)	133.3333333	0	5	1.75	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$9	Plastic LHS	1000	1.166666667	1000	200	466.6666667
$F$10	Chrome LHS	866.6666667	0	1200	1E+30	333.3333333
$F$11	Stainless Steel LHS	2000	0.8	2000	555.5555556	333.3333333

1) The optimal weekly product mix is: 700 ballpoints, 0 pencils, and 133.33 fountain pens. We could round the fountain pens off to 133, but we shall not, in order to avoid rounding problems. The profit is $2,766.67.

2)

3) Yes, the answer is unique, because there is no zero in the allowable increase and allowable decrease of adjustable cells. To tell whether an LP problem has multiple optimal solutions, we look at the "Variable Cells" (or "Adjustable Cells") part of the sensitivity report, that is, the top part of the sensitivity report.

If	**then**
There is no zero in the "allowable increase" and "decrease column" for all decision variables	The optimal solution is unique
There exist zero(s) in in the "allowable increase" and "decrease column" for at least one decision variable	There are multiple optimal solutions. Solver simply returns one of the optimal solutions.

4) The marginal values for one more unit of each of the resources are: $1.17 for plastic, 0 for chrome, and $0.80 for stainless steel. These are the shadow prices for our three resources.

5) Because 500 is less than the maximum allowable increase of 555.56 ounces, the shadow price for stainless steel can be used to assess the value of the 500 ounces of stainless steel. Because the 80 cent shadow price for an additional ounce of stainless steel is more than the 60 cents the distributor is charging Parket (above which they ordinarily charge when the model was formulated), it pays to buy the 500 ounces they are willing to sell. Parket will "be ahead" 20 cents for each ounce they buy.

6) Because we are buying the 500 ounces, each ounce bought will increase our profit by 20 cents. Thus the 500 ounces will increase our profit by 500*$0.20 = $100. Our new profit is then $2766.67 + $100 = $2866.67. Once we change the RHS of a constraint, we generally cannot tell what the new product mix will be. To get that, we must solve the following LP problem:

$$\text{MAX: } 3.0\,X_1 + 3.0\,X_2 + 5.0\,X_3 - 500*0.6$$
$$\text{Subject to:}$$
$$\text{Plastic: } 1.2\,X_1 + 1.7\,X_2 + 1.2\,X_3 \le 1{,}000$$
$$\text{Chrome: } 0.8\,X_1 + 0\,X_2 + 2.3\,X_3 \le 1{,}200$$
$$\text{Stainless Steel: } 2.0\,X_1 + 3.0\,X_2 + 4.5\,X_3 \le 2{,}500$$

7) Five hundred additional ounces of plastic is above the allowable increase of 200 ounces. That is, we are guaranteed that the shadow prices given will remain the same at 1.167/ounce so long as we have no more than 1200 (=1000 + 200) ounces of plastic. Above that value, the shadow prices are different than those given. To evaluate the marginal value of the 300 ounces on top of 200 ounces, we need to reformulate the LP by changing the RHS value of the plastic constraint to 1200, and solve the new problem. Obtain the shadow price and allowable increase of plastic in

the new sensitivity report. The new shadow price tells you the marginal profit contribution of additional ounces of plastic in the new allowable increase range. If the new shadow price is greater than $1.00, then Parket sisters should accept at least 200 +new allowable increase (ounces) of plastic, and you can repeat the LP reformulation-resolve process until shadow price drops below $1.00.

An alternative approach is provided in the answers to question 8.

8) We can buy as many as 200 additional ounces of plastic before we "run out of information" from the computer output. Parket ordinarily pays $5.00 per ounce, and now the distributor is charging $6.00 per ounce, or $1.00 more than Parket paid. Still, the shadow price for plastic is $1.17 (or $1.1667 to be exact) which is more than the $1.00 the distributor is "overcharging." Parket will still be ahead about 17 cents (or 16.6667 cents) for each ounce they buy. Buying the 200 ounces will increase their profit by (200)($0.17) = $34 (or $33.33 to be exact). The new profit will thus be $2766.67 + $33.33 = $2800.00.

To find the optimal number of lots of additional plastic to purchase, we need to reformulate LP by introducing a new decision variable y = # of additional lots of plastic to purchase. The new objective function becomes

MAX 3x1 +3x2 +5x3 -100 y

We also need to change the *plastic constraint* to

$$1.2\,X1 + 1.7\,X2 + 1.2\,X3 \le 1{,}000+100y$$

Other constraints remain the same. Solving the new LP yields x1=1000, x2=x3=0, y=2. That is, it is optimal for us to purchase only 2 lots of plastic.

9) Because 300 ounces of plastic is within the allowable decrease of plastic is 466.67, we can use the shadow price of plastic to assess the impact of reducing 300 ounces of plastic. Selling an ounce of plastic reduces Parket's profit by $1.17 (or $1.1667 to be exact), yet the company is willing to pay $6.50 per ounce. This represents a yield of $1.50 above what Parket ordinarily pays (Parket pays $5.00 per ounce, if you recall). Thus Parket will be ahead by ($1.50 - $1.1667)*300 = $100 if they sell the plastic. The new profit will be $2766.67 + $100 = $2866.67. Once again, we cannot tell what the new product mix will be unless we reformulate and resolve the problem.

The reformulation requires two changes:

Change objective to: MAX $3.0\,X_1 + 3.0\,X_2 + 5.0\,X_3 + 1.50*300$

Change plastic constraint to: $1.2\,X_1 + 1.7\,X_2 + 1.2\,X_3 \le 700$

Other parts of the LP do not change.

Solve the new problem, compare the new optimal profit with the old optimal profit to decide whether to accept this offer.

10) Parket had ordered 1200 ounces of chrome, yet the optimal solution uses only 866.67 ounces. There are 333.33 ounces left over. That is why the shadow price for chrome is zero. If only 1000 ounces are delivered, Parket will still have 133.33 ounces of chrome left over. Thus the new profit will not change and remain at $2766.67. We *do not* have to solve the problem again here to get the new product mix. The product mix stays the same as: 700 ballpoints, zero pencils, and 133.33 fountain pens. The only difference is that the slack for chrome is now 133.33, instead of the original 333.33.

11) Price out the new design. First, producing a unit of this new design will need 1.1 ounce of plastic, 2.0 ounce of chrome, and 2.0 ounce of stainless steel, and the shadow prices for these materials are: $1.17 for plastic, $0 for chrome, $0.80 for stainless steel. Since 1.1/466.67 +2/333.33 + 2/333.33 <100%, we can use those shadow prices to estimate the impact of taking away of those resources from current production by producing one unit of the new design. It is Marginal profit – marginal cost = $3 - [(1.1)($1.17) + (2)($0) + (2)($0.80)] = $0.12, approximately. Because this value is positive, Parket should give the go-ahead for this new design.

12) If the profit on ballpoints were to decrease to $2.50, down from the original assumed $3.00, this $0.50 decrease is still less than the allowable decrease of $0.78 for ballpoints. The optimal solution stays the same, but the total profit now drops by ($3.00 - $2.50)(700 ballpoints) = $350. The new profit is $2416.67.

13) Price out the new felt tip pen. First, producing a unit of the felt tip pen will need 1.8 ounces of plastic, 0.5 ounces of chrome, and 1.3 ounces of stainless steel. Since 1.8/466.67 + 0.5/333.33 +0.8/333.33 <100%, we can use their shadow prices to estimate the impact of producing one unit of the felt tip pen. It is Marginal profit – Marginal cost = Marginal profit - [(1.8)($1.167) + (0.5)($0) + (1.3)($0.80)] = Marginal profit -3.14. If we are to produce the felt tip then this amount should be nonnegative. Thus, Profit -3.14≥ 0, or Profit≥ $3.14. At exactly $3.14, we have alternative optima and would be indifferent between producing or not producing the felt tip pens.

14) It should be at least $4.38 ($3.00 plus the reduced cost of $1.38).

15) Currently it does not pay to make pencils; the profit of $3.00 is not high enough. 20 mechanical pencils consume 1.7*20=34 ounce plastic, and 3*20=60 ounce stainless steel. Because 34/466.67+60/333.33< 100%, we can use reduce cost to calculate the impact of producing 20 mechanical pencils (recall the reduced cost of mechanical pencil is calculated using shadow prices). That is, if Parket insists on making 20 pencils per week, their profit would be down by (20)($1.383) = $27.66. That is, they would have a total profit of $2766.67 - $27.66 = $2739.

16) At $6.75, or $1.75 more than the assumed $5.00, we are just at the limit of the allowable increase for the profit of fountain pens. We have alternative optima at this point. Still, the optimal solution stays the same. The new profit is $2766.67 + ($1.75)(133.33 fountain pens) = $3000.