Lecture 5

Sensitivity Analysis Review

Shadow Prices: we had two interpretations

1). Marginal gain in the objective per unit increase of the corresponding right hand side value
   - Tells us how much we are willing to pay for a unit increase in RHS

2). At market equilibrium, the price at which we could a unit of the corresponding resource in an outside market
   - whenever you make one unit of product j, it uses up a_{ij} units of resource i, which is valued at y_i on the market
   $\Rightarrow \sum_i a_{ij} y_i$ is the opportunity cost of producing each unit of product j

Reduced Cost of a product:

1). Difference between its marginal contribution to the objective function and the opportunity cost of its resources

$$c_i - \sum_i a_{ij} y_i \begin{cases} \leq 0 \text{ for products we don't produce} \\ = 0 \text{ for products we produce} \end{cases}$$

   - Tells us how much we have to increase the revenue of a product by in order to produce it

2). At a corner point, it's the slope of the objective "ramp in directions that we can move.

Modified Bake Sale Example:

$$\max \quad 100x_1 + 60x_2$$

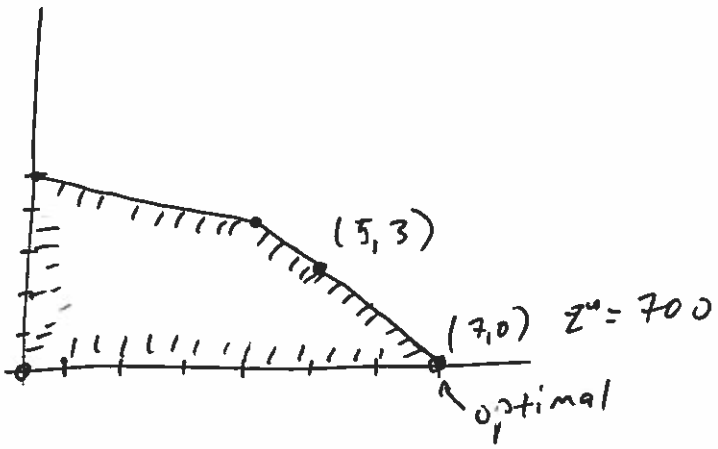
s.t.

$$x_1 + 4x_2 \leq 20 \quad (a_{1,2})$$

$$9x_1 + 6x_2 \leq 63 \quad (a_{2,1})$$

$$1x_1 + x_2 \leq 8$$

$$1x_1 + 1x_2 \leq 10 \quad (a_{2,1},\ a_{4,2})$$



Reduced cost of $x_1 = 0$

Reduced cost of $x_2 = 60 - [4(10) + 6(11.11) + 1(10) + 1(10)]$

$= -6.66$

Consider moving from (7,0) to (5,3):

- Our objective goes from 700 to 680,
  a decrease of $20 = 3(6.67)$!