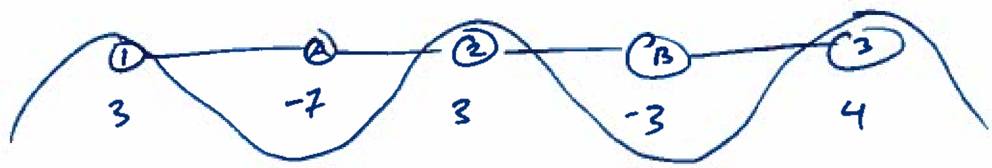
Lecture 3

Transportation Problems



Let x_{ij} = # of truckloads to move from highpoint i to lowpoint j

$i = \{1, 2, 3\}$ $\quad j = \{A, B\}$

Objective

$$\min \; 10x_{1A} + 29x_{1B} + 15x_{2A} + 4x_{2B} + 24x_{3A} + 5x_{3B}$$

Constraints

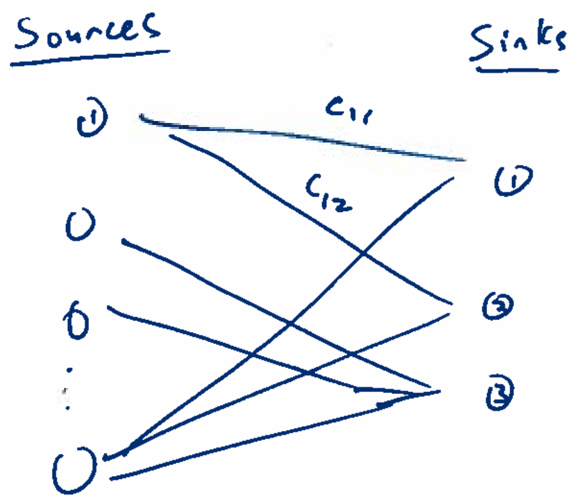
s.t.

$$\begin{aligned}
x_{1A} + x_{1B} &= 3 \\
x_{2A} + x_{2B} &= 3 \\
x_{3A} + x_{3B} &= 4 \\
x_{1A} + x_{2A} + x_{3A} &= 7 \\
x_{1B} + x_{2B} + x_{3B} &= 3
\end{aligned}$$

$$x_{ij} \geq 0$$

The Model

- Want to distribute stuff from any set of sources to any set of sinks/destinations
- Each source has a supply
  - Assume:
    - fixed
    - entire supply must be distributed
- Each sink has a demand
  - Assume:
    - fixed
    - entire demand must be met
- Cost c_{ij} per unit shipped from i to j
  - directly proportional to # of units shipped

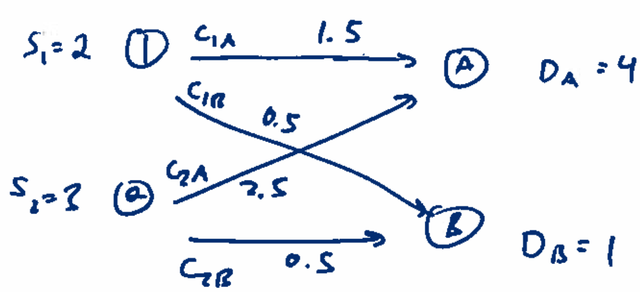


Special property of transportation problems

- ordinary LP's may have non-integer sol'ns
- transportation problems <u>guarantee</u> and integer opt. sol'n when supplies + demands are integer

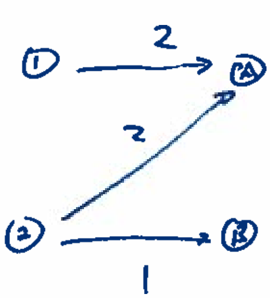
<u>Proof sketch</u>: Consider the following fractional flow

#1

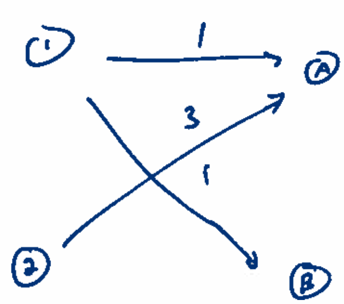


There are two feasible <u>integer</u> flows

#0



#1



$$\left.\begin{aligned} Cost_0 &= 2C_{1A} + 0C_{1B} + 2C_{2A} + 1C_{2B} \\ Cost_1 &= 1.5C_{1A} + 0.5C_{1B} + 2.5C_{2A} + 0.5C_{2B} \\ Cost_2 &= 1C_{1A} + 1C_{1B} + 3C_{2A} + 0C_{2B} \end{aligned}\right]$$

Note that $cost_1 = \frac{Cost_0 + Cost_2}{2}$

So $cost_1 \le \max\{cost_0, cost_2\}$

* Either fractional flow isn't optimal (b/c #0 /#2 optimal)
* or it is optimal, in which case $Cost_0 = Cost_1 = Cost_2$

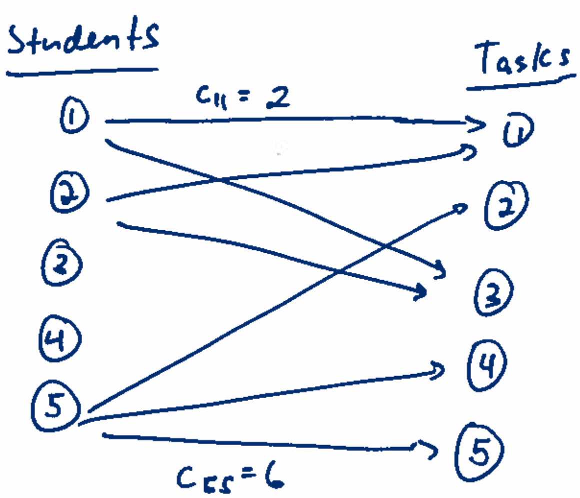
what if supply > demand

$\sum$ supply $= S_i \implies \le S_i$ or create dummy demand node w/ $D = \sum S_i - \sum D_i$ w/ zero cost to ship to

$\sum$ demand $= D_i$

II. Assignment Problems

* This is a transportation problem where all S_i's + D_i's are 1 (students are "shipping" their labor to the tasks)



This example actually doesn't have a feasible assignment!

III. Min Cost Flow Problems

* Formulate as an LP

$$\min \quad 2x_{13} + x_{14} + 3x_{21} + 6x_{23} + 4x_{34} + x_{35}$$

s.t.

$200 + x_{21} = x_{13} + x_{14}$ (Flow balance at node 1)

$500 = x_{21} + x_{23}$ (node 2)

$x_{13} + x_{23} = x_{34} + x_{35}$ (node 3)

$x_{14} + x_{34} = 300$ (node 4)

$x_{35} = 400$ (node 5)

$x_{ij} \geq 0$ $\quad x_{14} \leq 100$ $\quad x_{13} \leq 300$

$x_{21} \leq 100$ $\quad x_{35} \leq 400$

- Integrality still holds! (for integer S + D + UB)

IV. Shortest Path Problem

* MCF = min cost flow

Source: A - 1 unit of supply

Sink: F - 1 unit of demand

Do we have to put a capacity on the arcs?