

**SCREENSHOT 6-11B****Excel Layout and Solver Entries for Thermolock—NLP Solution 2**

**Thermolock (NLP Solution #2)**

	R	O
Number of pounds	10.00	10.00

  

	R	O	R <sup>2</sup>	O <sup>2</sup>	R <sup>3</sup>
Variable terms	10.00	10.00	100.00	100.00	1000.00
Cost	\$5	\$7			

  

Constraints:							
Hardness	3	4	0.25	0.3		125.00	>= 125
Tensile strength	13				1	1130.00	>= 80
Elasticity	0.7	1				17.00	>= 17
						LHS	Sign RHS

  

**Solver Parameters**

Set Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Variable Cells:

Subject to the Constraints:

GRG procedure used to solve NLP models.

Only cells B5 and C5 are the changing cells.

Optimal solution if initial values of R and O are set to five each.

GRG procedure used to solve NLP models.



File: 6-11.xls,  
sheet: 6-11B

Here again, we ask whether this is a local or global optimal solution. To verify this, let us solve the model again, using different starting values for R and O. When we start with values of 5 each for R and O, the final result obtained is shown in Screenshot 6-11B. The solution now is to use 10 pounds each of rubber and oil, at a total cost of \$120. Because this cost is higher than the \$119.33 cost in the earlier solution (Screenshot 6-11A), it is clear that this solution is a local optimal solution.

Are there any other solutions with a cost even lower than \$119.33? We urge you to use the Excel file 6-11.xls on the CD-ROM that accompanies this textbook and experiment with other starting values. If the only solutions found are the two shown in Screenshots 6-11A and 6-11B, the solution shown in Screenshot 6-11A could, in fact, be the global optimal solution.

## DISCUSSION QUESTIONS AND PROBLEMS

### Discussion Questions

- 6-1 Compare the similarities and differences of LP and GP.
- 6-2 Provide your own examples of five applications of IP.
- 6-3 What is the difference between pure and mixed IP models? Which do you think is most common, and why?
- 6-4 What is meant by *satisficing*, and why is the term often used in conjunction with GP?
- 6-5 What are deviation variables? How do they differ from decision variables in traditional LP problems?
- 6-6 If you were the president of the college you are attending and were employing GP to assist in decision making, what might your goals be? What kinds of constraints would you include in your model?
- 6-7 What does it mean to rank goals in GP? How does this affect the problem's solution?

- 6-8 Provide your own examples of problems where (a) the objective is nonlinear and (b) one or more constraints are nonlinear.
- 6-9 Explain in your own words why IP problems are more difficult to solve than LP problems.
- 6-10 Explain the difference between assigning weights to goals and ranking goals.
- 6-11 What does the term *quadratic programming* mean?
- 6-12 Which of the following are NLP models, and why? Are any of these quadratic programming models?

(a) Maximize profit =  $3X_1 + 5X_2 + 99X_3$

subject to the constraints

$$X_1 \geq 10$$

$$X_2 \leq 5$$

$$X_3 \geq 18$$

(b) Minimize cost =  $25X_1 + 30X_2 + 8X_1X_2$

subject to the constraints

$$X_1 \geq 8$$

$$X_1 + X_2 \geq 12$$

$$X_1 - X_2 = 11$$

(c) Maximize profit =  $3X_1 + 4X_2$

subject to the constraints

$$X_1^2 - 5X_2 \geq 8$$

$$3X_1 + 4X_2 \geq 12$$

(d) Minimize cost =  $18X_1 + 5X_2 + X_2^2$

subject to the constraints

$$4X_1 - 3X_2 \geq 8$$

$$X_1 + X_2 \geq 18$$

### Problems

- 6-13 Student Enterprises sells wall posters in two sizes: large and small. The profit from the sale of each large poster is \$3; each small poster earns \$2. Up to three large posters and five small posters can be sold each week. The firm consists of just one art student, Jan Meising, at the University of Kentucky. Because of her class schedule, Jan can spend only a maximum of 10 hours each week on posters. Each large poster requires 2 hours of work, and each small one needs 1 hour. Use Excel to find the integer solution that will maximize the firm's profit.
- 6-14 An airline owns an aging fleet of jet airplanes. It is considering a major purchase of up to 17 new model 7A7 and 7B7 jets. The decision must take into account numerous cost and capability factors, including the following: (1) The airline can finance up to \$400 million in purchases; (2) each 7A7 jet will cost \$35 million, and each 7B7 jet will cost \$22 million; (3) at least one-third of the planes purchased should

be the longer-range 7A7; (4) the annual maintenance budget is to be no more than \$8 million; (5) the annual maintenance cost per 7A7 is estimated to be \$800,000, and it is \$500,000 for each 7B7 purchased; and (6) each 7A7 can carry 125,000 passengers per year, and each 7B7 can fly 81,000 passengers annually. Formulate this as an IP problem to maximize the annual passenger-carrying capability. Solve using Excel.

- 6-15 Marketing Concepts is a telemarketing company that needs the following number of employees on the phones during the upcoming week: Monday 23, Tuesday 16, Wednesday 21, Thursday 17, Friday 20, Saturday 12, and Sunday 15. Each employee is entitled to 2 consecutive days off per week. How many telemarketers need to be scheduled each day of the week to begin their five-day work week? The objective is to minimize the total number of employees needed to fulfill the daily requirements.

(a) Solve as an IP model.

(b) Additional information is now available for Marketing Concepts. Daily pay from Monday through Friday is \$90, pay for Saturday is \$110, and Sunday workers earn \$125. In addition, up to four people can be hired who will work Friday, Saturday, and Sunday. Their pay for this three-day week is \$250. The new objective is to minimize total weekly labor costs. Revise the IP model and solve it.

- 6-16 The Beacon Company is a large manufacturer of automotive supplies. The company has decided to spend \$35 million next year to expand its manufacturing and warehouse facilities. Each warehouse will cost \$3.5 million and will contribute \$17,000 per month toward profitability. Each plant will cost \$5.5 million and will contribute \$36,000 per month toward profitability. Management does not want to build more than seven warehouses or four plants next year. How many plants and warehouses should be constructed to maximize profitability?

- 6-17 Formulate and solve an IP model that will decide how to make change for \$22.98, based on the availability of coins and bills shown in the following table:

DENOMINATION	AVAILABILITY
\$10 bill	1
\$5 bill	4
\$1 bill	1
Quarter (\$0.25)	8
Dime (\$0.10)	1
Nickel (\$0.05)	5
Penny (\$0.01)	6

The objective should be to minimize the total number of bills and coins used to make the change.

- 6-18 Stockbroker Susan Shader has made the following recommendations to her client:

INVESTMENT	COST (IN THOUSANDS)	EXPECTED RETURN (IN THOUSANDS)
Andover municipal bonds	\$ 500	\$ 50
Hamilton city bonds	\$1,000	\$100
East Power & Light Co.	\$ 350	\$ 30
Nebraska Electric Service	\$ 490	\$ 45
Southern Gas and Electric	\$ 700	\$ 65
Manuel Products Co.	\$ 270	\$ 20
Builders Paint Co.	\$ 800	\$ 90
Rest Easy Hotels Co.	\$ 400	\$ 35

The client agrees to this list but provides several conditions: (1) No more than \$3,000,000 can be invested, (2) the money is to be spread among at least five investments, (3) no more than one type of bond can be purchased, (4) at least two utility stocks must be purchased, and (5) at least two regular stocks must be purchased. Formulate this as a 0-1 IP problem for Ms. Shader to maximize expected return. Solve using Excel.

- 6-19 Peters Financial needs to develop an investment portfolio for Mrs. Charles from the following list of possible investments:

INVESTMENT	COST	EXPECTED RETURN
A	\$ 5,000	\$ 500
B	\$ 8,000	\$ 640
C	\$ 3,500	\$ 390
D	\$10,000	\$ 700
E	\$ 8,500	\$ 750
F	\$12,000	\$1,000
G	\$ 4,000	\$ 300

Mrs. Charles has a total of \$30,000 to invest. The following conditions must be met: (1) If investment F is chosen, then investment G must also be part of the portfolio, (2) at least four investments should be chosen, and (3) of investment A and B, exactly one must be included. What stocks should be included in Mrs. Charles's portfolio?

- 6-20 A truck with the capacity to load 2,000 cubic feet of cargo is available to transport items selected from the following table:

ITEM	VALUE	VOLUME (CUBIC FEET)
A	\$1,900	700
B	\$1,500	600
C	\$1,200	450
D	\$ 800	400
E	\$1,700	650
F	\$1,000	350
G	\$1,400	600

If selected, an item must be shipped in its entirety (i.e., partial shipments are not allowed). Of items B, C, and D, at least two items must be selected. If item B is selected, then item G cannot be selected. Which items should be selected to maximize the value of the shipment?

- 6-21 The Osaka Bombers have \$13 million available to sign free agent pitchers for the next season. The following table provides the relevant information for eight pitchers who are available for signing, such as whether each throws right or left handed, whether each is a starter or reliever, the cost in millions of dollars to sign each, and the relative value of each on the market on a scale of 1 to 10 (10 = highest):

PITCHER	THROWS	START/ RELIEF	COST (IN MILLIONS)	VALUE
A	R	S	\$2	4
B	R	S	\$4	5
C	R	S	\$5	6
D	R	R	\$3	5
E	R	R	\$6	8
F	L	S	\$5	5
G	L	S	\$8	7
H	L	R	\$9	8

The Bombers feel the following needs exist for next season: (1) at least two right-handed pitchers, (2) at least one left-handed pitcher, (3) at least two starters, and (4) at least one right-handed reliever. Who should the Bombers try to sign with the objective to maximize total value?

- 6-22 Allied Products has six R&D projects that are potential candidates for selection during the upcoming fiscal year. The table at the top of page 288 provides the expected net present value (NPV) and capital requirements over the next five years for each project.

The table also indicates the planned budget expenditures for the entire R&D program during each of the next five years. Which projects should be selected?

- 6-23 Tiger Airlines needs to decide which cities that it currently operates from will be designated as a hub. The candidate cities are shown in the table on page 288, along with distances between them.

Each hub will serve other cities within a 1,400 mile radius. Tiger would like to identify the minimum number of hubs required to cover all eight cities and also identify the cities that are covered by each hub.

- 6-24 Jefferson County has six communities that need to be served by fire stations. The number of minutes it takes

Table for Problem 6-22

PROJECT	NPV (IN THOUSANDS)	CAPITAL REQUIRED (IN THOUSANDS)				
		YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5
1	\$182	\$ 80	\$25	\$22	\$18	\$10
2	\$167	\$ 95	\$40	\$ 5	\$10	\$35
3	\$114	\$ 58	\$17	\$14	\$12	\$12
4	\$ 98	\$ 32	\$24	\$10	\$ 6	\$ 7
5	\$250	\$115	\$25	\$25	\$10	\$ 0
6	\$130	\$ 48	\$20	\$12	\$32	\$40
R&D Budget		\$225	\$80	\$60	\$50	\$50

Table for Problem 6-23

	ATLANTA	BOSTON	CHICAGO	DALLAS	DENVER	LA	PHILADELPHIA	SEATTLE
Atlanta	—	1,108	717	783	1,406	2,366	778	2,699
Boston		—	996	1,794	1,990	3,017	333	3,105
Chicago			—	937	1,023	2,047	767	2,108
Dallas				—	794	1,450	1,459	2,112
Denver					—	1,026	1,759	1,313
LA						—	2,723	1,141
Philadelphia							—	2,872

to travel between the communities is shown in the following table:

	A	B	C	D	E	F
A	—	10	6	9	10	4
B		—	3	3	4	6
C			—	5	5	3
D				—	6	5
E					—	8

The county would like to establish the minimum number of fire stations so that each community can be responded to in 5 minutes or less. How many stations will be needed, and what communities will each station serve?

- 6-25 Georgia Atlantic Corporation needs to decide on the locations for two new warehouses. The candidate sites are Philadelphia, Tampa, Denver, and Chicago. The following table provides the monthly capacities and the monthly fixed costs for operating warehouses at each potential site:

WAREHOUSE	MONTHLY CAPACITY (UNITS)	MONTHLY FIXED COST
Philadelphia	250	\$1,000
Tampa	260	\$ 800
Chicago	280	\$1,200
Denver	270	\$ 700

The warehouses will need to ship to three marketing areas: North, South, and West. Monthly requirements are 200 units for North, 180 units for South, and 120 units for West. The following table provides the cost to ship one unit between each location and destination:

WAREHOUSE	NORTH	SOUTH	WEST
Philadelphia	\$4	\$ 7	\$ 9
Tampa	\$6	\$ 3	\$11
Chicago	\$5	\$ 6	\$ 5
Denver	\$8	\$10	\$ 2

In addition, the following conditions must be met by the final decision: (1) A warehouse must be opened in

either Philadelphia or Denver, and (2) if a warehouse is opened in Tampa, then one must also be opened in Chicago. Which two sites should be selected for the new warehouses to minimize total fixed and shipping costs?

- 6-26 Furniture Unlimited has the capability to manufacture desks, cabinets, and chairs. In order to manufacture these products, it must rent the appropriate equipment at a weekly cost of \$2,000 for the desks, \$2,500 for the cabinets, and \$1,500 for the chairs. The labor and material requirements for each product are shown in the following table, along with the selling price and variable cost to manufacture:

	LABOR HOURS	LUMBER (SQ. FT.)	SALES PRICE	VARIABLE COST
Desks	3	10	\$110	\$82
Cabinets	4	12	\$135	\$97
Chairs	2	5	\$ 50	\$32

There are 2,500 labor hours and 4,000 square feet of lumber available each week. Determine the product mix that maximizes weekly profit.

- 6-27 PH Publishing needs to decide what textbooks to publish from the following table:

	DEMAND	FIXED COST	VARIABLE COST	SELLING PRICE
Book 1	5,000	\$15,000	\$30	\$52
Book 2	7,000	\$18,000	\$20	\$45
Book 3	9,000	\$12,000	\$19	\$40
Book 4	6,000	\$10,000	\$20	\$34
Book 5	8,000	\$21,000	\$28	\$60

For each book, the maximum demand, fixed cost of publishing, variable cost, and selling price are provided. PH only has the capacity to publish a total of 20,000 books. Which books should be selected, and how many of each should be published?

- 6-28 Geraldine Shawhan is president of Shawhan File Works, a firm that manufactures two types of metal file cabinets. The demand for her two-drawer model is up to 600 cabinets per week; demand for her three-drawer cabinet is limited to 400 per week. Shawhan has a weekly operating capacity of 1,300 hours, with the two-drawer cabinet taking 1 hour to produce and the three-drawer cabinet requiring 2 hours. Each two-drawer model sold yields a \$10 profit, and the profit for the three-drawer model is \$15. Shawhan has listed the following goals, in rank order:

Rank 1: Attain a profit as close to \$11,000 as possible each week.

Rank 2: Avoid underutilization of the firm's production capacity.

Rank 3: Sell as many two- and three-drawer cabinets as the demand indicates.

Set up and solve this problem as a GP problem.

- 6-29 Harris Segal, marketing director for North-Central Power and Light, is about to begin an advertising campaign promoting energy conservation. Each TV ad costs \$5,000, while each newspaper ad costs \$2,000. Segal has set the following goals, in rank order:

Rank 1: The total advertising budget of \$120,000 should not be exceeded.

Rank 2: There should be at least 10 TV ads and at least 20 newspaper ads.

Rank 3: The total number of people reached by the ads should be at least 9 million.

Each television spot reaches approximately 300,000 people. A newspaper advertisement is read by about 150,000 people. Set up and solve Segal's GP problem to find out how many of each type of ad to place.

- 6-30 Consider the staffing situation faced by Marketing Concepts in Problem 6-15. Ignore the salary and part-time information given in part (b) of that problem. Develop a schedule that meets the daily employee requirements and satisfies the following three weighted goals:

Goal 1: Use at most 27 total employees Weight = 50

Goal 2: Minimize the excess employees scheduled on Saturday Weight = 30

Goal 3: Minimize the excess employees scheduled on Sunday Weight = 20

- 6-31 White & Becker Tools (W&B) requires 2,000 electric motors next month for its product line of weed trimmers. Each motor is composed of three components: a coil, a shaft, and housing. W&B has the capability to produce these components or purchase them from an outside vendor. The costs of producing them and purchasing them are shown in the following table:

COMPONENT	PRODUCTION COST PER UNIT	PURCHASE COST PER UNIT
Coil	\$2.50	\$3.90
Shaft	\$1.75	\$2.95
Housing	\$1.25	\$2.00

The components that are produced by W&B must pass through three departments: fabrication, molding, and inspection. The number of hours each component requires in each department and the total number of hours available next month in each department are shown in the following table:

DEPARTMENT	COIL (HRS.)	SHAFT (HRS.)	HOUSING (HRS.)	AVAILABILITY (HRS.)
Fabrication	0.5	0.2	0.6	2,500
Molding	0.4	0.7	0.3	2,500
Inspection	0.2	0.3	0.4	1,600

In order to determine the number of components that will be produced and the number that will be purchased, W&B has set the following goals, in rank order:

Rank 1: The total costs to produce and purchase components next month should not exceed \$13,000.

Rank 2: Idle time in the fabrication department should be minimized.

Rank 3: At least 200 coils should be purchased from the vendor next month.

Determine the number of components produced and purchased next month according to these ranked goals.

- 6-32 Kent County has plans to develop several new recreational facilities that must be completed within the \$3.5 million budget. A survey of county residents has resulted in information about the type of facilities that county residents would like to see built that is described in the following table:

FACILITY	COST PER FACILITY	ACRES PER FACILITY	USAGE IN PEOPLE PER MONTH	ANNUAL MAINTENANCE
Basketball courts	\$300,000	3	700	\$3,000
Baseball fields	\$250,000	5	1,000	\$6,000
Playgrounds	\$ 75,000	2	800	\$3,000
Soccer fields	\$175,000	3	1,200	\$7,000

More specifically, this table provides the cost to construct and maintain each facility, the acres each facility will require, and the average monthly usage of each facility. The county has decided that at least 15 facilities will be built and has set aside 55 acres to be used for construction.

The county has also established the following list of ranked goals:

Rank 1: The county would like to spend the entire budget.

Rank 2: The county would like to build enough facilities so that 15,000 people or more each month can use them.

Rank 3: The county wants to avoid using more than the 55 acres that have been set aside for the project.

Rank 4: The county would like to avoid spending more than \$80,000 per year on maintenance costs for the new facilities.

How many of each type of facility should be constructed?

- 6-33 John Parson earns \$35,000 a year and has \$6,000 to invest in a portfolio. His investment alternatives and their expected returns are shown in the following table:

INVESTMENT	DESCRIPTION	EXPECTED RETURN
A	IRA (retirement)	6.5%
B	Employer's retirement plan	15.8%
C	Deferred income (retirement)	18.0%
D	Unity mutual fund	11.9%
E	Liberty mutual fund	9.5%
F	Money market	4.8%

John's investment goals are as follows and can be ranked according to the weights shown in parenthesis:

Goal 1: (25) Invest all funds available.

Goal 2: (20) Maximize the total annual return in dollars, with a target of \$1,200.

Goal 3: (15) Avoid investing less than 3% of salary in employer's retirement plan.

Goal 4: (15) Avoid investing less than 10% of the total investment in the money market.

Goal 5: (10) Avoid investing more than 25% of the total investment for retirement plans.

Goal 6: (10) Avoid investing less than 50% of the total investment to non-retirement plans.

Goal 7: (5) Avoid investing more than 50% of the total investment in mutual funds.

Which investments should be included in John's portfolio?

- 6-34 We need to make a fruit salad that contains at least 6,500 units of vitamin A and 1,800 units of vitamin C. Data on five available fruits are shown in the following table:

FRUIT TYPE	VITAMIN A (UNITS/LB.)	VITAMIN C (UNITS/LB.)	COST PER LB.	MAX LBS. AVAILABLE
Apple	330	36	\$1.49	No limit
Banana	367	41	\$0.49	11
Grape	453	18	\$1.69	8
Pear	91	18	\$0.99	No limit
Strawberry	122	254	\$2.99	14

It is estimated that at least 24 pounds of fruit salad will be necessary, but no more than 32 pounds. The following goals (in rank order) need to be considered for the mix:

Rank 1: The salad should cost no more than \$40.

Rank 2: At least 5 pounds of bananas should be in the salad.

Rank 3: At least 2 pounds of pears should be in the salad.

- 6-35 Major Bill Bligh, director of the Army War College's new six-month attaché training program, is concerned about how the 20 officers taking the course

spend their precious time while in his charge. Major Bligh recognizes that there are 168 hours per week and thinks that his students have been using them rather inefficiently. Bligh lets

- $X_1$  = number of hours  
of sleep needed per week
- $X_2$  = number of personal hours  
(eating, personal hygiene,  
handling laundry, and so on)
- $X_3$  = number of hours of class and studying
- $X_4$  = number of hours of social time  
off base (dating, sports,  
family visits, and so on)

He thinks that students should study 30 hours a week to have time to absorb material. This is his most important goal. Bligh feels that students need at most 7 hours of sleep per night on average and that this goal is number 2. He believes that goal number 3 is to provide exactly 20 hours per week of social time. Formulate this as a GP problem and solve using Excel.

6-36 Consider the following NLP problem:

$$\text{Maximize } 20X_1 + 40X_2 + 31X_3$$

subject to the constraints

$$\begin{aligned} X_1 + X_2 + X_3 &\leq 15 \\ X_1^2 + X_2^2 &\leq 49 \\ 2X_1 + X_3^3 &\leq 53 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

- (a) Set up and solve the model using Solver. Use a starting value of zero for all decision variables.
- (b) Is the solution obtained a local optimal or global optimal solution? How do you know?

6-37 Consider the following NLP problem:

$$\text{Maximize } 4X_1 + 2X_2 - 3X_3 + 2X_1X_2 + 8X_3^3$$

subject to the constraints

$$\begin{aligned} 2X_1 + 4X_2 + 3X_3 &\geq 29 \\ 3X_1 + X_2 &\geq 14 \\ X_1 + X_2 + X_3 &\leq 10 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

- (a) Set up and solve the model using Solver. Use a starting value of zero for all decision variables.
- (b) Is the solution obtained a local optimal or global optimal solution? How do you know?

6-38 Hinkel Rotary Engine, Ltd., produces four- and six-cylinder models of automobile engines. The firm's profit for each four-cylinder engine sold during its quarterly production cycle is  $\$1,800 - \$50X_1$ ,

where  $X_1$  is the number of four-cylinder engines sold. Hinkel makes  $\$2,400 - \$70X_2$  for each of the larger engines sold, with  $X_2$  equal to the number of six-cylinder engines sold. There are 5,000 hours of production time available during each production cycle. A four-cylinder engine requires 100 hours of production time, whereas six-cylinder engines take 130 hours to manufacture. Formulate this production problem for Hinkel and solve using Excel. Use several different starting values for the decision variables to try to identify a global optimal solution.

6-39 Motorcross of Wisconsin produces two models of snowmobiles, the XJ6 and the XJ8. In any given production-planning week, Motorcross has 40 hours available in its final testing bay. Each XJ6 requires 1 hour to test, and each XJ8 takes 2 hours. The revenue (in \$1,000s) for the firm is nonlinear and is stated as (no. of XJ6s)  $\times$  (4 - 0.1 no. of XJ6s) + (no. of XJ8s)  $\times$  (5 - 0.2 no. of XJ8s). Formulate this problem and solve using Excel. Use several different starting values for the decision variables to try to identify a global optimal solution.

6-40 Susan Jones would like her investment portfolio to be selected from a combination of three stocks—Alpha, Beta, and Gamma. Let variables  $A$ ,  $B$ , and  $G$  denote the percentages of the portfolio devoted to Alpha, Beta, and Gamma, respectively. Susan's objective is to minimize the variance of the portfolio's return, given by the following function:

$$3A^2 + 2B^2 + 2G^2 + 2AB - 1.1AG - 0.7BG$$

The expected returns for Alpha, Beta, and Gamma are 15%, 11%, and 9%, respectively. Susan wants the expected return for the total portfolio to be at least 10%. No individual stock can constitute more than 70% of the portfolio. Formulate this portfolio selection problem and solve using Excel. Use several different starting values for the decision variables to try to identify a global optimal solution.

6-41 Ashworth Industries would like to make a price and production decision on two of its products. Define  $Q_A$  and  $Q_B$  as the quantities of products A and B to produce and  $P_A$  and  $P_B$  as the price for products A and B. The weekly quantities of A and B that are sold are functions of the price, according to the following expressions:

$$\begin{aligned} Q_A &= 5500 - 200P_A \\ Q_B &= 4500 - 225P_B \end{aligned}$$

The variable costs per unit of A and B are \$18 and \$12, respectively. The weekly production capacity for A and B are 275 and 350 units, respectively. Each unit of A requires 1 hour of labor, while each unit of B requires 2 hours. There are 700 hours of labor available each week. What quantities and price of products A and B will maximize weekly profit? (*Hint:* Set up the objective function in terms of profit per unit multiplied by the number of units produced for both products.) Use several different starting values for the decision variables to try to identify a global optimal solution.

## CASE STUDY

### Schank Marketing Research

Schank Marketing Research has just signed contracts to conduct studies for four clients. At present, three project managers are free for assignment to the tasks. Although all are capable of handling each assignment, the times and costs to complete the studies depend on the experience and knowledge of each manager. Using his judgment, John Schank, the president, has been able to establish a cost for each possible assignment. These costs, which are really the salaries each manager would draw on each task, are summarized in the following table:

PROJECT MANAGER	CLIENT			
	HINES CORP.	NASA	GENERAL FOUNDRY	CBT TV
Gardener	\$3,200	\$3,000	\$2,800	\$2,900
Ruthman	\$2,700	\$3,200	\$3,000	\$3,100
Hardgraves	\$1,900	\$2,100	\$3,300	\$2,100

Schank is very hesitant about neglecting NASA, which has been an important customer in the past. (NASA has employed the firm to study the public's attitude toward the Space Shuttle

and proposed Space Station). In addition, Schank has promised to try to provide Ruthman a salary of at least \$3,000 on his next assignment. From previous contracts, Schank also knows that Gardener does not get along well with the management at CBT Television, so he hopes to avoid assigning him to CBT. Finally, because Hines Corporation is also an old and valued client, Schank feels that it is twice as important to assign a project manager immediately to Hines's task as it is to provide one to General Foundry, a brand-new client. Schank wants to minimize the total costs of all projects while considering each of these goals. He feels that all these goals are important, but if he had to rank them, he would put his concern about NASA first, his worry about Gardener second, his need to keep Hines Corporation happy third, his promise to Ruthman fourth, and his concern about minimizing all costs last.

Each project manager can handle, at most, one new client.

### Discussion Questions

1. If Schank were not concerned about non-cost goals, how would he formulate this problem so that it could be solved quantitatively?
2. Develop a formulation that will incorporate all five objectives.
3. Solve using Excel.

## CASE STUDY

### Oakton River Bridge

The Oakton River had long been considered an impediment to the development of a certain medium-sized metropolitan area in the southeast. Lying to the east of the city, the river made it difficult for people living on its eastern bank to commute to jobs in and around the city and to take advantage of the shopping and cultural attractions that the city had to offer. Similarly, the river inhibited those on its western bank from access to the ocean resorts lying one hour to the east. The bridge over the Oakton River had been built prior to World War II and was grossly inadequate to handle the existing traffic, much less the increased traffic that would accompany the forecasted growth in the area. A congressional delegation from the state prevailed upon the federal government to fund a major portion of a new toll bridge over the Oakton River, and the state legislature appropriated the rest of the needed monies for the project.

Progress in construction of the bridge has been in accordance with what was anticipated at the start of construction. The state highway commission, which will have operational jurisdiction over the bridge, has concluded that opening of the bridge for traffic is likely to take place at the beginning of the next summer, as scheduled. A personnel task force has been established to recruit, train, and schedule the workers needed to operate the toll facility.

The personnel task force is well aware of the budgetary problems facing the state. It has taken as part of its mandate the requirement that personnel costs be kept as low as possible. One particular area of concern is the number of toll collectors that will be needed. The bridge is scheduling three shifts of collectors: shift A from midnight to 8 A.M., shift B from 8 A.M. to 4 P.M., and shift C from 4 P.M. to midnight. Recently, the state employees' union negotiated a contract with the state which requires that all toll collectors be permanent, full-time employees. In addition, all collectors must work a five-on, two-off schedule on the same shift. Thus, for example, a worker could be assigned to work Tuesday, Wednesday, Thursday, Friday, and Saturday on shift A, followed by Sunday and Monday off. An employee could not be scheduled to work, say, Tuesday on shift A followed by Wednesday, Thursday, Friday, and Saturday on shift B or on any other mixture of shifts during a five-day block. The employees would choose their assignments in order of their seniority.

The task force has received projections of traffic flow on the bridge by day and hour. These projections are based on extrapolations of existing traffic patterns—the pattern of commuting, shopping, and beach traffic currently experienced, with growth projections factored in. Standards data from other state-operated toll facilities have allowed the task force to convert these traffic flows into toll collector requirements—that is, the minimum number of collectors required per shift, per day, to



handle the anticipated traffic load. These toll collector requirements are summarized in the following table:

MINIMUM NUMBER OF TOLL COLLECTORS REQUIRED PER SHIFT							
SHIFT	SUN.	MON.	TUE.	WED.	THU.	FRI.	SAT.
A	8	13	12	12	13	13	15
B	10	10	10	10	10	13	15
C	15	13	13	12	12	13	8

The numbers in the table include one or two extra collectors per shift to fill in for collectors who call in sick and to provide relief for collectors on their scheduled breaks. Note that each of the eight collectors needed for shift A on Sunday, for

example, could come from any of the A shifts scheduled to begin on Wednesday, Thursday, Friday, Saturday, or Sunday.

### Discussion Questions

1. Determine the minimum number of toll collectors that would have to be hired to meet the requirements expressed in the table.
2. The union had indicated that it might lift its opposition to the mixing of shifts in a five-day block in exchange for additional compensation and benefits. By how much could the numbers of toll collectors required be reduced if this were done?

Source: B. Render, R. M. Stair, and I. Greenberg. *Cases and Readings in Management Science*, 2/e. 1990, pp. 55–56. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.

## CASE STUDY

### Puyallup Mall

Jane Rodney, president of the Rodney Development Company, was trying to decide what types of stores to include in her new

shopping center at Puyallup Mall. She had already contracted for a supermarket, a drugstore, and a few other stores that she considered essential. However, she had available 16,000 square

**TABLE 6.9** Characteristics of Possible Leases, Puyallup Mall Shopping Center

TYPE OF STORE	SIZE OF STORE (1000s SQ. FT.)	ANNUAL RENT (IN THOUSANDS)	PRESENT VALUE (IN THOUSANDS)	CONSTRUCTION COST (IN THOUSANDS)
Clothing				
1. Men's	1.0	\$ 4.4	\$ 28.1	\$ 24.6
2. Women's	1.6	\$ 6.1	\$ 34.6	\$ 32.0
3. Variety (both)	2.0	\$ 8.3	\$ 50.0	\$ 41.4
Restaurants				
4. Fancy restaurant	3.2	\$24.0	\$162.0	\$124.4
5. Lunchroom	1.8	\$19.5	\$ 77.8	\$ 64.8
6. Cocktail lounge	2.1	\$20.7	\$100.4	\$ 79.8
7. Candy and ice cream shop	1.2	\$ 7.7	\$ 45.2	\$ 38.6
Hardgoods				
8. Hardware store	2.4	\$19.4	\$ 80.2	\$ 66.8
9. Cutlery and variety	1.6	\$11.7	\$ 51.4	\$ 45.1
10. Luggage and leather	2.0	\$15.2	\$ 62.5	\$ 54.3
Miscellaneous				
11. Travel agency	0.6	\$ 3.9	\$ 18.0	\$ 15.0
12. Tobacco shop	0.5	\$ 3.2	\$ 11.6	\$ 13.4
13. Camera store	1.4	\$11.3	\$ 50.4	\$ 42.0
14. Toys	2.0	\$16.0	\$ 73.6	\$ 63.7
15. Beauty parlor	1.0	\$ 9.6	\$ 51.2	\$ 40.0

feet of floor space yet to allocate. She drew up a list of the 15 types of stores she might consider (see Table 6.9), including the floor space required by each. Rodney did not think she would have any trouble finding occupants for any type of store.

The lease agreements Rodney used in her developments included two types of payment. The store had to pay a certain annual rent, depending on the size and type of store. In addition, Rodney was to receive a small percentage of the store's sales if the sales exceeded a specified minimum amount. The amount of annual rent from each store is shown in the second column of the table. To estimate the profitability of each type of store, Rodney calculated the present value of all future rent and sales percentage payments. These are given in the third column. Rodney wants to achieve the highest total *present value* over the set of stores she selects. However, she could not simply pick those stores with the highest present values, for there were several restrictions. The first, of course, was that she has available only 16,000 square feet.

In addition, a condition on the financing of the project required that the total annual rent should be at least as much as the annual fixed costs (taxes, management fees, debt service, and

so forth). These annual costs were \$130,000 for this part of the project. Finally, the total funds available for construction of this part of the project were \$700,000, and each type of store required different construction costs, depending on the size and type of store (fourth column in the table).

In addition, Rodney had certain requirements in terms of the mix of stores she considered best. She wanted at least one store from each of the clothing, hardgoods, and miscellaneous groups, and at least two from the restaurant category. She wanted no more than two from the clothing group. Furthermore, she did not want the number of stores in the miscellaneous group to exceed the total number of stores in the clothing and hardgoods groups combined.

### Discussion Question

Which tenants should be selected for the mall?

**Source:** Adapted from H. Bierman, C. P. Bonini, and W. H. Hausman. *Quantitative Analysis, 7/e.* (Homewood, IL: Richard D. Irwin, Inc.), pp. 467–468. Copyright © 1986. Reprinted by permission of McGraw-Hill Companies, Inc.

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