Let S_1 be some reasonable upper bound on ①'s supply, e.g. $S_1 = 11$

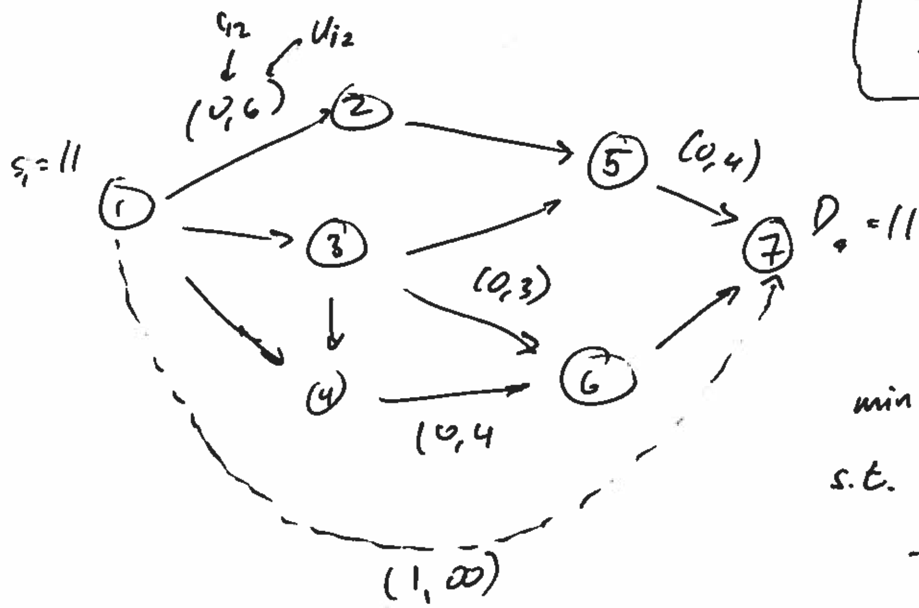
Let $D_7 = S_1$

Let $S_k = 0$ for all other nodes, $D_k = 0$

Let $c_{ij} = 0$ for all arcs in the original network

Keep capacities on the arcs

* Converting Max Flow to Min Cost Flow



$$\min \sum\sum c_{ij} x_{ij} = x_{17}$$

$$\text{s.t. } x_{12} + x_{13} + x_{14} + x_{17} = 11$$

$$-x_{57} - x_{67} - x_{17} = -11$$

$$\sum_j x_{kj} - \sum_i x_{ik} = 0$$

$$0 \le x_{ij} \le U_{ij}$$

Interpretation: send 11 units from ① to ⑦ using arc 1-7 as little as possible (i.e. by maximizing flow through original network)

Sensitivity Analysis

After you solve an L.P., you discover some of your data (parameters) were poorly estimated

- How does this affect the optimal solution

You're designing a system & don't know what values the parameters will take on

- what will be the optimal solution(s) for a range of param

(*) This is a crucial step in any modeling process, not just LP models, and one that many mathematicians, scientists and product managers overlook.

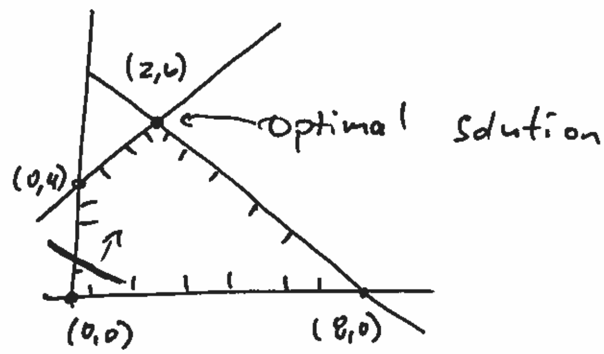
Introductory Example

$$\max \quad Z = X_1 + 3X_2$$

s.t. $X_1 + X_2 \le 8 = b_1$

$-X_1 + X_2 \le 4$

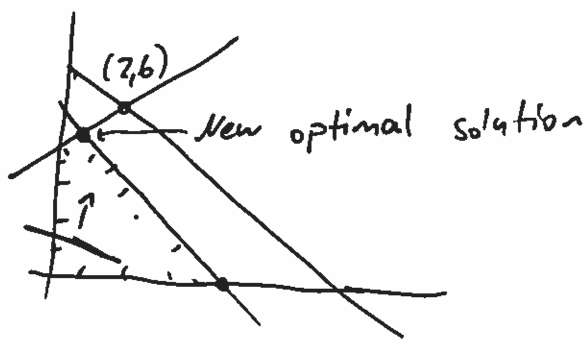
$X_1, X_2 \ge 0$



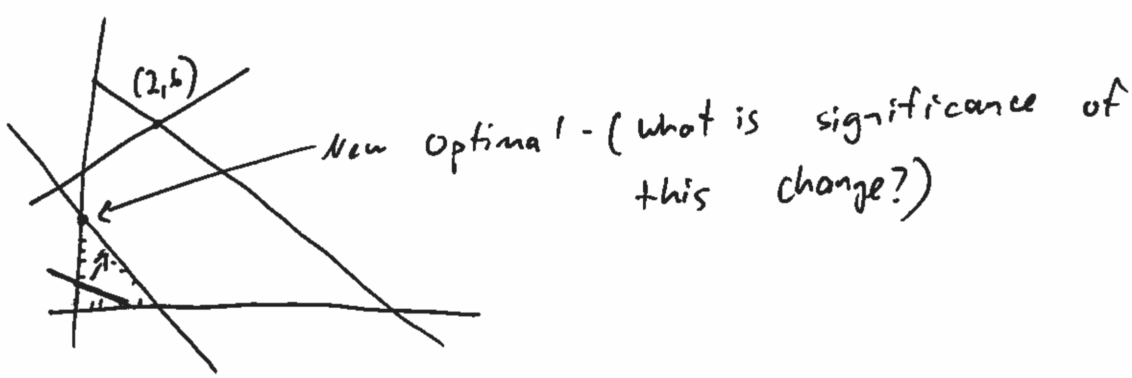
Consider one-by-one the impact of the following changes

(1a) b_1 changes from 8 to 6

Feasible region shrinks and (2,6) is no longer feasible

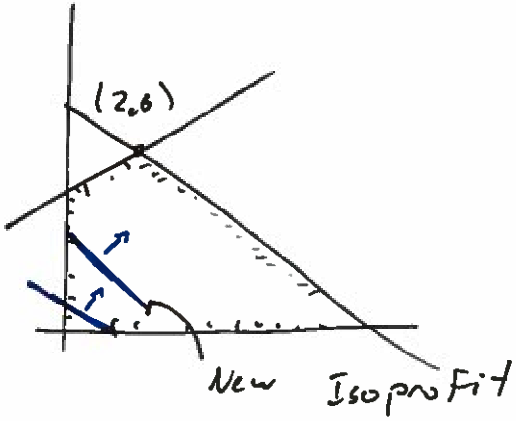


(1b) b_1 changes from 8 to 3

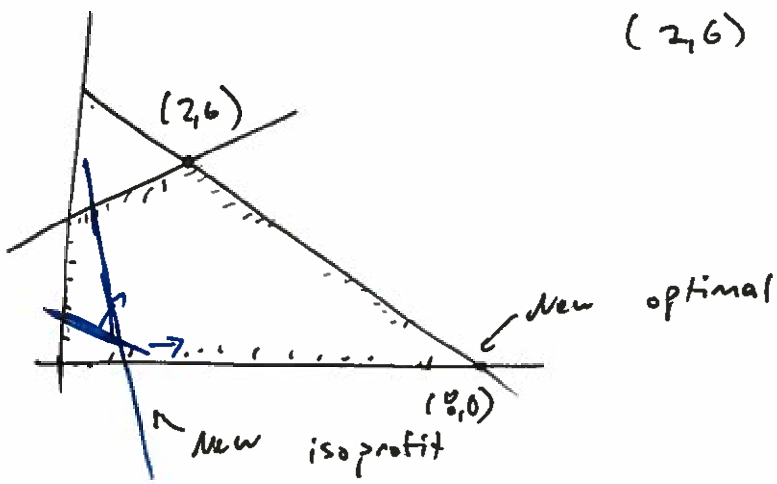


(2a) c_2 changes from 3 to 2

Feasibility is unaffected and (2, 6) is still optimal



(2b) c_2 changes from 3 to $1/2$ - Feasibility unaffected, but (2,6) no longer optimal



3) The constraint is added:

$$x_1 - x_2 \le 4$$

If previous optimal solution remains feasible it must still be optimal. why?