a. What was the original constraint before the slack variable was included?
b. What value of S is associated with each of the following points:
   i) $X_1 = 2, X_2 = 2$
   ii) $X_1 = 8, X_2 = 0$
   iii) $X_1 = 1, X_2 = 3$
   iv) $X_1 = 4, X_2 = 1$

26. Consider the following constraint, where S is a slack variable:

$$3X_1 + 4X_2 - S = 12$$

a. What was the original constraint before the slack variable was included?
b. What value of S is associated with each of the following points:
   i) $X_1 = 5, X_2 = 0$
   ii) $X_1 = 2, X_2 = 2$
   iii) $X_1 = 7, X_2 = 1$
   iv) $X_1 = 4, X_2 = 0$

CASE 4.1: PARKET SISTERS: A CASE IN LINEAR PROGRAMMING & SENSITIVITY ANALYSIS

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Computers and word processors notwithstanding, the art of writing by hand recently entered a boom era. People are buying fountain pens again, and mechanical pencils are becoming more popular than ever. Joe Script, the president and CEO of Parket Sisters, a small but growing pen and pencil manufacturer, wants to establish a better foothold in the market. The writing market is divided into two main sectors. One, dominated by Mont Blanc, Cross, Parker Brothers, Waterman, Schaffer, and a few others, caters to people who want writing instruments. The product lines from these companies consist of pens and pencils of elaborate design, lifetime warranty, and high price. At the other end of the market are manufacturers like BIC, Pentel, and many companies from the far east, offering good quality items, low price, few trims, and limited diversity. These pens and pencils are meant to be used for a limited time and disposed of when the ink in a ballpoint pen runs out, or when the lead in a mechanical pencil won't retract or extend. In short, these items are not meant for repair.

Joe thinks that there must be a middle ground, and that is where he wants to position his company. Parket Sisters makes high-quality items, with limited trim and diversity, but also offers lifetime warranties. Furthermore, its pens and pencils are ergonomically efficient. Joe knows that some people want the status of the Mont Blanc Meisterstuck pen, for example, but he has never met a person who said that writing with such a pen is enjoyable. The pen is too large and clumsy for smooth writing. Parket Sisters' products, on the other hand, have a reputation for working well, are easy to hold and use, and cause limited "writer's fatigue."

Parket Sisters makes only three items—a ballpoint pen, a mechanical pencil, and a fountain pen. All are available in just one color, black, and are sold mostly in specialty stores and from better catalog companies. The per-unit profit of the items is $3.00 for the ballpoint pen, $3.00 for the mechanical pencil, and $5.00 for

the fountain pen. These values take into account labor, the cost of materials, packing, quality control, and so on.

The company is trying to plan its production mix for each week. Joe believes that the company can sell any number of pens and pencils it produces, but production is currently limited by the available resources. Because of a recent strike and certain cash-flow problems, the suppliers of these resources are selling them to Parket Sisters in limited amounts. In particular, Joe can count on getting at most 1,000 ounces of plastic, 1,200 ounces of chrome, and 2,000 ounces of stainless steel each week from his suppliers, and these figures are not likely to change in the near future. Because of Joe's excellent reputation, the suppliers will sell Joe any amount (up to his limit) of the resources he needs when he requires them. That is, the suppliers do not require Joe to buy some fixed quantities of resources in advance of his production of pens and pencils; therefore, these resources can be considered variable costs rather than fixed costs for the pens and pencils.

Each ballpoint pen requires 1.2 ounces of plastic, 0.8 ounces of chrome, and 2 ounces of stainless steel. Each mechanical pencil requires 1.7 ounces of plastic, no chrome, and 3 ounces of stainless steel. Each fountain pen requires 1.2 ounces of plastic, 2.3 ounces of chrome, and 4.5 ounces of stainless steel. Joe believes LP could help him decide what his weekly product mix should consist of.

Getting his notes and notebooks, Joe grapples with the LP formulation. In addition to the constraints of the available resources, he recognizes that the model should include many other constraints (such as labor time availability and materials for packing). However, Joe wants to keep his model simple. He knows that eventually he'll have to take other constraints into account, but as a first-pass model, he'll restrict the constraints to just the three resources: plastic, chrome, and stainless steel.

With only these three constraints, Joe can formulate the problem easily as:

$$
\begin{aligned}
&\text{MAX:} && 3.0X_1 + 3.0X_2 + 5.0X_3 \\
&\text{Subject to:} && 1.2X_1 + 1.7X_2 + 1.2X_3 \leq 1{,}000 \\
& && 0.8X_1 + 0X_2 + 2.3X_3 \leq 1{,}200 \\
& && 2.0X_1 + 3.0X_2 + 4.5X_3 \leq 2{,}000 \\
& && X_1, X_2, X_3 \geq 0
\end{aligned}
$$

where:

X_1 = the number of ballpoint pens
X_2 = the number of mechanical pencils
X_3 = the number of fountain pens

Joe's knowledge of Excel and the Solver feature is limited so he asks you to enter and solve the problem for him, then answer the following questions. (Assume each question is independent unless otherwise stated.)

1. What should the weekly product mix consist of, and what is the weekly net profit?
2. Is the optimal solution to question 1 degenerate? Explain your response.
3. Is the optimal solution from question 1 unique, or are there alternate answers to this question? Explain your response.
4. What is the marginal value of one more unit of chrome? Of plastic?

5. A local distributor has offered to sell Parket Sisters an additional 500 ounces of stainless steel for $0.60 per ounce more than it ordinarily pays. Should the company buy the steel at this price? Explain your response.
6. If Parket Sisters buys the additional 500 ounces of stainless steel noted in question 5, what is the new optimal product mix and what is the new optimal profit? Explain your response.
7. Suppose that the distributor offers to sell Parket Sisters some additional plastic at a price of only $1.00 over its usual cost of $5.00 per ounce. However, the distributor will sell the plastic only in lot sizes of 500 ounces. Should Parket Sisters buy one such lot? Explain your response.
8. The distributor is willing to sell the plastic in lots of just 100 ounces instead of the usual 500-ounce lots, still at $1.00 over Parket Sisters' cost of $5.00 per ounce. How many lots (if any) should Parket Sisters buy? What is the optimal product mix if the company buys these lots, and what is the optimal profit?
9. Parket Sisters has an opportunity to sell some of its plastic for $6.50 per ounce to another company. The other company (which does not produce pens and pencils and, therefore, is not a competitor) wants to buy 300 ounces of plastic from Parket Sisters. Should Parket Sisters sell the plastic to the other company? What happens to Parket Sisters' product mix and overall profit if it does sell the plastic? Be as specific as possible.
10. The chrome supplier might have to fulfill an emergency order, and would be able to send only 1,000 ounces of chrome this week instead of the usual 1,200 ounces. If Parket Sisters receives only 1,000 ounces of chrome, what is the optimal product mix and optimal profit? Be as specific as possible.
11. The R&D department at Parket Sisters has been redesigning the mechanical pencil to make it more profitable. The new design requires 1.1 ounces of plastic, 2.0 ounces of chrome, and 2.0 ounces of stainless steel. If the company can sell one of these pencils at a net profit of $3.00, should it approve the new design? Explain your response.
12. If the per-unit profit on ballpoint pens decreases to $2.50, what is the optimal product mix and what is the company's total profit?
13. The Marketing department suggested introducing a new felt tip pen that requires 1.8 ounces of plastic, 0.5 ounces of chrome, and 1.3 ounces of stainless steel. What profit must this product generate in order to make it worthwhile to produce?
14. What must the minimum per-unit profit of mechanical pencils be in order to make them worthwhile to produce?
15. Management believes that the company should produce at least 20 mechanical pencils per week to round out its product line. What effect would this have on overall profit? Give a numerical answer.
16. If the profit on a fountain pen is $6.75 instead of $5.00, what is the optimal product mix and optimal profit?

Preparation questions (turn in before class starts):

1) Set up the LP formulation in Excel and run Solver to get the optimal product mix.
2) Answer questions 3 and 4 in the case.
3) Answer questions 5, 6, and 14 in the case (Use Excel's Sensitivity Report to answer as many questions as possible)

Please also turn in

- **Answer report** and **Sensitivity report**
- The **number version** and the **formula version** of the spreadsheet with **Row and Column Headings/Gridlines**.