DISCUSSION QUESTIONS AND PROBLEMS

Discussion Questions

4-1 Discuss the role of sensitivity analysis in LP. Under what circumstances is it needed, and under what conditions do you think it is not necessary?

4-2 Is sensitivity analysis a concept applied to LP only, or should it also be used when analyzing other techniques (e.g., break-even analysis)? Provide examples to prove your point.

4-3 Explain how a change in resource availability can affect the optimal solution of a problem.

4-4 Explain how a change in an OFC can affect the optimal solution of a problem.

4-5 Are simultaneous changes in input data values logical? Provide examples to prove your point.

4-6 Explain the 100% rule and its role in analyzing the impact of simultaneous changes in model input data values.

4-7 How can a firm benefit from using the pricing out procedure?

4-8 How do we detect the presence of alternate optimal solutions from a Solver Sensitivity Report?

4-9 Why would a firm find information regarding the shadow price of a resource useful?

Problems

4-10 We used a graphical approach to solve the following LP model in Problem 2-13:

$$\text{Maximize profit} = \$5X + \$3Y$$

subject to the constraints

$$\begin{aligned} 5X + 2Y &\le 40 \\ 3X + 6Y &\le 48 \\ X &\le 7 \\ 2X - Y &\ge 3 \\ X, Y &\ge 0 \end{aligned}$$

Now use the graphical solution to answer the following questions. Each question is independent of the others.

(a) If a technical breakthrough raises the profit per unit of Y to \$5, would this affect the optimal corner point? Does it change the optimal objective value?

(b) If the profit per unit of X decreases to only \$1.50, how would this affect the optimal corner point and profit?

(c) If the first constraint changes to $5X + 2Y \le 42$, how would this affect the optimal corner point and profit?

4-11 We used a graphical approach to solve the following LP model in Problem 2-14:

$$\text{Minimize cost} = \$1X + \$2Y$$

subject to the constraints

$$\begin{aligned} X + 3Y &\ge 90 \\ 8X + 2Y &\ge 160 \\ 3X + 2Y &\ge 120 \\ Y &\le 70 \\ X, Y &\ge 0 \end{aligned}$$

Now use the graphical solution to answer the following questions. Each question is independent of the others.

(a) Does the optimal solution change if the cost per unit of Y increases to \$4? What about the optimal cost?

(b) If the first constraint changes to $X + 3Y \ge 100$, does the optimal solution change? If so, how?

(c) If the fourth constraint changes to $Y \le 50$, does the optimal solution change? If so, how?

4-12 We used a graphical approach to solve the following LP model in Problem 2-15:

$$\text{Minimize cost} = \$4X + \$7Y$$

subject to the constraints

$$\begin{aligned} 3X + 7Y &\ge 231 \\ 10X + 2Y &\ge 200 \\ 2Y &\ge 45 \\ 2X &\le 75 \\ X, Y &\ge 0 \end{aligned}$$

Now use the graphical solution to answer the following questions. Each question is independent of the others.

(a) Does the optimal solution change if the cost per unit of X decreases to \$2? What about the optimal cost?

(b) If the first constraint changes to $3X + 7Y \ge 250$, does the optimal solution change? If so, how?

(c) If the third constraint changes to $2Y \ge 55$, does the optimal solution change? If so, how?

4-13 Consider the Win Big Gambling Club media selection example discussed in section 3.3 (page 88) of Chapter 3. Use the Sensitivity Report for this LP model (shown in Screenshot 4-6) to answer the following questions. Each question is independent of the others.

(a) What would be the impact if management approved spending \$200 more on radio advertising each week?

(b) Would it help Win Big if it could get out of the contractual agreement to place at least five radio spots each week?

(c) The radio station manager agrees to run the afternoon radio spots during some of the more popular programs. He thinks this will increase the audience reached per ad to 3,100. Will this change the optimal solution? Why or why not?

(d) There is some uncertainty in the audience reached per TV spot. For what range of values for this OFC will the current solution remain optimal?

4-14 Consider the MSA marketing research example discussed in section 3.3 (page 90) of Chapter 3. Use the Sensitivity Report for this LP model (shown in Screenshot 4-7) to answer the following questions. Each question is independent of the others.

(a) What is the maximum unit cost that will make it worthwhile to include in the survey persons 30 years of age or younger who live in a border state?

SCREENSHOT 4-6 **Solver Sensitivity Report for Problem 4-13: Win Big Gambling Club**

Microsoft Excel 10.0 Sensitivity Report
Problem 4-13. Win Big Gambling Club

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number of units TV spots	1.97	0.00	5000.00	1620.69	5000.00
C5	Number of units Newspaper ads	5.00	0.00	8500.00	1E+30	2718.75
D5	Number of units Prime-time radio spots	6.21	0.00	2400.00	1E+30	263.16
E5	Number of units Afternoon radio spots	0.00	-344.83	2800.00	344.83	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F8	Maximum TV	1.97	0.00	12.00	1E+30	10.03
F9	Maximum newspaper	5.00	2718.75	5.00	1.70	5.00
F10	Max prime-time radio	6.21	0.00	25.00	1E+30	18.79
F11	Max afternoon radio	0.00	0.00	20.00	1E+30	20.00
F12	Total budget	8,000.00	6.25	8000.00	8025.00	1575.00
F13	Maximum radio $	1,800.00	2.03	1800.00	1575.00	350.00
F14	Minimum radio spots	6.21	0.00	5.00	1.21	1E+30

(b) What is the impact if MSA wants to increase the sample size to 3,000?
(c) What is the impact if MSA insists on including people 31–50 years of age who do not live in a border state?
(d) What is the impact if we can reduce the minimum number of 30 or younger persons required to 900, provided that we raise the number of persons 31–50 years of age to 650?

4-15 Consider the Whole Food Nutrition Center diet problem example discussed in section 3.7 (page 107) of Chapter 3. Use the Sensitivity Report for this LP model (shown in Screenshot 4-8) to answer the

SCREENSHOT 4-7 **Solver Sensitivity Report for Problem 4-14: MSA Marketing Research**

Microsoft Excel 10.0 Sensitivity Report
Problem 4-14. MSA Marketing Research

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number of households <= 30 and border	0.00	0.60	7.50	1E+30	0.60
C5	Number of households 31-50 and border	600.00	0.00	6.80	0.45	0.82
D5	Number of households >= 51 and border	140.00	0.00	5.50	0.6	29.90
E5	Number of households <= 30 and not border	1000.00	0.00	6.90	0.6	0.92
F5	Number of households 31-50 and not border	0.00	0.45	7.25	1E+30	0.45
G5	Number of households >= 51 and not border	560.00	0.00	6.10	1.025	0.60

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H8	Total households	2300.00	5.98	2300.00	1E+30	700.00
H9	<= 30 households	1000.00	0.92	1000.00	700.00	1000.00
H10	31-50 households	600.00	0.82	600.00	700.00	493.75
H11	Border Mexico	740.00	0.00	0.00	395.00	1E+30
H12	<= 30 and not border	1000.00	0.00	0.00	500.00	1E+30
H13	>= 51 and border	140.00	-0.60	0.00	560.00	140.00

SCREENSHOT 4-8

Solver Sensitivity Report for Problem 4-15: Whole Food Nutrition Center

Microsoft Excel 10.0 Sensitivity Report
Problem 4-15. Whole Food Nutrition Center

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number of pounds Grain A	0.025	0.000	0.330	0.063	1E+30
C5	Number of pounds Grain B	0.050	0.000	0.470	1E+30	0.190
D5	Number of pounds Grain C	0.050	0.000	0.380	1E+30	0.073

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E8	Protein	3.000	0.038	3.000	0.000	0.250
E9	Riboflavin	2.350	0.000	2.000	0.350	1E+30
E10	Phosphorus	1.000	0.088	1.000	0.018	0.000
E11	Magnesium	0.425	0.000	0.425	0.000	1E+30
E12	Total Mix	0.125	-1.210	0.125	0.004	0.000

following questions. Each question is independent of the others.

(a) What is the impact if the daily allowance for protein can be reduced to 2.9 units?
(b) Whole Food believes the unit price of grain A could be 5% overestimated and the unit price of grain B could be 10% underestimated. If these turn out to be true, what are the new optimal solution and optimal total cost?
(c) What is the impact if the reduction in the daily allowance for protein in (a) requires Whole Food to simultaneously increase the daily allowance of riboflavin to 2.20 units?

4-16 Consider the Quitmeyer Electronics product mix problem first presented in Chapter 3 as Problem 3-3 on page 125. Use Solver to create the Sensitivity Report for this LP problem. Then answer the following questions using this report. Each question is independent of the others.

(a) Interpret the reduced costs for the products that are not currently included in the optimal production plan.
(b) Another part of the corporation wants to take 35 hours of time on test device 3. How does this affect Quitmeyer's optimal solution?
(c) Quitmeyer has the opportunity to obtain 20 additional hours on test device 1 at a cost of $25 per hour. Is this deal worthwhile?
(d) Quitmeyer has the opportunity to give up 20 hours of time on device 1 and obtain 40 hours of time on device 2 in return. Is this deal worthwhile? Justify your answer.

4-17 Consider Margaret Young's farm planning problem first presented in Chapter 3 as Problem 3-29 on page 132. Use Solver to create the Sensitivity Report for this LP problem. Then answer the following questions using this report. Each question is independent of the others.

(a) Is this solution a unique optimal solution? Why or why not?
(b) If there are alternate solutions, use Solver to identify at least one other optimal solution.
(c) Would it help Margaret's total profit if she could increase barley sales by 10%? If so, how?
(d) How would the availability of more water affect Margaret's total profit?

4-18 Consider the Battery Park Stables animal feed problem first presented in Chapter 3 as Problem 3-34 on page 133. Use Solver to create the Sensitivity Report for this LP problem. Then answer the following questions using this report. Each question is independent of the others.

(a) If the price of grain decreases by $0.01 per pound, will the optimal solution change?
(b) Which constraints are binding? Interpret the shadow price for the binding constraints.
(c) What would happen to the total cost if the price of mineral decreased by 20% from its current value?
(d) For what price range of oats is the current solution optimal?

4-19 Consider Kathy Roniger's diet problem first presented in Chapter 3 as Problem 3-35 on page 133. Use Solver to create the Sensitivity Report for this LP problem. Then answer the following questions using this report. Each question is independent of the others.

(a) Interpret the shadow prices for the carbohydrates and iron constraints.
(b) What will happen to total cost if Kathy insists on using milk in her diet?
(c) What will be the maximum amount Kathy could pay for beans to make it a cost-effective item for inclusion in her diet?
(d) Is the solution to this problem a unique optimal solution? Justify your answer.

SCREENSHOT 4-9

Solver Sensitivity Report for Problem 4-20

Microsoft Excel 10.0 Sensitivity Report
Problem 4-20

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Solution value X	2.00	0.00	4.00	1.00	1.50
C4	Solution value Y	4.00	0.00	5.00	3.00	1.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D7	Labor	10.00	1.00	10.00	2.00	2.00
D8	Material	36.00	0.50	36.00	4.00	6.00
D9	Storage	32.00	0.00	40.00	1E+30	8.00

4-20 Consider the following LP problem, in which X and Y denote the number of units of products X and Y to produce, respectively:

$$\text{Maximize profit} = \$4X + \$5Y$$

subject to the constraints

$X + 2Y \leq 10$ (labor available, in hours)
$6X + 6Y \leq 36$ (material available, in pounds)
$8X + 4Y \leq 40$ (storage available, in square feet)
$X, Y \geq 0$ (nonnegativity)

The Excel Sensitivity Report for this problem is shown in Screenshot 4-9. Calculate and explain what happens to the optimal solution for each of the following situations. Each question is independent of the other questions.

(a) You acquire 2 additional pounds of material.
(b) You acquire 1.5 additional hours of labor.
(c) You give up 1 hour of labor and get 1.5 pounds of material.
(d) The profit contributions for both products X and Y are changed to $4.75 each.
(e) You decide to introduce a new product that has a profit contribution of $2. Each unit of this product will use 1 hour of labor, 1 pound of material, and 2 square feet of storage space.

4-21 Consider the Mt. Sinai Hospital example first presented in Chapter 3 as Problem 3-45 on page 136. Use Solver to create the Sensitivity Report for this LP problem. Then answer the following questions using this report. Each question is independent of the others.

(a) What is the maximum revenue per year, how many medical patients per year are there, and how many surgical patients per year are there? How many medical beds and how many surgical beds of the 90-bed addition should be added?
(b) Are there any empty beds with this optimal solution? If so, how many empty beds are there? Discuss the effect of acquiring more beds, if needed.
(c) Are the laboratories being used to their capacity? Is it possible to perform more lab tests per year? If so, how many more? Discuss the effect of acquiring more lab space, if needed.
(d) Is the x-ray facility being used to its maximum? Is it possible to do more x-rays per year? If so, how many more? Discuss the effect of acquiring more x-ray facilities, if needed.
(e) Is the operating room being used to capacity? Is it possible to do more operations per year? If so, how many more? Discuss the effect of acquiring more operating room, if needed.

Source: Professor Chris Vertullo.

4-22 The Good-to-Go Suitcase Company makes three kinds of suitcases: (1) Standard, (2) Deluxe, and (3) Luxury styles. Each suitcase goes through four production stages: (1) cutting and coloring, (2) assembly, (3) finishing, and (4) quality and packaging. The total number of hours available in each of these departments is 630, 600, 708, and 135, respectively.

Each Standard suitcase requires 0.7 hours of cutting and coloring, 0.5 hours of assembly, 1 hour of finishing, and 0.1 hours of quality and packaging. The corresponding numbers for each Deluxe suitcase are 1 hour, 0.83 hours, 0.67 hours, and 0.25 hours, respectively. Likewise, the corresponding numbers for each Luxury suitcase are 1 hour, 0.67 hours, 0.9 hours, and 0.4 hours, respectively.

The sales revenue for each type of suitcase is as follows: Standard $36.05, Deluxe $39.50, and Luxury $43.30. The material costs are Standard $6.25, Deluxe $7.50, and Luxury $8.50. The hourly cost of labor for each department is cutting and coloring $10, assembly $6, finishing $9, and quality and packaging $8.

The Excel layout and LP Sensitivity Report of Good-to-Go's problem are shown in Screenshots 4-10A and 4-10B, respectively. Answer the following questions, each of which is independent of the others.

(a) What is the optimal production plan? Which of the resources are scarce?

SCREENSHOT 4-10A

Excel Layout for Problem 4-22: Good-to-Go Suitcase

	A	B	C	D	E	F	G	H
1	**Good-to-Go Suitcase Company**							
2								
3		Standard	Deluxe	Luxury				
4	Solution value	540.00	252.00	0.00				
5	Selling price per unit	$36.05	$39.50	$43.30	$29,421.00			
6	Material cost per unit	$6.25	$7.50	$8.50	$5,265.00			
7	Labor cost per unit	$19.80	$23.00	$25.30	$16,488.00			
8	Profit	$10.00	$9.00	$9.50	$7,668.00			
9	**Constraints**							Cost
10	Cutting & Coloring	0.70	1.00	1.00	630.00	<=	630	$10
11	Assembly	0.50	0.83	0.67	480.00	<=	600	$6
12	Finishing	1.00	0.67	0.90	708.00	<=	708	$9
13	Quality & Packaging	0.10	0.25	0.40	117.00	<=	135	$8
14					LHS	Sign	RHS	

SCREENSHOT 4-10B

Solver Sensitivity Report for Problem 4-22: Good-to-Go Suitcase

Microsoft Excel 10.0 Sensitivity Report
Problem P4-22. Good-to-Go Suitcase Company

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Solution value Standard	540.00	0.00	10.00	3.50	2.56
C4	Solution value Deluxe	252.00	0.00	9.00	5.29	1.61
D4	Solution value Luxury	0.00	-1.12	9.50	1.12	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E10	Cutting & Coloring	630.00	4.38	630	52.36	134.40
E11	Assembly	480.00	0.00	600	1E+30	120.00
E12	Finishing	708.00	6.94	708	192.00	128.00
E13	Quality & Packaging	117.00	0.00	135	1E+30	18.00

(b) Suppose Good-to-Go is considering including a polishing process, the cost of which would be added directly to the price. Each Standard suitcase would require 10 minutes of time in this treatment, each Deluxe suitcase would need 15 minutes, and each Luxury suitcase would need 20 minutes. Would the current production plan change as a result of this additional process if 170 hours of polishing time were available? Explain your answer.

(c) Now consider the addition of a waterproofing process in which each Standard suitcase would use 1 hour of time in the process, each Deluxe suitcase would need 1.5 hours, and each Luxury suitcase would require 1.75 hours. Would this change the production plan if 900 hours were available? Why or why not?

Source: Professors Mark and Judith McKnew, Clemson University.

4-23 Suppose Good-to-Go (Problem 4-22) is considering the possible introduction of two new products to its line of suitcases: the Compact model (for teenagers) and the Kiddo model (for children). Market research suggests that Good-to-Go can sell the Compact model for no more than $30, whereas the Kiddo model would go for as much as $37.50 to specialty toy stores. The amount of labor and the cost of raw materials for each possible new product are as follows:

Cost Category	Compact	Kiddo
Cutting and coloring (hr.)	0.50	1.20
Assembly (hr.)	0.75	0.75
Finishing (hr.)	0.75	0.50
Quality and packaging (hr.)	0.20	0.20
Raw materials	$5.00	$4.50

Use a pricing out strategy to check whether either model would be economically attractive to make.

4-24 The Strollers-to-Go Company makes lightweight umbrella-type strollers for three different groups of

children. The TiniTote is designed specifically for newborns who require extra neck support. The ToddleTote is for toddlers up to 30 pounds. Finally, the company produces a heavy-duty model called TubbyTote, which is designed to carry children up to 60 pounds. The stroller company is in the process of determining its production for each of the three types of strollers for the upcoming planning period.

The marketing department has forecast the following maximum demand for each of the strollers during the planning period: TiniTote 180, TubbyTote 70, and ToddleTote 160. Strollers-to-Go sells TiniTotes for $63.75, TubbyTotes for $82.50, and ToddleTotes for $66.00. As a matter of policy, it wants to produce no less than 50% of the forecast demand for each product. It also wants to keep production of ToddleTotes to a maximum of 40% of total stroller production.

The production department has estimated that the material costs for the TiniTote, TubbyTote, and ToddleTote strollers will be $4, $6, and $5.50 per unit, respectively. The strollers are processed through fabrication, sewing, and assembly workstations. The metal and plastic frames are made in the fabrication station. The fabric seats are cut and stitched together in the sewing station. Finally, the frames are put together with the seats in the assembly station. In the upcoming planning period, there will be 620 hours available in fabrication, where the direct labor cost is $8.25 per hour. The sewing station has 500 hours available, and the direct labor cost is $8.50 per hour. The assembly station has 480 hours available, and the direct labor cost is $8.75 per hour.

The standard processing rate for TiniTotes is 3 hours in fabrication, 2 hours in sewing, and 1 hour in assembly. TubbyTotes require 4 hours in fabrication, 1 hour in sewing, and 3 hours in assembly, whereas ToddleTotes require 2 hours in each station.

The Excel layout and LP Sensitivity Report for Strollers-to-Go's problem are shown in Screenshots 4-11A and 4-11B, respectively. Answer the following questions, each of which is independent of the others.

(a) How many strollers of each type should Strollers-to-Go make? What is the profit? Which constraints are binding?
(b) How much labor time is being used in the fabrication, sewing, and assembly areas?
(c) How much would Strollers-to-Go be willing to pay for an additional hour of fabrication time? For an additional hour of sewing time?
(d) Is Strollers-to-Go producing any product at its maximum sales level? Is it producing any product at its minimum level?

Source: Professors Mark and Judith McKnew, Clemson University.

4-25 Consider the Strollers-to-Go production problem (Problem 4-24).

(a) Over what range of costs could the TiniTote materials vary and the current production plan remain optimal? (*Hint*: How are material costs reflected in the problem formulation?)
(b) Suppose that Strollers-to-Go decides to polish each stroller prior to shipping. The process is fast and requires 10, 15, and 12 minutes, respectively, for TiniTote, TubbyTote, and ToddleTote strollers. Would this change the current production plan if 48 hours of polishing time were available?

4-26 Consider the Strollers-to-Go production problem (Problem 4-24).

(a) Suppose that Strollers-to-Go could purchase additional fabrication time at a cost of $10.50 per

SCREENSHOT 4-11A

Excel Layout for Problem 4-24: Strollers-to-Go

	A	B	C	D	E	F	G	H
1	**Strollers-to-Go Company**							
2								
3		TiniTote	TubbyTote	ToddleTote				
4	Solution value	100.00	35.00	90.00				
5	Selling price per unit	$63.75	$82.50	$66.00	$15,202.50			
6	Material cost per unit	$4.00	$6.00	$5.50	$1,105.00			
7	Labor cost per unit	$50.50	$67.75	$51.00	$12,011.25			
8	Profit	$9.25	$8.75	$9.50	$2,086.25			
9	**Constraints**							**Cost**
10	Fabrication	3.0	4.0	2.0	620.00	<=	620	$8.25
11	Sewing	2.0	1.0	2.0	415.00	<=	500	$8.50
12	Assembly	1.0	3.0	2.0	385.00	<=	480	$8.75
13	Tinitote demand	1.0			100.00	<=	180	
14	Tubbytote demand		1.0		35.00	<=	70	
15	Toddletote demand			1.0	90.00	<=	160	
16	Toddletote max prod ratio	-0.4	-0.4	0.6	0.00	<=	0	
17	Tinitote min prod	1.0			100.00	>=	90	
18	Tubbytote min prod		1.0		35.00	>=	35	
19	Toddletote min prod			1.0	90.00	>=	80	
20					**LHS**	**Sign**	**RHS**	

SCREENSHOT 4-11B

Solver Sensitivity Report for Problem 4-24: Strollers-to-Go

Microsoft Excel 10.0 Sensitivity Report
Problem 4-24. Strollers-to-Go Company

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Solution value TiniTote	100.00	0.00	9.25	5.00	3.33
C4	Solution value TubbyTote	35.00	0.00	8.75	4.10	1E+30
D4	Solution value ToddleTote	90.00	0.00	9.50	1E+30	3.33

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E10	Fabrication	620.00	3.60	620.00	110.50	43.33
E11	Sewing	415.00	0.00	500.00	1E+30	85.00
E12	Assembly	385.00	0.00	480.00	1E+30	95.00
E13	Tinitote demand	100.00	0.00	180.00	1E+30	80.00
E14	Tubbytote demand	35.00	0.00	70.00	1E+30	35.00
E15	Toddletote demand	90.00	0.00	160.00	1E+30	70.00
E16	Toddletote max prod ratio	0.00	3.85	0.00	13.00	8.67
E17	Tinitote min prod	100.00	0.00	90.00	10.00	1E+30
E18	Tubbytote min prod	35.00	-4.10	35.00	8.13	35.00
E19	Toddletote min prod	90.00	0.00	80.00	10.00	1E+30

hour. Should it be interested? Why or why not? What is the most that it would be willing to pay for an additional hour of fabrication time?

(b) Further suppose that Strollers-to-Go could only purchase fabrication time in multiples of 40-hour bundles. How many should it be willing to purchase in this case?

4-27 Suppose that Strollers-to-Go (Problem 4-24) is considering the production of TwinTotes for those families that were doubly blessed. Each TwinTote would require $5.75 in materials, 3.5 hours of fabrication time, 1.75 hours of sewing time, and only 1.5 hours to assemble. Would this product be economically attractive to manufacture if the sales price were $72.00? Why or why not?

4-28 The Classic Furniture Company is trying to determine the optimal quantities to make of six possible products—tables and chairs made of oak, cherry, and pine. The products are to be made using the following resources: labor hours and three types of wood. Minimum production requirements are as follows: at least 3 each of oak and cherry tables, at least 10 each of oak and cherry chairs, and at least 5 pine chairs.

The Excel layout and LP Sensitivity Report for Classic Furniture's problem are shown in Screenshots 4-12A and 4-12B, respectively. The OFCs in the screenshots refer to unit profit per item. Answer the following questions, each of which is independent of the others.

(a) What is the profit represented by the objective function, and what is the production plan?

(b) Which constraints are binding?

(c) What is the range over which the unit profit for oak chairs can change without changing the production plan?

(d) What is the range over which the amount of available oak could range without changing the combination of binding constraints?

(e) Does this Sensitivity Report indicate the presence of multiple optimal solutions? How do you know?

(f) After production is over, how many pounds of cherry wood will be left over?

(g) According to this report, how many more chairs were made than were required?

4-29 Consider the Classic Furniture product mix problem (Problem 4-28). For each of the following situations, what would be the impact on the production plan and profit? If it is possible to compute the new profit or production plan, please do so.

(a) Unit profit for oak tables increased to $83.

(b) Unit profit for pine chairs decreased by $15.

(c) Unit profit for pine tables increased by $15.

(d) Unit profit for cherry tables increased to $95.

(e) Company was required to make at least 20 pine chairs.

(f) Company was required to make no more than 55 cherry chairs.

4-30 Consider the Classic Furniture product mix problem (Problem 4-28). For each of the following situations, what would be the impact on the production plan and profit? If it is possible to compute the new profit or production plan, please do so.

SCREENSHOT 4-12A

Excel Layout for Problem 4-28: Classic Furniture

	A	B	C	D	E	F	G	H	I	J
1	**Classic Furniture Company**									
2										
3		Oak tables	Oak chairs	Cherry tables	Cherry chairs	Pine tables	Pine chairs			
4	Number of units	3.00	51.67	3.00	85.56	42.26	33.08			
5	Profit	$75	$35	$90	$80	$45	$20	$10,000.00		
6	**Constraints**									
7	Labor hours	7.5	3.5	9.0	6.0	4.5	2.0	1000.00	<=	1,000
8	Oak (pounds)	200	30					2150.00	<=	2,150
9	Cherry (pounds)			240	36			3800.00	<=	3,800
10	Pine (pounds)					180	27	8500.00	<=	8,500
11	Min oak tables	1						3.00	>=	3
12	Min cherry tables			1				3.00	>=	3
13	Min oak chairs		1					51.67	>=	10
14	Min cherry chairs				1			85.56	>=	10
15	Min pine chairs						1	33.08	>=	5
16								LHS	Sign	RHS

SCREENSHOT 4-12B

Solver Sensitivity Report for Problem 4-28: Classic Furniture

Microsoft Excel 10.0 Sensitivity Report
Problem 4-28. Classic Furniture Company

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Number of units Oak tables	3.00	0.00	75.00	0.00	1E+30
C4	Number of units Oak chairs	51.67	0.00	35.00	1E+30	0.00
D4	Number of units Cherry tables	3.00	0.00	90.00	0.00	1E+30
E4	Number of units Cherry chairs	85.56	0.00	60.00	1E+30	0.00
F4	Number of units Pine tables	42.26	0.00	45.00	88.33	0.00
G4	Number of units Pine chairs	33.08	0.00	20.00	0.00	13.25

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H7	Labor hours	1000.00	10.00	1000.00	373.30	37.21
H8	Oak (pounds)	2150.00	0.00	2150.00	316.93	1250.00
H9	Cherry (pounds)	3800.00	0.00	3800.00	223.25	2239.78
H10	Pine (pounds)	8500.00	0.00	8500.00	1488.33	5039.50
H11	Min oak tables	3.00	0.00	3.00	6.25	2.35
H12	Min cherry tables	3.00	0.00	3.00	11.33	1.20
H13	Min oak chairs	51.67	0.00	10.00	41.67	1E+30
H14	Min cherry chairs	85.56	0.00	10.00	75.56	1E+30
H15	Min pine chairs	33.08	0.00	5.00	28.08	1E+30

(a) Number of labor hours expanded to 1240.
(b) Amount of cherry wood increased to 4000.
(c) Number of labor hours decreased to 900.
(d) Company did not have a minimum requirement for oak chairs.

4-31 Consider the Classic Furniture product mix problem (Problem 4-28). For each of the following situations, what would be the impact on the production plan and profit? If it is possible to compute the new profit or production plan, please do so.

(a) OFCs for oak tables and cherry tables each decreased by $15.
(b) OFCs for oak tables and oak chairs were reversed.
(c) OFCs for pine tables and pine chairs were reversed.
(d) $OFC_{Pine\ Table}$ increased by $20, while at the same time $OFC_{Pine\ Chair}$ decreased by $10.
(e) Unit profit for all three types of chairs increased by $6 each.

4-32 Consider the Classic Furniture product mix problem (Problem 4-28). In answering each of the following

questions, be as specific as possible. If it is possible to compute a new profit or production plan, please do so.

(a) A part-time employee who works 25 hours per week decides to quit his job. How would this affect the profit and production plan?
(b) Classic has been approached by the factory next door, CabinetsRUs, which has a shortage of both labor and oak. CabinetsRUs proposes to take one full-time employee (who works 35 hours) plus 1,200 pounds of oak. It has offered $880 as compensation. Should Classic make this trade?
(c) Classic is considering adding a new product, a cherry armoire. The armoire would consume 450 pounds of cherry wood and take 15 hours of labor. Cherry wood costs $6 per pound, and labor costs $12 per hour. The armoire would sell for $3,000. Should this product be made?
(d) What would happen to the solution if a constraint were added to make sure that for every table made, at least two matching chairs were made?

4-33 The Tiger Catering Company is trying to determine the most economical combination of sandwiches to make for a tennis club. The club has asked Tiger to provide 70 sandwiches in a variety to include tuna, tuna and cheese, ham, ham and cheese, and cheese. The club has specified a minimum of 10 each of tuna and ham and 12 each of tuna/cheese and ham/cheese. Tiger makes the sandwiches, using the following resources: bread, tuna, ham, cheese, mayonnaise, mustard, lettuce, tomato, packaging material, and labor hours.

The Excel layout and LP Sensitivity Report for Tiger Catering's problem are shown in Screenshots 4-13A and 4-13B, respectively. The OFCs in the screenshots refer to unit cost per item. Answer the following questions, each of which is independent of the others.

(a) What is the cost represented by the objective function, and what is the sandwich making plan?
(b) Which constraints are binding?
(c) What is the range over which the cost for cheese sandwiches could vary without changing the production plan?
(d) What is the range over which the quantity of tuna could vary without changing the combination of binding constraints?
(e) Does this Sensitivity Report indicate the presence of multiple optimal solutions? How do you know?
(f) After the sandwiches are made, how many labor hours remain?

4-34 Consider the Tiger Catering problem (Problem 4-33). For each of the following situations, what would be the impact on the sandwich making plan and total cost? If it is possible to compute the new cost or sandwich making plan, please do so.

(a) Unit cost for tuna sandwiches increased by $0.50.
(b) Unit cost for tuna and cheese sandwiches increased to $2.40.
(c) Unit cost for ham sandwiches increased to $3.75.
(d) Unit cost for ham and cheese sandwiches decreased by $0.60.
(e) Club did not want any more than 14 ham sandwiches.
(f) Unit cost for cheese sandwiches decreased to $2.10.

4-35 Consider the Tiger Catering problem (Problem 4-33). For each of the following situations, what would be

SCREENSHOT 4-13A

Excel Layout for Problem 4-33: Tiger Catering

Tiger Catering Company

	Tuna	Tuna/Ch	Ham	Ham/Ch	Cheese			
Number to make	10.00	30.00	10.00	12.00	8.00			
Cost	$2.42	$2.12	$3.35	$3.02	$2.36	$176.42		
Constraints								
Bread (slices)	2	2	2	2	2	140.00	<=	140
Tuna (oz.)	4	3				130.00	<=	130
Ham (oz.)			4	3		76.00	<=	100
Cheese (oz.)		1		1	4	74.00	<=	80
Mayo (oz.)	1.2	0.9	0.5	0.5	0.5	54.00	<=	72
Mustard (oz.)			0.2	0.2		4.40	<=	8
Lettuce (oz.)	0.25	0.25	0.25	0.25	0.25	17.50	<=	20
Tomato (oz.)	0.5	0.5	0.5	0.5	0.5	35.00	<=	40
Package (unit)	1	1	1	1	1	70.00	<=	72
Labor (hrs)	0.08	0.08	0.08	0.08	0.08	5.60	<=	8
Min total	1	1	1	1	1	70.00	>=	70
Min Tuna	1					10.00	>=	10
Min Tuna/Ch		1				30.00	>=	12
Min Ham			1			10.00	>=	10
Min Ham/Ch				1		12.00	>=	12
						LHS	Sign	RHS

SCREENSHOT 4-13B

Solver Sensitivity Report for Problem 4-33: Tiger Catering

Microsoft Excel 10.0 Sensitivity Report
Problem 4-33. Tiger Catering Company

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Number to make Tuna	10.00	0.00	2.42	1E+30	0.38
C4	Number to make Tuna/Ch	30.00	0.00	2.12	0.24	1E+30
D4	Number to make Ham	10.00	0.00	3.35	1E+30	0.99
E4	Number to make Ham/Ch	12.00	0.00	3.02	1E+30	0.66
F4	Number to make Cheese	8.00	0.00	2.36	0.66	0.24

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G7	Bread (slices)	140.00	0.00	140.00	1E+30	0.00
G8	Tuna (oz.)	130.00	-0.08	130.00	24.00	6.00
G9	Ham (oz.)	76.00	0.00	100.00	1E+30	24.00
G10	Cheese (oz.)	74.00	0.00	80.00	1E+30	6.00
G11	Mayo (oz.)	54.00	0.00	72.00	1E+30	18.00
G12	Mustard (oz.)	4.40	0.00	8.00	1E+30	3.60
G13	Lettuce (oz.)	17.50	0.00	20.00	1E+30	2.50
G14	Tomato (oz.)	35.00	0.00	40.00	1E+30	5.00
G15	Package (unit)	70.00	0.00	72.00	1E+30	2.00
G16	Labor (hrs)	5.60	0.00	8.00	1E+30	2.40
G17	Min total	70.00	2.36	70.00	0.00	8.00
G18	Min Tuna	10.00	0.38	10.00	13.50	10.00
G19	Min Tuna/Ch	30.00	0.00	12.00	18.00	1E+30
G20	Min Ham	10.00	0.99	10.00	6.00	1.50
G21	Min Ham/Ch	12.00	0.66	12.00	8.00	2.00

the impact on the sandwich making plan and total cost? If it is possible to compute the new cost or sandwich making plan, please do so.

(a) Quantity of tuna available increased to 150 ounces.
(b) Quantity of ham available decreased to 85 ounces.
(c) Quantity of cheese available decreased to 72 ounces.
(d) Tiger is required to deliver a minimum of 18 tuna sandwiches.
(e) Tiger is required to deliver only a minimum of 5 tuna and cheese sandwiches.
(f) Tiger is asked to bring a minimum of only 65 sandwiches.

4-36 Consider the Tiger Catering problem (Problem 4-33). For each of the following situations, what would be the impact on the sandwich making plan and total cost? If it is possible to compute the new cost or sandwich making plan, please do so.

(a) Cost of ham sandwiches and cost of ham and cheese sandwiches each decreased by $0.35.
(b) Cost of both ham and cheese sandwiches and cheese sandwiches increased by $0.60.
(c) Cost of tuna decreased by $0.10 per ounce. (*Hint:* Note that tuna sandwiches use 4 ounces of tuna and tuna/cheese sandwiches use 3 ounces of tuna.)
(d) Availability of tuna increases by 10 ounces and availability of ham decreases by 10 ounces.
(e) A 16-ounce jar of mustard is sent by mistake, instead of a 16-ounce jar of mayonnaise. (*Hint:* This would decrease the quantity of mayonnaise by 16 ounces and increase the quantity of mustard by 16 ounces.)

4-37 Consider the Tiger Catering problem (Problem 4-33). In answering each of the following questions, be as specific as possible. If it is possible to compute a new cost or sandwich making plan, please do so.

(a) An additional pound of tuna can be obtained for a premium of $1.50. Should this tuna be purchased?
(b) The tennis club is willing to accept fewer ham and ham and cheese sandwiches. How many of these sandwiches would Tiger try to substitute with other types before it would not be able to predict its new total cost?
(c) The tennis club wants to include a dill pickle slice with each meat sandwich order. If Tiger finds an average of 18 slices in a 2-pound pickle jar, how many jars should be included with the club's order?

CASE STUDY

Coastal States Chemicals and Fertilizers

In December 2005, Bill Stock, general manager for the Louisiana Division of Coastal States Chemicals and Fertilizers, received a letter from Fred McNair of the Cajan Pipeline Company, which notified Coastal States that priorities had been established for the allocation of natural gas. The letter stated that Cajan Pipeline, the primary supplier of natural gas to Coastal States, might be instructed to curtail natural gas supplies to its industrial and commercial customers by as much as 40% during the ensuing winter months. Moreover, Cajan Pipeline had the approval of the Federal Power Commission (FPC) to curtail such supplies.

Possible curtailment was attributed to the priorities established for the use of natural gas:

First priority:	residential and commercial heating
Second priority:	commercial and industrial users that use natural gas as a source of raw material
Third priority:	commercial and industrial users whereby natural gas is used as boiler fuel

Almost all of Coastal States' uses of natural gas were in the second and third priorities. Hence, its plants were certainly subject to brownouts, or natural gas curtailments. The occurrence and severity of the brownouts depended on a number of complex factors. First, Cajan Pipeline was part of an interstate transmission network that delivered natural gas to residential and commercial buildings on the Atlantic coast and in northeastern regions of the United States. Hence, the severity of the forthcoming winter in these regions would have a direct impact on the use of natural gas.

Second, the demand for natural gas was soaring because it was the cleanest and most efficient fuel. There were almost no environmental problems in burning natural gas. Moreover, maintenance problems due to fuel-fouling in fireboxes and boilers were negligible with natural gas systems. Also, burners were much easier to operate with natural gas than with oil or coal.

Finally, the supply of natural gas was dwindling. The traditionally depressed price of natural gas had discouraged new exploration for gas wells; hence, shortages appeared imminent.

Stock and his staff at Coastal States had been aware of the possibility of shortages of natural gas and had been investigating ways of converting to fuel oil or coal as a substitute for natural gas. Their plans, however, were still in the developmental stages. Coastal States required an immediate contingency plan to minimize the effect of a natural gas curtailment on its multiplant, operations. The obvious question was, what operations should be curtailed, and to what extent could the adverse effect upon profits be minimized? Coastal States had approval from the FPC and Cajan Pipeline to specify which of its plants would bear the burden of the curtailment if such cutbacks were necessary. McNair, of Cajan Pipeline, replied, "It's your 'pie': we don't care how you divide it if we make it smaller."

The Model

Six plants of Coastal States Louisiana Division were to share in the "pie." They were all located in the massive Baton Rouge-Geismar-Gramercy industrial complex along the Mississippi River between Baton Rouge and New Orleans. Products manufactured at those plants that required significant amounts of natural gas were phosphoric acid, urea, ammonium phosphate, ammonium nitrate, chlorine, caustic soda, vinyl chloride monomer, and hydrofluoric acid.

Stock called a meeting of members of his technical staff to discuss a contingency plan for allocation of natural gas among the products if a curtailment developed. The objective was to minimize the impact on profits. After detailed discussion, the meeting was adjourned. Two weeks later, the meeting reconvened. At this session, the data in Table 4.3 were presented.

Coastal States' contract with Cajan Pipeline specified a maximum natural gas consumption of 36,000 cu ft z 10^3 per day

TABLE 4.3 **Contribution to Profit and Overhead**

PRODUCT	CONTRIBUTIN PER TON($)	CAPACITY (TONS PER DAY)	MAXIMUM PRODUCTION RATE (PERCENT OF CAPACITY)	NATURAL GAS CONSUMPTION (1,000 CU FT PER TON)
Phosphoric acid	60	400	80	5.5
Urea	80	250	80	7.0
Ammonium Phosphate	90	300	90	8.0
Ammonium nitrate	100	300	100	10.0
Chlorine	50	800	60	15.0
Caustic soda	50	1,000	60	16.0
Vinyl chloride monomer	65	500	60	12.0
Hydrofluoric acid	70	400	80	11.0

for all six member plants. With these data, the technical staff proceeded to develop a model that would specify changes in production rates in response to a natural gas curtailment. (Curtailments are based on contracted consumption and not current consumption.)

Discussion Questions

1. Develop a contingency model and specify the production rates for each product for
   (a) a 20% natural gas curtailment.
   (b) a 40% natural gas curtailment.
2. What impact will the natural gas shortage have on company profits?
3. Develop the Sensitivity Report for the 20% natural gas curtailment model. Use this report to answer the following questions. Each question is independent of the others.
   (a) Interpret the shadow prices for the natural gas availability constraint and for the two constraints that limit the maximum phosphoric acid and chlorine that Coastal can produce.
   (b) Brenda Lamb, Bill Stock's marketing manager, believes that due to increased competition she may have to decrease the unit profit contributions for all products by 3.5% each. What is the impact of this decrease on the production values, and on the total profit?
   (c) Jose Fernandez, Bill Stock's production manager, thinks that he can increase the maximum production rate for Chlorine and Vinyl chloride monomer to 80% of capacity. For all other products, he thinks he can increase the maximum production rate to 100% of capacity. What would be the impact of this change on the total profit?
   (d) Bill Stock thinks he can persuade Coastal's Mississippi Division to give him 1,000,000 cu ft of its allotment of natural gas from Cajan Pipeline. However, due to the Mississippi Division's pricing contract with Cajan Pipeline, this additional amount of natural gas will cost Stock an additional $1.50 per 1,000 cu ft (over current costs). Should Stock pursue this option? If so, what is the impact of this additional gas on his total profit? What is the impact if Bill Stock can persuade the Mississippi Division to give him 3,000,000 cu ft of its allotment of natural gas from Cajan Pipeline?
4. Redo question 3 using the Sensitivity Report for the 40% natural gas curtailment model. In addition, interpret the reduced cost for caustic soda.

Source: Professor Jerry Kinard, Western Carolina University.

BIBLIOGRAPHY

See the Bibliography at the end of Chapter 2.