

Robot Mapping

Grid-Based FastSLAM

Cyrill Stachniss



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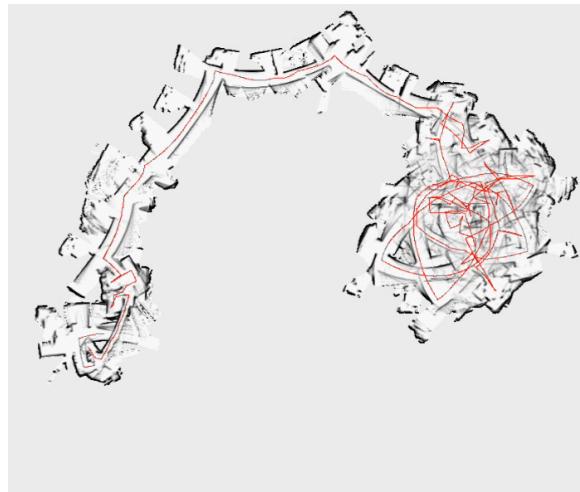
Motivation

- So far, we addressed landmark-based SLAM (KF-based SLAM, FastSLAM)
- We learned how to build grid maps assuming “known poses”

Today: SLAM for building grid maps

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Mapping With Raw Odometry



Courtesy: Dirk Hähnel

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Observation

- **Assuming known poses fails!**

Questions

- Can we solve the SLAM problem if no pre-defined landmarks are available?
- Can we use the ideas of FastSLAM to build grid maps?

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Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

$$p(x_{0:t}, m | z_{1:t}, u_{1:t})$$

poses map observations & movements

First introduced for SLAM by Murphy in 1999

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Rao-Blackwellization for SLAM

- Factorization of the SLAM posterior

$$\begin{aligned} p(x_{0:t}, m | z_{1:t}, u_{1:t}) \\ = p(x_{0:t} | z_{1:t}, u_{1:t}) p(m | x_{1:t}, z_{1:t}) \end{aligned}$$

poses map observations & movements

path posterior
(particle filter) map posterior
(given the path)

First introduced for SLAM by Murphy in 1999

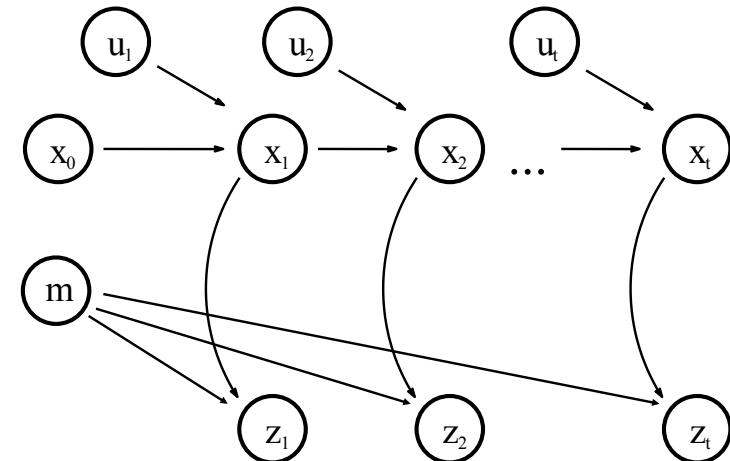
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Grid-Based SLAM

- As with landmarks, the map depends on the poses of the robot during data acquisition
- If the poses are known, grid-based mapping is easy (“mapping with known poses”)

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A Graphical Model for Grid-Based SLAM



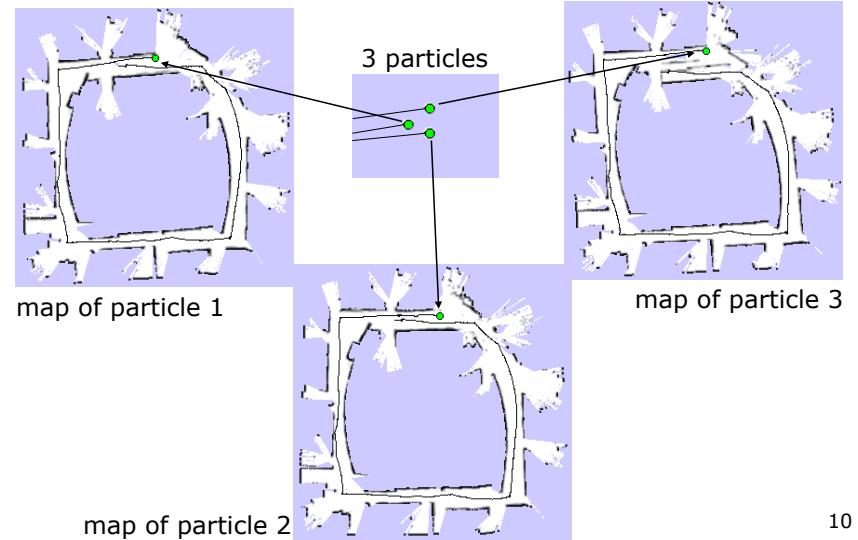
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Grid-Based Mapping with Rao-Blackwellized Particle Filters

- Each particle represents a possible trajectory of the robot
- Each particle maintains its own map
- Each particle updates it upon “mapping with known poses”

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Particle Filter Example



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Performance of Grid-Based FastSLAM 1.0



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Problem

- Too many samples are needed to sufficiently model the motion noise
- Increasing the number of samples is difficult as each map is quite large
- **Idea:** Improve the pose estimate **before** applying the particle filter

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Pose Correction Using Scan-Matching

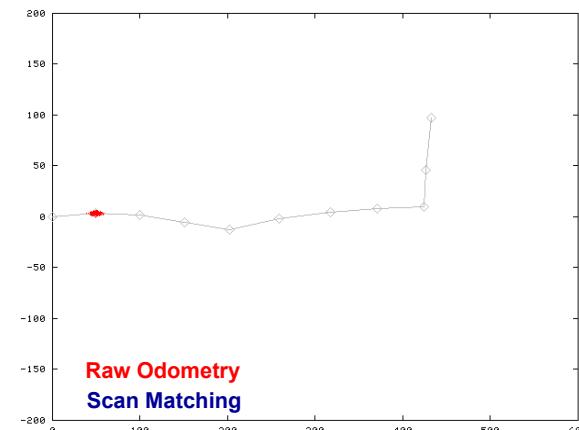
Maximize the likelihood of the **current** pose and map relative to the **previous** pose and map

$$x_t^* = \operatorname{argmax}_{x_t} \{ p(z_t | x_t, m_{t-1}) p(x_t | u_t, x_{t-1}^*) \}$$

current measurement robot motion
map constructed so far

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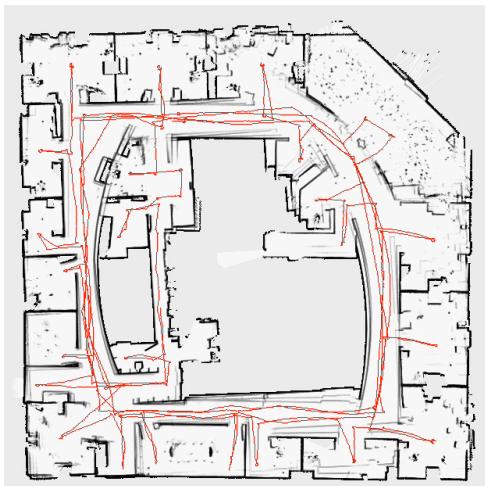
Motion Model for Scan Matching



Courtesy: Dirk Hähnel

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Mapping using Scan Matching



Courtesy: Dirk Hähnel

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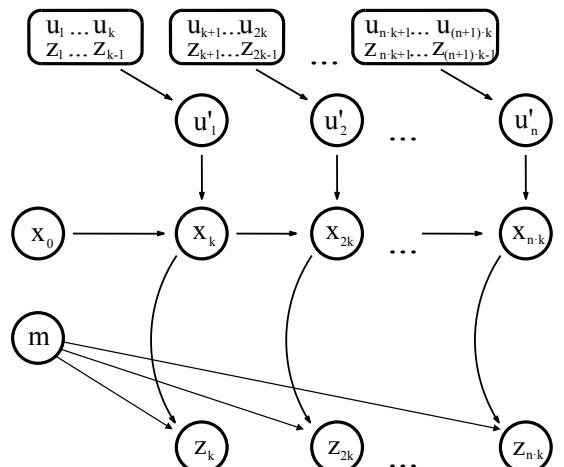
Grid-Based FastSLAM with Improved Odometry

- Scan-matching provides a **locally consistent** pose correction
- Pre-correct short odometry sequences using scan-matching and use them as input to FastSLAM
- Fewer particles are needed, since the error in the input is smaller

[Hähnel et al., 2003]

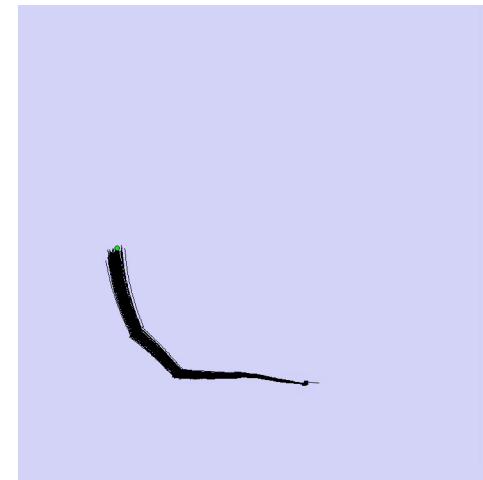
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Graphical Model for Mapping with Improved Odometry



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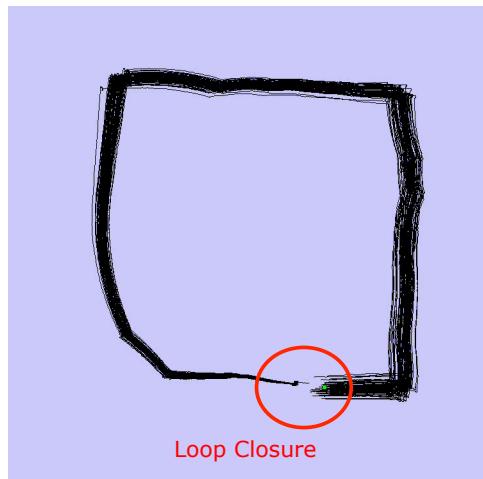
Grid-Based FastSLAM with Scan-Matching



Courtesy:
Dirk Hähnel

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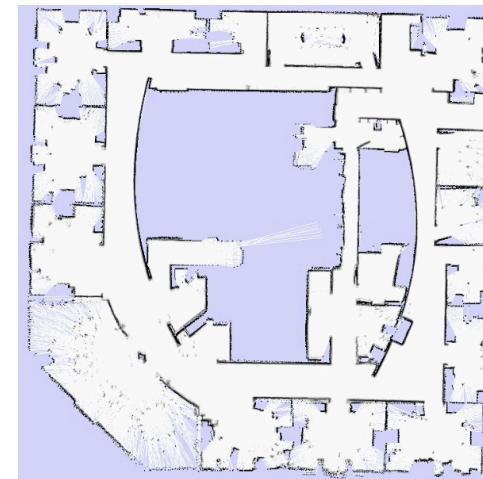
Grid-Based FastSLAM with Scan-Matching



Courtesy:
Dirk Hähnel

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Grid-Based FastSLAM with Scan-Matching



Courtesy:
Dirk Hähnel

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Summary so far ...

- Approach to SLAM that combines scan matching and FastSLAM
- Scan matching to generate virtual 'high quality' motion commands
- Can be seen as an ad-hoc solution to an improved proposal distribution

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What's Next?

- Compute an improved proposal that considers the most recent observation

$$x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t})$$

Goals:

- More precise sampling
- More accurate maps
- Less particles needed

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The Optimal Proposal Distribution

[Arulampalam et al., 01]

$$p(x_t \mid x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) = \frac{p(z_t \mid x_t, m^{[i]}) p(x_t \mid x_{t-1}^{[i]}, u_t)}{p(z_t \mid x_{t-1}^{[i]}, m^{[i]}, u_t)}$$



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Proposal Distribution

$$p(x_t \mid x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) = \overbrace{\frac{p(z_t \mid x_t, m^{[i]}) p(x_t \mid x_{t-1}^{[i]}, u_t)}{p(z_t \mid x_{t-1}^{[i]}, m^{[i]}, u_t)}}^{\tau(x_t)}$$

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Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) = \frac{\overbrace{p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)}^{\tau(x_t)}}{p(z_t | x_{t-1}, m^{[i]}, u_t)}$$

$$p(z_t | x_{t-1}^{[i]}, m^{[i]}, u_t) = \int p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t) dx_t$$

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Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) = \frac{\overbrace{p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)}^{\tau(x_t)}}{p(z_t | x_{t-1}, m^{[i]}, u_t)}$$

$$p(z_t | x_{t-1}^{[i]}, m^{[i]}, u_t) = \int p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t) dx_t$$

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) = \frac{\tau(x_t)}{\int \tau(x_t) dx_t}$$

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Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t)$$

$$= \frac{\overbrace{p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)}^{\tau(x_t)}}{\int \overbrace{p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)}^{} dx_t}$$

locally limits the area over which to integrate (measurement)

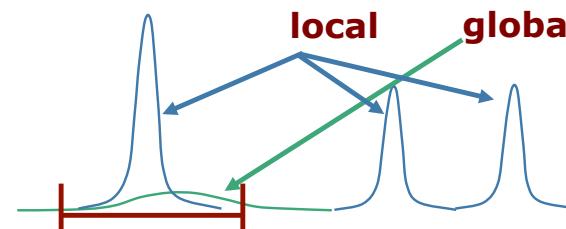
globally limits the area over which to integrate (odometry)

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Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t)$$

$$= \frac{\overbrace{p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)}^{\tau(x_t)}}{\int \overbrace{p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)}^{} dx_t}$$



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Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) \simeq \frac{\tau(x_t)}{\int_{\{x_t | \tau(x_t) > \epsilon\}} \tau(x_t) dx_t}$$

with $\tau(x_t) = p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t)$

How to sample from this term?

Gaussian approximation:

$$\tau(x_t) \simeq \mathcal{N}(\mu^{[i]}, \Sigma^{[i]})$$

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Estimating the Parameters of the Gaussian for Each Particle

$$\mu^{[i]} = \frac{1}{\eta} \sum_{j=1}^K x_j \tau(x_j)$$

$$\Sigma^{[i]} = \frac{1}{\eta} \sum_{j=1}^K (x_j - \mu^{[i]})(x_j - \mu^{[i]})^T \tau(x_j)$$

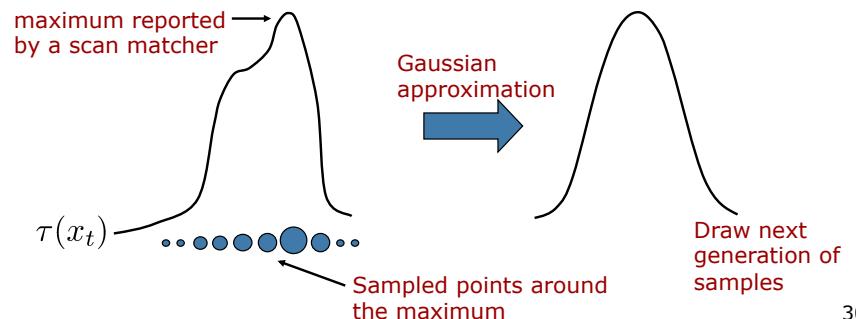
x_j are the points sampled around the result of the scan matcher

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Gaussian Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) \simeq \frac{\tau(x_t)}{\int_{\{x_t | \tau(x_t) > \epsilon\}} \tau(x_t) dx_t}$$

Approximate by a Gaussian:



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Gaussian Proposal Distribution

$$p(x_t | x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) \simeq \frac{\tau(x_t)}{\int_{\{x_t | \tau(x_t) > \epsilon\}} \tau(x_t) dx_t}$$

$$\begin{aligned} \tau(x_t) \simeq \mathcal{N}(\mu^{[i]}, \Sigma^{[i]}) &\quad \begin{matrix} \xrightarrow{\mu^{[i]}} \\ \xrightarrow{\Sigma^{[i]}} \end{matrix} \begin{aligned} \mu^{[i]} &= \frac{1}{\eta} \sum_{j=1}^K x_j \tau(x_j) \\ \Sigma^{[i]} &= \frac{1}{\eta} \sum_{j=1}^K (x_j - \mu^{[i]})(x_j - \mu^{[i]})^T \tau(x_j) \end{aligned} \end{aligned}$$

$$\int_{\{x_t | \tau(x_t) > \epsilon\}} \tau(x_t) dx_t \simeq \sum_{j=1}^K \tau(x_j)$$

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The Importance Weight

$$w_t^{[i]} = \frac{\text{target}(x_t^{[i]})}{\text{proposal}(x_t^{[i]})}$$

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The Importance Weight

$$\begin{aligned} w_t^{[i]} &= \frac{\text{target}(x_t^{[i]})}{\text{proposal}(x_t^{[i]})} \\ &\propto \frac{p(z_t | m^{[i]}, x_t^{[i]}) p(x_t^{[i]} | x_{t-1}^{[i]}, u_t)}{\pi(x_t^{[i]} | m^{[i]}, x_{t-1}^{[i]}, z_t, u_t)} \\ &\quad \frac{p(x_{1:t-1}^{[i]} | z_{1:t-1}, u_{1:t-1})}{\pi(x_{1:t-1}^{[i]} | z_{1:t-1}, u_{1:t-1})} \end{aligned}$$

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The Importance Weight

$$\begin{aligned} w_t^{[i]} &= \frac{\text{target}(x_t^{[i]})}{\text{proposal}(x_t^{[i]})} \\ &\propto \frac{p(z_t | m^{[i]}, x_t^{[i]}) p(x_t^{[i]} | x_{t-1}^{[i]}, u_t)}{\pi(x_t^{[i]} | m^{[i]}, x_{t-1}^{[i]}, z_t, u_t)} \\ &\quad \frac{p(x_{1:t-1}^{[i]} | z_{1:t-1}, u_{1:t-1})}{\pi(x_{1:t-1}^{[i]} | z_{1:t-1}, u_{1:t-1})} \\ &= \left(\frac{p(z_t | m^{[i]}, x_t^{[i]}) p(x_t^{[i]} | x_{t-1}^{[i]}, u_t)}{\int p(z_t | m^{[i]}, x_t) p(x_t | x_{t-1}^{[i]}, u_t) dx_t} w_{t-1}^{[i]} \right) \end{aligned}$$

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The Importance Weight

$$\begin{aligned} w_t^{[i]} &= \frac{\text{target}(x_t^{[i]})}{\text{proposal}(x_t^{[i]})} \\ &\propto \frac{p(z_t | m^{[i]}, x_t^{[i]}) p(x_t^{[i]} | x_{t-1}^{[i]}, u_t)}{\pi(x_t^{[i]} | m^{[i]}, x_{t-1}^{[i]}, z_t, u_t)} \\ &\quad \frac{p(x_{1:t-1}^{[i]} | z_{1:t-1}, u_{1:t-1})}{\pi(x_{1:t-1}^{[i]} | z_{1:t-1}, u_{1:t-1})} \\ &= \frac{p(z_t | m^{[i]}, x_t^{[i]}) p(x_t^{[i]} | x_{t-1}^{[i]}, u_t)}{\int p(z_t | m^{[i]}, x_t) p(x_t | x_{t-1}^{[i]}, u_t) dx_t} w_{t-1}^{[i]} \\ &= w_{t-1}^{[i]} \int p(z_t | m^{[i]}, x_t) p(x_t | x_{t-1}^{[i]}, u_t) dx_t \end{aligned}$$

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The Importance Weight

$$w_t^{[i]} = w_{t-1}^{[i]} \int p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t) dx_t$$

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The Importance Weight

$$\begin{aligned} w_t^{[i]} &= w_{t-1}^{[i]} \int p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t) dx_t \\ &\simeq w_{t-1}^{[i]} \int_{\{x_t | \tau(x_t) > \epsilon\}} \tau(x_t) dx_t \end{aligned}$$

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The Importance Weight

$$\begin{aligned} w_t^{[i]} &= w_{t-1}^{[i]} \int p(z_t | x_t, m^{[i]}) p(x_t | x_{t-1}^{[i]}, u_t) dx_t \\ &\simeq w_{t-1}^{[i]} \int_{\{x_t | \tau(x_t) > \epsilon\}} \tau(x_t) dx_t \\ &\simeq w_{t-1}^{[i]} \sum_{j=1}^K \tau(x_j) \end{aligned}$$

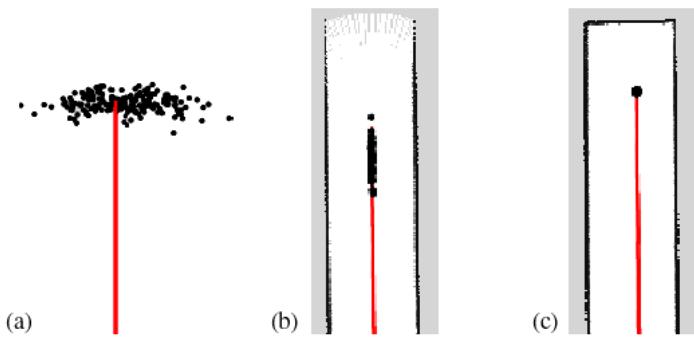
**Already computed
for the proposal!**

Sampled points around the maximum of the likelihood function found by scan-matching

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Improved Proposal

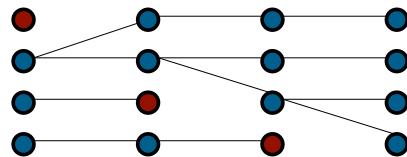
- The proposal adapts to the structure of the environment



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Resampling

- Resampling at each step limits the “memory” of our filter
- Suppose we loose each time 25% of the particles, this may lead to:



- Goal: Reduce the resampling actions

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Selective Resampling

- Resampling is necessary to achieve convergence
 - Resampling is dangerous, since important samples might get lost (“particle depletion”)
 - Resampling makes only sense if particle weights differ significantly
- **Key question: When to resample?**

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Number of Effective Particles

- Empirical measure of how well the target distribution is approximated by samples drawn from the proposal

$$n_{eff} = \frac{1}{\sum_i (w_t^{[i]})^2}$$

- n_{eff} describes “the inverse variance of the **normalized** particle weights”
- For equal weights, the sample approximation is close to the target

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Resampling with n_{eff}

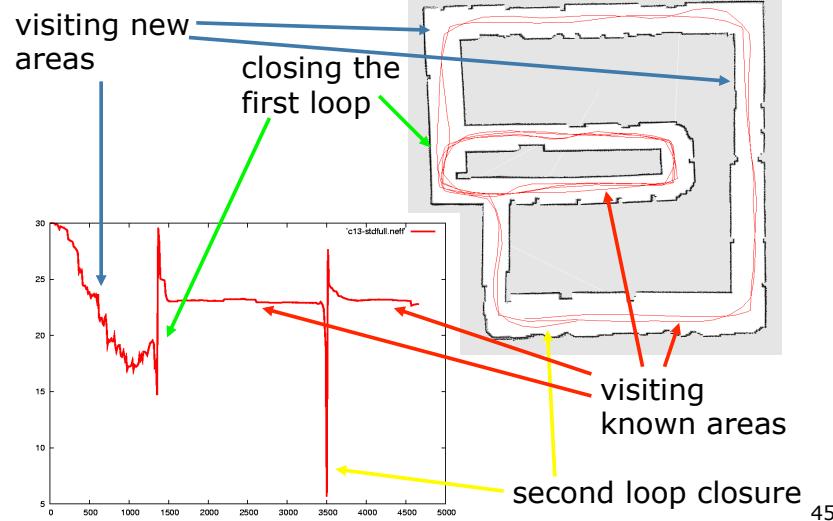
- If our approximation is close to the target, no resampling is needed
- We only resample when n_{eff} drops below a given threshold ($N/2$)

$$\frac{1}{\sum_i (w_t^{[i]})^2} < N/2$$

- Note: weights need to be normalized
[Doucet, '98; Arulampalam, '01]

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Typical Evolution of n_{eff}

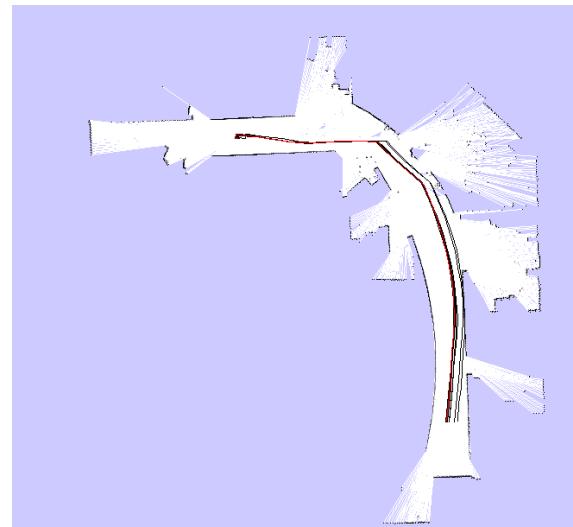


Intel Lab



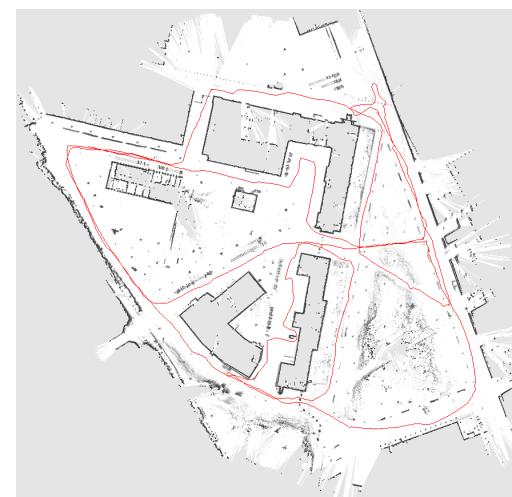
- **15 particles**
- four times faster than real-time P4, 2.8GHz
- 5cm resolution during scan matching
- 1cm resolution in final map

Intel Lab



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Outdoor Campus Map



- **30 particles**
- 250x250m²
- 1.75 km (odometry)
- 30cm resolution in final map

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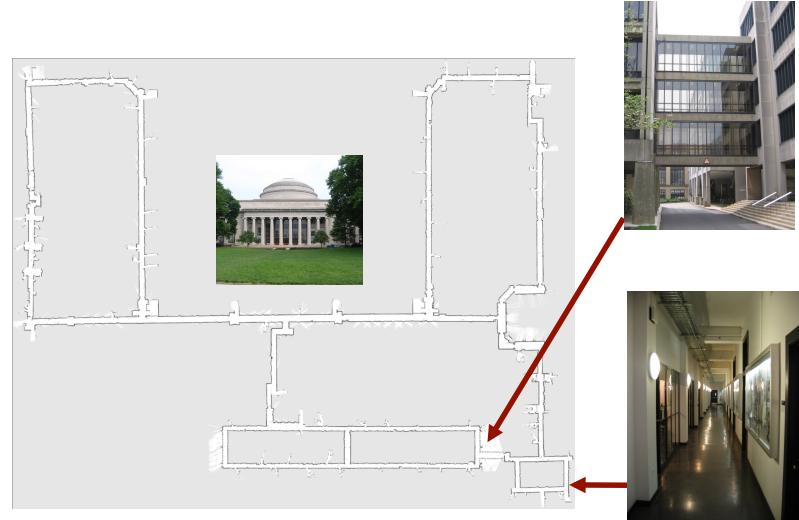
MIT Killian Court



- The “infinite-corridor-dataset” at MIT

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MIT Killian Court



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MIT Killian Court – Video



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Real World Application

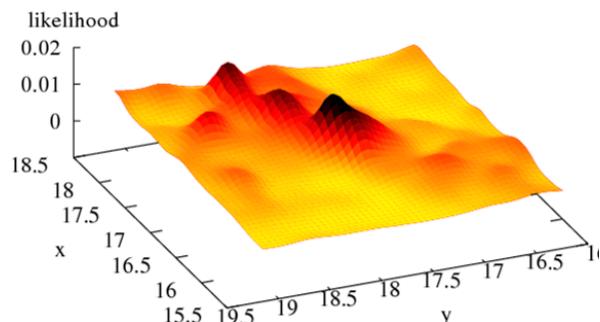
- This guy uses a similar technique...



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Problems of Gaussian Proposals

- Gaussians are uni-modal distributions
- In case of loop-closures, the likelihood function might be multi-modal



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Is a Gaussian an Accurate Choice for the Proposal?

Dataset	Gauss	Non-Gauss; 1 mode	Multi-modal
Intel Research Lab	89.2%	7.2%	3.6%
FHW Museum	84.5%	10.4%	5.1%
Belgioioso	84.0%	10.4%	5.6%
MIT CSAIL	78.1%	15.9%	6.0%
MIT Killian Court	75.1%	19.1%	5.8%
Freiburg Bldg. 79	74.0%	19.4%	6.6%

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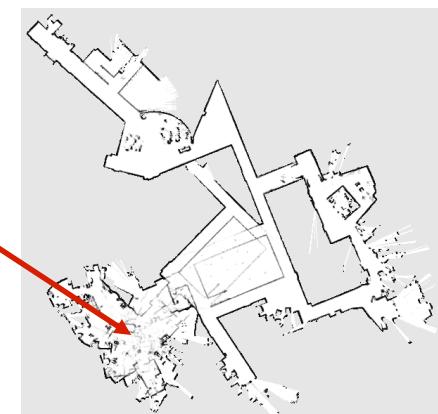
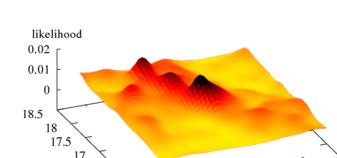
Gaussian or Non-Gaussian?

- Statistical test to check whether or not sample a generated from a Gaussian
- Anderson-Darling test (based on the cumulative density function)
- Difference between the Gaussian and the optimal proposal via KLD

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Problems of Gaussian Proposals

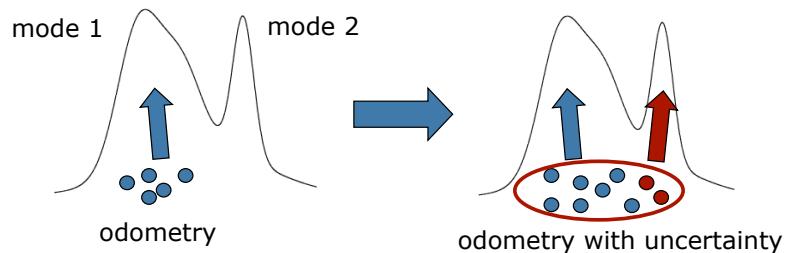
- Multi-modal likelihood function can cause filter divergence



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Efficient Multi-Modal Sampling

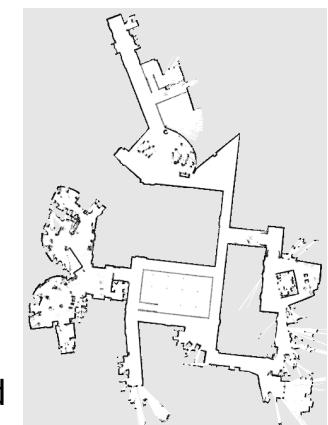
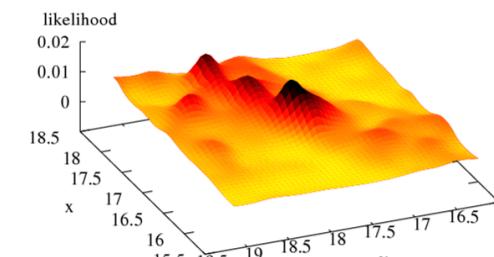
- Approximate the likelihood in a better way!



- Sample from odometry first and the use this as the start point for scan matching

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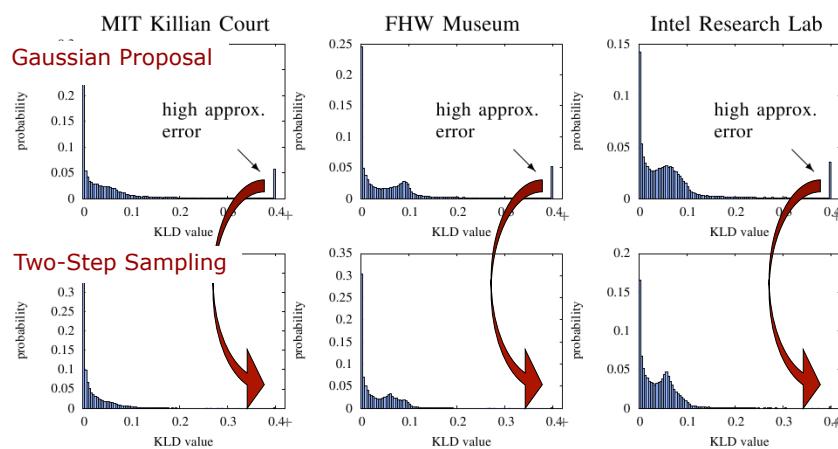
The Two-Step Sampling Works!



...with nearly zero overhead

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Proposal Error Evaluation



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Effect of Two-Step Sampling

- Allows for better modeling multi-modal likelihood functions (high KLD values do not occur)
- For uni-modal cases, identical results
- Minimal computational overhead

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Gaussian Proposal: Yes or No?

- Gaussian allow for efficient sampling
- Problematic in multi-model cases
- Laser-Based SLAM: 3-6% multi-modal distribution (for the datasets here)
- Gaussian proposals can lead to divergence
- Two-step sampling process overcomes this problem effectively and efficiently

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Conclusion

- The ideas of FastSLAM can also be applied in the context of grid maps
- Improved proposals are essential
- Similar to scan-matching on a per-particle base
- Selective resamples reduces the risk of particle depletion
- Substantial reduction of the required number of particles

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Literature

Grid-FastSLAM with Improved Proposals

- Grisetti, Stachniss, Burgard: Improved Techniques for Grid Mapping with Rao-Blackwellized Particle Filters, 2007
- Stachniss, Giorgio, Burgard, Roy. Analyzing Gaussian Proposal Distributions for Mapping with Rao-Blackwellized Particle Filters, 2007

Grid-FastSLAM & Scan-Matching

- Hähnel, Burgard, Fox, Thrun. An efficient FastSLAM Algorithm for Generating Maps of Large-Scale Cyclic Environments from Raw Laser Range Measurements, 2003

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GMapping

- Efficient open source implementation of the presented method (2005-2008)
- C++ Code available via
`svn co https://svn.Openslam.org/data/svn/gmapping`

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