CMPSCI 403: Introduction to Robotics: Perception, Mechanics, Dynamics, and Control

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October 3, 2021

Abstract

Your abstract.

1 Degrees of Freedom

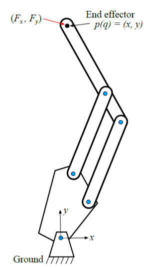

Figure 1: The mechanism in question.

The mechanism possesses two degrees of freedom. The first joint connected to the base allows for one degree of freedom, while the linkage to the arm with the end effector is a parallelogram, introducing another degree of freedom. In the kinematic equation, I am assuming that the arms are infinitely thin, the linkage is infinitely close together, etc, such that the second arm and joints in the linkage only serve to restrict the degrees of freedom of the arm with the end effector in relation to the arm connected to ground. (Maintains the parallelogram shape, where the arm with the end effector is governed by the angles of the parallelogram)

2 Motor Position

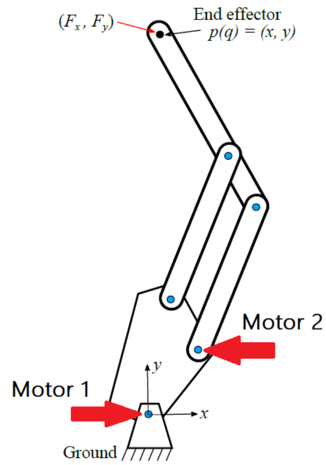

Figure 2: The mechanism with motors labelled.

Since the mechanism only has two degrees of freedom, we only need two motors to fully control the position of the end effector, one at the joint connected to the base and one at any of the joints of the parallelogram. The image shows one of four possible motor configurations. This will be used for the rest of this assignment.

3 Forward Kinematics

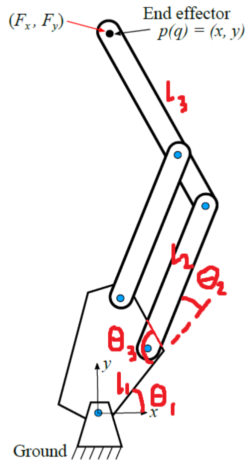

Figure 3: The mechanism with joint angles and lengths.

Notice thatθ_3 is not a joint angle, but the angle between l_1 and the line between the two joints on the arm connected to ground that determines the parallelogram. Therefore, if an angle between the line of l_2 and l_3 would be $\frac{\pi}{2} - \theta_3 - \theta_2$. The forward kinematics are then:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\theta_1 + \theta_2 + (\frac{\pi}{2} - \theta_3 - \theta_2)) \\ l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \theta_2) + l_3 * sin(\theta_1 + \theta_2 + (\frac{\pi}{2} - \theta_3 - \theta_2)) \end{bmatrix}$$

Which can be simplified to

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\frac{\pi}{2} + \theta_1 - \theta_3) \\ l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \theta_2) + l_3 * sin(\frac{\pi}{2} + \theta_1 - \theta_3) \end{bmatrix}$$

4 Jacobian

The Jacobian is the matrix representing all the partial derivatives of the forward kinematics equations:

```
syms a_1 a_2 a_3 l_1 l_2 l_3;
J = jacobian([l_1*cos(a_1)+l_2*cos(a_1+a_2)+l_3*cos(pi/2+a_1-a_3), ...
    l_1*sin(a_1)+l_2*sin(a_1+a_2)+l_3*sin(pi/2+a_1-a_3)], [a_1 a_2]);
```

produces (where a is θ)

```
J =
[- l_2*sin(a_1 + a_2) - l_3*sin(a_1 - a_3 + pi/2) - l_1*sin(a_1), -l_2*sin(a_1 + a_2)]
[  l_2*cos(a_1 + a_2) + l_3*cos(a_1 - a_3 + pi/2) + l_1*cos(a_1),  l_2*cos(a_1 + a_2)]
```

or

$$J(\theta)\dot{\theta} = \begin{bmatrix} -l_2 * sin(\theta_1 + \theta_2) - l_3 * sin(\theta_1 - \theta_3 + \frac{\pi}{2}) - l_1 * sin(\theta_1) & -l_2 * sin(\theta_1 + \theta_2) \\ l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\theta_1 - \theta_3 + \frac{\pi}{2}) + l_1 * cos(\theta_1) & l_2 * cos(\theta_1 + \theta_2) \end{bmatrix}$$

5 Force Vector

As per Chapter 5 of the textbook, the force vector f_{tip} can be expressed in terms of the joint torque vector τ with $f_{tip} = J^{-T}(\theta)\tau$ assuming that negligible force is required to move the robot due to the conservation of power.

MatLab evaluates this using (where t is τ):

```
syms t;
F = transpose(inv(J))*t;
```

to

```
F =

[-(t*cos(a_1 + a_2))/(l_1*cos(a_1 + a_2)*sin(a_1) - l_1*sin(a_1 + a_2)*cos(a_1) ...
+ l_3*cos(a_1 + a_2)*sin(a_1 - a_3 + pi/2) - l_3*cos(a_1 - a_3 + pi/2)*sin(a_1 + a_2)), ...
(t*(l_2*cos(a_1 + a_2) + l_3*cos(a_1 - a_3 + pi/2) ...
+ l_1*cos(a_1)))/(l_2*l_3*cos(a_1 + a_2)*sin(a_1 - a_3 + pi/2) ...
- l_2*l_3*cos(a_1 - a_3 + pi/2)*sin(a_1 + a_2) + l_1*l_2*cos(a_1 + a_2)*sin(a_1) ...
- l_1*l_2*sin(a_1 + a_2)*cos(a_1))]
[-(t*sin(a_1 + a_2))/(l_1*cos(a_1 + a_2)*sin(a_1) - l_1*sin(a_1 + a_2)*cos(a_1) ...
+ l_3*cos(a_1 + a_2)*sin(a_1 - a_3 + pi/2) - l_3*cos(a_1 - a_3 + pi/2)*sin(a_1 + a_2)), ...
(t*(l_2*sin(a_1 + a_2) + l_3*sin(a_1 - a_3 + pi/2) ...
+ l_1*sin(a_1)))/(l_2*l_3*cos(a_1 + a_2)*sin(a_1 - a_3 + pi/2) ...
- l_2*l_3*cos(a_1 - a_3 + pi/2)*sin(a_1 + a_2) + l_1*l_2*cos(a_1 + a_2)*sin(a_1) ...
- l_1*l_2*sin(a_1 + a_2)*cos(a_1))]
```

or

$$f_{tip} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

where

$$\begin{multline} A = -(t * cos(\theta_1 + \theta_2))/(l_1 * cos(\theta_1 + \theta_2) * sin(\theta_1) \\ - l_1 * sin(\theta_1 + \theta_2) * cos(\theta_1) + l_3 * cos(\theta_1 + \theta_2) * sin(\theta_1 - \theta_3 + \frac{\pi}{2}) \\ - l_3 * cos(\theta_1 - \theta_3 + \frac{\pi}{2}) * sin(\theta_1 + \theta_2)) \tag{1} \end{multline}$$

$$\begin{multline} B = (t * (l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\theta_1 - \theta_3 + \frac{\pi}{2}) + l_1 * cos(\theta_1)))/(l_2 * l_3 * cos(\theta_1 + \theta_2) * sin(\theta_1 - \theta_3 + \frac{\pi}{2}) \\ - l_2 * l_3 * cos(\theta_1 - \theta_3 + \frac{\pi}{2}) * sin(\theta_1 + \theta_2) + l_1 * l_2 * cos(\theta_1 + \theta_2) * sin(\theta_1) - l_1 * l_2 * sin(\theta_1 + \theta_2) * cos(\theta_1)) \tag{2} \end{multline}$$

$$\begin{multline} C = -(t * sin(\theta_1 + \theta_2))/(l_1 * cos(\theta_1 + \theta_2) * sin(\theta_1) \\ - l_1 * sin(\theta_1 + \theta_2) * cos(\theta_1) + l_3 * cos(\theta_1 + \theta_2) * sin(\theta_1 - \theta_3 + \frac{\pi}{2}) \\ - l_3 * cos(\theta_1 - \theta_3 + \frac{\pi}{2}) * sin(\theta_1 + \theta_2)) \tag{3} \end{multline}$$

$$\begin{multline} D = (t * (l_2 * sin(\theta_1 + \theta_2) + l_3 * sin(\theta_1 - \theta_3 + \frac{\pi}{2}) + l_1 * sin(\theta_1)))/(l_2 * l_3 * cos(\theta_1 + \theta_2) * sin(\theta_1 - \theta_3 + \frac{\pi}{2}) \\ - l_2 * l_3 * cos(\theta_1 - \theta_3 + \frac{\pi}{2}) * sin(\theta_1 + \theta_2) + l_1 * l_2 * cos(\theta_1 + \theta_2) * sin(\theta_1) - l_1 * l_2 * sin(\theta_1 + \theta_2) * cos(\theta_1)) \tag{4} \end{multline}$$