Nondeterministic Finite Automata

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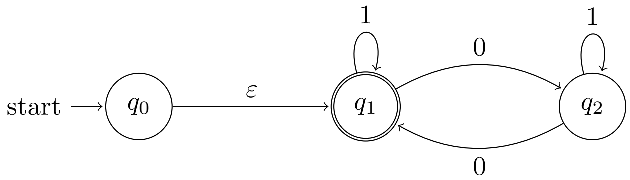
The language $\{w \in \{0,1\}^* \mid \text{the number of 0s in } w \text{ is even}\}$.

1 State Diagram

Below is the state diagram of the NFA that accepts strings with an even number of 0s:



2 Formal Description

The NFA can be described as a 5-tuple:

$$M = (Q, \Sigma, \delta, q_0, F)$$

where:

$Q = \{q_0, q_1, q_2\}$	the set of states
$\Sigma = \{0, 1\}$	the input alphabet
q_0	the start state
$F = \{q_1\}$	the set of accepting states
δ	the transition function:

δ	ε	0	1
q_0	q_1	$\emptyset$	$\emptyset$
q_1	$\emptyset$	q_2	q_1
q_2	$\emptyset$	q_1	q_2

How the NFA Works

The NFA starts in state q_0, which transitions to q_1 via an ε-transition. From q_1, reading a 1 causes the automaton to stay in q_1, and reading a 0 transitions it to q_2. From q_2, reading a 1 causes it to stay in q_2, and reading a 0 transitions it back to q_1. Thus, the automaton toggles between q_1 and q_2 on every 0. The NFA accepts if it ends in state q_1, which corresponds to having read an even number of 0s.

Example Strings and Traces

Accepted Strings ε, `1`, `11`, `0011`, `1010`
Rejected Strings: `0`, `10`, `011`, `000`

Trace for `1010`:

$$q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_2 \xrightarrow{0} q_1 \quad \text{Accepted}$$

3 Context-Free Grammar (CFG)

The CFG that generates the same language (even number of 0s) is defined as:

$$G = (V, \Sigma, R, S)$$

where:

$$\begin{array}{ll} V = \{S, A\} & \text{the set of variables} \\ \Sigma = \{0, 1\} & \text{the set of terminal symbols} \\ S & \text{the start symbol} \\ R & \text{the set of production rules} \end{array}$$

Production rules:

$$\begin{aligned} S &\to 1S \mid 0A \mid \varepsilon \\ A &\to 1A \mid 0S \end{aligned}$$

This CFG generates all strings over $\{0,1\}^*$ with an even number of 0s.

4 Regular Expression

Since the language is regular, it can also be described by the following regular expression:

$$(1^*(01^*01^*)^*)^*$$

This expression generates all strings with pairs of 0s, possibly separated by 1s.