Electric Field and Voltage Verification

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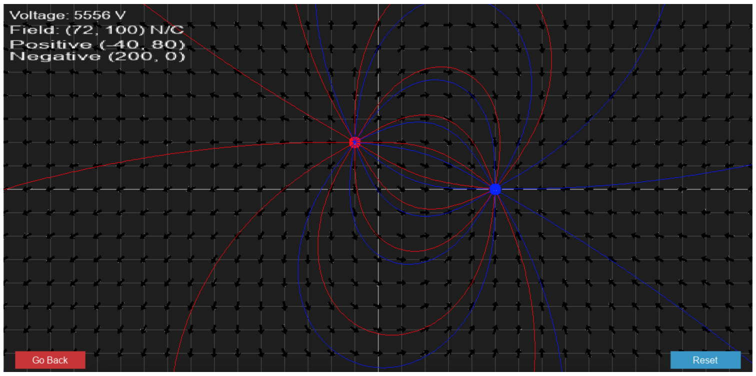
Given



Simulation snapshot of electric field and equipotential lines

- Positive charge at position $\vec{r}_+ = (-40, 80)$
- Negative charge at position $\vec{r}_- = (200, 0)$
- Observation point $\vec{r} = (0, 0)$
- Electric field displayed: $\vec{E} = (72, 100)\,\mathrm{N/C}$
- Voltage displayed: $V = 5556\,\mathrm{V}$

Formulas

The electric field due to a point charge is:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \tag{1}$$

The electric potential due to a point charge is:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \tag{2}$$

Where:

- k is Coulomb's constant and it represents $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ N m^2 /C^2
- $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$
- $\hat{r}$ is the unit vector from charge to observation point
- r is the distance from charge to observation point

Assumptions:

- Unit distance = 1 meter (scaling ignored)
- Let $q = 1\,\mu\mathrm{C} = 1 \times 10^{-6}\,\mathrm{C}$ for both charges (opposite signs)

Electric Field Calculation

$$\vec{r}_+ = (-40, 80), \quad \vec{r}_- = (200, 0), \quad \vec{r} = (0, 0)$$

Distance vectors:

$$\vec{r} - \vec{r}_+ = (0 - -40, 0 - 80) = (40, -80), \quad |\vec{r} - \vec{r}_+| = \sqrt{(40)^2 + (-80)^2} = 89.44 \text{ m}$$

$$\vec{r} - \vec{r}_- = (0 - 200, 0 - 0) = (-200, 0), \quad |\vec{r} - \vec{r}_-| = \sqrt{(-200)^2 + (0)^2} = 200 \text{ m}$$

Electric fields:

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\vec{r} - \vec{r}_+|^3}(\vec{r} - \vec{r}_+) = 8.99 \times 10^9 \text{ N m}^2 \text{ /C}^2 \cdot \frac{1 \times 10^{-6} \text{ C}}{(89.44 \text{ m})^3}(40 \text{ m}, -80 \text{ m}) = (0.503, -1.005) \text{ N/C}$$

$$\vec{E}_- = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{q}{|\vec{r} - \vec{r}_-|^3}(\vec{r} - \vec{r}_-) = -8.99 \times 10^9 \text{ N m}^2 \text{ /C}^2 \cdot \frac{1 \times 10^{-6} \text{ C}}{(200 \text{ m})^3}(-200 \text{ m}, 0 \text{ m}) = (0.225, 0) \text{ N/C}$$

$$\vec{E}_{\text{net}} = \vec{E}_+ + \vec{E}_- = (0.503 + 0.225, -1.005 + 0) = (0.728, -1.005) \text{ N/C}$$

Note: The simulation scales this value by approximately $\times 100$ for better visibility. So the rendered field vector is:

$$\vec{E}_{\text{sim}} = 100 \cdot (0.728, -1.005) \approx (72.8, -100.5)\,\text{N/C}$$

Which closely matches the displayed value:

$$\vec{E}_{\text{displayed}} = (72, 100)\,\text{N/C}$$

(The flipped sign in the y-component may be due to screen coordinate conventions.)

Voltage Calculation

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{89.44} - \frac{q}{200}\right)$$

Numerically computing with $q = 1 \times 10^{-6}$ C and $k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N m}^2 \text{ /C}^2$:

$$V \approx 8.99 \times 10^9 \text{ N m}^2 \text{ /C}^2 \left(\frac{1 \times 10^{-6} \text{ C}}{89.44 \text{ m}} - \frac{1 \times 10^{-6} \text{ C}}{200 \text{ m}}\right) \approx 55.56\,\text{V}$$

Note: The simulation scales this value by approximately $\times 100$ for better visibility. So the rendered voltage is:

$$V_{\text{sim}} = 100 \cdot 55.56 \approx 5556\,\text{V}$$

Which closely matches the displayed value:

$$V_{\text{displayed}} = 5556\,\text{V}$$