The Logarithmic Right Triangle

What is a Logarithmic Right Triangle?

A **logarithmic right triangle** is a geometric configuration where the side lengths are defined by natural logarithms, often expressed as $\ln(x)$, $\ln(2x)$, and $\ln(3x)$.

Pythagorean Theorem

To prove that a logarithmic right triangle is valid, the pythagorean theorem is used:

$$a^2 + b^2 = c^2$$

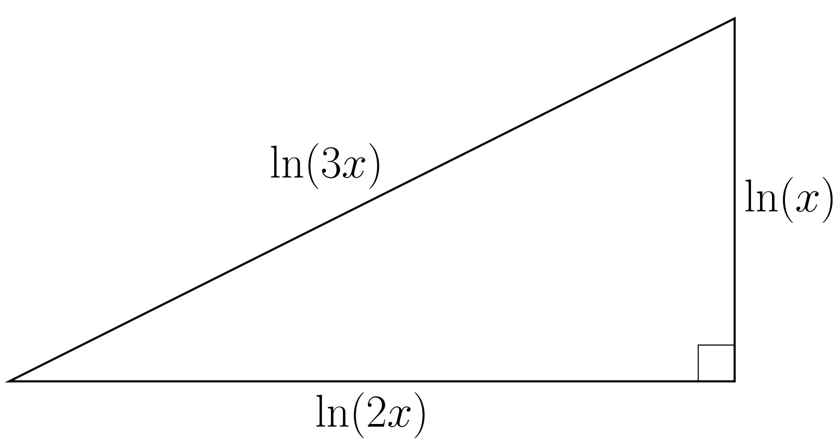
For a $3 - 4 - 5$ right triangle:

$$\begin{gathered} 3^2 + 4^2 = 5^2 \\ 9 + 16 = 25 \\ 25 = 25 \end{gathered}$$

The sides that are perpendicular to each other, when squared and added together, the resulting numerical sum should equal the square of the resultant side that connects the tail of the first side to the head of the second side.

The Logarithmic Right Triangle

The logarithmic right triangle looks like the following, in the image provided below:



Proving the Logarithmic Right Triangle

$$
\begin{aligned}
\ln(x)^2 + \ln(2x)^2 &= \ln(3x)^2 \\
2\ln(x) + 2\ln(2x) &= 2\ln(3x) \\
2\ln(x) + 2\ln(2) + 2\ln(x) &= 2\ln(3) + 2\ln(x) \\
-2\ln(x) &= \qquad\quad -2\ln(x) \\
2\ln(2) + 2\ln(x) &= 2\ln(3) \\
-2\ln(2) &= \qquad -2\ln(2) \\
2\ln(x) &= 2\ln(3) - 2\ln(2) \\
2\ln(x) &= 2\ln\left(\frac{3}{2}\right) \\
\ln(x) &= \ln\left(\frac{3}{2}\right) \\
e^{\ln(x)} &= e^{\ln\left(\frac{3}{2}\right)} \\
x &= \frac{3}{2}
\end{aligned}
$$

Now we will validate whether the all the sides work when plugging in $x = \frac{3}{2}$ into all of the logarithms.

$$
\begin{aligned}
\ln\left(\frac{3}{2}\right)^2 + \ln\left((2)\cdot\left(\frac{3}{2}\right)\right)^2 &= \ln\left((3)\cdot\left(\frac{3}{2}\right)\right)^2 \\
2\ln\left(\frac{3}{2}\right) + 2\ln\left((2)\cdot\left(\frac{3}{2}\right)\right) &= 2\ln\left((3)\cdot\left(\frac{3}{2}\right)\right) \\
2\ln\left(\frac{3}{2}\right) + 2\ln(3) &= 2\ln\left(\frac{9}{2}\right) \\
2\ln(3) - 2\ln(2) + 2\ln(3) &= 2\ln(9) - 2\ln(2) \\
+2\ln(2) \qquad\qquad &= \qquad\qquad +2\ln(2) \\
2\ln(3) + 2\ln(3) &= 2\ln(9) \\
4\ln(3) &= 2\ln(9) \\
\ln(3^4) &= \ln(9^2) \\
\ln(81) &= \ln(81)
\end{aligned}
$$

Quod Erat Demonstrandum