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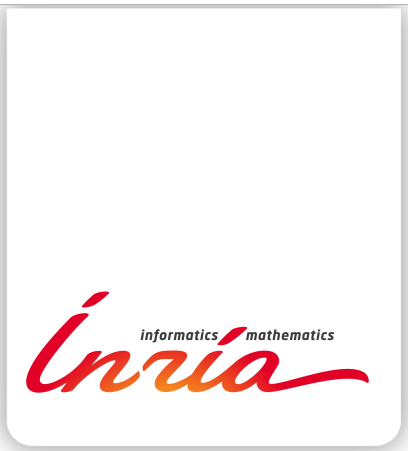

Component-based implicit parallelism model for multi-stencil numerical simulations

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1 Introduction

2 Multi-stencil programs

To numerically solve a set of PDEs, iterative methods are frequently used to approximate the solution by a step by step phenomena. Thus, the continuous time and space domains are discretized so that a set of numerical computations are iteratively (time discretization) applied on a mesh (space discretization). In other words, the PDEs are transformed to a set of numerical computations applied at each time step on all elements of the discretized space domain. Among the numerical computations is found a set of numerical schemes, also called *stencil computations*. A formal definition of a *stencil program*, and the parallelization of such program, are presented in this section.

2.1 Definitions

A mesh $\mathcal{M}$ defines the discretization of the continuous space domain Ω of a set of PDEs.

Definition 1 *A mesh is a connected undirected graph without bridges.*

E_i denotes a set of elements constructed by a function $elem_i$ which defines a precise association of nodes and edges of a mesh. If we denote *nodes* the function to get nodes of a mesh and *edges* the same for edges, the funtion $elem_i$ is defined as

$$elem_i : nodes(\mathcal{M}) \times edges(\mathcal{M}) \rightarrow E_i$$

For example, the set of cells E_0 in a Cartesian 2D mesh could be defined by exactly four nodes and four edges connected as a cycle. But we could also define another set of elements E_1 as the simple set of nodes etc.

A mesh can be structured (as Cartesian or curvilinear meshes), unstructured, regular or irregular (without the same topology for each element) and hybrid if mixing those.

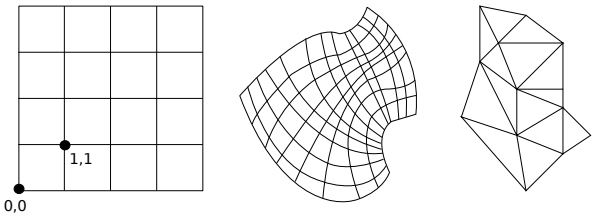


Figure 1: From left to right, Cartesian, curvilinear and unstructured meshes.

The discretization of the continuous time domain $\mathcal{T}$ is denoted T. T is responsible for the iteration time steps of the numerical simulation.

In a numerical simulation a set of data, or quantities, are applied onto the mesh and represent the set of values to compute, or to use, for computation. We denote the set of data applied on the mesh by Δ, such that $\delta \in \Delta$ is a function which associates each element $el \in E_i$ to a value $v \in V$,

$$\delta : E_i \to V$$

One can notice that in applied mathematics, the signature of δ would be $\delta : E_i \times T \to V$, however when programming a numerical simulation it is not wise to store all values of each time iteration.

Finally, in a numerical simulation are performed a set of ordered numerical computations denoted Γ. A numerical computation in Γ is denoted

$$c(R, w, D, e).$$

In this definition $R \subset \Delta, w \in \Delta$ are respectively the data read and written to compute the numerical expression e (a single data is written in a single computation). D is one of the subsets $E_i \subset \mathcal{M}$. It has to be noticed that at each time iteration, all the elements of a mesh are computed. However, it happens that the computation of the mesh elements is splitted in different computations (for example the computation of the physical border). In this case additional E_i can be specified for the mesh $\mathcal{M}$. The numerical expression e computes the data w from a set of data R.

$$e : R \to w$$

A *multi-stencil program* is defined by the quadruplet

$$\mathcal{MSP}(T, \mathcal{M}, \Delta, \Gamma), \tag{1}$$

If the number of computations in Γ is $card(\Gamma) = m$, such that $\bigcup_{i=0}^{m-1} c_i = \Gamma$, then $\bigcup_{i=0}^{m-1} R_i \cup w_i \subseteq \Delta$.

A computation $c \in \Gamma$ can be of two different types. The first type is called a *stencil computation* and involves an additional information denoted $\mathcal{N}$ which represents a neighborhood function around an element

$$\mathcal{N} : E_i \to E_k \times E_k \times ... \tag{2}$$

, where E_k is a set of elements of $\mathcal{M}$ which could be equal to E_i or not. In other words, from an element $el \in E_i$ the function $\mathcal{N}$ returns a set of elements of E_k which represents the

neighborhood of *el*. The function $\mathcal{N}$ is also sometimes called the *stencil shape*, or the *stencil* in applied mathematics.

A *stencil computation* is a quintuplet

$$s(R, w, D, e, \mathcal{N}), \tag{3}$$

where $R \subset \Delta, w \in \Delta$ are read and written in the numerical expression e, and D is one the subsets E_i. In a stencil computation s, $\forall d \in D$, the stencil numerical expression e is applied such that $w(d) = e(R(d), R(\mathcal{N}(d))$. In this work, a stencil computation $s(R, w, D, e, \mathcal{N})$ always verifies $R \cap w = \emptyset$, otherwize an implicit numerical scheme has to be solve which is over the scope of this paper.

Figure **??** gives an example of a stencil computation $s(R, w, D, e, \mathcal{N})$, on a two dimensional Cartesian mesh, where $R = \{A, B\}$, $w = C$, and where for $(x, y) \in D$

$$e(R(x, y), R(\mathcal{N}(x, y)) = A(x, y) + B(x, y + 1) + B(x, y - 1) + B(x + 1, y) + B(x - 1, y).$$

Finally, the second type of numerical computation is a local computation where e does not involve a neighborhood function $\mathcal{N}$

$$l(R, w, D, e) \tag{4}$$

2.2 Parallelization

3 Component-based parallelism model

3.1 A primitive component model

Definitions of a primitive component (ports, interfaces), a component instanciation, an assembly + simple semantics on the behavior of a primitive component. Cite L2C as an implementation of such a model + introduction of notations (use,provide,use-multiple,sync).

3.2 Control components

Definition of three control components, and of the kernel component. Explain the behavior.

3.3 Transformation to a parallel assembly

Transformation from a parallel algorithm using the directives: PARALLEL, PARALLEL SECTIONS, SECTION and FORALL, to a component assembly

4 Component Stencil Language

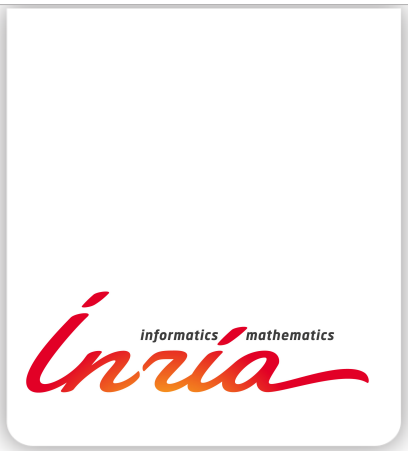
4.1 CSL language and example

4.2 CCSL compiler and example

4.3 Evaluation

5 Related work

6 Conclusion



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